▸ PAOLO LIPPARINI, *An ordering on ultrafilters.*

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If T is a class of topological spaces, let us define the *Comfort (pre-)order relative to* T as follows. For ultrafilters D and E, we write $E \leq_{T,C} D$ if and only if every D-compact topological space belonging to T is also E-compact. If T is the class of all topological spaces, we shall omit it.

Makowsky and Shelah defined the notion of an ultrafilter *related* to a logic. Throughout, we shall refer to the improved definition by Caicedo [1]. We shall say that a logic $\mathcal{L}$ is *D-compact*, in place of saying that D is related to $\mathcal{L}$.

DEFINITION 1. We define as follows the Caicedo-Makowsky-Shelah (pre-)order.
$E \leq_{CMS} D$ means that every D-compact logic is E-compact.

As an immediate corollary of results from [1], for every pair of ultrafilters D and E, if $E \leq_C D$, then $E \leq_{CMS} D$.

THEOREM 2. *Suppose that λ is an infinite cardinal, D is not (λ, λ)-regular, and E is (λ, λ)-regular. Then $E \not\leq_C D$. More generally:*

1. *$E \not\leq_{T,C} D$, where T is the class of Hausdorff normal topological spaces.*
2. *$E \not\leq_{T,C} D$, where T is the class of Tychonoff topological Boolean Algebras.*
3. *$E \not\leq_{CMS} D$ (even if we restrict the order to logics generated by at most 2 cardinality quantifiers)*

Theorem 2 strongly suggests the hypothesis that the study of compactness properties both of logics and of (products of) topological spaces actually deals with properties of the Comfort and related orders, and that problems about (transfer of) compactness are best stated as problems about these orders.

See [2] for more details and even more general notions.

[1] X. CAICEDO, *The Abstract Compactness Theorem Revisited*, ***Logic and Foundations of Mathematics***, (A. Cantini et al. editors), (Florence, 1995) Kluwer Academic Publishers, 1999, pp. 131–141.

[2] P. LIPPARINI, *Some more problems about orderings of ultrafilters*, arXiv:1005.2590 (2010).