# Can renewable energy grant sustainable development in an oil fed economy?

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**Abstract**

Conditions for optimality and sustainable growth that can be reconciled with natural resource use have been discussed in the literature since the first oil shocks. In the wake of the climate change debate, the innovation is this paper is to add a renewable energy source to the Dasgupta-Heal type of framework, also including for pollution. This paper presents an intertemporal planning horizon problem in the presence of an exhaustible resource, a renewable resource and pollution. Pollution is introduced as a stock that causes disutility. The renewable energy source does not increase emissions and the amount available is not limited but, compared to the depletable resource, it is more costly. In this framework we investigate the growth rate of the economy and whether strong sustainability is achievable. We obtain a closed form solution and a stringent condition on pollution that has to be met in order achieve strong sustainability. This does not hold in the long run.

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## Introduction

As more countries develop and energy demand increases, the limits to growth are probably no longer those traditionally put forward in the literature. The most serious environmental threat for economic wellbeing today is climate change thus moving away from the traditional argument of resource scarsity toward that of clean air scarsity, or more specifically, the accumulation of pollution stocks in the environment.

Concern for the environment and implications for economic growth have been widely discussed by economists over time. Ever since the times of Malthus, there has been concern regarding resource availability and the limits to growth caused by an ever increasing population and the consequent pressure on natural resources. During the second half of the twentieth century, the concern for the environment started to move up the political and global agenda, beginning with Rachel Carson's Silent Spring (1962) and Boulding's Spaceship Earth (1966). The formidable demand on natural resource that had been witnessed during the Second World War, had sparked off the debate on resource scarcity, resulting in the famous study by Barnett and Morse (1963).The enquiry presented put forward the "compelling case that scarcity of the resources...did not yet, probably would not soon, and conceivably might not ever halt economic growth". Although in the views of Barnett and Morse natural resources would not pose constraints to growth, in the early 1970s energy prices soared following the oil price shocks and the issue of energy supply became one of concern. Another group of academics responded and in 1972 the Club of Rome published its famous volume "Limits to Growth", also known as the Meadows report. The view of the authors was more cumbersome and "predicted that such limits were fast approaching and that global society ignored them at its collective peril" (Pearce 2002, Simpson et al. 2004).

All this prompted trepidation regarding fossil fuel dependent economies and led to a symposium on the Economics of Exhaustible Resources (1974), at which formal academic analysis was presented including the studies by Solow (1974), Dasgupta and Heal (1974) and Stiglitz (1974) which analyzed different issues related to economies with single exhaustible resources. The importance of the Hotelling's rule for optimal exploitation of the non-renewable resource came back on the scene in conjunction with the message that, in order to achieve non-declining consumption, reinvesting resource rents in capital formation was crucial (Hotelling 1931, Hartwick, 1977). Although the concern for the environment and for how much human intervention it could withheld had been raised, the focus of the analysis discussed in the energy fora had mainly been on the limits to growth posed by fossil fuels, namely by the non-renewability of the resource as an essential input for production. Much of the research had concluded that through technical progress or reinvestment of the rents, economic growth was feasible.

In the following years a more holistic approach was put forward, arguing that natural capital had to be accounted for, defining the idea of sustainable development. In 1987 the United Nations Commission on Environment and Development (the Bruntland Commission) published the Brundtland report which stressed the importance of focusing on sustainable development, defined as *development that meets the needs of the present without compromising the ability of future generations to meet their own needs.* Authors also distinguished between two levels of sustainability, namely weak and strong sustainability. Weak sustainability allowed for substitution between man-made capital and natural capital so that an economy could continuously accumulate physical capital at the expense of the natural capital without jeopardizing its sustainability. On the other hand strong sustainability focused on the stock of natural capital and claimed that this had to be maintained intact since the functions it performs cannot be duplicated by man-made capital (Pearce, Markandya, Barbier, 1989,Pearce 1993, Pezzey 1992).

The increase in CO2 concentrations of today have shown that limits to growth will likely not be due to lack of fossil fuel resources but to pollutants accumulation in the environment. In fact, as correctly argued by the authors cited, resource depletability has been overcome by technological innovation, but pollution damages have been an unexpected companion of fossil-fuel based economic development. The need to find alternative more sustainable energy sources for the world is becoming more and more a pressing issue and renewable energy technologies, such as wind and solar energy, are possible options that grant low or zero emission levels. The innovation in this paper is to build on the findings of Dasgupta and Heal (1974), Withagen (1998), Smulders (2000) and Pittel (2002) and add a renewable energy source to the framework.

There has been a large amount of research on renewable resources (see for example Chichilnisky et al 1998 and Titenberg 2005) but this has mostly been in the fields of forestry and fishery resources. The very nature of this type of renewable resources when compared to energy renewable resources are drastically different. Forestry and fishery resources are renewable in the sense that they can re-generate over time but they are available in a fixed amount which can be over harvested. Sun energy is potentially available in unlimited amounts, and likewise for wind energy (Azar 2005).

Literature has discussed some forms of clean alternatives in the form of technological innovation and back stop technology. Valente (2005) accounts for an exhaustible resource with some part of renewability deriving from a resource augmenting technology. The author finds that, in the case of constant returns to scale technology, optimal paths can be sustainable only if the social discount rate is less than the resource regeneration and augmentation rates. Schou (2000) analyses flow pollution, human capital and growth finding that economic growth is feasible but for the reasons given in Withagen (1995) and the issue at hand, namely climate change and the damages caused by pollution accumulation, we model pollution as a stock.

Building on the literature components discussed, this paper adds renewable energy to the modelling framework accounting for the pollution impacts of depletable resource use. More specifically, the paper will assess if a strongly sustainable path, as defined in Pittel (2002), is attainable when a renewable energy resource is available in addition to fossil fuel and if this availability is sufficient to grant long run sustainability. The two energy resources are differentiated based on cost, contribution to pollution, availability and technology. The representative agent gains from consumption but is damaged by pollution and society faces three constraints: scarcity of clean air, finiteness of the non-renewable energy stock and capital accumulation. The use of the non-depletable resource increases pollution although part of the pollution generated is naturally absorbed by the environment according to its absorption capacity. The standard constraint on the non depletable resources applies.

**The model**

We now study the long run behaviour of an economy that seeks to maximize utilitarian welfare in a representative agent framework. Utility is a function of both consumption and pollution. The pollution stock accumulates over time, as, for example fossil fuel emissions do in the world's atmosphere. Society can use a renewable and a non-renewable energy resource which have key differentiating features, in the production of energy which is used to produce output. Society faces a constraint on pollution, a constraint on the depletable resources and a constraint on physical capital. Details of each building block are now provided.

**Renewable and depletable resources and production**[1]

Energy is produced with a mixture of renewable, R(t), and non-renewable, Z(t), energy resources. As stated, contribution to pollution, availability of the resource, technology and cost are the four differentiating features of these environmental resources. We discuss these one by one.

As development has progressed, emissions from exhaustible resources have increased leading to expected severe impacts on the environment and human kind, while the emissions from renewable energy sources are potentially close to, if not zero. For the purpose of this model, the limit case scenario of zero emissions from renewable energy is considered. This entails that use of the depletable resource in energy production, and consequently in output production, contributes to the accumulation of the pollution stock.

By definition, depletable resources are available in a limited amount, namely the total amount of the resource is fixed. Consequently, as the resource is used, the stock of the depletable resource is exhausted. The renewable resource, on the other hand, by definition is available in an unlimited amount so that society faces no quantity constraint when using this resource.

Today, renewable resources cost considerably more than fossil fuel energy, taken to the extreme we could imagine that fossil fuels have a zero cost attached to them therefore we include the cost differential in a very simple way by assuming that the extraction costs of the

[1] Non renewable energy sources, also referred to as depletable, are those type of energy resources that are available in a fixed amount. This paper considers all types of fossil fuels in this category. On the other hand, renewable energy sources are not limited by availability. This paper mainly refers to solar energy and wind energy in this case.

depletable resources are zero while the renewable resource extraction costs are equal to a parameter g.

Finally, renewable energy, as opposed to the more traditional and well established fossil fuel technology, offers large technical progress potential due to the abundance of the resource and the little use society makes of it today. The innovation potential in the renewable resource entails that as time elapses the amount of output obtained per unit of renewable energy input would be increasing or equivalently that a smaller amount of renewable energy is required to obtain the same amount of final output. As outlined in Stiglitz (1974) and Valente (2005), we introduce exogenous technical progress in the form of resource augmenting technology., thus a 'technical progress' parameter, m, is included in the production function, augmenting the amount of renewable resource available, so that exogenous technical progress (ETP) takes the form of $m=e^{\omega t}$, where $\omega$ is the rate of exogenous technical progress.

Using a Cobb Douglas[2] production function is a necessary starting point in this case to be able to obtain a closed form solution. This is a standard intial assumption as for example in Schou (2000), Valente (2005) and Pittel (2002). Depletable and non depletable resources are used to generate energy, EN(t), through a Cobb Douglas production function

$$EN(t) = [Z(t)^{\beta} (mR(t))^{1-\beta}] \quad (1)$$

where $\beta$ is the share of the depletable resource.

The economy uses natural and physical capital, K(t), in production, where natural capital is used in energy production as shown in (1), in a Cobb Douglas functional form

$$F(K(t), Z(t), R(t)) = [K(t)^{1-\alpha} EN(t)^{\alpha}] = \left\{K(t)^{1-\alpha} Z(t)^{\alpha\beta} [mR(t)^{\alpha(1-\beta)}]\right\} \quad (2)$$

Assuming Cobb Douglas production functional forms entails that all inputs are essential in production, and more specifically, that both fossil fuels and renewable energy resources are

[2] Due to this, the standard restrictions deriving from a Cobb Douglas production function apply such as constant market shares and elasticity of substitution. A possible extension of this problem will be to introduce a CES production function.

needed to produce energy, thus $F(0,Z,R) = F(K,0,0) = 0$[3]. The standard Inada conditions are satisfied.

In sum, renewable energy sources are a costly input into production while depletable resources, available at no cost, pollute and are thier in a fixed amount.

**Consumption**

Agents enjoy consumption but suffer from pollution (Smulders (2000), Pittel (2002)), therefore the welfare function of the representative agent is a function of both consumption and pollution, which causes disutilty, (3).

$$u(C(t), P(t)) = \ln C(t) + \ln(\overline{P} - P(t)) \tag{3}$$

where C(t) is consumption and P(t) is the pollution stock[4], the amount of pollution that accumulates in the economy through use of Z(t) and utility is concave in both its arguments. Since consumers benefit from consumption, the marginal utility is positive, but at a decreasing rate $U_C > 0$ and $U_{CC} < 0$. On the other hand, pollution harms consumers so increasing pollution reduces utility, but at a decreasing rate $U_{PP} < 0$.. We take $\overline{P}$ to be the upper bound for pollution, namely the uppermost pollution that society can withstand, beyond which society would cease to exist. We assume that this threshold level of pollution can never be reached using all existing non-renewable resources at once[5].

[3] The question of whether production with no fossil fuel input in the long run is possible, namely $F(K,0,R) = 0$, and within what time horizon this may possible remains an issue open for discussion and is not discussed here but is extensively treated in Dasgupta and Heal (1974).

[4] As outlined in the introduction, we consider pollution to be a stock since we refer to fossil fuels and the disutility generated through the consequent accumulation of carbon dioxide pollution. This is inline with the discussion in Withagen (1995).

[5] The assumption of separability in the utility function entails that the level of pollution has no impact on $U_C$ and viceversa that consumption has no impact on $U_P$. Separability is a standard assumption as in for example Pittel (2002).

**The pollution, resource depletion and capital accumulation constraints of the economy**

Society is constrained by the amount of pollution that accumulates, the limited availability of depletable resources and physical capital accumulation in its strive to maximize the welfare function of the representative agent over time.

As shown in Dasgupta and Heal (1974), Hotelling (1931) and Stiglitz (1974), the stock of exhaustible resource, S(t), decumulates by the amount used at time t by society, namely

$$\dot{S}(t) = -Z(t) \tag{4}$$

Two opposing effects impact the pollution stock, Pittel (2002): on the one hand the use of the non-renewable resource increases pollution, on the other, the environment is capable to absorb a portion of the pollution generated based on the environment's absorption capacity, γ. We take γ to vary between 0 and 1, where 1 would mean that the environment has the capacity to absorb all emissions arising from the use of fossil fuels. Since renewable energy does not contribute to pollution it is not included in the pollution accumulation constrain and the net variation of the pollution stock is due to depletable resource use net of the absorption capacity of the environment, namely

$$\dot{P}(t) = Z(t) - \gamma P(t) \tag{5}$$

Capital is accumulated through net investment as standard including for renewable energy costs, g,

$$\dot{K}(t) = F[K(t), mR(t), Z(t)] - C(t) - gR(t) - \delta K(t) \tag{6}$$

where δ is the depreciation rate of physical capital

**The balanced growth path and the strong sustainability criterion**

Balanced growth is defined as the condition in which all variables grow at a constant rate and the rate of growth of output, consumption and capital are the same. We add a further constraint to this problem since we want to investigate the conditions under which Strong Sustainability holds. Strong sustainability is defined as the condition in which the stock of natural capital does not change and remains constant. As discussed earlier, in this paper we identify natural capital or the environment as green air due to the focus of the discussion on pollution and fossil versus renewable energy. Thus, according to Pittel (2002) and in the framework described in this paper, the minimum requirement for balanced growth to be strongly sustainable is that the stock of pollution stays constant over time, thus

$$\dot{P}(t) = 0 \Rightarrow \frac{\dot{P}(t)}{P(t)} = 0 \tag{7}$$

Therefore we look for the condition under which economic growth can be achieved using the depletable resource but keeping the pollution stock constant over time. In other words, the depletable resource would have to be used at the rate at which the environment can naturally counterbalance its use.

**The benevolent social planner's problem**

The maximization problem is that of a social planner that maximizes the present value of utility, as a function of consumption and pollution, subject to net emissions accumulation in the atmosphere, depletion of fossil fuel sources and net physical capital investment constraints. More specifically, the planner has to decide the optimal level of consumption, the optimal level of renewable resource use and how much of the exhaustible resource to optimally deplete. The structural form of the problem is as follow from (8) to (11):

$$\max_{\{C(t),R(t),Z(t)\}} \int_0^T u(C(t), P(t)) \cdot e^{-\rho t} dt \tag{8}$$

subject to

$$\dot{P}(t) = Z(t) - \gamma P(t) \tag{9}$$

$$\dot{S}(t) = -Z(t) \tag{10}$$

$$\dot{K}(t) = F[K(t), mR(t), Z(t)] - C(t) - gR(t) - \delta K(t) \tag{11}$$

where ρ is the rate of time preference and T is the end of the time horizon[6]. All quantities are positive and the initial conditions are as follows:

$$C(0) = C_0, K(0) = K_0, R(0) = R_0, P(0) = P_0, Z(0) = Z_0$$
$$C(t) \geq 0, K(t) \geq 0, R(t) \geq 0, P(t) \geq 0, Z(t) \geq 0$$

The current value Hamiltonian for this problem is as follow:

$$H(t) = U(C(t), P(t)) + \xi(t)[-Z(t)] + \lambda(t)[Z(t) - \gamma P(t)]$$
$$+ \mu(t)\{F[K(t), (mR(t)), Z(t)] - C(t) - gR(t) - \delta K(t)\} \tag{12}$$

where the shadow price of the exhaustible resource is ξ(t), the shadow price of pollution is λ(t), and the shadow price of net investment in capital is μ(t).

Deriving the Hamiltonian with respect to consumption we obtain:

$$\Rightarrow 1/C = \mu \tag{13}$$

which is the standard condition on utility according to which marginal benefits from consumption have to equal marginal costs along the optimal path or in other words the shadow price of consumption has to equal the marginal utility.

Under balanced growth and by loglinearizing and time differentiating the expression in (13) we obtain

$$\frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = -\frac{\dot{\mu}}{\mu} \tag{13'}$$

[6] We thank C. Withagen and C. Roseta-Palma for pointing out the need to limit out attention to a finite time horizon in our framework.

so that along the optimal path, consumption, production and physical capital all grow at the same rate, ultimately equal to the opposite of the growth rate of the shadow price of consumption.

From the first order condition with respect to the renewable resource we find

$$F_R[K,R,Z]=g \tag{14}$$

or in other words, along the optimal path, the marginal benefit accruing from production thanks to an additional unit of renewable resource is equal to its cost. By loglinearizing and time differentiating this expression we find that in the long run the output and renewable energy resources' growth rates coincide:

$$\frac{\dot{Y}}{Y}=\frac{\dot{R}}{R} \tag{15}$$

The derivation of the current value Hamiltonian with respect to the non-renewable resource yields

$$F_z[k,R,Z]=\frac{\xi-\lambda}{\mu} \tag{16}$$

Interpretation of this result is less straightforward. This condition implies that the marginal cost of the depletable resource due both to its exhaustibility and to its contribution to pollution must equal the marginal (shadow) benefits related to the use of Z in production.

By loglinearizing and time differentiating the expression in (16) we obtain the growth rate of the non-renewable resource which is a function of the growth rate of shadow price of pollution and of the shadow price of the non-renewable resource as shown in (16')

$$\frac{\dot{Z}}{Z}=\frac{\dot{\lambda}-\dot{\xi}}{\lambda-\xi} \tag{16'}$$

To achieve optimality, a set of co-state equations as listed in (17), (18) and (19).

The costate equation for the stock of pollution P(t) is equal to

$$\frac{\dot{\lambda}}{\lambda} = \rho - (\frac{u_P}{\lambda} - \gamma) \tag{17}$$

which shows that the growth rate of the shadow price of pollution must equal the social discount rate net of a *pollution related factor*. The pollution factor is equal to the ratio of the marginal utility of pollution divided by the benefit from pollution net of the environment's absorption capacity.

The costate equation with respect to the stock of non-renewable resource is

$$\frac{\dot{\xi}}{\xi} = \rho \tag{18}$$

which is the standard Hotelling rule whereby the optimal use of the non-renewable resource is defined as the path along which the growth rate of the resource's shadow price is equal to the rate of time preference.

The costate equation with respect to the stock of physical capital K(t) is equal to

$$\frac{\dot{\mu}}{\mu} = \rho - (F_K(\cdot) - \delta) \tag{19}$$

Thus, along the optimal path, the rate of growth of the capital's shadow price is equal to the difference between the the discount rate and output's productivity net of its depreciation which, as standard, leads to the Ramsey Golden Rule for consumption.

By rearranging equation (9) we can write the rate of growth of the stock of pollution as follows

$$\frac{\dot{P}}{P} = \frac{Z}{P} - \gamma \tag{20}$$

As discussed above, under the assumption of balanced growth, a minimum requirement for strong sustainability is that the stock of pollution be constant or in other words that $\frac{\dot{P}}{P}$ equal zero. Using this definition of strong sustainability and applying it to (20), infers that the ratio of Z to P is constant and equal to γ. , For this to be true, namely for the ratio of Z to P to remain constant, the growth rates of P and Z have to be the same, and since we know that $\frac{\dot{P}}{P}$ is equal to zero it follows that also $\frac{\dot{Z}}{Z}$ has to equal zero, as shown in (21).

$$\frac{\dot{P}}{P} = \frac{\dot{Z}}{Z} = 0 \tag{21}$$

We now use this condition in equation in equation (16') also find that this implies that changes in the shadow price of pollution are equal to changes in the shadow price of the non-renewable resource over time, so we conclude that

$$\frac{\dot{\lambda} - \dot{\xi}}{\lambda - \xi} = 0 \Rightarrow \dot{\lambda} = \dot{\xi} \text{ if } \lambda \neq \xi \tag{22}$$

From the conditions listed above and through the manipulations described, a set of equations have been derived that describe the long run optimal paths in our economy.

$$\frac{\dot{C}}{C} = \frac{\dot{R}}{R} = \frac{\dot{K}}{K} = \frac{\dot{Y}}{Y} = -\frac{\dot{\mu}}{\mu} = (F_K(\cdot) - \delta) - \rho \tag{23}$$

$$\frac{\dot{P}}{P} = \frac{\dot{Z}}{Z} = 0 \tag{24}$$

Equation (23) is the long run growth rate for the output, consumption, capital and the renewable resource, which coincide. Equation (24) is the long run optimal growth rate for pollution and fossil energy which is equal to zero. Therefore, along the optimal path, the economy will find it optimal to have output, consumption, capital and the renewable resource grow at a positive rate when the net marginal productivity of capital is larger than the social discount rate. On the other hand, for the optimal path to be strongly sustainable, the stock of pollution will remain constant, and in order to ensure this, the flow of the depletable resource will be constant as well over time.

Now, by log-linearizing the production function and using (23) and (24), we obtain the growth rate of the economy under exogenous technical progress, $\frac{\dot{Y}}{Y}^{EXO}$ and the strong sustainability condition:

$$\frac{\dot{Y}}{Y}^{EXO} = \frac{(1-\beta)}{\beta}\omega \tag{25}$$

which leads to the following proposition I

*Proposition I: In the case of an economy that dislikes pollution and uses a depletable and renewable energy resource, the long run strongly sustainable growth rate of the economy will be higher the larger the share of renewable energy resources and the larger the rate of exogenous technical progress. The growth rate of the economy will decrease the larger the share of exhaustible resources.*

The result in (25) is in line with some of the results in the literature. The emphasis on technology is standard as in Solow (1953) and Stiglitz (1974). The innovation here is to see a clear role for the renewable resource in contributing to output growth. This exogenous growth rate shows that three factors influence the growth rate of the economy, namely the share of renewable resource, the share of exhaustible resources and the ETP rate. As the share of the renewable resources increase, the economy growth rate will increase. On the contrary, as the share of non renewables increases, the economy growth rate will decrease. Higher rates of ETP will have a positive influence on the economy's growth rate. Furthermore, if the ETP were to equal zero, ie. no technical progress, the growth rate of the economy would be constant and equal to zero. This is in line with the classical results of the Solow Model in which exogenous technical progress is the driver of the economy's growth. When ω=1, or in other words when the rate of technical progress is equal to unity, (25) simplifies further. In this particular case the growth rate of the economy is equivalent to the ratio of the share of renewable resources to the share of depletable resources, thus as the share of renewable resources in the energy production increases the growth of the economy increases in the same proportion.

Conditions (23, 24, 25) define the sustainable balanced growth path: the longer the time horizon the smaller the amount of non-renewable resource available per period, and the stronger the substitution requirements between natural resources. This implies, intuitively, that in an economy with renewable and non-renewable resources, assuming an exogenous technical progress parameter, sustainable balanced growth in the long run is only possible if the two resources are almost perfectly substitutable (i.e. it is possible to produce output using only renewable resources). We can prove an even stronger result:

*Proposition* 2*: In an economy characterized by renewable and non renewable energy resources, Cobb Douglas technology and exogenous technical progress on the renewable resource, strongly sustainable balanced growth implies that pollution satisfies the following condition:* $P(T) = \overline{P} - \frac{1}{\rho \xi_0 e^{\rho T}}$

The proof for this can be attained through the set of transversality conditions that have to hold in order to ensure that the problem converges. As standard these conditions need to hold to ensure that as time draws in, either the stock of pollution or the capital asset will be exhausted, be it natural or physical, or that alternatively these assets will no longer have any value to society. The conditions are as follows:

$$e^{-\rho T} \xi(T) S(T) = 0 \tag{26}$$

$$e^{-\rho T} \mu(T) K(T) = 0 \tag{27}$$

$$e^{-\rho T}\lambda(T)P(T)=0 \quad (28)$$

Condition (26) is the transversality condition on the stock of depletable resource. In order for this condition to hold, at time T the stock of depletable resource is to be exhausted or the shadow price of the resource in T has to be zero. Based on (18) we know that the value of the resource can never be zero unless the initial value of it is zero. Thus in t=T the stock has to be exhausted for the transversality condition to hold.

Condition (27) has to hold for physical capital. Since we know from (23) that the shadow price of physical capital can never be zero, at time T the stock of physical capital will have to be fully exhausted for the optimality condition to be met.

Condition (28) is for pollution and states that when time elapses the discounted value of the stock of pollution has to be equal to zero, this can be due to either a final zero shadow price or the zero stock of pollution. Manipulation of (17), (18) and (22) yields the following for the shadow price of pollution

$$\lambda=\frac{u_p+\rho\xi_0 e^{\rho t}}{\gamma+\rho} \quad (29)$$

By substituting (29) in (28) when t=T we obtain

$$\left(\frac{u_p(T)+\rho\xi_0 e^{\rho T}}{\gamma+\rho}\right)e^{-\rho T}P(T)=0 \quad (30)$$

By deriving the utility function and manipulating (30) we obtain

$$P(T)=\overline{P}-\frac{1}{\rho\xi_0 e^{\rho T}}=\overline{P}-\frac{1}{\rho\xi(T)} \quad (31)$$

where $\overline{P}$ is the threshold maximum pollution level acceptable.

Since $\gamma + \rho > 0$[7], condition (31) states that the terminal stock of pollution, and thus, following from the definition of strong sustainability, the level of pollution in the economy in the long run, must be less than the threshold level. The optimal "terminal" (i.e. time T) pollution level under the strong sustainability constraint will be higher when the initial value of the depletable resource is large and when ρ and/or T are large. In particular:

- as is reasonable, when individuals are more impatient, namely with a higher $\rho$, the sustainable level of pollution is higher.
- the longer the time horizon the more pollution in time T approaches the threshold level of pollution

Result summed up in Proposition 2 leads us to the following.

*Corollary 1: If the time horizon were to extend to infinity, a strongly sustainable balanced growth path is not attainable.*

The proof is straightforward: from (31) as $T \to \infty$ we get $P(T) = \overline{P}$. This shows that in the very long run the mere existence of both renewable and non renewable energy sources, do not allow an economy to be strongly sustainable. The violation of the transversality[8] conditions lead us to conclude that a sustainable balanced growth path is solely attainable if, in the long run, renewable energy resources can fully substitute depletable resources. The mere existence of the renewable resource and an exogenous technical change process are not sufficient for the economy to be sustainable in the long run[9].

[7] If γ+ρ=0, this would mean that either γ or ρ would have to negative but neither γ nor ρ can ever take on negative values so will condition will always hold.
[8] Note that if the time horizon of this maximization problem were to run from zero to infinity, it can be shown that the transversality condition on the pollution stock variable does not hold and thus the economy is unsustainable.
[9] If a constant zero pollution level were possible, renewable resources would have to substitute fossil fuels in the long run for sustainable development to be achievable.

***Discussion and conclusions***

We presented a model which accounts for the disutility caused by pollution and the option of exogenous technical progress linked to renewable energy in an economy constrained by limited non-renewable energy stocks and the negative effects of pollution on welfare. This society can choose whether to use a renewable energy resource or a depletable one. The renewable energy source does not cause emissions when used, is not constrained by being available in a limited amount and has technical progress potential, but, compared to the depletable resource, it is more costly. The framework used in the paper is that of a social planner that seeks to maximize social welfare under the a pollution, fossil fuel and physical capital constraint. In the model, as it stands, technological progress is introduced exogenously.

The findings in this paper bring to two main conclusions. On the one side, when overtly distinguishing between renewable and depletable energy resources, we show that economic growth is positively linked to the share of renewable energy and inversely proportional to the share of fossil fuels. Therefore policies that allow the size of renewable energy resources to grow will also foster economic growth. On the other hand, when looking for a strongly sustainable path, which in our set up implies a path along which pollution is constant (as for example would be the 'safe' stabilization target levels advocated by the IPCC 2007), optimal growth is only achievable if a condition on the admissible pollution level is met. Further, we show that strong sustainability turns out to be unfeasible if we extend the planning horizon to infinity.

Our model relies on a specific set of assumptions on functional forms in order to obtain a closed form solution and represents a departure point for further research. Nonetheless, we deem the implications of our results as relevant for the current energy related debate. Indeed, our results suggest that renewable resources, by themselves, might not be enough to guarantee that the economy evolve along a strongly sustainable development path.

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