New proposals for the quantification of qualitative survey data

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Abstract

In this paper we deal with several issues related to the quantification of business surveys. In particular, we propose and compare new ways of scoring the ordinal responses concerning the qualitative assessment of the state of the economy, such as the spectral envelope and cumulative logit unobserved components models, and investigate the nature of seasonality in the series. We conclude with an evaluation of the type of business cycle fluctuations that is captured by the qualitative surveys.

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1   Introduction

An important set of indicators on current economic conditions arises from the monthly business survey conducted by various national institutions. Their relevance stems from the fact that they provide timely information on economic variables that are either difficult to measure, such as expectations or capacity utilisation, or whose measurement on a quantitative scale is more expensive and time consuming (turnover and production in volume).

The data collected are mostly categorical or ordinal and timeliness is achieved by a suitable survey design. Survey questions are kept to a minimum and bear on the direction of the trend in an economic variable, as perceived by the respondent. For instance, with respect to orders and the level of production, the respondent is asked whether they are low, normal, or high, abstracting from seasonal fluctuations. The individual data are finally aggregated into a single time series by subtracting the percentage of responses falling in the below normal category from the percentage of the above normal. These differences are called balances and are often used for the quantification of the survey responses, insofar as qualitative information is translated into a quantitative scale. See Pesaran and Weale (2006) for a general exposition and review of alternative quantification methods.

With reference to the assessment of the current level of production, this paper discusses two alternative quantification methods; the first is based on the notion of the spectral envelope and originates a signal extraction filter which has solely cross-sectional dimension, i.e. only contemporaneous values are employed. As a result the quantification suffers from excess roughness. The second is based on a dynamic cumulative logit model for the time series of ordered proportions; the signal extraction filter for the underlying latent cycle is nonlinear and has also a time series dimension. The paper also addresses explicitly whether a particular quantification adheres to a specific notion of business cycles. According to the classical definition the business cycle is a recurrent, but not necessarily periodic, sequence of expansions and contractions in the aggregate level of economic activity (see Burns and Mitchell, 1946, p. 3). The growth cycle is defined instead in terms of the deviation from trend or potential output, and thus within an additive or multiplicative trend-cycle decomposition. The third definition is concerned with the cyclical upswings and downswings in the growth rate of economic activity at a given horizon . Hence, a recession is defined as a prolonged and sustained decline in underlying growth. See Artis et al. (2003) for further details on the measurement issue related to each definition. The paper also discusses the presence of seasonality and calendar components in the business survey indicators.

The individual survey data take the form of a categorical time series, $y_t, t = 1, \ldots, T$, with k ordered response categories identified by the labels $c_1, \ldots, c_k$. For algebraic manipulation it is often convenient to represent the response categories introducing the $k \times 1$ vectors $\mathbf{e}_j, j = 1, \ldots, k,$, where $\mathbf{e}_j$ has the value 1 in the j-th position and zero elsewhere. We thus define a multinomial vector time series, $\mathbf{Y}_t$, taking the value $\mathbf{Y}_t = \mathbf{e}_j$ if $y_t = c_j$,

that is if the j-th category is selected. In the sequel we shall denote $\pi_{jt} = P(\mathbf{Y}_t = \mathbf{e}_j) = P(y_t = c_j), \sum_j \pi_{jt} = 1$. The unconditional mean and covariance matrix of $\mathbf{Y}_t$ are $E(\mathbf{Y}_t) = \boldsymbol{\pi}_t = (\pi_{1t}, \dots, \pi_{kt})'$ and $\text{Var}(\mathbf{Y}_t) = \text{diag}(\boldsymbol{\pi}_t) - \boldsymbol{\pi}_t \boldsymbol{\pi}_t'$, respectively.

Given n_t independent observations, $\mathbf{Y}_{it}, i = 1, \dots, n$, interest often centers on analysing the number of responses in each category, $\mathbf{Y}_{.t} = \sum_i \mathbf{Y}_{it}$. We assume throughout that sampling is such that at any given time t, $\mathbf{Y}_{.t}$ has a multinomial distribution, that is it takes the values $\mathbf{n}_t = (n_{1t}, \dots, n_{jt}, \dots, n_{kt})', \sum_j n_{jt} = n_t, n_{jt} = \sum_i \mathbf{e}_{ij}$, with probability

$$P(\mathbf{Y}_{.t} = \mathbf{n}_t) = \frac{n_t!}{n_{1t}! \cdots n_{jt}! \cdots n_{kt}!} \pi_{1t}^{n_{1t}} \cdots \pi_{jt}^{n_{jt}} \cdots \pi_{kt}^{n_{kt}}, \quad n_t = \sum_j n_{jt}, 1 = \sum_j \pi_{jt}.$$

Typically, the total sample size n_t does not change with time, although nonresponse affects it. We assume anyway that nonresponse is fully ignorable, that is it only affects the sample through a reduction of the sample size. Further, we define $\mathbf{p}_t = (p_{1t}, \dots, p_{kt})' = n_t^{-1} \mathbf{Y}_{.t}$, the proportion of responses in category j. The latter is such $\mathbf{i}_k' \mathbf{p}_t = 1$, where $\mathbf{i}_k$ is a $k \times 1$ vector of 1s, and has a scaled multinomial distribution with mean $\boldsymbol{\pi}_t$ and covariance matrix $n_t^{-1}(\text{diag}(\boldsymbol{\pi}_t) - \boldsymbol{\pi}_t \boldsymbol{\pi}_t')$.

Finally, in the contemporaneous aggregation of the individual responses across groups (e.g branches and sectors of economic activity), weights can be used that stands for the relative importance of the group. Often the data are made available to the public in the form $\mathbf{Y}_{.t}^* = \sum_{is} w_{st} \mathbf{Y}_{ist}$ where s denotes the group to which unit i belongs, and $w_{st}, \sum_s w_{st} = 1$, is the group weight (e.g. the share of gross domestic product or employment, or a measure of size). The aggregate series can be written $\mathbf{Y}_{.t}^* = \sum_s w_{st} \mathbf{Y}_{.st} = \sum_s w_{st} n_{st} \mathbf{p}_{st}$, where $\mathbf{p}_{st}$ is the vector containing the proportions of the responses of each category in group s and n_{st} are the number of respondents in the same group. The scaled series is thus $\mathbf{p}_t^* = \sum_s w_{st}^* \mathbf{p}_{st}$, where the group weights are $\frac{w_{st} n_{st}}{\sum_s w_{st} n_{st}}$. More often, the series $\sum_s w_{st} \mathbf{p}_{st}$ are made available (European Commission). In the sequel we will ignore the complications that arise due to the weighted aggregation of the responses and will continue to assume that the observed counts or proportions arise from a multinomial distribution.

A number of methods have been proposed in the literature for converting these proportions into aggregate measures of perceived business conditions and expectations. The paper evaluates some novel quantification methods based on the notion of spectral envelope (section **??**) and cumulative logit unobserved components models (section **??**). Some issues related to seasonality in survey data are also presented (section **??**) and finally the evaluation of the type of business cycle fluctuations captured by the qualitative surveys is attempted.

2 Quantification through balances

When the original survey question is balanced, the individual data are sometimes aggregated into a single time series subtracting the proportion of responses falling in the two

most extreme categories. These differences are called balances. This section illustrates the conditions under which the balances are a "sufficient" summary of the information provided by the original proportions.

Let $\mathbf{p}_t = (p_{1t}, p_{2t}, p_{3t})'$, $\mathbf{i}'\mathbf{p}_t = 100$, denote the vector containing the percentages of the responses falling in category $j = 1, 2, 3$ (respectively low, normal and high, if we refer to the assessment of the level of production). Defining the contrasts $d_{1t} = p_{2t} - p_{1t} = (-1, 1, 0)'\mathbf{p}_t$ and $d_{2t} = p_{3t} - p_{1t} = (-1, 0, 1)'\mathbf{p}_t$, we can rewrite the original series as a linear combination of the two above contrasts:

$$\mathbf{p}_t = \frac{100}{3}\mathbf{i} + \boldsymbol{\theta}_1 d_{1t} + \boldsymbol{\theta}_2 d_{2t}, \boldsymbol{\theta}_1 = \frac{1}{3}\begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}, \quad \boldsymbol{\theta}_2 = \frac{1}{3}\begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}.$$

The contrasts d_{2t} are used in the quantification of the survey results. They can be justified from the following scoring of the response categories: low: -1, normal:0, high: +1.

It is immediate to realize that they provide a complete summary of the information contained in $\mathbf{p}_t$ if and only if the other contrast d_{1t} is linearly dependent on d_{2t}. In this case there is an additional deterministic linear combination $(\delta_1 - 1, 1, -\delta_1)\mathbf{y}_t = \delta_0$, so that the three series are generated by the common cycle d_{2t}.

In general, d_{2t} is a complete summary for l-step-ahead prediction if we can express $d_{1t} = \delta_0 + \delta_1 d_{2t} + \epsilon_t$, where ϵ_t is a moving average process, orthogonal to d_{2t}, of order $l - 1$. As a matter of fact,

$$\mathbf{p}_t = \frac{100}{3}\mathbf{i} + \delta_0\boldsymbol{\theta}_1 + (\delta_1\boldsymbol{\theta}_1 + \boldsymbol{\theta}_2)d_{2t} + \delta_1\boldsymbol{\theta}_1\epsilon_t,$$

implies that $\mathrm{E}(\mathbf{p}_{t+l}|\mathbf{p}_t, \ldots, \mathbf{p}_1)$ depends only on $\mathrm{E}(d_{2,t+l}|\mathbf{p}_t, \ldots, \mathbf{p}_1)$.

For the level of production the balance is plotted in the last panel of figure **??**. The graph highlights that the balances display sizable seasonal and high-frequency components. The issue of seasonality will be considered in a later section.

Alternative static quantifications could be based on a common cycles analysis of the transformed proportions. For time series of proportions or percentages, that are bounded from high and low, there are several parametric transformations that map the (0,1) range to the real interval. Atkinson (1985) discusses the folded power transformation $u_t(\lambda) = p_t^\lambda + (1 - p_t)^\lambda$, where $0 < p_t < 1$ denotes a generic proportion, which yields the untransformed observations for $\lambda = 1$ and the logit transformation for λ approaching 0, $u_t(0) = \ln(p_t/(1 - p_t))$. One serious drawback is that the transformation is not invertible, that is y_t is an implicit function of u_t. Guerrero and Johnson (1982) proposed to apply the Box-Cox transformation to the odds ratio $o_t = p_t/(1 - p_t)$, i.e. $u_t(\lambda) = (o_t^\lambda - 1)/\lambda$, which yields the logit transformation for $\lambda = 0$ and $1/(1 - p_t)$ for $\lambda = 1$. The inverse transformation can be calculated explicitly, but the fact that $u_t(\lambda)$ fails to give the untransformed observations for any value of λ can be seen as a limitation.

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Aranda-Ordaz (1981, AO henceforth) proposed a class of transformation that does not suffer from the above drawbacks, being defined as:

$$u_t(\lambda) = \frac{2}{\lambda} \frac{p_t^\lambda - (1 - p_t)^\lambda}{p_t^\lambda + (1 - p_t)^\lambda} = \frac{2}{\lambda} \frac{o_t^\lambda - 1}{o_t^\lambda + 1}. \tag{1}$$

For $\lambda \to 0$ it yields the logit transformation, $u_t(0) = \ln(p_t/(1 - p_t))$, and the untransformed series for $\lambda = 1$, $u(1) = 2(2p_t - 1)$. The reverse transformation is:

$$p_t = \frac{1}{1 + \exp(-v_t)}, v_t = \begin{cases} \frac{1}{\lambda} \ln\left(\frac{2 + \lambda u_t}{2 - \lambda u_t}\right), & \lambda \neq 0, \\ u_t, & \lambda = 0 \end{cases}$$

Other types of transformations and generalizations are considered in Stukel (1988).

Given a trivariate time series of proportions pertaining to ordered response categories let

$$u_{1t} = \frac{2}{\lambda} \frac{p_{1t}^\lambda - (1 - p_{1t})^\lambda}{p_{1t}^\lambda + (1 - p_{1t})^\lambda}, u_{2t} = \frac{2}{\lambda} \frac{(p_{1t} + p_{2t})^\lambda - (1 - p_{1t} - p_{2t})^\lambda}{(p_{1t} + p_{2t})^\lambda + (1 - p_{1t} - p_{2t})^\lambda}$$

denote the AO transformation of the cumulative proportions, we can write down an encompassing bivariate measurement model for the pair (u_{1t}, u_{2t}) that nests the standard quantification using the balances in the particular case in which $\lambda = 1$ and a common cycle enters both series with a vector of loadings that is proportional to the unit vector.

3   Seasonality

Although the respondent is explicitly asked to abstract from seasonal movement in forming his/her judgement, a well known common feature of business survey indicators is the presence of seasonality. The seasonal dynamics in the business survey indicators reflect the seasonality in the underlying quantitative indicators (orders, turnover and industrial production) as far as the location of seasonal peaks and troughs within the year is concerned. This evidence has been advocated in support of the notion that seasonal fluctuations are not independent of the trend-cycle, which implies that economic time series are not decomposable (Franses, 1996).

The presence of seasonality can be illustrated from the month-by-month plots of the Industrial production series and the percentages p_{it} of responses high and low for the level of production, which are presented in figure **??**. While Industrial production displays a very deep seasonal trough in August and a minor one in December, the percentage of low and high or no change (transformed into logits) display seasonal peaks in correspondence.

This descriptive evidence can be supported by formal statistical tests, such as the Canova - Hansen (1995) and Busetti and Harvey (2003) test, concerning the presence and the nature of the seasonal movements. A related issue is whether the responses are affected by the number of working days in the month and any other calender effect, such as the length of the month and Easter.

These issues can be addressed in an unobserved components framework, according to which a univariate time series, y_t, is decomposed according to the following model:

$$y_t = \mu_t + \gamma_t + \varepsilon_t \qquad t = 1...T, \qquad (2)$$

where μ_t is the level component, $\mu_t = \mathbf{x}_t'\boldsymbol{\delta}$, $\mathbf{x}_t$ is a vector of linearly independent deterministic regressors, e.g. $\mathbf{x}_t = [1, t - (T+1)/2]'$ for a linear trend, γ_t denotes the seasonal component and $\varepsilon_t \sim \text{NID}(0, \sigma_\varepsilon^2)$. Busetti and Harvey (2003) derive the locally best invariant test of the null that there is no seasonality against a permanent seasonal component, that can be either deterministic or stochastic, or both. The seasonal component is decomposed into a deterministic term, a linear combination with fixed coefficients of sines and cosines defined at the seasonal frequencies $\lambda_j = 2\pi j/s$, $j = 1...[s/2]$, where s is the number of seasons in a year (e.g. 12 for monthly time series), and $[s/2]$ is the nearest integer resulting from the division in the argument, and a nonstationary stochastic term, a linear combination of the same elements with random coefficients:

$$\gamma_t = \gamma_t^D + \gamma_t^S.$$

Defining $\mathbf{z}_t = \left[\cos\omega_{1t}, \sin\omega_{1t}, ..., \cos\omega_{jt}, \sin\omega_{jt}, ..., \cos\omega_{[s/2]t}, \sin\omega_{[s/2]t}\right]'$, $\gamma_t^D = \mathbf{z}_t'\gamma_0$, γ_0 fixed coefficients. The stochastic component is $\gamma_t^S = \mathbf{z}_t' \sum_{i=1}^t (k_i)$ where k_t is a vector of serially independent disturbances with zero mean and covariance matrix $\sigma_k^2 W$, independently of ε_t.

The null hypothesis is then formulated as $H_0 : \gamma_0 = 0, \sigma_k^2 = 0$; a permanent seasonal component is present under the two alternatives: $H_a : \gamma_0 \neq 0, \sigma_k^2 = 0$ (deterministic seasonality), $H_b : \gamma_0 = 0, \sigma_k^2 > 0$ (stochastic seasonality). The test statistic proposed by Busetti and Harvey (2003) is consistent against both alternative hypotheses, and it is computed as follows:

$$\varpi = \sum_{j}^{[s/2]} \varpi_j, \quad \varpi_j = \frac{a_j}{T^2\sigma^2} \sum_{t=1}^{T} \left[\sum_{i=1}^{t}(e_i \cos\omega_j i)^2 + \sum_{i=1}^{t}(e_i \sin\omega_j i)^2\right], \qquad (3)$$

where e_i are the OLS residuals obtained from the regression of y_t on the explanatory variables $\mathbf{x}_t$. Under the null ϖ is asymptotically distributed according to a Cramér von Mises (CvM) distribution with $s - 1$ degrees of freedom.

The test of the null that the seasonal component is deterministic (H_a) against the alternative that it evolves according to a nonstationary seasonal process (H_b), i.e. characterised by the presence of unit roots at the seasonal frequencies ω_j, is based on the Canova-Hansen test statistic which is (**??**), with e_i replaced by the OLS residuals that are obtained by including the trigonometric functions in $\mathbf{z}_t$ as additional explanatory variables along with $\mathbf{x}_t$. Under the null of deterministic seasonality, the test has again a Cramér von Mises distribution, with $s - 1$ degrees of freedom.

In both cases the statistic ϖ_j is used to test the hypothesis that the series display (nonstationary, if $\mathbf{z}_t$ is included in the set of regressors) seasonality at a particular frequency.

The null distribution of the test is CvM with 2 degrees of freedom if $j = 1, ..., (s-1)/2$ and 1 degree of freedom for $j = s/2$, for an even s. If $\omega_j = 0$ the test statistic ϖ_0 is the usual KPSS test of stationarity at the long–run frequency.

The test statistics in (**??**) require an estimate of σ^2. A nonparametric estimate is obtained by rescaling by 2π the estimate of the spectrum of the sequence e_t at the frequency ω_j, using a Bartlett window. Canova and Hansen (1995) further allow for seasonal heteroscedasticity.

The same framework can be used to test for the presence of calendar effects in the series. Provided that it is well known that the underlying quantitative indicators (production, turnover) are highly affected by calendar effects, it seems reasonable to detect the presence of such component also in the qualitative surveys. For this purpose the test statistic ϖ_j in (**??**) can be applied at the specific frequencies $.348 \times 2\pi$, $.432 \times 2\pi$, $.304 \times 2\pi$, which Cleveland and Devlin (1982) have shown to be corresponding to the spectral peaks induced by the trading days effect.

In the sequel we discuss the empirical evidence arising from the application of seasonality and stationarity tests on the first differences of the proportions p_{it} emerging from the ISAE survey for Italy. The results are invariant to the adoption of a transformation, such as the logit transformation. The results are reported in the tables **??**– **??**.

Table 1: Harvey-Busetti general tests for seasonality

	overall test ϖ	single frequencies						
		0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
Industrial production	7816.82	0.399	44.341	601.34	2306.1	1302.7	1517.7	2044.3
Low	171.79	0.049	3.531	11.097	50.207	14.804	53.772	38.331
Normal	156.48	0.052	4.530	12.666	48.338	15.956	54.804	20.132
High	134.36	0.043	7.570	6.566	38.069	23.934	23.892	34.287
Balance	165.34	0.046	4.524	9.565	48.200	16.779	45.580	40.650

Note: Critical values (see tab. 1 in Nyblom (1989)) are respectively:
CvM(11)=9.03 for overall test, CvM(1)=1.65 for 0 and π and CvM(2)=2.63 for other frequencies

The null of no permanent seasonal effects is always rejected for all the three response categories "low", "normal", "high", either for each single frequencies or for all of them (see tab. **??**). As for the nature of the seasonal pattern, for some of the seasonal frequencies the null of a deterministic seasonal pattern cannot be rejected and the overall evidence is against the null for all the series considered (see tab. **??**). The test results for the survey data do not always mirror those concerning the Industrial production series.

Finally, as far as the trading-days effects are concerned, the overall test is significant for all the response categories as well as for the balance series, although the test statistic takes a much smaller value with respect to the Industrial production series (see tab. **??**). More generally, the evidence is that calendar effects constitute a much less relevant source of variation for the business survey series.

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Table 2: Canova-Hansen test for stationary seasonality with correction for heteroschedasticity

	overall test ϖ	single frequencies						
		0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
Industrial production	3.496	0.093	0.596	0.714	0.785	0.353	0.879	0.076
Low	3.90	0.045	0.937	0.681	0.820	0.472	0.444	0.499
Normal	4.04	0.041	1.096	0.538	1.157	0.307	0.661	0.241
High	5.14	0.048	0.611	0.940	1.020	0.483	1.425	0.609
Balance	4.53	0.047	0.748	0.941	0.665	0.544	0.743	0.838

Note: Critical values (see tab. I(a) in Harvey (2001)) are respectively:
CvM(11)=2.739 for overall test, CvM(1)=.461 for 0 and π and CvM(2)=.748 for other frequencies

Table 3: Harvey-Busetti test for calendar effects

	overall test ϖ	single frequencies		
		$.348 * 2\pi$	$.432 * 2\pi$	$304 * 2\pi$
Industrial production	367.39	222.469	128.584	16.335
Low	11.23	8.931	2.020	0.283
Normal	11.36	3.672	6.504	1.179
High	18.76	16.090	2.227	0.441
Balance	13.58	12.123	1.165	0.290

Note: Critical values (see tab. 1 in Nyblom (1989)) are respectively:
CvM(6)=5.68 for overall test and CvM(1)=.461 for single frequencies

4   The spectral envelope

Stoffer, Tyler and McDougall (1993) proposed a frequency domain approach to scaling categorical time series which can be applied to the quantification of business surveys. They introduced the notion of spectral envelope for a categorical time series $\mathbf{Y}_t$, which can be defined at any angular frequency $\omega \in [0, \pi]$ as the spectral density of the univariate synthetic time series $\mathbf{b}(\omega)'\mathbf{Y}_t$ (or $\mathbf{b}(\omega)'\mathbf{p}_t$ for time series of proportions) that is obtained when the optimal scores $\mathbf{b}(\omega)$ are applied to the categories. The notation $\mathbf{b}(\omega)$ stresses the fact that the optimal scores vary with the frequency. The optimality lies on the fact that $\mathbf{b}(\omega)$ provide the greatest evidence for periodicity at frequency ω, in that the scaled time series has the greatest relative power at that frequency.

Since the distribution of the vector time series $\mathbf{Y}_t$ (or $\mathbf{p}_t$) is singular ($\mathbf{i}'\mathbf{Y}_t = 1$ with probability one), one of the series is redundant and can be assigned a score equal to zero. Let $\mathbf{Z}_t = \mathbf{A}'\mathbf{Y}_t$ be a linearly independent subset of series, where $\mathbf{A}$ is a fixed selection matrix, and let $\mathbf{\Gamma}(\tau)$ denote the crosscovariance matrix at lag τ of $\mathbf{Z}_t$; then, $\mathbf{F}(\omega) = (2\pi)^{-1}\sum_{\tau=-\infty}^{\infty}\mathbf{\Gamma}(\tau)\exp(-\imath\omega\tau)$ is then the spectral density at frequency ω and let $\mathbf{F}^r(\omega) == (2\pi)^{-1}\sum_{\tau=-\infty}^{\infty}\mathbf{\Gamma}(\tau)\cos(\omega\tau)$ is its real part. In our case study concerning

the assessment of the level of production we can assign a zero score to the central category and choose $\mathbf{A} = (\mathbf{e}_1, \mathbf{e}_3)$.

Defining $\mathbf{b}^*(\omega) = \mathbf{A}'\mathbf{b}(\omega)$ the vector of scores attached to the selected series, the quantification $\mathbf{b}^{*'}(\omega)\mathbf{Z}_t$ takes place by choosing the scores so as to emphasize the periodic features in the series. so as to maximize the power at frequency ω relative to the total power. In particular, the spectral density of the scaled series is $(2\pi)^{-1}\mathbf{b}^{*'}(\omega)\mathbf{F}^r(\omega)\mathbf{b}^*(\omega)$, whereas $\mathbf{b}^{*'}(\omega)\mathbf{\Gamma}_Y(0)\mathbf{b}^*(\omega) = \int_0^\pi (2\pi)^{-1}\mathbf{b}^{*'}(\omega)\mathbf{F}^r(z)\mathbf{b}^*(\omega)dz$ is the variance of the scaled series. Thus, $\mathbf{b}^*(\omega)$ is chosen so as to maximize the ratio:

$$\frac{\mathbf{b}^{*'}(\omega)\mathbf{F}^r(\omega)\mathbf{b}^*(\omega)}{\mathbf{b}^{*'}(\omega)\mathbf{\Gamma}_Y(0)\mathbf{b}^*(\omega)}. \tag{4}$$

Differentiating with respect to $\mathbf{b}^*(\omega)$, and denoting by $\lambda(\omega)$ the supremum of **??**, the first order conditions lead to the system of equations $\mathbf{F}^r(\omega)\mathbf{b}^*(\omega) = \lambda(\omega)\mathbf{\Gamma}(0)\mathbf{b}^*(\omega)$. Hence, $\lambda(\omega)$ is the largest eigenvalue of $\mathbf{\Gamma}(0)^{-1/2}\mathbf{F}^r(\omega)\mathbf{\Gamma}(0)^{-1/2}$, $\mathbf{\Gamma}(0)^{1/2}\mathbf{\Gamma}(0)^{1/2} = \mathbf{\Gamma}(0)$ (in practice, the symmetric square root matrix is constructed from the spectral decomposition of $\mathbf{\Gamma}(0)$) and $\mathbf{b}^*(\omega) = \mathbf{\Gamma}(0)^{-1/2}\mathbf{v}(\omega)$, where $\mathbf{v}(\omega)$ is the corresponding eigenvector. The scalar $\lambda(\omega)$ is the spectral envelope at frequency ω.

It should be noticed that the scores are not a monotonic function of the category index j, which may be regarded as a drawback; the estimation of a vector of scores β_j that is monotonic in j is left to future research. It can however be argued that the monotonic solution is bounded from above by the standard solution.

Given a realization of $\mathbf{Y}_t$ or $\mathbf{p}_t$, the spectral envelope and the associated optimal scores can be estimated using a nonparametric estimator of the real part of the cross-spectrum. In the application below we use a Parzen lag window with truncation parameter at 60.

Figure **??** plots the spectral envelope for the assessment of the level of production. Its main features are the concentration of power at the long run and business cycle frequencies, along with the presence of spectral peaks at the seasonal frequencies. The estimated optimal scaling corresponding to the spectral envelope are plotted against the angular frequency in the central panel of figure **??**. It is very interesting to notice that the scores have different signs, i.e. produce a contrast, only at a subset of the business cycle frequencies, ranging from 2.5 to 5 years (shaded area). Since we have assigned the score 0 to the central category (normal) the optimal scores are a monotonic function of the index j. The bottom panel of figure **??** displays the scaled series using (linearly transformed so as to match the mean and standard deviation of the d_{2t} series). Essentially, for our particular application, the spectral envelope validates the use of the balance for quantification.

5 Cumulative logit model

The quantification method proposed in this section can be regarded as a dynamic version of the probability approach initiated by Carlson and Parkin (1975), described, among

others, in Pesaran and Weale (2006), which postulates the existence of a common latent variable with known distribution function. The method, which is based on a dynamic cumulative logit model, see Fahrmeir (1992) and Fahrmeir and Tutz (1994 sec. 3.3), overcomes the independence assumption for the latent variable. As a consequence, the quantification will be based on a dynamic nonlinear combination of the observed proportions.

The most popular approach to the analysis of ordinal responses is to assume the existence of a latent continuous response variable, ς_t such that $\mathbf{Y}_t = \mathbf{e}_j$, i.e. the individual response is category i if $q_{i-1} < \varsigma_t \le q_i$, where q_i is a threshold $q_0 = -\infty < q_1 < \cdots < q_k = \infty$. As we shall see shortly, the natural choice for the latent variable is a stochastic cycle, but we need to take into account also the presence of seasonality by introducing a seasonal feature into the latent variable.

The process ς_t is a linear Gaussian time series process that is parameterised according to the state space model

$$\varsigma_t = \mathbf{z}'\boldsymbol{\alpha}_t + \epsilon_t, \qquad \boldsymbol{\alpha}_{t+1} = \mathbf{T}\boldsymbol{\alpha}_t + \mathbf{H}\boldsymbol{\eta}_t$$

where ϵ_t has logistic distribution function $F(\epsilon) = 1/(1 + \exp(-\epsilon))$.

The most popular approach to modelling the series y_{it} uses logits of cumulative probabilities, also termed cumulative logits. These can be related to the latent signal as follows:

$$P(y_t \le c_i) = \sum_{j=1}^{i} \pi_{jt} = P(\varsigma_t \le q_i | \boldsymbol{\alpha}_t) = P(\epsilon_t \le q_i - \mathbf{z}'\boldsymbol{\alpha}_t) = \left(\frac{1}{1 + \exp(\mathbf{z}'\alpha_t - q_i)} \right).$$

Consider now the assessment of the level of production and let $i = 1, 2, 3$ label the three response categories. Then,

$$\ln \left[\frac{P(y_t \le c_i)}{1 - P(y_t \le c_i)} \right] = \ln \left[\frac{P(\varsigma_t \le q_i)}{1 - P(\varsigma_t \le q_i)} \right] = q_i - \mathbf{z}'\boldsymbol{\alpha}_t.$$

Hence, we can express the cumulative probabilities

$$\pi_{1t} = \frac{\exp(\theta_{1t})}{1 + \exp(\theta_{1t})}, \quad \pi_{1t} + \pi_{2t} = \frac{\exp(\theta_{2t})}{1 + \exp(\theta_{2t})},$$

and write $\theta_{1t} = \text{logit}(\pi_{1t}), \theta_{2t} = \text{logit}(\pi_{1t} + \pi_{2t})$.

Assuming that, conditionally on $\boldsymbol{\alpha}_t$, the observations are independent, the multinomial density of the counts n_{it} is

$$p(\mathbf{n}_t | \boldsymbol{\pi}_t) = \sum_{t=1} \left\{ \ln K_t + n_t \ln \pi_{kt} + \sum_{i=1}^{k-1} n_{it} \left[\ln \pi_{jt} - \ln \pi_{kt} \right] \right\}$$

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where K_t denotes the multinomial coefficient $n_t!/\left(\prod_{i=1}^{k} n_{it}!\right)$. The likelihood can be expressed in terms of the cumulative logits $\theta_{it} = q_i - \mathbf{z}'\boldsymbol{\alpha}_t$. In particular, when $k = 3$:

$$
\begin{aligned}
p(n_{1t}, n_{2t}|\theta_{1t}, \theta_{2t}) = {} & \sum_{t=1} \{\ln K_t + n_{1t}\left[\theta_{1t} + \ln(1 + \exp(\theta_{2t}))\right] + \\
& n_{2t}[\exp(\theta_{2t}) - \exp(\theta_{1t})] - (n_{1t} + n_{2t})\ln(1 + \exp(\theta_{1t}))\}.
\end{aligned}
\tag{5}
$$

When the sample size is constant, $n_t = n$, we can reexpress the likelihood in terms of the proportions $p_{jt} = n_{jt}/n_t$ and the parameters $\theta_{it}, i = 1, 2$.

$$
\begin{aligned}
p(p_{1t}, p_{2t}|\theta_{1t}, \theta_{2t}) = {} & \sum_{t=1} \{\ln K_t^* + p_{1t}\left[\theta_{1t} + \ln(1 + \exp(\theta_{2t}))\right] + \\
& p_{2t}[\exp(\theta_{2t}) - \exp(\theta_{1t})] - (p_{1t} + p_{2t})\ln(1 + \exp(\theta_{1t}))\}.
\end{aligned}
\tag{6}
$$

As for as the specification of the linear and Gaussian state space model for the latent component, we have

$$
\mathbf{z}'\boldsymbol{\alpha}_t = \psi_t + \gamma_t,
$$

where ψ_t is a stochastic cycle, generated by the dynamic equation:

$$
\begin{bmatrix} \psi_t \\ \psi_t^* \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c \end{bmatrix} \begin{bmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{bmatrix} + \begin{bmatrix} \kappa_t \\ \kappa_t^* \end{bmatrix}
$$

where $\kappa_t \sim \text{NID}(0, \sigma_\kappa^2)$ and $\kappa_t^* \sim \text{NID}(0, \sigma_\kappa^2)$ and $\text{E}(\kappa_t\kappa_t^*) = 0$. The seasonal component is parameterised according to the trigonometric seasonal model (see Harvey, 1989), which results from the sum of $[s/2]$ nonstationary stochastic cycles defined at the seasonal frequencies: $\gamma_t = \sum_{j=1}^{[s/2]} \gamma_{jt}$,

$$
\begin{bmatrix} \gamma_{jt} \\ \gamma_{jt}^* \end{bmatrix} = \begin{bmatrix} \cos\lambda_j & \sin\lambda_j \\ -\sin\lambda_j & \cos\lambda_j \end{bmatrix} \begin{bmatrix} \gamma_{j,t-1} \\ \gamma_{j,t-1}^* \end{bmatrix} + \begin{bmatrix} \chi_{jt} \\ \chi_{jt}^* \end{bmatrix}, \quad j = 1, ..., [s/2],
$$

with χ_{jt} and χ_{jt}^* being a set of serially and mutually uncorrelated WN sequences with common variance σ_χ^2.

The cumulative logit model is a particular instance of dynamic generalised linear model. Inference on the unknown parameters and the underlying cycle can be made using the approach based on importance sampling Durbin and Koopman (2001). The linear and Gaussian approximating model is derived as follows. Let us suppose that we are able to start from trial values $\tilde{\theta}_{it} = q_i - \mathbf{z}'\tilde{\boldsymbol{\alpha}}_t$, $i = 1, 2$, and set $\tilde{\boldsymbol{\theta}}_t = [\tilde{\theta}_{1t}, \tilde{\theta}_{2t}]'$. Consider the first order Taylor expansion of the gradient of the logarithm of the multinomial density, denoted $\mathbf{g}(\boldsymbol{\theta}_t)$, with respect to the $\theta_{it}, i = 1, 2$.

$$
\mathbf{g}(\boldsymbol{\theta}_t) = \mathbf{g}(\tilde{\boldsymbol{\theta}}_t) + \mathbf{D}(\tilde{\boldsymbol{\theta}}_t)(\boldsymbol{\theta}_t - \tilde{\boldsymbol{\theta}}_t), \mathbf{g}(\boldsymbol{\theta}_t) = \frac{\partial p(\mathbf{n}_t|\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}, \quad \mathbf{D}(\boldsymbol{\theta}) = \left[\frac{\partial^2 p(\mathbf{n}_t|\boldsymbol{\theta}_t)}{\partial\boldsymbol{\theta}_t\partial\boldsymbol{\theta}_t'}\bigg|_{\boldsymbol{\theta}_t=\tilde{\boldsymbol{\theta}}_t}\right].
\tag{7}
$$

Equating (**??**) to zero and rearranging yields the pseudo-linear approximating state space model

$$\tilde{\mathbf{y}}_t = \mathbf{q_i} - \mathbf{i}_2 \mathbf{z}' \boldsymbol{\alpha}_t + \mathbf{u}_t, \quad \mathbf{u}_t \sim \text{NID}(\mathbf{0}, \mathbf{D}^{-1}(\tilde{\boldsymbol{\theta}}_t)), \quad \boldsymbol{\alpha}_{t+1} = \mathbf{T}\boldsymbol{\alpha}_t + \mathbf{H}\boldsymbol{\eta}_t$$

where the pseudo-observations are $\tilde{\mathbf{y}}_t = \tilde{\boldsymbol{\theta}}_t - \mathbf{D}^{-1}(\tilde{\boldsymbol{\theta}}_t)\mathbf{g}(\tilde{\boldsymbol{\theta}}_t)$, $\mathbf{q} = [q_1, q_2]'$. The linear Gaussian approximating model that is used for simulation is found by iterating the Kalman filter and Smoother on the linearised model, that is, starting from $\tilde{\boldsymbol{\alpha}}_t$ (and thus $\tilde{\boldsymbol{\theta}}_t$) the KFS is applied to the pseudo observations to get a new estimated of the state vector $\tilde{\boldsymbol{\alpha}}_t$, and of $\tilde{\boldsymbol{\theta}}_t$, which are in turn replaced into the gradient and the hessian so as to give a new set of pseudo observations and to update the system matrices of the linear Gaussian approximating model. This process, iterated until convergence, yields the estimates of the posterior mode of $\boldsymbol{\theta}_t$ and $\boldsymbol{\alpha}_t$, conditional on the available data. The Gaussian density is used to draw samples and to estimate functions of the state by means of importance sampling techniques, and a Monte Carlo estimate of $\text{E}(\boldsymbol{\alpha}_t|\mathbf{n}_t)$ is available. Importance sampling is used also for Monte Carlo estimation of the likelihood, which can be maximised using a quasi-Newton method.

6   Application to the Level of Production

6.1   Description of the available data

Business surveys are a well established source of timely information on the current state of the economy and its future developments. In the European Union the *Joint Harmonised EU Programme of Business and Consumer Surveys* (see European Commission, 1997) has contributed to enhance the comparison and the harmonisation both in terms of sampling design and questionnaire.

The *Manufacturing Survey* for Italy is carried out on a monthly basis by ISAE from 1962. Several modifications and improvements on the sampling design and survey questions have occurred through time in order to improve the accuracy and to enforce the harmonisation with other EU countries. More recently, particular attention has been devoted to the issue of weighting the individual responses to produce sectoral aggregates. After the last revision which took place in 2003, the current weights are based on economic activity (3-digit l NACE Rev 1.1 classification), 20 administrative regions and firm size.

The questionnaire is divided into three parts, the first concerning the assessment of the situation for the current month and the last relating to the expectations for the next 3 months. The central part is devoted to a specific question formulated in terms of the variation with respect to the previous month. The survey refers to the main aspect of economic activity: level of production and orders, unsold stocks, liquidity, selling prices, employees. In addition, in the last month of each quarter the questionnaire is supplemented with

questions that relate to capacity utilization, raw materials, worked hours, export and the presence of economic constraints that hindered growth. For each question the respondent firm should provided a qualitative answer on a ordinal balanced closed scale, with 3 or 5 response categories, always formulated with a central neutral category.

As for the assessment of the level of production, which is the focus of this paper, the survey question is formulated as follows: "Excluding seasonal patterns, the level of production is: high, normal, low?". No attempt at providing a detailed explanation of the reference, or "normal", value is made. The number of firms participating to the survey has increased steadily: from about 2,500 in 1991 to almost 4,000 in 2005. The selection of the sample is made by ISAE purposively, according to the representativeness of the firm (an implicit measure of size is used for sample selection), and the dataset takes the form of a panel, even though only 14 firms have a continuous participation record from 1991 to 2005; this number does not increase considerably if we restrict our analysis to the last decade (from 1996 to 2005 only 59 complete records are available). Partial nonresponse is widespread and may be detrimental to the quality of the survey especially in August, which is the traditional holiday period in Italy. In the following application we use a monthly sample of firms from 1991.1 to 2005.12 and the number of responses is weighted by using firm size and sector, following the ISAE aggregation scheme (see tab. 5 pag 27 in Malgarini *et al*, 2005). Proportions of responses for the level of production survey question are shown in Figure **??**.

6.2   Main results

In the sequel no account is made for the fact that the responses refer to the same units; we will postulate that at each time point an independent sample of manufacturing units is drawn, and that the autocorrelation of the responses is fully explained by a latent variable which expresses the state of the economy. We leave to future research the modelling and the investigation accounting for a panel time series.

For the ISAE survey question related to the level of production estimation was carried out using Ssfpack 3.0 beta by Koopman et al. (1999). The maximum likelihood estimates of the threshold parameters resulted $\hat{\mathbf{q}} = [-1.4, 2.1]$, the asymptotic standard error obtained by the delta method resulted 0.07 for both parameters. The estimates of the posterior mean of the latent cyclical factor, $\mathrm{E}(\psi_t|\mathbf{n}_t)$, obtained via importance sampling using 1000 replications and an antithetic variable (see Durbin and Koopman, 2001, p. 205), are displayed in Figure **??**, along with an approximated 95% confidence interval. The estimated cycle has a period of 69 months, that is about 5 years and half, and is a persistent component with an estimated damping factor equal to 0.97; moreover, seasonality is a smaller source of variation, with $\hat{\sigma}_\kappa^2 = 0.004\hat{\sigma}_\chi^2$.

In order to asses the accuracy of the model we provide some graphic diagnostics both for residuals than for the importance sampler. The Pearson's residuals, presented in the top left panel in Figure **??**, show a not desirable cyclical pattern in the first years of the

sample. On the other hand survey data in the late '90s referred to a different sample design and size, which perhaps is one of the motivation for a different composition of proportions as shown by the Figure **??**. The remaining graphs in Figure **??** are dedicated to importance sampling weights diagnostics. The weights for a simulation of 1.000 replications are presented, together with the largest 100 weights. In addition, the bottom right panel shows the recursive estimation of the standard deviation, which provide evidence that the variance exists and this is not seriously affect by outliers.

Figure **??** compares the estimated latent cycle with the cycle in the corresponding quantitative indicator, the index of Industrial production for the Italian manufacturing sector, and with the balances $p_{3t} - p_{1t}$, appropriately rescaled. For computing the balances we considered the seasonally adjusted proportions. The deviation cycle in Industrial production was estimated by the Harvey and Jaeger (1993) model with seasonality, using the time series produced by ISTAT for the sample period 1990.1–2005.12. Accordingly, the series is decomposed into a local linear trend (see Harvey, 1989), a stochastic cycle with the same representation as ψ_t above and a trigonometric seasonal component including trading days effect correction.

As mentioned before survey data has become more reliable and homogeneous in the last decade, fact that appear also by the inspection of Figure **??**. Therefore we applied the same model to the sample: (1996.1-2005.12). Results, shown in Figure **??**, are very similar to the previous: although the Pearson's residuals show a better pattern for the resticted sample (Figure **??**), the survey data cycle on the whole sample is more to close to the balance series (Figure **??**).

6.3   Time varying thresholds

In the model presented above we implicitly assume thresholds constant over time, although is likely to expect that during periods with an high growth rate of production, the upper threshold in the indifference interval $\mathbf{q} = [q_1, q_2]'$ would increase (ad so for a decrease in low rate periods of production). This restriction could be relaxed in order to provide more generality. In the State Space Form it is straightforward adding the following equation in the state vector: $q_{it} = q_{it-1} + \vartheta_t$ with $\vartheta_t \sim NID(0, \sigma_\vartheta^2)$ and independent from other errors in the model. The estimated thresholds, shown in Figure **??**, suggest that the indifference interval has moved to the right part of the real axe during the last ten years. Diagnostics are very similar to the fixed thresholds model, although the comparison with the Industrial production deviation cycle is slightly improved (compare Figure **??** and Figure **??**).

The log-likelihood for the two model is very similar (-85.055 and -85.096 respectively with fixed and time varying thresholds), which suggests not significant difference between the two models, nevertheless any test of model choice based on the likelihood, like the Likelihood Ratio test (LR), needs some additional considerations. If the order of integration do not change between one specification and the other, the LR has a distribution that

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is a mixture of Chi-squared, otherwise we will end to a Cramèr-von-Mises distribution. We omit the presentation for the time varying thresholds model on the restricted sample from 1996 because this is a degenerate case that could be included in the time constant thresholds model shown in Figure **??**.

Several interesting considerations emerge: first and foremost, our model based quantification can be seen as a smoothed version of the balances. The amount of smoothing is dictated by the parameters of the model; the turning points are more clearly identifiable and the assessment of cyclical stance is made much easier. Secondly, the latent cycle is highly coherent with that in Industrial production which was estimated independently.

7   Conclusions

This paper has investigated several issues related to the quantification and the analysis of business survey variables. The presence and nature of seasonal fluctuations and trading days variation was addressed and two novel quantification methods were proposed and investigated. The first is based on a purely cross–sectional filter derived from the notion of a spectral envelope. The second is based on a dynamic cumulative logit model, which extracts the latent cycle from the available time series of proportions and yields signal extraction filters that have both time series and cross-sectional dimensions. As a result the quantification has a smoother appearance, which is more amenable for the identification of turning points and for the characterization of the perceived cyclical stance.

The underlying cycle is highly coherent with the deviation cycle in the corresponding quantitative indicator, the index of Industrial production. This raises the important issue of understanding what notion of business cycle (deviation or growth rate) the economic agents have in mind when they answer the qualitative survey question. We leave to future research this important issue.

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Figure 1: Monthplots.

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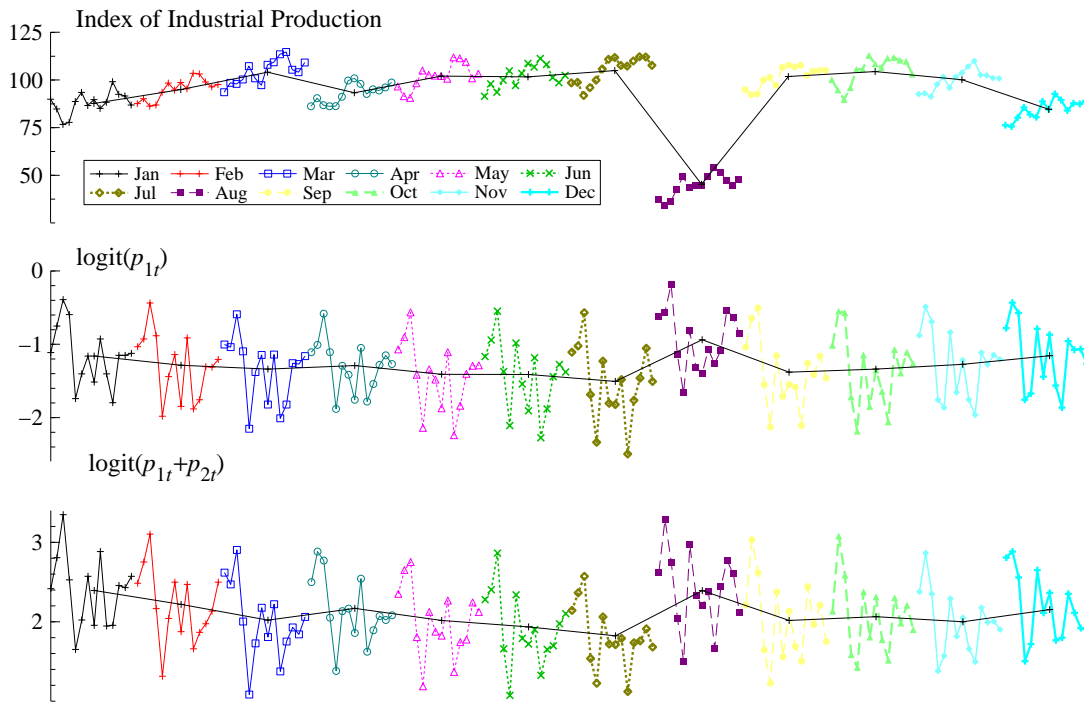
Figure 2: Spectral envelope, scores and quantification of survey questions on the level of production.
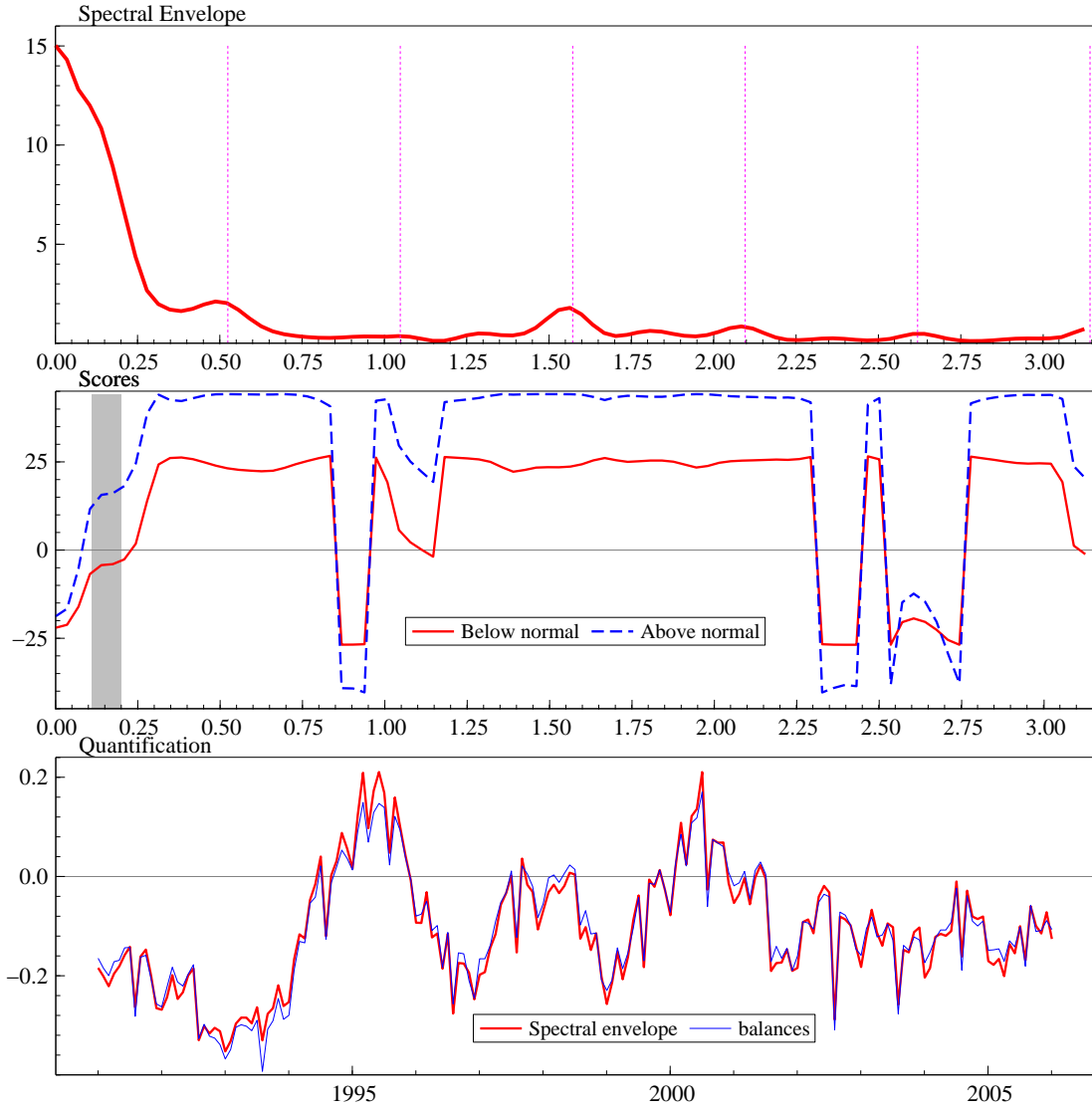
Figure 3: Proportion of responses for the assessment of the level of production, ISAE Survey on Business Tendency for Italy
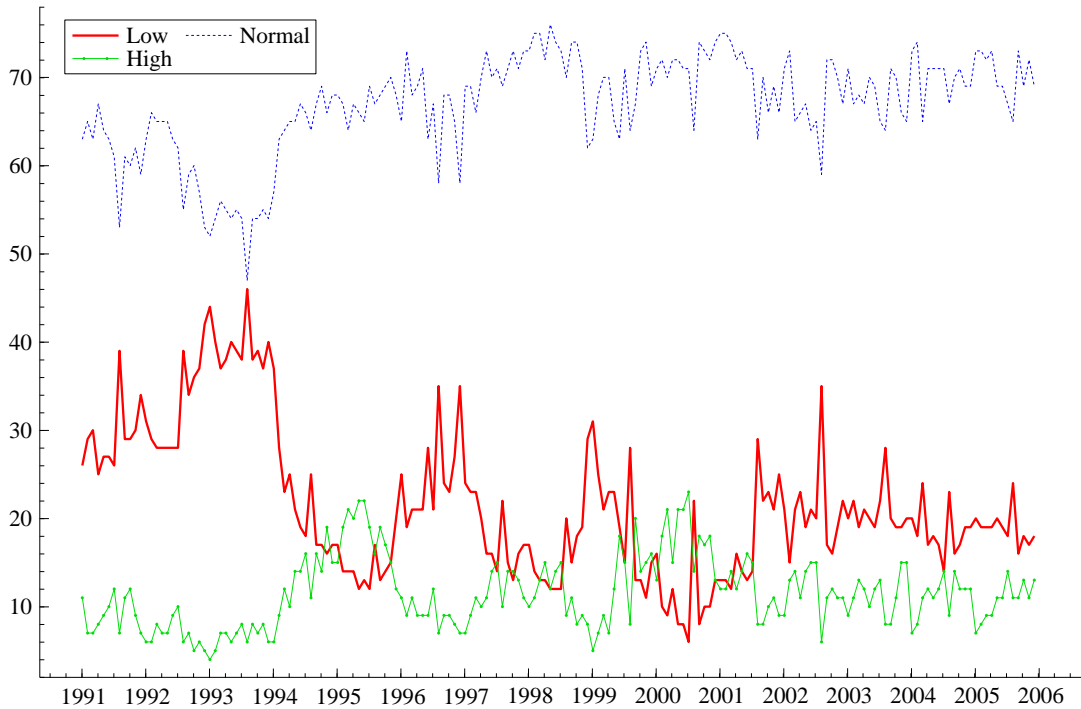
Figure 4: Dynamic cumulative logit model: estimates of the conditional mean of the latent cyclical factor, $E(\psi_t|\mathbf{n}_t)$ obtained via importance sampling using 1000 replications.
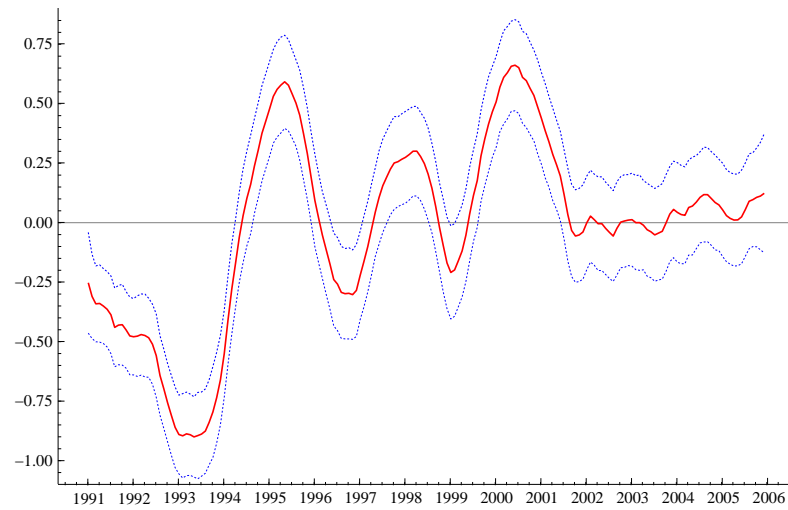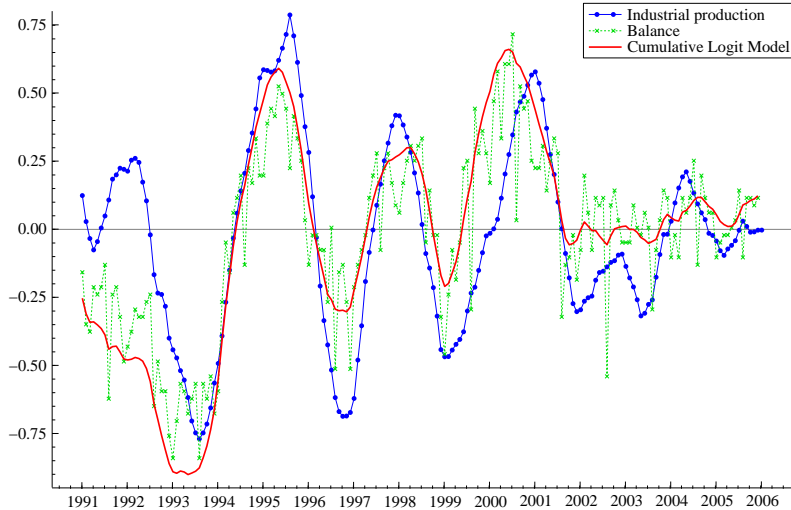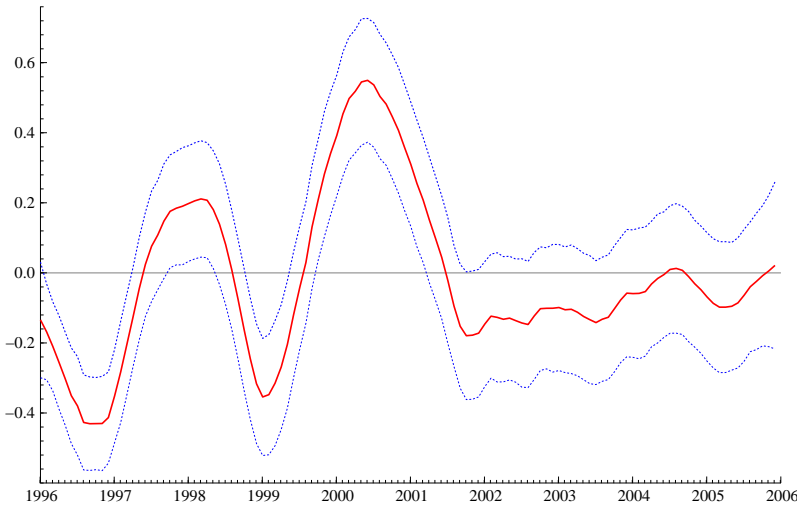

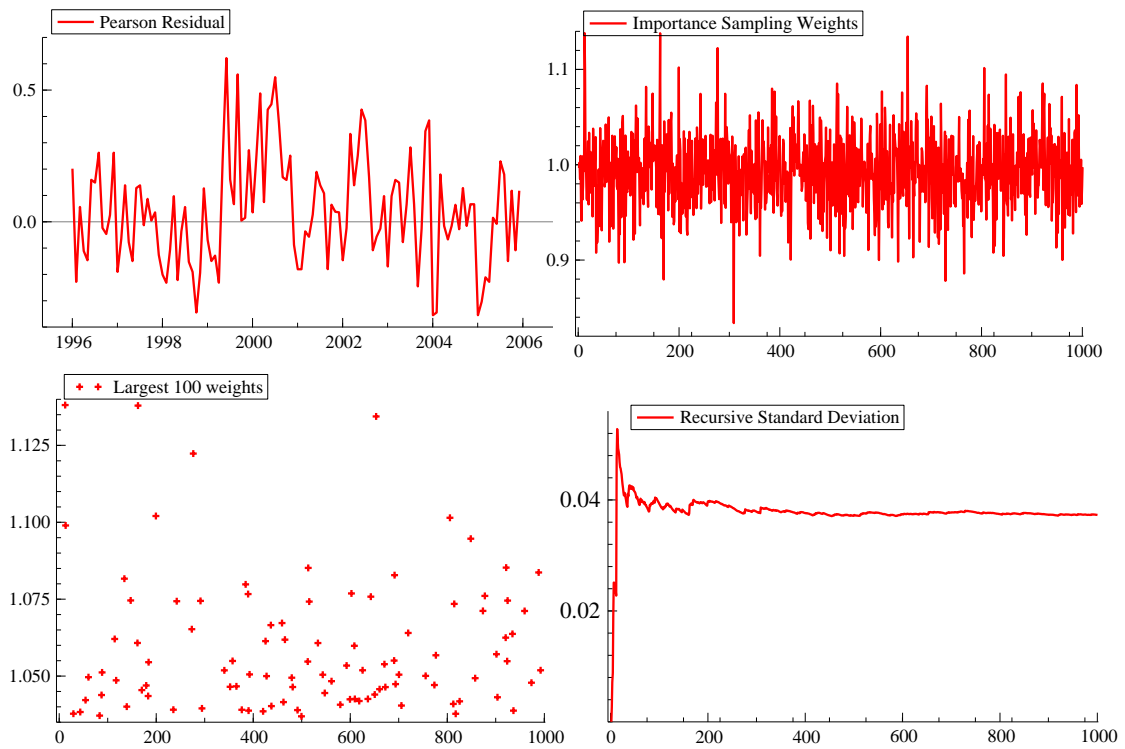Figure 5: Pearson's residuals and importance sampling diagnostics for the cumulative logit model
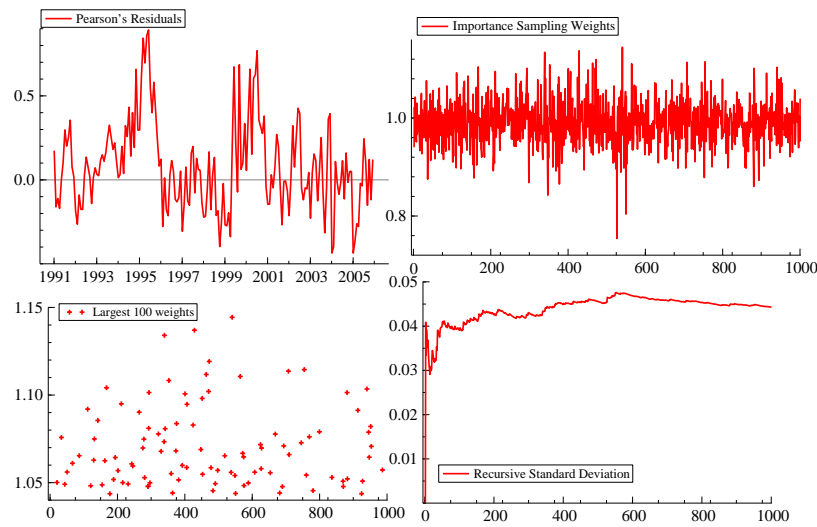
Figure 6: Comparison of Industrial production deviation cycle, survey data latent cycle with constant thresholds, quantification through balances

Figure 7: Dynamic cumulative logit model: estimates of the conditional mean of the latent cyclical factor, $E(\psi_t|\mathbf{n}_t)$ obtained via importance sampling using 1000 replications- (1996-2005).

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Figure 8: Pearson's residuals and importance sampling diagnostics for the cumulative logit model (1996-2005)

Figure 9: Comparison of Industrial production deviation cycle, survey data latent cycle with constant thresholds, quantification through balances-(1996-2005)
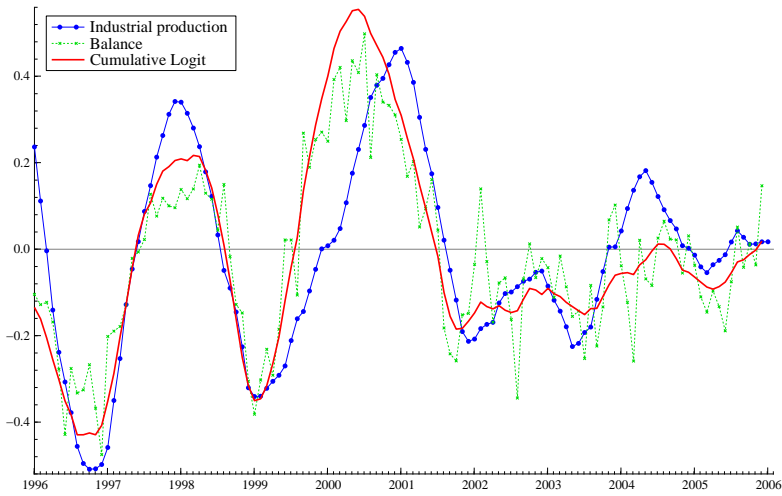

Figure 10: Dynamic Cumulative Logit model with time varying thresholds: estimates of the conditional mean of the latent cyclical factor, $E(\psi_t|\mathbf{n}_t)$ obtained via importance sampling using 1000 replications.
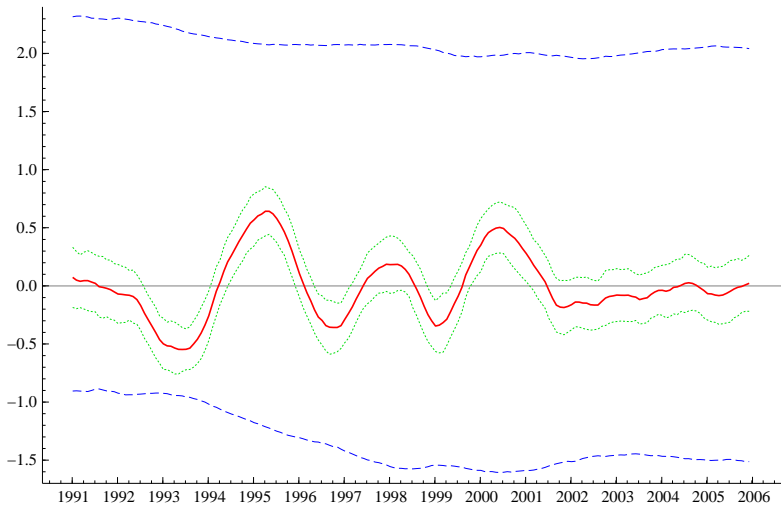
Figure 11: Pearson's residuals and importance sampling diagnostics for the cumulative logit model with time varying thresholds
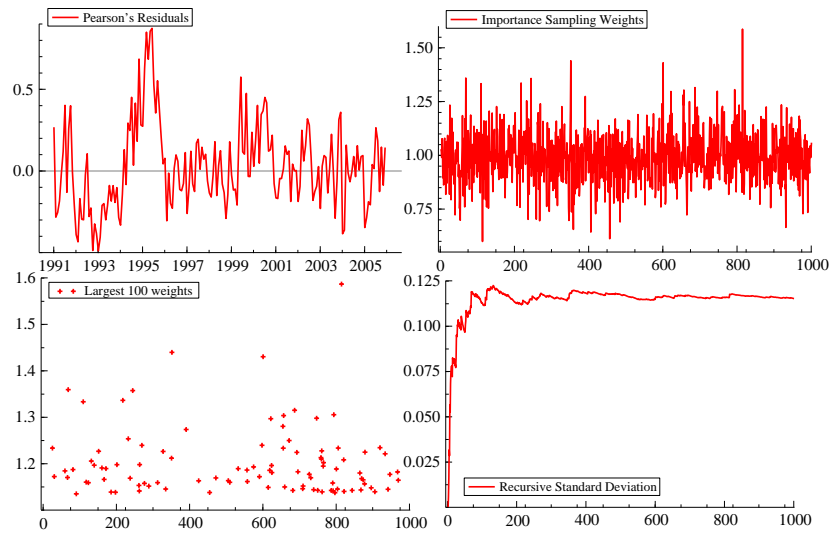

Figure 12: Comparison of Industrial production deviation cycle, survey data latent cycle with time varying thresholds, quantification through balances

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