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A Quantitative Assessment

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# Monetary Policy and Potential Output Uncertainty: A Quantitative Assessment <sup>\*</sup>

Simona Delle Chiaie<sup>†</sup>

University of Rome, Tor Vergata

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## Abstract

This paper contributes to the recent literature that studies the quantitative implications of the imperfect information about potential output for the conduct of monetary policy. By means of Bayesian techniques, a small New Keynesian model is estimated taking explicitly account of the imperfect information problem. The estimation of the structural parameters and of the monetary authorities' objectives is key in assessing the quantitative relevance of the imperfect information problem and in evaluating the robustness of previous exercises based on calibration. Finally, the model allows us to analyse the usefulness of unit labor costs as monetary policy indicator.

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<sup>†</sup>Department of Economics, University of Rome 'Tor Vergata', via Columbia 2, Rome, e-mail: delle.chiaie@economia.uniroma2.it

# 1 Introduction

In recent years, central banks' inability to observe the potential output in real-time has received attention as having important implications for the conduct of monetary policy. For example, Orphanides (2000, 2001), Lansing (2000), Cukierman and Lippi (2005) have highlighted how the significant misperception of potential output, following the productivity slowdown of the early 1970s, may have contributed to the rise of U.S. inflation.

Ehrmann and Smets (2003, ES henceforth) provide a description of the economic mechanisms by which the misperception of potential output would have affected inflation. Since the central bank has imperfect information about potential output, it cannot discern to what extent fluctuations in output and inflation are due to the different structural shocks. For instance, following a negative potential output shock, the central bank only observes a fall in output and a rise in price but it cannot perfectly distinguish if these effects are caused by a negative potential output or a positive cost-push shock (or a combination of both). As a consequence of this "information problem", ES argue that, in response to a negative potential output shock, the central bank is forced to assign some probability to the fact that this could be a positive cost-push shock. Hence, even if the central bank uses the best forecasting procedure, it over-estimates the potential output and under-estimates the output gap. Since, in real-time, the output gap is perceived as negative (while it is actually positive) the central bank will lower the interest rate. In particular, monetary policy will result too loose in comparison to a full information benchmark and this will lead to a higher inflation.

The analysis of ES is conducted within a calibrated dynamic stochastic

model for the euro area. Even though they contribute to clarify in which way potential output uncertainty may affect monetary policy and welfare, however, their quantitative findings are driven by the particular set of calibrated parameter values. For instance, the crucial result of the persistence of output gap forecast errors is clearly a function of the relative variance of potential output and cost-push shocks. Common welfare measures, such as the central bank expected losses and its ability to control inflation, the output gap and the interest rate adjustments also depend on the covariance matrix of the shocks as well as weights attached to the central bank's objective function. For these reasons, this paper presents an estimation of the structural parameters and of the monetary authorities' objectives which is crucial in appraising the quantitative importance of the potential output uncertainty and thus, in evaluating the robustness of these previous calibrations.

By means of Bayesian techniques, this paper estimates a new keynesian dynamic stochastic general equilibrium (DSGE) model which explicitly accounts for the imperfect information about the state of the economy. Once the model is estimated, the structural estimates allows us to revisit the issue discussed, among others, by Cukierman and Lippi (2005) and Ehrmann and Smets (2003).

This paper reveals that the quantitative implications of the potential output uncertainty substantially change if different assumptions on the information set available to the agents are made. In particular, we compare the case in which agents in the economy only use the detrended GDP to infering the output gap level with the alternative situation in which also the real unit labor costs are included into the vector of observables.

Following a potential output shock, the central bank makes a large and persistent error (about 40 quarters) in forecasting the output gap. This error leads optimal monetary policy to deviate from its benchmark value of full information causing an effect on inflation which is completely absent in the case of complete information. On the contrary, when the central bank also observes the real unit labor cost to estimating the output gap, the forecast error is quantitatively negligible. As a consequence, the optimal policy does not deviate from its benchmark of full information as well as the inflation dynamics are no longer affected by the potential output uncertainty.

Finally, this paper shows that the real unit labor cost plays an important role for monetary policy. Since such indicator provides information about potential output, it strongly improves the central bank's ability in controlling the output gap target.

The paper proceeds as follows. Next section reviews the model. Section 3 presents the estimation details and comments the results. Section 4 analyses the quantitative effects of potential output uncertainty on monetary policy and the role of unit labor cost as monetary policy indicator. Section 5 concludes.

## 2 The model economy

The model, taken from ES, consists of the following equations:

$$(1) \quad y_t = \delta y_{t-1} + (1 - \delta) E_t y_{t+1} + \sigma(i_t - E_t \pi_{t+1}) + u_{y,t},$$

$$(2) \quad \pi_t = \alpha \pi_{t-1} + (1 - \alpha) E_t \pi_{t+1} + \kappa(y_t - \bar{y}_t) + u_{\pi,t},$$

$$(3) \quad \bar{y}_t = \rho \bar{y}_{t-1} + u_{\bar{y},t}.$$

where  $\pi_t$ ,  $y_t$ ,  $\bar{y}_t$ , and  $i_t$  denote, respectively, inflation, output, potential output and the nominal short term interest rate. The preference shock  $u_{y,t}$ , the cost - push shock  $u_{\pi,t}$  and the potential output shock  $u_{\bar{y},t}$  are i.i.d. innovations with zero mean and covariance matrix  $\Sigma_u^2$ .

Since in this specification the dynamics of output and inflation depend on both lagged and expected future values, the model is considered as an hybrid version of more traditional backward looking models such as in Svensson (1997a, 1997b) and purely forward looking models such as in Rotemberg and Woodford (1997) and Woodford (1999). As a matter of fact, the hybrid approach arises mostly for empirical reason; in fact while the new generation of Keynesian models may be theoretically more appealing because based on much stronger microfoundations, they cannot explain the persistence existing in the data. In order to account for these features, the presence of the lagged term in the aggregate demand equation has been explicitly motivated by introducing an external habit variable in the household's utility function while, the lagged inflation in the aggregate supply curve has been justified by assuming, in model with staggered prices and wages, partial indexation to past inflation rates of prices that cannot be freely set (see e.g. Christiano et al., 2005; Smets and Wouters, 2003).

The model is closed by assuming that the central bank chooses a path for the short-term interest rate minimizing the intertemporal loss function (4) which is over three policy goals: inflation, output gap and the change in the short term nominal interest rate.

$$(4) \quad E_t \sum_{\tau=0}^{\infty} \beta^{\tau} [\pi_{t+\tau}^2 + \lambda(y_t - \bar{y}_t)^2 + \nu(i_{t+\tau} - i_{t+\tau-1})^2].$$

The relative weights  $\lambda$  and  $\nu$  synthesize the preferences of the policymaker over the related policy targets.

It is assumed that the policymaker observes contemporaneous but noisy measures of output, inflation and real unit labor cost which are represented by the following vector of measurables:

$$(5a) \quad y_t^{\circ} = y_t + v_{y,t},$$

$$(5b) \quad \pi_t^{\circ} = \pi_t + v_{\pi,t},$$

$$(5c) \quad c_t^{\circ} = c_t + v_{c,t}.$$

The measurement errors in the vector  $v$  are assumed to be i.i.d. with covariance matrix  $\Sigma_v^2$  and they are uncorrelated with the vector of innovations  $u$ .

According to the New Keynesian paradigm, the firms' inability to adjust prices optimally every period creates the existence of a wedge between output and its natural level (output gap). As shown, among others, by Rotemberg and Woodford (1997), the output gap is proportional to deviations of real marginal cost from steady state. Hence, a measure of real marginal cost can be used to approximate (up to a scalar factor) the true, or model-based, output gap. In line with these results, it is finally assumed that:

$$(6) \quad c_t = \mu(y_t - \bar{y}_t).$$

where  $c_t$  represents the actual value of real unit labor cost.

Agents and policymaker in the economy have symmetric information both on the model parameters,  $\Omega \equiv [\alpha, \beta, \delta, \kappa, \lambda, \mu, \nu, \rho, \sigma, \Sigma_u^2, \Sigma_v^2]$  and on the whole history of the observables, therefore the information set  $I_t$  at period  $t$  is represented by  $I_t \equiv \{Z_\tau, \tau \leq t; \Omega\}$ .

For estimation purposes it is necessary solving the model. Restricting the attention on the case in which central bank operates in absence of commitment, following Svensson and Woodford (2003), the equilibrium (i.e. Markov perfect) under discretion is characterized by the optimal policy rule being a linear function of the current estimate of the predetermined variables. The equilibrium law of motion of the state, forward-looking and indicator variables as well as the optimal predictor of the state vector, are given by:

$$(7a) \quad i_t = FX_{t|t},$$

$$(7b) \quad X_{t+1} = HX_t + JX_{t|t} + C_u u_{t+1},$$

$$(7c) \quad Z_t = LX_t + MX_{t|t} + v_t,$$

$$(7d) \quad x_t = GX_{t|t} + G^1(X_t - X_{t|t}),$$

$$(7e) \quad X_{t|t} = X_{t|t-1} + K[L(X_t - X_{t|t-1}) + v_t].$$

The matrices  $F, G, G^1, H, J, K, L$  and  $M$  are defined in Svensson and Woodford (2003) and depend on the parameters in  $\Omega$ , whereas  $X_t' \equiv [y_{t-1} \ \pi_{t-1} \ \bar{y}_t \ u_{y,t} \ u_{\pi,t} \ i_{t-1}]$ ,  $x_t' \equiv [y_t \ \pi_t]$ ,  $u_t' \equiv [u_{\pi,t} \ u_{y,t} \ u_{\bar{y},t}]$  and  $Z_t' \equiv [y_t^\circ \ \pi_t^\circ \ c_t^\circ]$  stand for, respectively, the predetermined state variables, the forward-looking variables, the structural shocks, the observables and, finally,  $i_t$  is the central bank's policy instrument.



As in recent papers by Schorfheide (1999), Smets and Wouters (2003) and Fernández-Villaverde and Rubio-Ramírez (2004), the model is estimated using Bayesian methods which have been built around the likelihood function derived from DSGE models. Next section presents the estimation details and comments the results.

### 3 Bayesian analysis

In a Bayesian framework, the sample information represented by the likelihood function is combined with *a priori* informations that we may have about model and parameters. Adding a proper prior may down-weight regions of the parameter space that are at odds with out-of-sample information and, in which, the structural model becomes uninterpretable. Moreover, even a weakly informative prior may add curvature to a likelihood function that is nearly flat in some dimensions of the parameter space clearly facilitating numerical maximization procedures (An and Schorfheide, 2005)<sup>1</sup>.

#### 3.1 Estimation methodology

The Bayesian analysis requires to transform the solution of the model into a state space form. So, jointly to the equilibrium law of motion of the state, we define a measurement equation that relates the elements of the states vector

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<sup>1</sup>A well known result in Bayesian econometrics (i.e. Poirier, 1998) is that the prior distribution is not updated in directions of the parameters space in which the likelihood function is flat.

to the following set of observables:  $f_t' \equiv [Z_t' i_t]$  (see equation 13 in appendix).

The vector  $f_t$  collects the four series of observations used in this analysis: output, inflation, real unit labor cost and short-term nominal interest rate.

Output is measured by the log of seasonally-adjusted real gross domestic product in chained 2000 dollars, the inflation rate is provided by the log of the quarterly changes in the seasonally adjusted GDP implicit price deflator, real unit labor cost is represented by the series of the log of labor income share in the non-farm business sector and finally, the three-month U.S. Treasury bill rate provides the measure of the nominal interest rate (expressed in percentages per quarter). The data are quarterly, run from 1947:1 through 2005:4 and they are linearly detrended before the estimation.

It is worthwhile to note that while only two shocks are present in the measurement equation (13), the covariance matrix of the endogenous variables is singular. Despite it is not essential for applying the Bayesian techniques however we append an additional measurement error on the interest rate inasmuch it is helpful in computationally reducing the singularity problem.

The Kalman filter is then applied to the state space system in order to obtain the prediction error decomposition of the likelihood (see, appendix for the analytical derivation). The latter is then combined with a prior distributions of the model parameters to form the posterior density function. Since the analytical solution of the posterior is impossible, Monte-Carlo Markov-Chain (MCMC) sampling methods are used. In particular, a random walk version of the Metropolis Hasting algorithm with small uniform errors is used to generate a Markov chain with stationary distribution that correspond to

the posterior distribution of interest<sup>2</sup>. It is important to stress that the choice of the joint prior distribution of the model parameters influences the posterior shape hence, it is important to know what features of the posterior are generated by the prior rather than the likelihood. A direct comparison of priors and posteriors can often provide valuable insights about the extent to which data provide information about the parameters of interest. For these reasons, in the following sections, we discuss the choice of prior distributions, we compare relevant moments of prior and posterior distributions and finally, we check the robustness of posterior estimates to changes in the prior distributions.

## 3.2 Priors

Table 1 presents prior distributions of the model parameters. For convenience, it is typically assumed that all parameters are a priori independent. Prior distributions are centered around standard calibrated values of the parameters used in the literature while standard errors are chosen in order to cover the range of existing estimates and to avoid to put too much structure on the data. Since priors are loose, the exact form of the densities is chosen for computation convenience. For the parameters  $\alpha, \beta, \delta$  and  $\rho$  which must lie in the interval  $[0,1)$  Beta distributions are chosen. All the variances of shocks are assumed to be distributed as a Gamma distribution because it assures a positive variance with a rather large domain. Gamma distribution

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<sup>2</sup>Variance of errors is set in order to obtain an acceptance rate of about 35-40%. The cumulative sum of draws (CUMSUM) statistics is then used for checking convergence.

is also used for the inflation elasticity to the output gap  $\kappa$  in order to include in its domain the wide range of estimated and calibrated parameter values suggested by the literature. Finally, normal distribution is chosen for remaining parameters.

*Table 1 - Prior distributions*

<i>Name</i>	<i>Domain</i>	<i>Density</i>	<i>mean</i>	<i>st.dev.</i>
$\alpha$	[0,1)	Beta	0.5	0.25
$\delta$	[0,1)	Beta	0.5	0.25
$\sigma$	R	Normal	-0.20	0.075
$\kappa$	R	Gamma	0.2	0.14
$\lambda$	R	Normal	0.5	0.05
$\nu$	R	Beta	0.75	0.0104
$\rho$	[0,1)	Beta	0.9	0.0012
$\beta$	[0,1)	Beta	0.97	0.017
$\mu$	R	Normal	1	0.10
$\sigma_{u,i}^{-1} i = \pi, y, \bar{y}$	R <sup>+</sup>	G	2.5	1.76
$\sigma_{v,i}^{-1} i = \pi, y, r, \varsigma$	R <sup>+</sup>	G	2.5	1.76

### 3.3 Findings

The Bayesian analysis produces reasonable posterior distributions for the model parameters. Except for the discount factor  $\beta$ , data are informative in the sense that posterior distributions of the model parameters are more concentrated and rather shifted relative to the priors. Table 2 presents relevant statistics of the posterior distributions of the model parameters. Focusing

on the posterior distribution of the backward-looking component of the New-Keynesian Phillips curve, one observes that, in spite of a relatively loose prior on this parameter, the posterior distribution has a small dispersion with a range that goes from 0.38 to 0.41. These findings imply a degree of inflation inertia greater than in Gali and Gertler (1999), Gali, Gertler and Lopez-Salido (2001), supporting the view that both backward looking and forward looking behaviors are important in shaping the U.S. inflation dynamics. The posterior distribution of the Phillips curve slope suggests a significant effect of real activity on inflation. The posterior mean is 0.07 and it is close to the estimates of obtained by Gali and Gertler (1999) and Gali, Gertler and Lopez - Salido (2005) using GMM<sup>3</sup>.

Regarding the structural parameters of the aggregate demand equation, the estimation suggests that both backward and forward looking components are significant in explaining output dynamic. The posterior mean of the elasticity of output to the real interest rate is -0.30 and it is quite consistent with other previous results found in the Real Business Cycle literature. The analysis also delivers plausible estimates for the parameters describing the preferences of the monetary authority. The posterior mean of the weight attached to the output gap  $\lambda$  is 0.47 suggesting that the policymaker looks out for the deviation of output from its natural level. The estimates of  $\nu$

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<sup>3</sup>In a first version of this paper, the model was estimated by excluding the unit labor cost indicator. The estimation results showed that the Phillips curve slope estimate was close to zero and moreover, the standard deviations of the cost-push shock and the measurement errors were very high. These findings suggested to re-estimate the model taking into account a better proxy for the output gap rather than the detrended GDP (i.e. Gali and Gertler (1999)).

(0.58) provides evidence of a substantial degree of interest rate smoothing. Anyway, the posterior distribution of  $v$  is bimodal so that care should be exercised in using the posterior mean as a measure of location. The estimates of the structural shocks ( $\sigma_u$ ) show that cost-push and potential output shocks have the largest standard deviation.

*Table 2 - Posterior distributions*

<i>Name</i>	<i>Relevant statistics</i>				
	<i>mean</i>	<i>median</i>	<i>st.d.</i>	<i>min</i>	<i>max</i>
$\alpha$	0.4059	0.4126	0.0105	0.382	0.414
$\delta$	0.7171	0.6833	0.0593	0.656	0.904
$\sigma$	-0.2957	-0.2933	0.0051	-0.314	-0.282
$\kappa$	0.0712	0.0167	0.1417	0.0002	0.608
$\lambda$	0.4689	0.4653	0.0084	0.456	0.501
$\nu$	0.5764	0.5699	0.0167	0.554	0.632
$\rho$	0.9291	0.9763	0.0782	0.757	0.999
$\mu$	1.606	1.6977	0.1722	1.083	1.773
$\beta$	0.9411	0.9473	0.0644	0.459	0.999
$\sigma_{u,\bar{y}}$	1.4271	1.7334	0.4952	0.126	1.752
$\sigma_{u,y}$	0.3555	0.4169	0.0985	0.099	0.419
$\sigma_{u,\pi}$	1.472	1.7464	0.4543	0.1	1.758
$\sigma_{v,y}$	0.1619	0.1830	0.0401	0.031	0.189
$\sigma_{v,\pi}$	0.2731	0.2735	0.0595	0.017	0.373
$\sigma_{v,r}$	1.4323	1.6702	0.0039	0.048	1.706
$\sigma_{v,\varsigma}$	0.0592	0.0609	0.4272	0.045	0.067

*Table 3 - Asymptotic variance decomposition*

	<i>Fundamental shocks</i>			<i>Measurement errors</i>		
	<i>pot</i>	<i>dem</i>	<i>cos</i>	<i>out</i>	<i>infl</i>	<i>ulc</i>
Output	59.09	11.03	29.48	0.04	0.35	0.01
Output gap	59.03	11.02	29.46	0.05	0.43	0.02
Inflation	1.26	0.35	98.28	0	0.10	0
Interest changes	37.23	19.05	40.66	1.40	1.65	0.02

The variance decomposition of Table 3 indicates that cost-push shocks are the main source of fluctuations for inflation and they also explain a large part of the interest rate volatility. Demand shocks explain a substantial part of the variance of output and interest rate. Finally, potential output shocks explain more than one half of the output gap volatility. The estimates also show that the measurement error concerning real unit labor cost is rather small. Instead, the measurement errors of output and inflation are significant (0.16 and 0.27, respectively), even if the variance decomposition analysis of Table 3 indicates that they have a marginal role in explaining the variables' fluctuations. Figure 1 describes the model fit by plotting the one-step ahead predictions for each of the four variables used in estimation. The figure shows that the model forecasting performance is rather accurate for nominal variables. The correlation between the one-step-ahead prediction and actual values is 0.78 and 0.76 for inflation and nominal interest rate respectively, indicating that the model is able in predicting high frequency inflation movements. The model forecasting performance is more modest with respect to

output (0.58) and real unit labor cost (0.59).

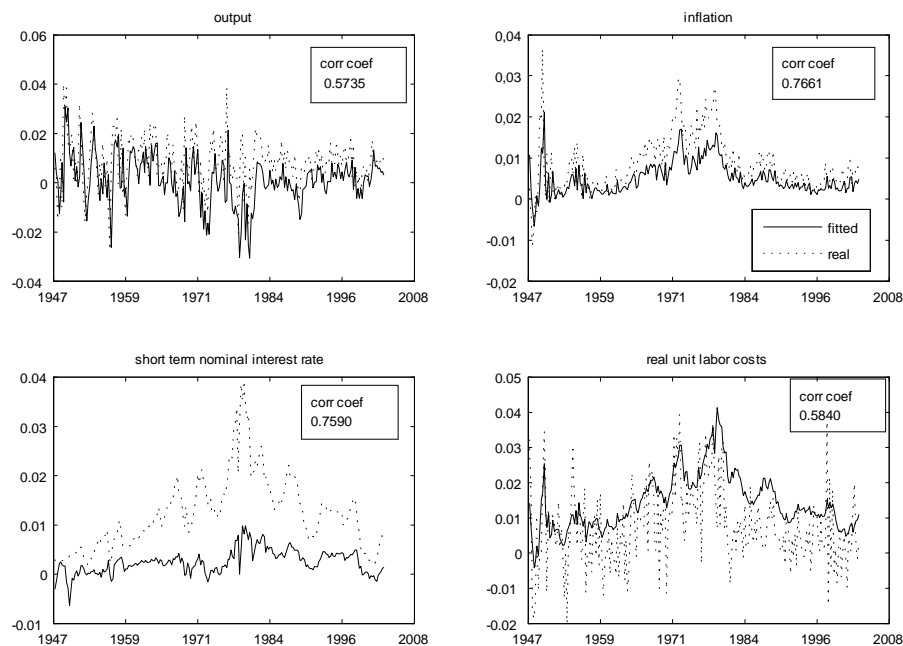


Figure 1 - Data (dotted line) and one-step ahead forecasts (solid line)

Finally, we analyse the robustness of posterior estimates to changes in the prior distribution. Table 4 reports the posterior moments of the prior and the posterior in the baseline case and the posterior moments in two alternative specifications, obtained making the prior progressively more informative. In particular, we maintain the same measures of location but the probabilities



density are rescaled by reducing the prior ranges by 10 and 20 percents.

*Table 4 - Robustness analysis*

<i>name</i>	<i>posterior (90% spread)</i>			<i>posterior (80% spread)</i>		
	<i>mean</i>	<i>median</i>	<i>sd</i>	<i>mean</i>	<i>median</i>	<i>sd</i>
$\alpha$	0.4593	0.4683	0.0142	0.4593	0.4683	0.0142
$\delta$	0.7156	0.7234	0.0250	0.7156	0.7234	0.0250
$\sigma$	-0.2109	-0.2124	0.0031	-0.1279	-0.1313	0.0073
$\kappa$	0.0672	0.0125	0.1365	0.063	0.0086	0.1304
$\lambda$	0.4965	0.4885	0.0181	0.5226	0.5077	0.0332
$\nu$	0.5607	0.5506	0.0257	0.5158	0.4935	0.0563
$\rho$	0.9287	0.9311	0.0685	0.9599	0.9222	0.0306
$\mu$	1.1186	1.0063	0.2528	1.0562	1.0000	0.1928
$\beta$	0.8909	0.8366	0.2040	0.8539	0.8286	0.4121
$\sigma_{u,\bar{y}}$	1.3137	1.5714	0.4274	1.1773	1.3789	0.3497
$\sigma_{u,y}$	0.3722	0.4412	0.1118	0.3917	0.4698	0.1273
$\sigma_{u,\pi}$	1.3537	1.5811	0.3863	1.2115	1.3852	0.3087
$\sigma_{v,y}$	0.1514	0.1770	0.0482	0.1405	0.1685	0.0511
$\sigma_{v,\pi}$	0.2828	0.2842	0.0697	0.2945	0.2958	0.0779
$\sigma_{v,r}$	1.3234	1.5237	0.3699	1.1912	1.3476	0.3039
$\sigma_{v,\varsigma}$	0.0267	0.0284	0.0038	0.0099	0.0109	0.0022

Following, Geweke (1998), posterior draws from the new distribution are obtained reweighting the posterior draws obtained in the baseline case with  $w(\alpha) = \frac{g^i(\alpha)}{g^B(\alpha)}$  where the  $g^i(\alpha)$  is the new prior and  $g^B(\alpha)$  is the baseline one. Table 4 shows that the posterior results are reasonably invariant to changes

in the prior specification.

## 4 Potential Output Uncertainty and Monetary Policy

In this section the estimated model is used to analyse the quantitative effects of the imperfect information about potential output and also to assess the usefulness of unit labor cost as monetary policy indicator. Throughout this section, we compare outcomes of two different situations. The first situation is one with complete information (CI) which implies that all agents in the economy perfectly observe current output, inflation, real unit labor cost and nominal interest rate as well as current potential output. In the second more realistic situation, the central bank and the private sector are subject to incomplete information (II) about potential output. This implies that agents do not observe potential output directly and moreover, it is supposed that they only have noisy indicators. From the demand equation, it is clear that the central bank and the private sector will be able to estimate the demand shock perfectly, whereas they will face a signal extraction problem in trying to distinguish cost-push shocks from potential output shocks. This problem may create a misperception of the potential output even though we present how different scenarios may emerge if the real unit labor cost indicator is available to the agents or not.

Figure 2 presents the responses of output, inflation, actual and perceived output gap, nominal interest rate and the output gap forecast error following a positive shock to potential output when the real unit labor cost indicator

is removed from the central bank's vector of observables.

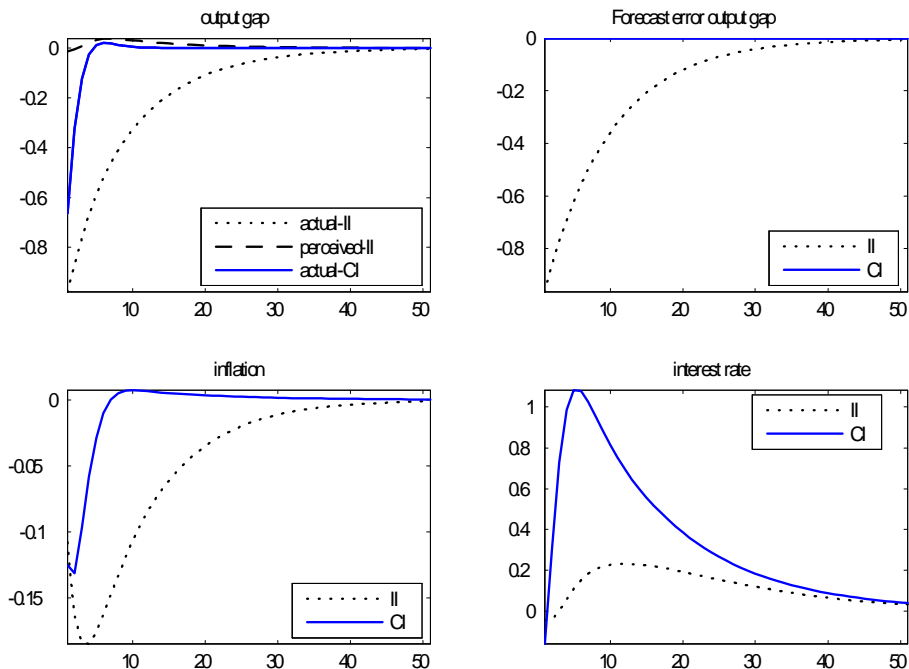


Figure 2 - Complete vs Incomplete information (positive potential output shock)

One may observe that the central bank makes a large and persistent error in forecasting the output gap. The reasons for such an error hinge on the above signal extraction problem. Following a positive potential output shock, the central bank just observes a rise in output and a fall in price but it does not perfectly recognize if those effects are caused by a positive potential output or a negative cost-push shock (or a combination of both). As a result,

it assigns some probability to the fact that this is actually a negative cost-push shock in this way causing an under-prediction of the potential output.

The upper part of the figure shows that for about 40 quarters the central bank perceives a positive output gap while it is actually negative. The output gap forecast error leads the optimal interest rate to deviate from the benchmark value under perfect information causing a larger fall in the output gap and a larger fall in inflation.

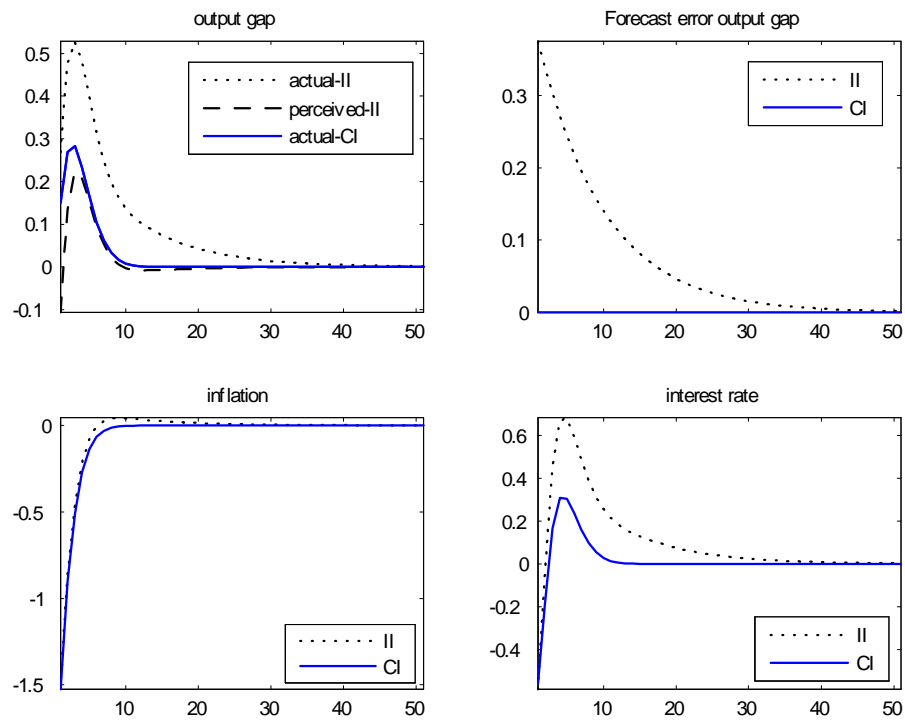


Figure 3 - Complete vs Incomplete information (negative cost-push shock)

The opposite occurs in response to a negative cost-push shock. In this case the central bank assigns some probability that this is actually a positive potential output shock under-predicting the output gap. As a result, it will lower the interest rate by more than it would otherwise have done. This leads to a larger response in the output gap, and a smaller fall in inflation.

Figure 4 compares the responses of the variables of interest following a positive potential output when the central bank can infer the level of potential output based on output and real unit labor cost, too.

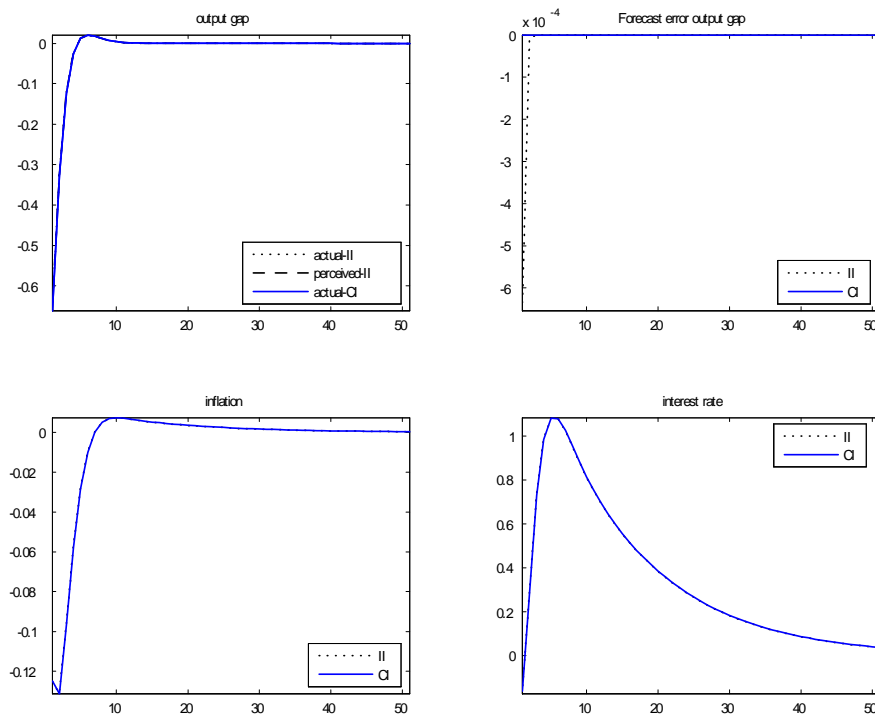


Figure 4 - Effects of a positive potential output shock (agents observe real unit labor cost)

It is important to note that, in this case, the error in forecasting the output gap is quantitatively negligible. As a result, the certainty equivalent policy rule tracks the one we would expect if information was perfect and the inflation dynamics are not affected by the potential output uncertainty.

Figure 5 confirms the above conclusions for the case of a negative cost-push shocks: The forecast error is tiny and the dynamics of all variables of interest completely overlap their benchmarks of full information.

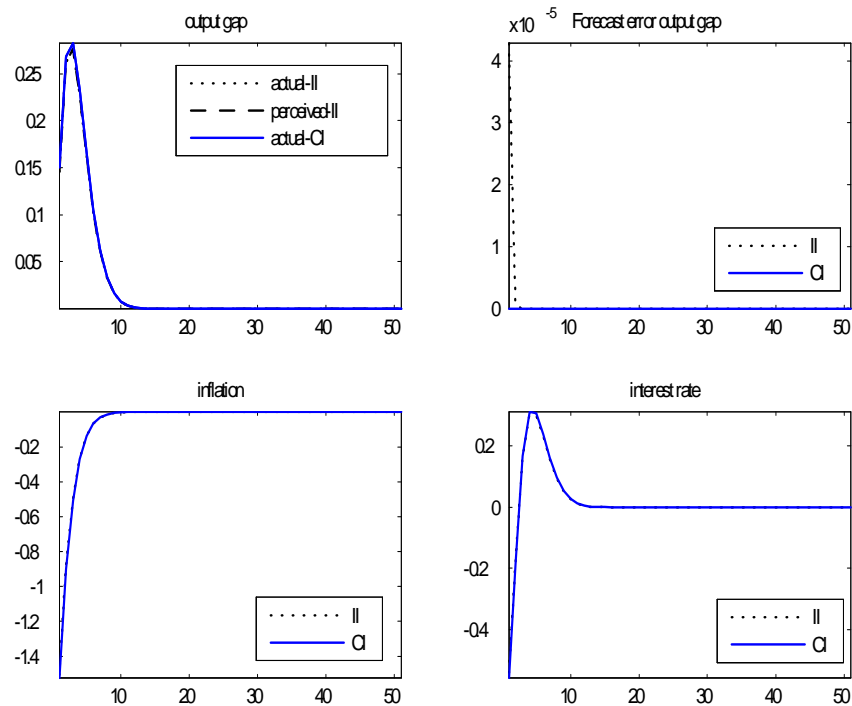


Figure 5 - Effects of a negative cost-push shock (agents observe real unit labor cost)

The finding that the forecast error is very small when real unit labor cost is employed in estimating the output gap suggests that such indicator contains useful information on potential output. At the same time, this result confirms the objection raised by Gali and Gertler (1999), Gali and Gertler and Lopez-Salido (2001, 2005) in using the detrended GDP (the deviations of log GDP from a smooth trend) as a proxy for the output gap in empirical applications.

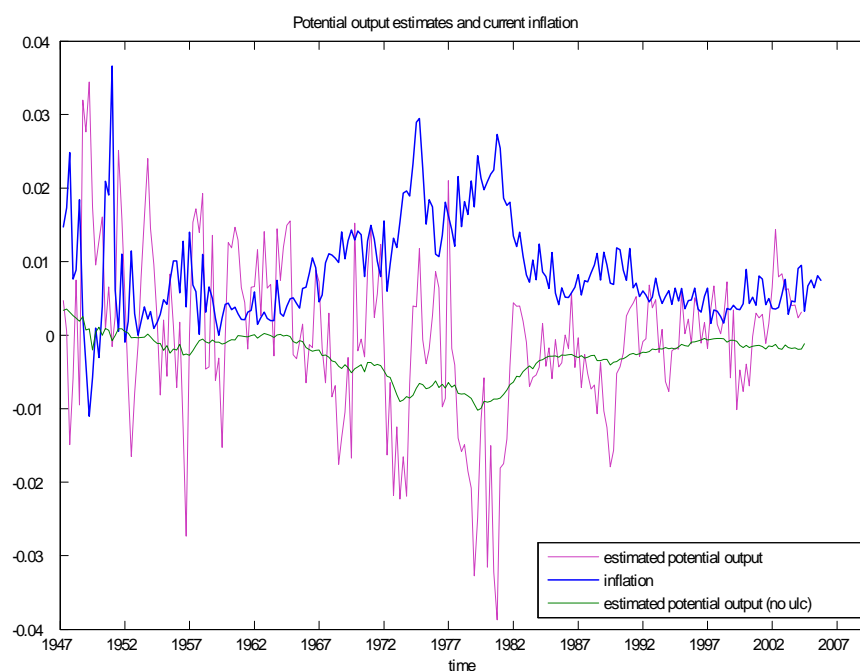


Figure 6 - Potential output estimates and current inflation

Figure 6 presents an informal assessment of this point based on the patterns of cross-correlations between two alternative estimates of potential out-

put and inflation. The violet line corresponds to the potential output estimate obtained using both output and real unit labor cost indicators. The green line corresponds to the counterfactual estimate obtained by removing the unit labor cost indicator from the central bank’s vector of observables. This visual experiment shows that when the output gap indicator is not available, the estimated potential output is a smooth series and it is clear that no obvious correlation among this estimate of potential output and the inflation rate (blu line) exists.

*Table 5 - Welfare effects of observing unit labor cost*

	<i>Indicators</i>	
	<i>inflation, output, unit labor costs</i>	<i>no unit labor costs</i>
output gap	1.38	3.52
inflation	2.74	2.81
interest changes	1.73	1.69
expected losses	238.75	369.29

Finally, we study the usefulness of unit labor cost indicator through the effects it produces on some welfare measures. Once again, we analyse how economic performance is affected by the removal of such indicator from the vector of observables. Table 5 reports the standard deviation of target variables (output gap, inflation and interest rate changes) and central bank expected losses. The first column considers the case in which all indicators are available to the central bank. The second one instead shows the values



of these variables in the case in which unit labor costs are taken away from the central bank's information set.

This exercise shows that expected losses significantly increase when that indicator is removed from the vector of observables. This effect is mainly due to the raise of the standard deviation of the output gap and inflation. On the contrary, the volatility of the interest rate changes has a little decline. This last result means that when unit labor costs are taken away from the information set, the greater uncertainty concerning the estimate of potential output causes a reduction in monetary policy activism.

## 5 Conclusions

This paper contributes to the recent literature that studies the quantitative implications of the imperfect information about potential output for the conduct of monetary policy. For this purpose, a small New Keynesian model which explicitly accounts for the imperfect information problem, is estimated by means of Bayesian techniques.

The Bayesian analysis produces reasonable posterior distributions for the model parameters. In particular, the posterior distribution of the Phillips curve slope suggests a significant effect of real activity on inflation and it is consistent with those of Gali and Gertler (1999), Sbordone (1999), Gali, Gertler and Lopez-Salido (2005).

Using the estimates of the structural parameters and of the monetary authority's objectives, this paper analyses the quantitative relevance of the imperfect information about potential output for monetary policy.

When the information set available to the agents only includes noisy measures of output and inflation, this work corroborates the Ehrmann and Smets (2003) conclusion that following a potential output shock, the central bank makes a large and persistent error in forecasting the output gap. This error leads the optimal policy to deviate from the benchmark value of full information creating an effect on inflation which is completely absent in the case of perfect information. On the contrary, we show that when the real unit labor cost indicator is available to the agents, following a shock to potential output, the output gap forecast error is quantitatively negligible. As a consequence, the optimal policy does not deviate from its benchmark of full information as well as the inflation dynamics are not affected by the potential output uncertainty.

Finally, this paper shows the relevance of the real unit labor cost as monetary policy indicator. Our findings suggest that real unit labor cost contains information on potential output and this, in turn, improves the central bank's ability in making stabilization policy more effective.

## Appendix

Using (7e) and (7b) I get

$$(8) \quad X_{t+1} = (H + JKL)X_t + J(I - KL)X_{t|t-1} + JKv_t + C_u u_{t+1},$$

taking expectations and using (7e) we obtain

$$(9) \quad X_{t+1|t} = (H + J)(I - KL)X_{t|t-1} + (H + J)KLX_t + (H + J)Kv_t.$$

we can rewrite (8) and (9) as follows

$$(10) \quad S_{t+1} = AS_t + Be_{1,t+1},$$

by defining  $S_{t+1} \equiv \begin{bmatrix} X_{t+1} \\ X_{t+1|t} \end{bmatrix}$

$$A = \begin{bmatrix} (H + JKL) & J(I - KL) \\ (H + J)KL & (H + J)(I - KL) \end{bmatrix}, \quad B = \begin{bmatrix} C_u & JK \\ 0 & (H + J)K \end{bmatrix}$$

and

$$e_{1,t+1} = \begin{bmatrix} u_{t+1} \\ v_t \end{bmatrix}$$

In the same way, substituting (7e) into (7a) and (7c) we get

$$(11) \quad Z_t = (L + MKL)X_t + M(I - KL)X_{t|t-1} + (I + MK)v_t,$$

$$(12) \quad i_t = (F - FKL)X_{t|t-1} + FKLX_t + FKv_t.$$

Finally, we can rewrite (11) and (12) in state-space form:

$$(13) \quad f_t = CS_t + Dv_t,$$

$$\text{where } f'_t \equiv [Z'_t i_t], \quad C = \begin{bmatrix} L + MKL & M(I - KL) \\ FKL & F(I - KL) \end{bmatrix}, \quad D = \begin{bmatrix} I + MK \\ FK \end{bmatrix}$$

adding a vector of measurement errors we get

$$(14) \quad f_t = CS_t + e_{2,t},$$

where  $e_{2,t} \equiv Dv_t + \xi_t$ . is the vector of measurement errors and  $\xi_t \equiv \begin{bmatrix} 0 & 0 & \xi_{i,t} \end{bmatrix}'$ .

$$(15) \quad V_1 \equiv E(e_{1,t+1}e'_{1,t+1}) = \begin{bmatrix} \Sigma_u^2 & 0 \\ 0 & \Sigma_v^2 \end{bmatrix},$$

$$(16) \quad V_2 \equiv E(e_{2,t}e'_{2,t}) = D\Sigma_v^2D' + E(\xi_t\xi'_t),$$

$$(17) \quad V_3 \equiv E(e_{1,t+1}e'_{2,t}) = \begin{bmatrix} 0 \\ \Sigma_v^2D' \end{bmatrix}.$$

The Kalman filter is then applied to the state space model (10) and (14). That filter takes the observations of  $f_t$  for  $t = 1, 2, \dots, T$  and works recursively to construct a series of forecast errors as follows:

$$(18) \quad w_t = f_t - f_{t|t-1} = f_t - CS_{t|t-1},$$

where  $f_{t|t-1}$  is the prediction of the observable variables given the information available at period  $t$  and the forecast error covariance matrix is given by:

$$(19) \quad \begin{aligned} \Xi_t &\equiv E(w_t w_t') = E(f_t - f_{t|t-1})(f_t - f_{t|t-1})' = \\ &= E[(CS_t + e_{2,t} - CS_{t|t-1})(CS_t + e_{2,t} - CS_{t|t-1})'] = \\ &= E\{[C(S_t - S_{t|t-1}) + e_{2,t}][C(S_t - S_{t|t-1}) + e_{2,t}]\}' = \\ &= E[C((S_t - S_{t|t-1})(S_t - S_{t|t-1})' C') + E(e_{2,t} e_{2,t}') = C\Sigma_{t|t-1}^2 C' + V_2. \end{aligned}$$

where  $\Sigma_{t|t-1}^2 = E(S_t - S_{t|t-1})(S_t - S_{t|t-1})'$ .

Since by construction, the forecast error  $w_t$  is serially uncorrelated and normally distributed for all  $t = 1, 2, \dots, T$  with mean zero and covariance matrix  $\Xi_t$ , then log-likelihood function is given by:

$$(20) \quad \ln L = -\frac{nT}{2} \log(2\pi) - \frac{1}{2} \sum_{t=1}^T \ln |\Xi_t| - \frac{1}{2} \sum_{t=1}^T w_t \Xi_t^{-1} w_t'.$$

The optimal predictor of the states vector using the Kalman filter is given by:

$$(21) \quad S_{t+1|t} = AS_{t|t-1} + K_t w_t, \text{ where}$$

$$(22) \quad K_t \equiv (A\Sigma_{t|t-1}^2 C' + BV_3)(C\Sigma_{t|t-1}^2 C' + V_2)^{-1},$$

$$(23) \quad \Sigma_{t+1|t}^2 = (A\Sigma_{t|t-1}^2 A' + BV_1 B') - K_t (A\Sigma_{t|t-1}^2 C' + BV_3)'.$$

where the matrix  $K_t$  is the Kalman gain and  $\Sigma_{t+1|t}^2 \equiv E(S_{t+1} - S_{t+1|t})(S_{t+1} - S_{t+1|t})'$ .

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