Fiscal Policy and Exchange Rates*

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Abstract

This paper examines the dynamics of the nominal exchange rate and fiscal deficits in a continuous time optimising general equilibrium model with finite horizon. It is shown that alternative financing modes of budget deficits imply different patterns of adjustment along the transitional path towards the steady state equilibrium. Fiscal policy may provide the nominal anchor for the exchange rate and the respect of public solvency without money financing is not sufficient to avoid the depreciation of the exchange rate in the long-run after a fiscal expansion.

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1 Introduction

Monetary variables and equilibrium in international financial markets typically are the crucial variables determining exchange rate dynamics. Recent contributions have challenged this view, stressing the importance of fiscal variables in determining the time pattern of exchange rates. In particular, the fiscal theory of price determination has been extended to open economy models. The effects of fiscal policies on real economic variables, on the other hand, have been analyzed in a variety of models with forward-looking agents and finite horizons. Notably, Frenkel and Razin (1986) study the effects of tax cuts on the world interest rates, on consumption and on the current account in a two country interdependent economy set-up. Helpman and Razin (1987) analyze the dependence of exchange rate management on the time pattern of fiscal and monetary policies. Daniel (1993) studies the effects of a tax cut in an economy where there is uncertainty regarding the time of a future budget-balancing tax increase. Kawai and Maccini (1995) examine the effects of fiscal deficits on a small open economy with flexible exchange rates and finite horizons. The fiscal deficit is assumed to be financed by selling bonds and and it is anticipated to be financed in the future by seignorage, tax increases or some combination of the two. The effects of a permanent change in the price of the imported materials on the current account are examined by Matsuyama (1987) in small open economy models with capital accumulation, while the steady state effects of monetary policy are studied by Giovannini (1988). Van der Ploeg (1991) analyzes the implications of monetary policy in a two-country optimizing model with capital accumulation.

More recently, the fiscal theory of price determination has been extended to the open economy with infinite horizon agents, showing the implications for exchange rate systems and common currency areas. The fiscal theory of price determination has become increasingly popular following the contributions of Woodford (1995) and Sims (1994) and has its roots in the paper originally presented by Sargent and Wallace (1981). Canzoneri, Cumby and Diba (2001) show that more severe fiscal discipline is required in common currency areas where national fiscal authorities enjoy less autonomy in pursuing their objectives. They distinguish between *Ricardian* and *non-Ricardian* regimes, following the terminology coined by Woodford (1995); in the first case the nominal anchor is provided by monetary policy and the exchange rate is determined according to standard theories, while in the second case fiscal policy serves as the nominal anchor and determines the exchange rate. In other words, in a *non-Ricardian* regime prices and not deficits adjust to satisfy the intertemporal budget constraint of the government[1]. Dupor (2000) studies the determi-

[1] The fiscal theory of the price level uses the term *Ricardian* to indicate the irrelevance of a fiscal regime for the determination of the price level, i.e. the intertemporal budget constraint of the government necessarily holds regardless of the time path of the price level. Conversely, according to Aiyagari and Gertler (1985) a fiscal regime can be defined as *Ricardian* only when there is no money financing and the value of the public debt equalizes the present discounted value of future taxes. The validity of the fiscal theory of the price level has recently been seriously questioned by Buiter (2002).

nation of the exchange rate in a two-country cash-in-advance model and shows that the exchange rate cannot univocally be determined when governments are assumed to peg the nominal interest rate on domestic bonds. Daniel (2001a) extends the fiscal theory of the price level to an open economy and demonstrates how the exchange rate is determined to ensure intertemporal fiscal solvency of the public sector, showing how indeterminacy of prices and of exchange rates can be avoided. In a different contribution Daniel (2001b) provides a fiscal theory of currency crises, showing that the crisis cannot be avoided when the present discounted value of primary surpluses is less than the present value of government debt at the pegged level of exchange rate. The monetary authorities are forced to abandon the peg, in order to extract the necessary seignorage revenues to restore public sector solvency.

The main purpose of this paper is to build a general framework describing the implications of fiscal policy in the determination of the nominal exchange rate when there are finite horizons. The fiscal authorities may close the deficits by reducing public expenditure or by raising taxes while the monetary authorities independently set the rate of money growth in order to provide the monetary anchor for the exchange rate. Alternatively, the authorities may elect to extract seignorage revenues to close the deficits and therefore monetary policy is not independent, since the rate of money growth is chosen in order to satisfy the intertemporal budget constraint of the government. The present paper emphasises the role of fiscal policy in the determination of the nominal exchange rate.

We present a small open economy of perpetually young consumers with flexible exchange rates and forward-looking agents, in which Ricardian equivalence does not hold. The aim is to analyze the effects of a fiscal deficit, followed by later surpluses, according to a tax rule increasing in public debt, so as to always satisfy the intertemporal budget constraint of the government. In particular, this paper studies the effects of a current and unexpected lump-sum tax cut which initially implies a sequence of budget deficits. As debt accumulates the government is assumed to increase taxes, use seignorage or employ some combinations of the two to close the deficit[2].

The analysis shows how fiscal variables determine the dynamics of the exchange rate. We demonstrate that after a fiscal expansion the respect of public solvency without money financing is not sufficient to avoid the depreciation of the exchange rate. In particular, it is shown that for any financing mode a fiscal expansion would lead on impact to an appreciation of the domestic currency, an increase in consumption and in real money holdings and to a depreciation of the exchange rate in the long run. On the other hand, alternative financing modes determine different effects on the level of net financial wealth of households in the long run: tax financed budget deficits reduce the level of wealth, while money financed bud-

[2]Kawai and Maccini (1995) present a model where current fiscal deficits are anticipated to be closed in the future by a tax increase, money creation or some combination of the two. Similarly, but in a different set-up with infinite horizon agents, Drazen and Helpman (1987) describe how different stabilization policies affect the dynamics of economic variables.

get deficits increase it.

The paper is organized as follows. Section II presents the optimising general equilibrium model; Section III analyzes the short and long-run effects of a once-for-all lump-sum tax cut. Conclusions are summarized in Section IV.

2 The general equilibrium optimising model

Consider a small open economy composed by two types of agents: households and the government. To keep the presentation simple, assume that it is a one-good world so that purchasing power parity (PPP) holds , $P_t^* E_t = P_t$, where E_t is the nominal exchange rate, P_t is the domestic price level and $P_t^*$is the foreign price level normalized to unity. There is perfect capital mobility and domestic and foreign assets are perfect substitutes, that is uncovered interest parity (UIP) is always verified. The real interest rate r_t^* on foreign assets is exogenously given and assumed to be constant over time. The two conditions on price levels and nominal interest rates imply real interest rate parity, so that the domestic real interest rate is $r_t = r^*$.

2.1 Households

The demand side of the economy is an extended version of the Yaari (1965)-Blanchard (1985) model where forward looking agents have finite horizons, lifetime is uncertain and there is no bequest motive. Agents are assumed to be identical and face the same probability of death δ in each period of time.

The birth and death rates are the same. For convenience the size of each period generation of agents is normalized to δ and total population is equal to unity.

Households are assumed to maximize the discounted value of their expected utility subject to the appropriate budget constraint. The instantaneous utility function is logarithmic in total consumption and real money balances[3]. Individuals hold their financial wealth in government bonds, real money balances and foreign real assets. Domestic residents are assumed to hold the entire stock of government bonds. The supply of labor at time t of individuals born at time $s \leq t$, $l(s,t)$, is inelastic and normalized to unity.

The representative agent of generation s is assumed to

[3]Entering real money balances directly into the utility function allows to account for the services provided by money holdings. The idea behind this approach is that money yields direct utility to the consumers. It can be shown that, under some regularity conditions, the procedure of introducing money in the utility function is equivalent to the maximization problem with money modelled by means of a cash-in-advance constraint *à la* Clower or by liquidity costs (see Feenstra, 1986).

The introduction of real money balances as an argument of the utility function within a Blanchard-Yaari overlapping generations framework is common to several papers including Marini and van der Ploeg (1988), van der Ploeg (1991), Daniel (1993), Kawai and Maccini (1995).

solve the following optimisation problem

$$\max_{c(s,t),\ \varphi(s,t)} \int_t^{\infty} \log\left[c(s,v)^{\xi}\varphi(s,t)^{1-\xi}\right] e^{-(\beta+\delta)(v-t)} dv \qquad (1)$$

subject to the individual consumer's flow budget constraint

$$\begin{aligned} \frac{da(s,t)}{dt} &= (r^{*}+\delta)a(s,t) \qquad (2) \\ &\quad +\omega(s,t)l(s,t)-\tau(s,t)-c(s,t)-i(t)\varphi(s,t) \end{aligned}$$

and to the usual transversality condition

$$\lim_{t\to\infty}[a(s,t)]e^{-(r^{*}+\delta)t} = 0 \qquad (3)$$

where $0 < \xi < 1$, $\beta < r^{*}$[4] is the constant rate of time preference, $i(t)$ is the nominal interest rate at time t, $\varphi(s,t) = m(s,t)/E(t)$ represents real money balances, $c(s,t)$, $\omega(s,t)$, $m(s,t)$, $a(s,t)$ and $\tau(s,t)$, denote consumption, wage rate, nominal money balances, total real non-human wealth and lump-sum taxes, of the generation born at time s, respectively. Total real non-human wealth consists of real money holdings, government bonds, $b(s,t)$, and foreign assets, $f(s,t)$.

The effective discount rate of consumers is given by $(\beta+\delta)$, where $\beta+\delta > r^{*}$[5]. Each individual is assumed to receive for

[4]This condition ensures that consumers are relatively patient, in order that the steady state-level of aggregate financial wealth $A(t)$ is positive (see Blanchard (1985) and Matsuyama, (1987)).

[5]This condition ensures that savings are a decreasing function of wealth and that a steady-state value of aggregate consumption $C(t)$ exists. See Blanchard (1985).

every time period of his life an actuarial fair premium equal to $\delta a(s,t)$ by a life insurance company. At the time of his death the individual's net wealth goes to the insurance company.

The solution to the consumer's maximization problem yields the following demand functions[6]

$$c(s,t) = \frac{\beta+\delta}{1+\eta}\left[a(s,t)+h(s,t)\right] \tag{4}$$

$$\frac{m(s,t)}{E(t)} = \frac{\eta c(s,t)}{r^{*}+\pi(t)} \tag{5}$$

where, $\eta \equiv \frac{1-\xi}{\xi}$ and $\eta < 1$, by assumption; $\pi(t) = \frac{\dot{E}(t)}{E(t)}$ is the rate of depreciation of the national currency and $h(s,t)$ is the human wealth defined as the present discounted value of labor income net of taxes

$$h(s,t) = \int_{t}^{\infty}\left[\omega(s,v)-\tau(s,v)\right]e^{-(r^{*}+\delta)(v-t)}dv \tag{6}$$

Equation (4) shows that consumption is linear in total wealth, while equation (5) is the portfolio balance condition

[6]Given that preferences are intertemporally separable and the utility function is homothetic, the consumer's problem can be solved by using a two-stage budgeting procedure. Defining the total consumption as the sum of consumption plus the interest foregone on money holdings, the procedure works as follows. In the first stage, one determines the optimal mix of consumption and real money holdings, conditional upon a given level of total consumption. In the second stage, one derives the optimal time path of total consumption, given the budget constraint and the solvency condition. (Marini and van der Ploeg, 1988).

equalizing the marginal rate of substitution between consumption and real money holdings to the nominal interest rate.

Assuming that the wage rate and taxes are independent of age and that non-human wealth for newly born agents is zero, $a(t,t) = 0$, after aggregation over all the cohorts of the consumers one obtains

aggregate consumption at time t

$$C(t) = \frac{\delta + \beta}{1 + \eta} \left[H(t) + A(t) \right] \tag{7}$$

the portfolio balance condition

$$\frac{\eta C(t) E(t)}{M(t)} = \frac{\dot{E}(t)}{E(t)} + r^* \tag{8}$$

total non-human wealth

$$A(t) = F(t) + \frac{M(t)}{E(t)} + B(t) \tag{9}$$

and total human-wealth

$$H(t) = \frac{\omega}{r^* + \delta} - \int_t^{\infty} T(v) e^{-(r^* + \delta)(v - t)} dv \tag{10}$$

where $C(t)$, $T(t)$, $A(t)$, $F(t)$, $M(t)$ and $B(t)$ denote aggregate consumption, taxes, non-human wealth, foreign assets, real money balances and government bonds at time t, respectively. Macroeconomic variables are obtained by aggregating over all the generations as follows

$$X(t) = \int_{-\infty}^{t} x(s,t)\delta e^{\delta(s-t)} ds \tag{11}$$

for $x(s,t) = c(s,t)$, $\tau(s,t)$, $a(s,t)$, $f(s,t)$, $m(s,t)$ and $b(s,t)$.

The aggregate non-human wealth accumulation equation is

$$\begin{aligned} \overset{\bullet}{A}(t) &= r^*[B(t) + F(t)] + \\ &\quad +\omega - T(t) - C(t) - \pi(t)\frac{M(t)}{E(t)} \end{aligned} \tag{12}$$

and the dynamic equation of aggregate human wealth is

$$\overset{\bullet}{H}(t) = (r^* + \delta)H(t) - \omega + T(t) \tag{13}$$

2.2 Government

The infinitely lived government can finance public expenditures and interest payments by seignorage, lump-sum tax and bond issues. The intertemporal budget constraint is given by

$$\overset{\bullet}{B}(t) = r^* B(t) + G(t) - T(t) - \frac{\mu(t)M(t)}{E(t)} \tag{14}$$

where G is total government spending and μ is the rate of nominal money growth. The government must remain solvent and respect the usual condition precluding Ponzi games

$$\lim_{t \to \infty} B(t) e^{-r^* t} = 0 \tag{15}$$

Forward integration of equation (14), given the transversality condition (15), yields the intertemporal budget constraint of the government

$$B(t) = \int_t^\infty \left[T(v) - G(v) + \mu(v) \frac{M(t)}{E(t)} \right] e^{-r^*(v-t)} dv \tag{16}$$

Equation (16) states that the government debt is equal to the present discounted value of future budget surpluses.

The authorities adopt the policy rules

$$T(t) = \psi \alpha B(t) - Z(t) \tag{17a}$$

$$Z(t) = \begin{cases} 0 \text{ for } t < 0 \\ Z \text{ for } t \geq 0 \end{cases} \tag{17b}$$

$$\mu(t) \frac{M(t)}{E(t)} = (1 - \psi) \alpha B(t) \tag{17c}$$

$$0 \leq \psi \leq 1 \tag{17d}$$

$$B_0 = 0 \tag{17e}$$

$$G(t) = 0 \text{ for } t \geq 0 \tag{17f}$$

where Z is a once-for-all lump-sum tax cut, α is a positive parameter which links the taxes and the seignorage revenues to the level of public debt, B_0 is the initial condition for the stock of public debt, while government spending is set equal to zero for simplicity[7]. Taxes and seignorage are increasing functions of the public debt. The rate of nominal money growth $\mu(t)$ is endogenously determined. The parameter ψ is the weight of tax finance in closing the deficits, when $\psi = 0$ budget deficits are entirely financed by seignorage and for $\psi = 1$ only by lump-sum taxes. Under pure tax finance the fiscal authority is assumed to adjust the sequence of taxes to satisfy its intertemporal budget constraint (16). The policy experiment is the following. At time $t = 0$ there is an unexpected and once-for-all increase of Z from zero, creating a sequence of deficits and an increase in government debt according to equation (16), that can be rewritten as

$$\overset{\bullet}{B}(t) = (r^* - \alpha)B(t) + Z \tag{18}$$

The transversality condition on the debt level (15) is satisfied if and only if $\alpha \geq r^*$. For simplicity it is assumed that the condition $\alpha > r^*$ holds the time.

Following Blanchard (1985), the effects produced by current and anticipated fiscal changes on wealth, real exchange rate and consumption can be summarized by the fiscal policy index

$$d(t) = \frac{\delta + \beta}{1 + \eta} \left\{ B(t) - \int_t^\infty T(v) e^{-(\delta + r^*)(v - t)} dv \right\} \tag{19}$$

[7]The government is assumed to adopt a *Ricardian regime*, as defined by Woodford (1995), Canzoneri et al. (2001) and Daniel (2001a). When $\psi = 1$ the policy is *Ricardian* in the sense of Aiyagari and Gertler (1985).

Fiscal policy affects aggregate consumption through two channels. Public debt is part of the non-human wealth and lump-sum taxes reduce disposable labor income. Money finance reduces instead the real value of money balances via the inflation tax and increases the opportunity costs of money holdings. Integrating the equation of motion (18) for the time path of B and T , and substituting the result in (19) yields

$$d(t) = \frac{\delta+\beta}{1+\eta}\frac{1}{(\alpha-r^*)}\left[\frac{\delta+\alpha}{\delta+r^*} - \psi\frac{\alpha}{\delta+r^*} - (1-\psi\frac{\alpha}{\alpha+\delta})e^{-(\alpha-r^*)t}\right]Z \tag{20}$$

Differentiating equation (20) with respect to time one obtains

$$\overset{\bullet}{d}(t) = (r^* - \alpha)d(t) + \frac{(\delta+\beta)}{(r^*+\delta)(1+\eta)}[\delta+\alpha(1-\psi)]Z \tag{21}$$

and

$$d_0 = \frac{\delta+\beta}{1+\eta}\left[\frac{1}{\delta+r^*} - \psi\frac{\alpha}{(\delta+r^*)(\alpha+\delta)}\right]Z \tag{22a}$$

$$\widetilde{d} = \frac{(\delta+\beta)}{(r^*+\delta)(1+\eta)(\alpha-r^*)}[\delta+\alpha(1-\psi)]Z > d_0 \tag{22b}$$

The value of the fiscal index depends on the level of the initial tax cut and on the degree of future tax finance ψ. During the adjustment process, as debt accumulates, the government budget goes from deficit to surplus, but the net effect is positive; hence, the index increases from d_0 to $\widetilde{d}$.

2.3 The Basic Model

The rate of growth of real money balances is given by

$$\overset{\bullet}{RM}(t) = \left(\frac{\overset{\bullet}{M}(t)}{E(t)} \right) = [\mu(t) - \pi(t)] \frac{M(t)}{E(t)} \tag{23}$$

which combined with the portfolio budget condition (8) yields the equation of motion of real money balance $RM(t)$

$$\overset{\bullet}{RM}(t) = [r^* + \mu(t)] \, RM(t) - \eta C(t) \tag{24}$$

By substituting the equation of the government budget constraint (18) and (23) into the dynamic equation of non-human wealth of households (10), one obtains

$$\overset{\bullet}{F}(t) = r^* F(t) + \omega - C(t) \tag{25}$$

which is the current account of the balance of payments and denotes the dynamic evolution of the economy's net external assets.

The basic model can then be described by the following set of equations

$$C(t) = \frac{\delta + \beta}{1 + \eta} \left[\frac{\omega}{r^* + \delta} + F(t) + \frac{M(t)}{E(t)} \right] + d(t) \tag{26a}$$

$$\overset{\bullet}{E}(t) = \frac{\eta C(t) E(t)^2}{M(t)} - r^* E(t) \tag{26b}$$

$$\frac{\overset{\bullet}{M}(t)}{E(t)} = (1-\psi)\frac{\alpha Z}{(\alpha - r^*)}\left[1 - e^{-(\alpha - r^*)t}\right] \qquad (26c)$$

$$\overset{\bullet}{F}(t) = \omega + r^* F(t) - C(t) \qquad (26d)$$

$$\overset{\bullet}{d}(t) = (r^* - \alpha)d(t) + \frac{(\delta + \beta)}{(r^* + \delta)(1+\eta)}[\delta + \alpha(1-\psi)]Z \qquad (26e)$$

with

$$\lim_{t\to\infty} B(t)e^{-r^*t} = \lim_{t\to\infty} M(t)e^{-r^*t} = \lim_{t\to\infty} F(t)e^{-r^*t} = 0 \qquad (27)$$

$B(0) = B_0$, $F(0) = F_0$, $M(0) = M_0, d(0) = d_0$ and $E(0)$, $C(0) = free$.

The time path of real money balance is described by the following equation

$$R\overset{\bullet}{M}(t) = r^* RM(t) + (1-\psi)\frac{\alpha Z}{(\alpha - r^*)}\left[1 - e^{-(\alpha - r^*)t}\right] - \eta C(t) \qquad (28)$$

obtained by combining equation (24a) and (24c) given the time path of public debt[8].

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[8]Equation (26b) can be rewritten as

$$\frac{\overset{\bullet}{E}(t)}{E(t)} RM(t) = \eta C(t) - r^* RM(t) \qquad (1n)$$

3 Fiscal Deficits and Exchange Rates

In this Section we examine the short and long run effects of a current and once-for-all lump-sum tax cut. After the fiscal expansion the government may increase taxes, use seignorage or employ some combinations of the two to finance the sequence of deficits.

Firstly, we analyse the short-run comparative static properties of the model and then we consider the long-run effects of a fiscal expansion. We derive the stability properties of the system and the transitional dynamics for the three alternative financing modes of the budget deficit.

3.1 Short-Run Effects of a Fiscal Expansion

Suppose that the economy is initially in a steady state equilibrium with the steady-state values of the macrovariables denoted by C_0, M_0, K_0, F_0, d_0, RM_0 and E_0 and that there is an unexpected and permanent lump-sum tax cut Z. The impact effect of a permanent and unanticipated tax cut on consumption and the nominal exchange rate are obtained by

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which gives the inflation tax. Similarly, equation (26c) can be expressed as

$$\frac{\dot{M}(t)}{M(t)} RM(t) = (1-\psi)\frac{\alpha Z}{(\alpha - r^*)}\left[1 - e^{-(\alpha - r^*)t}\right] \tag{2n}$$

which is seignorage revenues.

combining equation (26a) and the portfolio balance condition

$$RM_0^+ = \frac{M_0}{E_0^+} = \frac{\eta C_0^+}{r^*} \tag{29}$$

where the plus superscript denotes the value of the jump variables after the shock[9]. Consumption and the exchange rate jump on impact and, as shown in detail in the Mathematical Appendix, the partial derivatives are

$$C = C(F, Z) \qquad C_F > 0; C_Z > 0; \tag{30a}$$

$$E = E(F, Z) \qquad E_F < 0; E_Z < 0; \tag{30b}$$

Nominal money holdings do not change and the nominal exchange rate jumps as consequence of the shock. Combining equations (26a) and (29) and solving for C and RM, one obtains the following partial derivatives describing the short-run behavior of real money holdings

$$RM = RM(F, Z) \qquad RM_F > 0; RM_Z > 0; \tag{31}$$

These results can be explained as follows.

An increase in the stock of foreign assets positively affects non-human wealth and stimulates consumption and real money holdings. The increased demand for money determines a nominal appreciation of the exchange rate.

[9] Equation (29) can be obtained from equation (26b) by setting $\dot{E}(t) = 0$ or from equation (28) by setting $\dot{RM}(t) = 0$.

A tax cut increases the human wealth of individuals and positively affects consumption[10]. The portfolio balance condition implies an increase in real money holdings and this excess demand for the domestic currency requires a real appreciation of the exchange rate to clear the money market. A tax cut leads on impact to an appreciation of the exchange rate and to an increase in both consumption and in real money holdings, independently of the financing mode, as shown in the Mathematical Appendix. However, the short-run effects on the jump macrovariables are shown to be larger when money is the main financing source of budget deficits.

3.2 Long-Run Effects of a Fiscal Expansion and Dynamics

Long-run macroeconomic equilibrium is reached when $\dot{RM}(t) = \dot{F}(t) = \dot{C}(t) = \dot{d}(t) = 0$, and it is described by the following set of equations

$$\widetilde{C} = \frac{\delta+\beta}{1+\eta}\left[\frac{\omega}{r^*+\delta} + \widetilde{F} + \widetilde{RM}\right] + \widetilde{d} \tag{32a}$$

$$\widetilde{RM} = \frac{\eta\widetilde{C}}{r^*} - (1-\psi)\frac{\alpha Z}{r^*(\alpha - r^*)} \tag{32b}$$

[10]In this model agents have finite horizon and the positive effect on total wealth produced by a tax cut on impact outweighs the negative effects of the anticipated future tax increase and of seignorage. All these effects are summarized by the index of fiscal policy computed at the time of the tax cut, d_0.

$$\widetilde{F} = \frac{\widetilde{C} - \omega}{r^*} \tag{32c}$$

$$\widetilde{d} = \frac{(\delta + \beta)}{\rho(r^* + \delta)(1 + \eta)(\alpha - r^*)}[\delta + \alpha(1 - \psi)]Z \tag{32d}$$

where $\widetilde{C}$, $\widetilde{RM}$, $\widetilde{F}$ and $\widetilde{d}$ denote the steady state values of the macrovariables.

In the steady state the exchange rate and the nominal stock of money grow at the same rate

$$\frac{\overset{\bullet}{M}(t)}{M(t)} = \frac{\overset{\bullet}{E}(t)}{E(t)} = \frac{\eta\widetilde{C}}{\widetilde{RM}} - r^* = (1 - \psi)\frac{\alpha Z}{(\alpha - r^*)}\frac{1}{\widetilde{RM}} \tag{33}$$

As shown in the Mathematical Appendix, the long-run relationships among the variables of interest can be obtained by solving equations (32a)-(32d) for $\widetilde{C}$, $\widetilde{RM}$ and $\widetilde{F}$.

3.2.1 Mix-Financed Fiscal Expansion , $0 < \psi < 1$

Consider the case when the government is assumed to use seignorage and to levy lump-sum taxes on households to close the deficits. The steady-state effects of once-for-all tax-cut on consumption, foreign assets and real money balances are

$$\widetilde{C} = \widetilde{C}(Z) \quad and\ \widetilde{C}_Z = \begin{cases} > 0 \ \text{ for } \psi < \bar{\psi} \\ = 0 \ \text{ for } \psi = \bar{\psi} \\ < 0 \ \text{ for } \psi > \bar{\psi} \end{cases} \tag{34}$$

$$\widetilde{F} = \widetilde{F}(Z) \quad and \quad \widetilde{F}_Z = \begin{cases} > 0 & \text{for } \psi < \bar{\psi} \\ = 0 & \text{for } \psi = \bar{\psi} \\ < 0 & \text{for } \psi > \bar{\psi} \end{cases} \tag{35}$$

$$\widetilde{RM} = \widetilde{RM}(Z) \quad \widetilde{RM}_Z \lesseqgtr 0 \tag{36}$$

and the rate of depreciation of the exchange rate is given by monetary growth, as described by equation (33).

The long-run effects of a fiscal expansion are shown to critically depend on the degree of tax finance, ψ. If seignorage is the primary source to finance budget deficits, consumption and foreign assets increase to reach a new equilibrium above the original level; conversely, if taxation is mainly used to close the deficits, consumption and foreign assets decline in the long-run. In particular, there is a threshold level of tax financing, $\bar{\psi} = \frac{\alpha - r^*}{\alpha}$, for which an increase in Z would not have any effect on the long-run values of consumption and of net external assets. After the initial jump the economy experiments an adjustment process towards the new steady state, but foreign assets and consumption return to their original level. The overall effect on real money balance is ambiguous depending in a complex fashion on the values of the parameters, while nominal variables all grow at a constant rate.

To examine the transitional dynamics of the economy after a once-for-all tax-cut, consider the following system, linearized around the new steady state

$$\begin{pmatrix} \overset{\bullet}{RM} \\ \overset{\bullet}{F} \\ \overset{\bullet}{d} \end{pmatrix} = \begin{pmatrix} r^* - \eta\Gamma & -\eta\Gamma & a_{13} \\ -\Gamma & r^* - \Gamma & -1 \\ 0 & 0 & r^* - \alpha \end{pmatrix} \begin{pmatrix} RM - \widetilde{RM} \\ F - \widetilde{F} \\ d - \widetilde{d} \end{pmatrix} \tag{37}$$

where $\Gamma \equiv \frac{\beta+\delta}{1+\eta}$ and $a_{13} \equiv \frac{(1-\psi)(\alpha+\delta)\alpha}{\Gamma[\alpha(1-\psi)+\delta]} - \eta$.

In order to have a unique convergent adjustment path the system must have one positive and two negative eigenvalues, since foreign assets F and the fiscal index d are predetermined, while real money balances RM jumps instantaneously. The system has this property and the two stable roots are $\lambda_1 = r^* - \alpha$ and $\lambda_2 = r^* - \beta - \delta$, as shown in the Mathematical Appendix.

The stable path towards the steady state is described by the following set of equations

$$RM(t) - \widetilde{RM} = \left[e^{\lambda_1 t} - \eta v_{21} e^{\lambda_2 t}\right] \frac{1}{v_{31}}(d_0 - \widetilde{d}) + \eta(F_0 - \widetilde{F})e^{\lambda_2 t} \tag{38a}$$

$$F(t) - \widetilde{F} = \frac{v_{21}}{v_{31}}(d_0 - \widetilde{d})(e^{\lambda_1 t} - e^{\lambda_2 t}) + (F_0 - \widetilde{F})e^{\lambda_2 t} \tag{38b}$$

$$d(t) - \widetilde{d} = (d_0 - \widetilde{d})e^{\lambda_1 t} \tag{38c}$$

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\begin{aligned}
C(t)-\widetilde{C} &= \frac{\delta+\beta}{1+\eta}\left(\frac{1}{v_{31}}+\frac{v_{21}}{v_{31}}+1\right)e^{\lambda_1 t}(d_0-\widetilde{d})+ \\
&\quad -(\delta+\beta)\frac{v_{21}}{v_{31}}(d_0-\widetilde{d})e^{\lambda_2 t}+(\delta+\beta)(F_0-\widetilde{F})e^{\lambda_2 t}
\end{aligned} \tag{38d}
$$

where v_{21} and v_{31} denote the components of the eigenvector associated to the root λ_1 and can be shown to depend on the degree of tax finance ψ.

According to equations (38a), (38b) and (38d) the dynamics of real money balances, external assets and aggregate consumption around the steady state can be split in two different components. The first component is produced by the changes in wealth, summarized by the index of fiscal policy; the second component shows the effect of changes in the level of external assets.

The intuition behind these results can be understood by focusing on two opposite modes of budget deficits financing: the pure money-financing and pure tax-financing.

3.2.2 Pure money-financed fiscal expansion, $\psi = 0$

When deficits are entirely financed by using seignorage the new steady state is characterized by the following partial derivatives

$$\widetilde{C} = \widetilde{C}(Z) \qquad \widetilde{C}_Z > 0 \tag{39a}$$

$$\widetilde{RM} = \widetilde{RM}(Z) \qquad \widetilde{RM}_Z \lesseqgtr 0 \tag{39b}$$

$$\widetilde{F} = \widetilde{F}(Z) \qquad \widetilde{F}_Z > 0 \tag{39c}$$

In the long-run foreign assets and consumption are above their original level. This result can be explained by the adjustment dynamics followed by the system after the fiscal shock. In a first stage, there is a decumulation of external assets due to the excess of domestic absorption and at the same time households start to decrease consumption since the inflation tax reduces the real value of their money balances. In a second stage, the level of domestic absorption has decreased in such a way that the economy starts to run current account surpluses. At the end of the adjustment process the level of external assets and the financial wealth of households are above the original level[11]. The intuition behind this result is that in the case of money finance, inflation tends to redistribute wealth from current to future generations which fully benefit from the tax cut. On the other hand, the net effect on real money balances cannot be unambiguously be determined. As shown in the Mathematical Appendix, the partial derivative of real money holdings depends in a complex fashion on the parameters of the model.

[11]These results are consistent with the "unpleasant fiscal arithmetic" obtained by Kawai and Maccini (1995) in the case of pure money finance.

In the steady state the exchange rate depreciates at a constant rate proportional to the initial tax-cut and the equilibrium inflation tax corresponds exactly to the seignorage revenues.

The dynamics of the economy towards the steady state is described by the following set of equations

$$RM(t) - \widetilde{RM} = -\frac{1+\eta}{\beta+\delta}(d(t) - \widetilde{d}) + \eta(F(t) - \widetilde{F}) \qquad (40a)$$

$$F(t) - \widetilde{F} = (F_0 - \widetilde{F})e^{\lambda_2 t} \qquad (40b)$$

$$d(t) - \widetilde{d} = (d_0 - \widetilde{d})e^{\lambda_1 t} \qquad (40c)$$

$$C(t) - \widetilde{C} = (\beta + \delta)(F(t) - \widetilde{F}) \qquad (40d)$$

Equations (40) show that the transitional path of real money balances depends on the changes in human wealth and in foreign assets; conversely, in the neighborhood of the new steady state, foreign assets are shown to converge to $\widetilde{F}$ independently of the time path of the fiscal index d. The dynamics of consumption,on the other hand, is entirely explained by the time path of foreign assets. The fiscal index affects aggregate consumption through two channels: it directly enters the consumption function and indirectly influences the level of consumption through its effects on real money balances. These two effects cancel out and the time path of consumption is entirely explained by the dynamics of external assets [12].

3.2.3 Pure tax-financed fiscal expansion, $\psi = 1$

When deficits are financed only by taxation the stock of nominal money does not change and the long-run effects of a fiscal shock are described by the following partial derivatives

$$\widetilde{C} = \widetilde{C}(Z) \qquad \widetilde{C}_Z < 0 \tag{41a}$$

$$\widetilde{E} = \widetilde{E}(Z) \qquad \widetilde{E}_Z > 0 \tag{41b}$$

$$\widetilde{F} = \widetilde{F}(Z) \qquad \widetilde{F}_Z < 0 \tag{41c}$$

In the new steady state consumption, real money balances and foreign assets are below their original levels, while the exchange rate is above its original level, reflecting a nominal depreciation of the exchange rate. After the fiscal shock, current generations profit from the present lump sum tax cut, since they share the burden of future increase in taxation with future generations. However, during the adjustment process the economy runs current account deficits and there is a decumulation of external assets[13]. In the long run total non-human wealth and the level of consumption are below their original

[12]This effect follows from the particular form of the utility function which is logarithmic in consumption and real money balances.

[13]The displacement of foreign assets by government debt and the twin deficits phenomenon are typical results of finite horizons models. See Blanchard (1985), Kawai and Maccini (1995) and Piersanti (2001).

level. The decrease in the demand for domestic currency determines the depreciation of the exchange rate[14].

To examine the dynamics of the economy after a once-for-all tax-cut, consider the system of equations (26b), (26d) and (26e) and linearize it around the new steady state. The stable solution is of the form

$$E(t)-\widetilde{E} = \frac{1}{\upsilon_{31}}(d(t)-\widetilde{d})e^{\lambda_1 t}+\upsilon_{21}\eta\widetilde{E}^2(d_0-\widetilde{d})e^{\lambda_2 t} -(F(t)-\widetilde{F})\eta\widetilde{E}^2 e^{\lambda_2 t} \quad (42a)$$

$$F(t)-\widetilde{F} = \frac{\upsilon_{21}}{\upsilon_{31}}(e^{\lambda_1 t}-e^{\lambda_2 t})(d_0-\widetilde{d})-(F_0-\widetilde{F})e^{\lambda_2 t} \quad (42b)$$

$$d(t)-\widetilde{d} = (d_0-\widetilde{d})e^{\lambda_1 t} \quad (42c)$$

where υ_{21} and υ_{31} denote the components of the eigenvector associated to the stable root λ_1. Equation (42a) describes the time path of the exchange rate and its dynamics is explained by the time evolution of the fiscal index and of the current account. The exchange rate is shown to depreciate when d rises

[14] In the Mathematical Appendix it is shown that a fiscal expansion is neutral with respect to the short and the long-run values of consumption, foreign assets and exchange rate when deficits are entirely financed by lump-sum taxation and under the hypothesis that agents have infinite life. In this case the discipline of a *Ricardian regime* and the absence of money financing guarantee the ineffectiveness of a fiscal expansion.

(provided that $\upsilon_{31} > 0$ and this is the case) and to appreciate when the economy accumulates external assets.

As the economy moves towards the new steady state and the budget goes from deficits to surplus, consumption begins to decline while the exchange rate rises. During the adjustment process the economy will experience a current account deficit, so that foreign assets end up to a lower level in the steady state.

4 Conclusions

In a optimising general equilibrium model with finite lives, we have analysed how a fiscal expansion affects the time path of the exchange rate and of relevant macroeconomic variables for alternative financing modes of budget deficits.

Our major finding is that when the government is assumed to satisfy the intertemporal budget constraint without resorting to seignorage revenues, a once-for-all tax-cut would still lead to a depreciation of the exchange rate in the long run. After the fiscal shock, the exchange rate appreciates on impact and then starts to depreciate along the time path towards the new steady state. During the adjustment process the economy runs current account deficits and there is a decumulation of foreign assets. In the long run total financial wealth and aggregate consumption are below their initial level. On the other hand, with money finance the non-indexed financial wealth of current generations starts to reduce after the tax cut, so that consumption starts to decrease and the economy runs current account surpluses. During the adjustment process, there is first a decumulation of external assets due to the excess of domestic absorption and a decrease in consumption as a consequence of the inflation tax which reduces the real value of money balances. The process of decumulation of foreign assets stops when domestic absorption has decreased to a point where the economy starts to run current account surpluses.

Money and tax financing generate different wealth effects and opposite intergenerational reallocation of resources. Hence, in the case of mixed finance the net effect of a fiscal expansion on financial wealth crucially depends on the degree of tax finance in closing the deficits. It is shown that there is a degree of tax finance for which a fiscal expansion is neutral with respect to the long-run values of consumption and foreign assets.

In a set-up in which Ricardian equivalence does not hold a fiscal expansion financed entirely by lump-sum taxation determines a depreciation of the exchange rate in the long-run. On the other hand, in the new long-run equilibrium after a mix-financed fiscal expansion the exchange rate depreciates steadily at a constant rate. These results further narrow the concept of fiscal discipline, in the sense that the respect of fiscal solvency without money finance is not sufficient to avoid a worsening of the fundamentals and the depreciation of the exchange rate. Independence of the monetary authorities thus appears to be a necessary but not a sufficient condition to avoid the depreciation of the exchange rate.

Mathematical Appendix

Short-run Equilibrium

Assuming that the economy is initially in a steady state, after the fiscal shock at time $t = t_0^+$ the economy is described by the following system of equations

$$C_0^+ = \frac{\delta+\beta}{1+\eta}\left[\frac{\omega}{r^*+\delta} + F_0 + \frac{M_0}{E_0^+}\right] + + \frac{\delta+\beta}{1+\eta}\left[\frac{1}{\delta+r^*} - \psi\frac{\alpha}{(\delta+r^*)(\alpha+\delta)}\right] Z \tag{1A}$$

$$RM_0^+ = \frac{M_0}{E_0^+} = \frac{\eta C_0^+}{r^*} \tag{2A}$$

The system can be solved for C and RM and by total differentiating one obtains the following partial derivatives

$C_Z = \frac{\partial C}{\partial Z} = \frac{r^*(\beta+\delta)[\alpha(1-\psi)+\delta]}{(\alpha+\delta)(r^*+\delta)\Delta} > 0$

$C_F = \frac{\partial C}{\partial F} = \frac{r^*(\beta+\delta)}{\Delta} > 0$

$RM_Z = \frac{\partial RM}{\partial Z} = \frac{\eta(\beta+\delta)[\alpha(1-\psi)+\delta]}{(\alpha+\delta)(r^*+\delta)\Delta} > 0$

$RM_F = \frac{\partial RM}{\partial F} = \frac{\eta(\beta+\delta)}{\Delta} > 0$

with $\Delta \equiv r^*(1+\eta) - \eta(\beta+\delta) > 0$.

The system of equations (1A)-(2A) can be solved for C and E and by total differentiating one obtains the impact effect on the exchange rate of a change in Z and F, respectively

$$E_Z = \frac{\partial E}{\partial Z} = -\frac{\eta(\beta+\delta)[\alpha(1-\psi)+\delta]E^2}{(\alpha+\delta)(r^*+\delta)\Delta} < 0$$

$$E_F = \frac{\partial E}{\partial F} = -\frac{\eta(\beta+\delta)E^2}{M\Delta} < 0$$

with $\Delta \equiv r^*(1+\eta) - \eta(\beta+\delta) > 0$

Remark 1 *When $\psi = 0$ the short-run effects on the jump macrovariables, C, RM and E, are larger.*

Remark 2 *When $\psi = 1$ and $\delta = 0$ the fiscal expansion is neutral with respect to the short-run values of C, RM and E.*

Long-run Equilibrium

To analyze the long-run responses to the tax cut consider the system of equations (32) in the text. The equilibrium system can be solved for $\widetilde{C}$, $\widetilde{RM}$ and $\widetilde{F}$ and then total differentiated in order to obtain the following partial derivatives

$$\widetilde{C}_Z = \frac{\partial \widetilde{C}}{\partial Z} = \frac{(\beta+\delta)[(1-\psi)\alpha - r^*]\delta}{\Lambda} \begin{cases} > 0 & \text{for } \psi < \bar{\psi} \\ = 0 & \text{for } \psi = \bar{\psi} \\ < 0 & \text{for } \psi > \bar{\psi} \end{cases}$$

$$\widetilde{F}_Z = \frac{\partial \widetilde{F}}{\partial Z} = \frac{(\beta+\delta)[(1-\psi)\alpha - r^*]\delta}{\Lambda r^*} \begin{cases} > 0 & \text{for } \psi < \bar{\psi} \\ = 0 & \text{for } \psi = \bar{\psi} \\ < 0 & \text{for } \psi > \bar{\psi} \end{cases}$$

$$\widetilde{RM}_Z = \frac{\partial \widetilde{RM}}{\partial Z} = \frac{(1+\eta)(1-\psi)(1-\beta)\alpha r^* - (\beta+\delta)(\eta r^* + 1 - \psi)\delta}{\Lambda r^*} \lesseqgtr 0$$

where $\Lambda \equiv (1+\eta)(\beta+\delta-r^*)(\alpha-r^*)(\delta+r^*) > 0$.

When $\psi = 1$ the equilibrium system (32) can be solved for $\widetilde{C}$, $\widetilde{E}$ and $\widetilde{F}$. Differentiating it with respect to Z yields the following partial derivative

$$\widetilde{E}_Z = \frac{\partial \widetilde{E}}{\partial Z} = \frac{(\beta+\delta)\eta\delta\widetilde{E}^2}{\Lambda} > 0$$

Remark 3 *When $\psi = 1$ and $\delta = 0$ the fiscal expansion is neutral with respect to the long-run values of C, RM, F and E.*

Stability and Transitional Dynamics

The system (37) is such that the Jacobian matrix is block recursive, which makes it straightforward to find the eigenvalues. The characteristic equation is

$$(\lambda - r^* + \alpha)[\lambda^2 + (\beta+\delta-2r^*)\lambda - r^*(\beta+\delta-r^*)] = 0 \quad (3A)$$

and the roots are $\lambda_1 = r^* - \alpha < 0$, $\lambda_2 = r^* - \beta - \delta < 0$ and $\lambda_3 = r^* > 0$. The eigenvector $v_i = (1\ v_{2i}\ v_{3i})'$ associated with the negative root $i = 1, 2$ satisfies

$$\begin{pmatrix} r^* - \eta\Gamma - \lambda_i & -\eta\Gamma & a_{13} \\ -\Gamma & r^* - \Gamma - \lambda_i & -1 \\ 0 & 0 & r^* - \alpha - \lambda_i \end{pmatrix} \begin{pmatrix} 1 \\ v_{2i} \\ v_{3i} \end{pmatrix} = 0 \quad (4A)$$

Solving the system one obtains

$$v_1 = \begin{pmatrix} 1 \\ v_{21} \\ v_{31} \end{pmatrix} \text{ and } v_2 = \begin{pmatrix} 1 \\ 1/\eta \\ 0 \end{pmatrix}$$

where

$$v_{21} = \begin{cases} 1/\eta & \text{if } \psi = 1 \\ \frac{(\beta+\delta)\delta\psi}{\delta(\beta+\delta)(1+\eta-\psi)-(1+\eta)(1-\psi)\alpha(\alpha-\beta)} & \text{if } 0 < \psi < 1 \\ 0 & \text{if } \psi = 0 \end{cases}$$

and

$$v_{31} = \begin{cases} -(\beta+\delta-\alpha)/\eta & \text{if } \psi = 1 \\ \frac{(\beta+\delta)[\alpha(\beta+\delta-\alpha)\psi+\alpha(\alpha-\beta)-\delta(\beta+\delta)]}{\delta(\beta+\delta)(1+\eta-\psi)-(1+\eta)(1-\psi)\alpha(\alpha-\beta)} & \text{if } 0 < \psi < 1 \\ -(\beta+\delta)/(1+\eta) & \text{if } \psi = 0 \end{cases}$$

The components of the eigenvector associated to the stable root $\lambda_1$vary with the degree of tax finance ψ. The stable general solution is of the form

$$RM(t) - \widetilde{RM} = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$$

$$F(t) - \widetilde{F} = v_{21} A_1 e^{\lambda_1 t} + \tfrac{1}{\eta} A_2 e^{\lambda_2 t}$$

$$d(t) - \widetilde{d} = v_{31} A_1 e^{\lambda_1 t}$$

where A_1 and A_2 are the arbitrary constants which can be obtained from the initial conditions F_0 and d_0

$A_1 = \frac{(d_0 - \tilde{d})}{v_{31}}$

$A_2 = \eta(F_0 - \tilde{F}) - \eta\frac{v_{21}}{v_{31}}(d_0 - \tilde{d})$

The transition equations of the model are

$$RM(t) - \tilde{RM} = \frac{1}{v_{31}}(d_0 - \tilde{d})e^{\lambda_1 t} + \eta(F_0 - \tilde{F})e^{\lambda_2 t} + -\eta\frac{v_{21}}{v_{31}}(d_0 - \tilde{d})e^{\lambda_2 t} \quad (5A)$$

$$F(t) - \tilde{F} = \frac{v_{21}}{v_{31}}(d_0 - \tilde{d})(e^{\lambda_1 t} - e^{\lambda_2 t}) + (F_0 - \tilde{F})e^{\lambda_2 t} \quad (6A)$$

$$d(t) - \tilde{d} = (d_0 - \tilde{d})e^{\lambda_1 t} \quad (7A)$$

The transition equation of consumption is obtained by linearizing around the steady state equation (26a) and combining with (5A)-(7A)

$$C(t) - \tilde{C} = \frac{\delta + \beta}{1 + \eta}\left[\frac{1}{v_{31}} + \frac{v_{21}}{v_{31}} + 1\right](d_0 - \tilde{d})e^{\lambda_1 t} + -(1 + \eta)\frac{v_{21}}{v_{31}}(d_0 - \tilde{d})e^{\lambda_2 t} + (\delta + \beta)(F_0 - \tilde{F})e^{\lambda_2 t} \quad (8A)$$

The slope of the stable path in the space F, d is obtained by differentiating with respect to time (6A) and (7A) and by combining the results

$$\frac{\partial F(t)}{\partial d(t)} = \frac{\lambda_1 \frac{v_{21}}{v_{31}}(d_0-\tilde{d})e^{\lambda_1 t} - \lambda_2 \frac{v_{21}}{v_{31}}(d_0-\tilde{d})e^{\lambda_2 t} + \lambda_2(F_0-\tilde{F})e^{\lambda_2 t}}{\lambda_1(d_0-\tilde{d})e^{\lambda_1 t}}$$

The slope of the stable trajectory changes over time. The direction in which the path approaches the new steady state is determined by the dominant eigenvector, which is associated to the larger of the stable eigenvalues (see Calvo,1987). Let $\lambda_1 > \lambda_2$, then

$$\lim_{t\to\infty} \frac{\partial F(t)}{\partial d(t)} = \frac{v_{21}}{v_{31}}$$

where

$$\frac{v_{21}}{v_{31}} = \begin{cases} -1/(\beta+\delta-\alpha) & \text{if } \psi = 1 \\ \frac{\delta\psi}{[\alpha(\beta+\delta-\alpha)\psi+\alpha(\alpha-\beta)-\delta(\beta+\delta)]} & \text{if } 0 < \psi < 1 \\ 0 & \text{if } \psi = 0 \end{cases}$$

When $\psi = 1$ the transition equations are

$$RM(t) - \tilde{RM} = \eta(F(t) - \tilde{F}) \tag{9A}$$

$$F(t)-\tilde{F} = \frac{1}{\alpha-\beta-\delta}(d_0-\tilde{d})(e^{\lambda_1 t}-e^{\lambda_2 t})+(F_0-\tilde{F})e^{\lambda_2 t} \tag{10A}$$

$$d(t) - \tilde{d} = (d_0 - \tilde{d})e^{\lambda_1 t} \tag{11A}$$

$$C(t) - \tilde{C} = (\beta+\delta)(F(t) - \tilde{F}) + (d_0 - \tilde{d})e^{\lambda_1 t} \tag{12A}$$

When $\psi = 0$ the transition equations are

$$RM(t) - \widetilde{RM} = -\frac{1+\eta}{\beta+\delta}(d_0 - \widetilde{d})e^{\lambda_1 t} + \eta(F_0 - \widetilde{F})e^{\lambda_2 t} \quad (13A)$$

$$F(t) - \widetilde{F} = (F_0 - \widetilde{F})e^{\lambda_2 t} \quad (14A)$$

$$d(t) - \widetilde{d} = (d_0 - \widetilde{d})e^{\lambda_1 t} \quad (15A)$$

$$C(t) - \widetilde{C} = (\beta+\delta)(F(t) - \widetilde{F}) \quad (16A)$$

Alternatively, when $\psi = 1$ one can studies the dynamic stability by focusing on the following system linearized around the new steady state

$$\begin{pmatrix} \dot{E} \\ \dot{F} \\ \dot{d} \end{pmatrix} = \begin{pmatrix} r^* - \eta\Gamma & -\eta\Gamma\widetilde{E}^2 & \eta\widetilde{E}^2 \\ -\Gamma/\widetilde{E}^2 & r^* - \Gamma & -1 \\ 0 & 0 & r^* - \alpha \end{pmatrix} \begin{pmatrix} E - \widetilde{E} \\ F - \widetilde{F} \\ d - \widetilde{d} \end{pmatrix} \quad (17A)$$

where M_0 has been normalized to unity for simplicity.

The Jacobian matrix is block recursive and the system presents the same eigenvalues of system (37). Given the initial condition the solution of the system is

$$E(t) - \widetilde{E} = (d_0 - \widetilde{d})\frac{1}{\upsilon_{31}}(e^{\lambda_1 t} + \upsilon_{21}\eta\widetilde{E}^2 e^{\lambda_2 t}) + -(F(t) - \widetilde{F})\eta\widetilde{E}^2 e^{\lambda_2 t} \quad (18\text{A})$$

$$F(t) - \widetilde{F} = \frac{\upsilon_{21}}{\upsilon_{31}}(e^{\lambda_1 t} - e^{\lambda_2 t})(d_0 - \widetilde{d}) - (F_0 - \widetilde{F})e^{\lambda_2 t} \quad (19\text{A})$$

$$d(t) - \widetilde{d} = (d_0 - \widetilde{d})e^{\lambda_1 t} \quad (20\text{A})$$

where
$\upsilon_{21} = \frac{2\eta(\beta+\delta)-(1+\eta)\alpha}{\eta\widetilde{E}^2[2(\beta+\delta)-\alpha(1+\eta)]}$ $\upsilon_{31} = \frac{(1+\eta)(\beta+\delta-\alpha)\alpha}{\eta\widetilde{E}^2[2(\beta+\delta)-\alpha(1+\eta)]}$

The slope of the stable trajectory in the space E, d is obtained by differentiating (18A) and (20A) with respect to time, combining the results we obtain

$$\frac{\partial E(t)}{\partial d(t)} = \frac{\lambda_1 \frac{1}{\upsilon_{31}}(d_0-\widetilde{d})+\lambda_2 \frac{\upsilon_{21}}{\upsilon_{31}}(d_0-\widetilde{d})\eta\widetilde{E}^2 e^{(\lambda_2-\lambda_1)t}+\lambda_2(F(t)-\widetilde{F})\eta\widetilde{E}^2 e^{(\lambda_2-\lambda_1)t}}{\lambda_1(d_0-\widetilde{d})}$$

The slope of the stable path is time varying. The direction in which the path approaches the new steady state is determined by the dominant eigenvector, which is associated to the larger of the stable eigenvalues. Let again $\lambda_1 > \lambda_2$, then the slope of the stable trajectory in the neighborhood of the steady state is

$$\lim_{t\to\infty} \frac{\partial E(t)}{\partial d(t)} = \frac{1}{\upsilon_{31}} > 0$$

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