# Managing the Risk of Loans with Basis Risk
# Sell, Hedge or Do Nothing

by

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Individual loans contain a bundle of risks including credit, and interest rate risk. Moreover, credit risk may depend on national macroeconomic, regional economic and borrower-specific factors. This paper focuses on the issue of banks' management of these various risks. Why is risk management important? Among the reasons are to preserve the value of a bank's charter, to avoid costly regulatory intervention, to minimize tax obligations in the presence of convex tax rates, and to reduce the compensation demanded by risk-averse management and suppliers.[1] Different ways in which banks manage the risks is by reducing them via costly evaluation of the borrower before making the loan and by costly monitoring and collection after making the loan, by selling the right to the proceeds to a third party while retaining the responsibility to service the loan, and by hedging some of the risks.

Historically, banks have held most of the loans that they originate in their portfolio and managed the entire bundle of risks. Holding the loan in their portfolio has the advantage for banks of maximizing their incentive to engage in costly underwriting and

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[1] See Keeley (1990) on bank charter value, Buser, Chen and Kane (1981) on costly supervisory intervention and Smith and Stulz (1985) on the benefits of hedging in reducing taxes and compensation to risk averse parties that contract with a firm.

monitoring. The disadvantage is that the bank is fully exposed to any remaining risks in the loan and bears any related costs.

An increasingly common alternative to holding the loans is to sell the loans to a third party. If the loan is sufficiently large, then such a sale may consist of all or part the loan. However, if the loan is small then it may be securitized. Securitization involves bundling the loan together with other similar loans, creating new securities that are claims on parts of the cash flow from the package of loans. Among the types of loans that are commonly securitized are home mortgages, and credit card receivables. Another alternative is to use a credit derivative written on the loan or pool of loans held in a bank's portfolio. An advantage of selling or fully hedging with a derivative is that the bank is able to profit from any comparative advantage that it has in finding borrowers without incurring any risks. A drawback of sales is that doing so may reduce the lender's incentive to engage in costly evaluation and monitoring of loans. Consequently, many loan sales are structured in such a way that the seller retains a substantial part of the risk.[2] Such transactions may reduce financing costs by reducing regulatory capital requirements and providing collateral to the banks' creditors, but these transactions cannot be fairly characterized

[2] Pennacchi (1988) models bank loan sales and includes a section on loan sales without recourse. However, in these "sales" the optimal contract is one in which the bank sells a debt-like claim on the loans and retains an equity interest. Such a sale reduces the moral hazard and adverse selection associated with loan sales but leaves the bank with most of the risk. Jones (1999) discusses a variety of ways in which banks structure loan sales to reduce regulatory capital requirements without significantly reducing their risk exposure.

as important tools for managing the riskiness of a bank's loan portfolio.

Banks may hedge some types of risks using derivatives, but not others. For example, a bank may use interest rate futures to manage interest rate exposure. Alternatively, a bank may use credit derivatives written on claims that are correlated with the loan portfolio's performance but where the return is independent of the bank's underwriting and monitoring efforts. An advantage of hedging is that it reduces risk exposure but not the incentive to undertake costly underwriting and monitoring.

This paper analyzes a bank's decision to hold a loan unhedged, hold the loan and hedge, or sell the loan. Loan sales in our model consist of a transfer of all the risk to the buyer. In order to focus the discussion, monitoring is used as the only costly tool for banks managing risk. Further, the model focuses on convex taxes as the sole rationale for risk management, although any imperfection that generates concavity of firm value with respect to net revenues would be sufficient for our purposes.

The first part of the paper provides analytic results for the case with no basis risk. If a bank perfectly and costlessly hedges the part of loan risk that is uncorrelated with its costly monitoring then hedging always dominates holding a loan unhedged. The bank reduces its expected tax payments by hedging some of the risks and retains the full incentive to monitor the borrower's performance. Whether hedging dominates loan sales depends on whether the net gain from costly monitoring exceeds the potential tax savings from fully hedging the loan. We also include several comparative static results emanating from changes in the distribution of loan returns, interest rates and taxes.

If hedging is costly or subject to basis risk then a policy of hedging will not necessarily dominate a policy of holding the loan unhedged. Copeland and Copeland (1999) analyze the case of hedging with transaction costs and conclude that fully hedging (in the sense of minimizing the variance of returns) is not always optimal. The second part of our paper provides a specific scenario in which hedging is dominated by no hedging due to basis risk. For example, the bank may be hedging its risk exposure on a mortgage portfolio with derivatives written on Treasury securities. Analytic results for the case of basis risk would be easy to generate, but would be difficult to interpret given that basis risk adds several free parameters to the model. In contrast, analysis of a specific scenario facilitates discussion of the exact form of basis risk needed to eliminate the gains from hedging in the context of our model.

In the paper the first section develops the model. The second section presents analytical results comparing the no hedging alternative with hedging without basis risk, and with selling the loan. The third section presents a numerical example where hedging yields a higher expected return than hedging with basis risk or selling the loan. The fourth section outlines empirical implications. Section five concludes the paper.

## *1. The Model*

The model analyzes a bank that invests in a fixed pool of loans.[3] All parties in the model are risk neutral. The owner/manager (banker) must decide: 1) whether not to hedge, hedge or sell the risk associated with the loans and 2) whether to engage in a low level of monitoring or to undertake maximum monitoring efforts. The bank may obtain equity capital from the banker at an opportunity cost equal to the riskless, gross rate of return, $r$. The bank may only invest in loans and it has access to a pool of positive net present value loans of fixed amount $L$. The rate of return earned on these loans depends on two types of risk. One type of risk is perfectly correlated with some observable index, such as an interest rate index or an index of loan defaults. This risk earns a rate of return $o_g$ with probability $p$ in the good state. The rate of return in the bad state, $o_b$, occurs with probability $(1\text{-}p)$. The other risk is the part of credit risk that is uncorrelated with any observable index. The rate of return on this risk is $c_g$ in the good state and $c_b$ in the bad state. The probability that the bank will earn the good rate of return depends on the banker's monitoring effort, $j$. If the banker undertakes the low amount of monitoring effort, $l$, then the good return state will occur with probability $q_l$ and the bad state will occur with probability $(1\text{-}q_l)$. The banker may undertake the maximum monitoring effort, $m$. If the banker undertakes the maximum monitoring effort then the probability of the good return state is $q_m$ and the probability of the bad return state is $(1\text{-}\ q_m)$ with $q_m > q_l$.

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[3] . The assumption that the bank invests in a fixed pool of loans eliminates scale considerations from the analysis. Note, however, that the addition of a risk-free asset would not fundamentally change the analysis since insuring a portfolio of loans provides a risk-free rate of return in our model.

Loan monitoring imposes a non-pecuniary cost on the banker which is equivalent to a financial cost of $mc_j$ where $j = l, m$. In order to simplify the analysis the non-pecuniary cost of the low monitoring effort, $mc_l$ , is normalized to zero.

The bank may completely eliminate its risk exposure by selling the loan while retaining responsibility to collect payments from the borrower, or the bank may enter into a derivative-based contract to hedge the observable part of its risk exposure. The markets are competitive and have no transactions costs implying that the expected value of the payments on both types of risk management are equal to their respective expected receipts. Contracting on monitoring effort $j$ in return for paying an amount equal to the bank's receipts on the loan the bank receives a guaranteed fixed payment of $s$:[4]

$$s = po_g + (1-p)o_b + q_jc_g + (1-q_j)c_b > 0. \tag{1}$$

The derivative may take two forms: a perfect hedge and one with basis risk. The guaranteed payment to the bank under the perfect hedge, $h$, is

$$h = po_g + (1-p)o_b . \tag{2}$$

The cash flows from the derivative with basis risk are not perfectly correlated with the observable risk, but will reduce the variance of the bank's cash flow.

[4] The payment in return for selling the loan in the model appears to be collected at the end of the period. However, this formulation is equivalent to assuming that the bank receives the payments at the beginning of the period and invests the proceeds at the risk neutral rate during the period.

The bank finances the loans with a combination of deposits and equity capital. The government requires banks to hold capital that at least equals the bank's losses in the worst possible state of the world. The only risk is the risk associated with the bank's loans. Thus, an unlimited amount of deposits is available at the gross, risk-free rate of $r$. If the bank neither hedges nor sells its loan portfolio, the capital requirement, $RK^{no}$, is

$$RK^{no} = -L(o_b + c_b) + (L-RK^{no})r = -L(o_b + c_b - r)/(1+r). \quad (3)$$

The capital requirement for a bank that only hedges its credit risk exposure, $RK^{hg}$ , is

$$RK^{hg} = -L(h + c_b - r)/ (1+r). \quad (4)$$

The capital requirement if the bank insures, $RK^s$, is assumed to be zero.[5]

The bank is taxed at a rate $t$ on positive income with no carryforward or carryback provisions for negative income.[6] If the

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[5]. The assumption that the regulators accurately measure the riskiness of the loans is a reasonable approximation of the method of calculating capital for the "trading book" of banks but not their "banking book." However, the Basel Committee on Banking Supervision has recognized some weaknesses in the current standards and published the consultative paper *A New Capital Adequacy Framework* in June 1999 which discusses various ways to make the capital adequacy guidelines more sensitive to the bank's credit risk. The consultative paper may be obtained from the BIS website: http://www.bis.org/publ/index.htm as Paper number 50 under the Basel Committee on Banking Supervision.

bank sells the loan, it will have positive income in all four return states. If the bank does not sell the loan, it has positive income if good return states obtain for both the observable and credit risk. The bank is also assumed to have negative income in both states with the bad credit risk return: (1) good observable return ($o_g$) and bad credit risk return ($c_b$), and (2) bad observable return ($o_b$) and bad credit risk return ($c_b$). Whether the bank has positive returns in the fourth state, [$o_b$, $c_g$], depends on whether it hedges: $[L(o_b + c_g) - (L\text{-}RK^{no})r] < 0$, but $[L(h + c_g) - (L\text{-}RK^{hg})r] > 0$. The expected cash flow from operations depends on the banker's monitoring effort, and the bank's hedging decision. The expected cash flow if the banker engages in monitoring level $j$ ($j = l, m$) and does nothing, $E(C^{j,no})$, is:

$$E(C^{j,no}) = pq_j[L(o_g+c_g)-(L\text{-}RK^{no})r](1\text{-}t) + (1\text{-}p)q_j[L(o_b+c_g)-(L\text{-}RK^{no})r] + p(1\text{-}q_j)[L(o_g+c_b)-(L\text{-}RK^{no})r] + (1\text{-}p)(1\text{-}q_j)[L(o_b+c_b)-(L\text{-}RK^{no})r] \quad (5)$$

The expected net income if the banker engages in monitoring level $j$ and hedges the credit exposure, $E(C^{j,hg})$, is:

$$E(C^{j,hg}) = q_j[L(h + c_g) - (L\text{-}RK^{hg})r](1\text{-}t) + (1\text{-}q_j)[L(h + c_b) - (L\text{-}RK^{hg})r] \quad (6)$$

The expected cash flow from the bank's operations if it sells the loan, $E(C^{j,in})$, is

$$E(C^{j,s}) = [Ls - Lr](1\text{-}t). \quad (7)$$

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[6] . These assumptions approximate for the possibility that the bank may experience losses in excess of its recent earnings and, thus, lose at least some of the time value of the payments.

The value of the bank to the banker after incorporating the opportunity cost of the banker's equity investments is $U^{l,z}$, where

$$U^{l,z} = E(C^{l,z}) - RK^{z}\, r \tag{8}$$

The utility of the banker with maximum monitoring effort, *m,* is

$$U^{m,z} = E(C^{m,z}) - RK^{z} r - mc_{m}\ . \tag{9}$$

*2. No basis risk*

Prior to deciding whether to hedge or sell the loan the firm must determine whether it should hedge or not hedge the loan. Proving that a firm should hedge rather than do nothing is straightforward.[7] Hedging may affect the bank's value through the value of the tax shield of debt and the level of monitoring. Hedging increases the probability that the firm will have positive net income and, hence, positive value for the tax shield of debt. Further, the set of parameters under which the bank engages in additional monitoring if it does nothing is a subset of the parameters for additional monitoring, if it hedged.[8] Thus, hedging both increases the tax shield and may result in additional monitoring. However, parameter values decide whether hedging or selling the loan dominates. The tax benefit of debt is greater if the firm sells the loan. The gains from hedging with additional monitoring, however, may exceed those from selling.

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[7] The proof is available upon request from the authors.
[8] The proof is available from the authors upon request.

**Proposition** **If the bank does not engage in maximum monitoring effort, *m*, under a hedging strategy, then selling the loan is always superior. If the bank does engage in maximum monitoring effort, *m*, when it hedges then selling the loan maximizes firm value only if**

$$(U^{l,s} - U^{m,hg}) = (E(C^{l,s}) - mc_l) - (E(C^{m,hg}) - RK^{hg}r - mc_m)$$
$$= (1-q_l)[L(h+c_b) - Lr)(-t)] + tq_lRK^{hg}r - [(q_m - q_l)(L(h+c_g)+ -(L-RK^{hg})r)(1-t)$$
$$+ (q_l - q_m)(L(h+c_b)-(L-RK^{hg})r)] - mc_m] > 0 \qquad (10)$$

If the bank sells the loan it will not engage in additional monitoring because the banker bears some positive cost, yet all of the benefits go to the buyer of the loan. If the bank undertakes the minimum monitoring effort when it hedges, the two strategies differ only in terms of their tax implications. If the bank sells the loan then it will fully finance with debt and receive the tax shield benefits in all states. If the bank hedges, it will have a smaller tax shield of debt. Further, the bank will lose the tax shield benefits of debt in the unobservable bad credit risk state.

However, if a bank engaging in hedging undertakes maximum monitoring effort, *m*, then hedging may dominate.[9] Equation (10) provides the condition under which selling the loan dominates hedging. We compare selling the loan with hedging under maximum monitoring effort. Here, the bank incurs a non-pecuniary cost, and gets lesser benefit from the tax-shield of debt. If these costs are more than offset by the gains from the increased probability of obtaining the

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[9] If partial hedging is considered then the firm may choose to partially hedge.

higher return, hedging dominates selling the loan. Otherwise, selling the loan is the value maximizing policy for the banker.

Given that the proposition does not provide an unambiguous answer, further analysis of the model may provide interesting insight into the choice of hedging versus selling the loan. The following subsections provide comparative static results from analyzing the model. The presentation of these results may be simplified, without loss of generality, by dividing equation (10) through by loans, $L$. This requires two changes to equation (10) for terms that depend on $L$. First define the ratio of capital to loans, $k$, as
$k = RK^{hg}/L$. Second, assume a specific functional form for $mc_m$, $mc_m = fL$, where
$f$ is the non-pecuniary cost of maximum monitoring per dollar of loans. Thus, comparative statics are developed for the following equation:

$$X = (1-q_l)(h+c_b-r)(-t)+tq_l kr - [(q_l-q_m)\{h+c_b-(1-k)r\}+(q_m-q_l)\{h+c_g-(1-k)r\}(1-t)-f] \quad (11)$$

where $X = (U^{l,s} - U^{m,hg}) = (E(C^{l,s})-mc_l) - (E(C^{m,hg}) - RK^{hg} r - mc_m)$.

### 2.1 Default probabilities

Does an increase in the probability of the good state result in an increase in the value of hedging or selling the asset? If the probability of the good state is higher only if the bank engages in more monitoring, then the increase raises the value of hedging relative to selling the loan. The bank obtains the gains only if it hedges. If the probability is greater only if the bank does not engage in additional monitoring then that increases the benefit of selling the loan relative to hedging. The bank benefits from the higher probability only if it sells the loan. In Corollary 1, the probability of the good state increases whether the bank engages in additional monitoring or not.

**Corollary 1 An increase in the probability of the good state for both levels of monitoring reduces the benefit of selling the loan relative to hedging.**

$\frac{\partial X}{\partial q_m} + \frac{\partial X}{\partial q_l} = -kt < 0$ The gain in operating cash flows due to hedging and maximum monitoring effort, $q_m$, is offset by the gain in operating cash flows arising from exerting the low monitoring effort under loan sale, $q_l$. Thus, the net effect depends solely on the effect of an increase in $q_l$ on the value of the tax shield under a loan sale. Part of the value of the tax shield arises because loan sale allows the bank to reduce income by all of the losses it would otherwise incur in the bad state. An increase in $q_l$ reduces the value of this part of the tax shield. The other part of the tax shield arises because the bank requires less capital. An increase in $q_l$ results in an increase in the value of this part of the tax shield. Netting the two effects, the effect of an increase in $q_l$ is to reduce the benefit of selling the loan.

## *2.2 Cash flows*

How would cash flow changes affect relative gains from hedging and loan sales? An increase in the good state cash flows , $c_g$, increases the gains from hedging, since the probability of the good state is higher if the bank hedges (thus, engages in maximum monitoring). The effect of an increase in cash flows in the bad state is more complicated.

**Corollary 2 An increase in the cash flows in the bad state, $c_b$, increases the value of selling the loan relative to hedging if**

$$\frac{\partial X}{\partial c_b} = (q_m - q_l) - \frac{q_m rt}{(1+r)} + (1-q_l)(-t) > \mathbf{0}$$

An increase in $c_b$ has three effects. The direct effect reduces the gains arising from maximum monitoring, thereby increasing the value of loan sales relative to hedging. This is merely the reverse of the effect of an increase in $c_g$. Two indirect effects offset the direct effect. First, an increase in $c_b$ reduces capital requirements resulting in a larger debt shield of taxes for firms that hedge. Second, an increase in $c_b$ reduces the losses that would be nondeductible in the bad state if the firm hedges, thus, decreasing the value of loan sales relative to hedging.

Although the effect of an increase in $c_b$ is ambiguous across the entire parameter space, it does have an unambiguous sign in the sections of the parameter space where, otherwise, the bank would be indifferent between hedging and selling the loan. If the bank would be indifferent then the sign is unambiguously positive if $c_b - r > c_g - c_b$.. This relationship is likely to hold for loans where the lender recovers almost all of the loan if the borrower goes bankrupt.[10] The next two corollaries consider what happens first, if cash flows increase in both states and then, if a mean preserving increase in the variance of cash flows occurs.

**Corollary 3 A simultaneous increase in $c_g$ and $c_b$ decreases the value of selling the loan relative to hedging.**

$$\frac{\partial X}{\partial c_g}+\frac{\partial X}{\partial c_b}=(1-q_l)(-t)+t(q_m-q_l)-\frac{q_m rt}{(1+r)}<0$$

[10] This relationship arises for a combination of three reasons. First, the indirect effects of $c_b$ on X are functions of the tax rate, if the tax rate is sufficiently low, these indirect effects will be dominated by the direct effect of reducing the gains from additional monitoring. Second, the effect of $c_b$ on X is interesting only if X is zero and ö is positive. If X is significantly greater or less than zero then small changes in $c_b$ will not change the sign of X and, hence, will not cause the firm to change its choice of hedging versus selling the loan. The value of X can be set to zero by choosing an appropriate value of ö. However, if ö is less than zero then the firm would hedge rather than insure only if it gets paid for engaging in additional monitoring. A necessary and sufficient condition for ö to be positive is that $c_b - r > c_g - c_b$. The proof is available as corollary 2A in an Appendix that is available upon request from the authors.

The increase in pre-tax cash flows due to the additional monitoring under hedging is offset by the increase in pre-tax cash flows in the bad state that boosts the value of selling the loan. Adding together the gain and loss to additional monitoring yields only a net tax term (the middle term in the corollary) which is smaller than the reduction in nondeductible losses in the bad state for hedging (the first term), given that $q_m < 1$. Thus, an increase in the cash flows in both states increases the relative value of hedging.

The concept of a mean preserving increase in the spread is not straightforward in this model. In order to analyze a mean-preserving increase in the spread, the cash flows
in one state must be adjusted by the relative probability of the two states so that the mean is held constant. However, such weighting is complicated by the fact that the relative probabilities of the two states depend on the level of monitoring effort ($l$ or $m$) which makes the weighting dependent upon the assumed level of monitoring. Thus, Corollary 4 examines the effect of a mean preserving spread using each of the monitoring levels. The weighting on the bad state cash flows required to maintain a constant mean is $w^j$:

**Corollary 4 An increase in $c_g$ and a simultaneous mean-preserving reduction in $c_b$ increases the value of loan sales relative to hedging if**

*For $j = l$,*

$$\frac{\partial X}{\partial c_g} - w^l \frac{\partial X}{\partial c_b} = -\frac{(q_m - q_l)}{(1-q_l)} + t(q_m - q_l) - \frac{q_l}{(1-q_l)}[(1-q_l)(-t) - \frac{q_m rt}{(1+r)}] > 0$$

For $j=m$,

Corollary 4 suggests that an increase in the riskiness of the cashflow

$$\frac{dX}{dc_g} - w^m \frac{dX}{dc_b} = -\frac{(q_m - q_l)}{(1-q_m)} + t(q_m - q_l) +$$
$$-\frac{q_m}{(1-q_m)}\left[(1-q_l)(-t) - \frac{q_m rt}{(1+r)}\right] > 0$$

implied by the mean-preserving spread has an ambiguous effect on the value of loan sales relative to that of hedging, for low monitoring as well as for maximum monitoring. The first two terms give the outcome of adding together the increased value of hedging due to greater benefit of monitoring associated with higher cash flows in the good state and the reduced gains from maximum monitoring associated with higher cash flows in the bad state. The first term is unambiguously negative and the second is positive. The third is the weighted sum of the reduced losses that would be nondeductible in the bad state if the firm hedges, and the lower taxes in the good state due to holding less capital if hedging. These terms are unambiguously positive. Thus, a mean-preserving increase in the spread has an ambiguous effect on gains from selling the loan relative to hedging.

The sign in corollary 4 is ambiguous because the sign of $\partial X/\partial c_b$ is ambiguous. If $\partial X/\partial c_b$ is unambiguously positive then an increase in either definition of the mean preserving spread has an unambiguously negative effect causing the firm to hedge rather than sell the loan. From the discussion of corollary 2, a sufficient condition for an unambiguously negative sign to hold for the interesting part of the parameter space in corollary 4 is that $c_b - r > c_g - c_b$.[11]

## 2.3 *Monitoring costs, interest rates and taxes*

The model also yields some insight into the effect of higher monitoring costs, interest rates, and taxes. Higher monitoring increases the value of loan sales relative to hedging. The effects of interest rates and taxes are equally intuitive.

**Corollary 5 An increase in the interest rate, *r*, increases the value of selling the loans relative to the value of hedging.**

$$\frac{dk}{dr} = t[(1-q_l) - (q_m - q_l)(1-k)] + tq_l k + tq_l r \frac{(1-k)}{(1-r)} +$$

$$+ (q_m - q_l)t \frac{(1-k)}{(1-r)} r > 0$$

An increase in $r$ causes an increase in the value of the tax shield associated with both loan sales and hedging. However, the increase in $r$ boosts the value of the tax shield in all states under loan sale but only when the good state is realized for the unobservable risk,

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[11] Exact conditions for an increase in the mean preserving spread to reduce the value of loan sale relative to hedging may be derived for corollary 4 in a manner similar to that discussed in footnote 10 for corollary 2. The results are that a sufficient condition under the maximum monitoring definition of a mean preserving spread is $c_b - r > q_m(c_g - c_b)$. Similarly, a sufficient condition under the low monitoring definition of a mean preserving spread is $c_b - r > q_l(c_g - c_b)$. These conditions are provided in an Appendix (available upon request) as corollaries 4A and 4B.

that is, $c_g$ is realized. Moreover, an increase in $r$ also increases the capital requirement if the bank hedges, which further reduces the tax shield gains.

**Corollary 6 An increase in the tax rate, *t*, increases the value of selling the loans relative to the value of hedging.**

$$\frac{\partial X}{\partial t} == -(s^{hg} + c_b - r) - q_l(c_b - c_g) + q_m\{s^{hg} + c_g - (1-k)r\} > 0$$

An increase in the tax rate increases the value of the tax shield associated with loan sales. An increase in the tax rate also reduces the value of after tax returns in the good return state, which reduces the expected benefit of additional monitoring.

*With basis risk*

Although hedging clearly dominates doing nothing if the hedge does not have any basis risk, this result need not hold in the presence of basis risk. An analytical result may easily be obtained showing that hedging using a derivative with the correct type of basis risk reduces the value of the bank. However, an analytical solution that results in a reduction in the value of the bank may implicitly require implausible values for the basis risk. As an alternative, basis risk is analyzed in the context of a numerical example, which also allows for some discussion of the sensitivity of the results to the parameters.

The common set of parameters used in the simulation are provided at the top of Table 1. The size of the loan is set to \$100. The gross return in the worst possible state of $c_l$ and $o_l$ would be 0.5 + 0.4= .9 or 90 percent. In the best possible state, the combination of $c_h$ and $o_h$, the bank earns a *gross* return of 0.7 + 0.5=1.2 or 120 percent. The gross required rate of return, r, over the period is 110% and the tax rate on positive income is 30%. The additional cost of maximum monitoring, $mc_m$, is \$0.1 for the \$100 in loans.

The first column in Table 1 provides the probabilities used as the base case: The good credit risk state occurs with a probability of 0.99, and the good state for the observable risk occurs with a probability of 0.50. Assuming the firm does not engage in any hedging and fully monitoring, the model may be solved to obtain the required amount of equity capital, the pre-tax and after tax rates of return, and the expected return in excess of the required rate of return on capital. The required amount of capital is approximately 9.524 percent. The expected return in excess of the cost of capital is \$0.103, or 0.103 percent and the variance of this excess return is 14.727 percent.

The results from hedging without basis risk are presented in the second column. A perfect hedge of the observable risk implies the receipt of hedge payments by the bank of \$5.0 in two states with low observable returns, $o_l$, and the payment of \$5.0 by the bank in the two states with high observable returns, $o_h$. If the bank could hedge the observable risk perfectly then hedging clearly dominates a 'do nothing' hedging strategy, with an expected return of 0.881 and a variance of 2.580.

However, if the bank cannot perfectly hedge the observable risk, what basis risk would result in the 'do nothing' strategy earning a higher expected rate of return? A limitation imposed on the basis risk is that the excess returns after hedging should have a lower variance than in the base case of retaining the loan. For one set of parameter values, Column 3 of the table demonstrates the hedging strategy having a lower expected return than the unhedged returns in column 1. The derivative pays: 1) nothing in the state $c_l$ and $o_l$, 2) 5 percent of the value of the underlying loan or \$5 in the state $c_h$ and $o_l$, and requires the bank to pay 3) 10 percent of the value of the underlying loan or $10 in the state $c_l$ and $o_h$, and 4) 4.899 percent in the state $c_h$ and $o_h$. The expected value of the derivative given the assumed probabilities is $0. The after-tax excess return would fall to $0.089 or .089 percent and its variance would fall to 4.12 percent. Thus, a hedge that is subject to basis risk exists, lowering expected returns and variance of expected returns.

The exact form of the basis risk in the derivative was dictated by several aspects of the model. The derivative pays nothing in state $c_l$ and $o_l$ because this is the state that determines the minimum capital requirements. If the state $c_l$ and $o_l$ had the largest losses after hedging, then a payment of even $1 would translate into an increase of expected returns in excess of capital of approximately 0.15 percent.[12] Second, the large payment in the state $c_l$ and $o_h$ is required to transfer income to a state where the bank does not pay taxes, which reduces the tax shield of debt. Finally, the high probability of the states with $c_h$ implies that reducing the variability in returns across these states will significantly reduce the variance of overall returns.

Although the unhedged expected returns are higher than the returns to a hedging strategy, they would not necessarily have a higher expected return than selling the loans. Indeed, if the monitoring level is unchanged then selling the loan results in an excess return of $3.26 or 3.26 percent.[13] However, selling the loan would result in reduced monitoring in the model. How far must the probability of the good

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[12] The increase in the tax shield of debt due to a lower capital requirements is partially offset by a drop in the shield due to the transfer of income from a state where the bank pays taxes to one where it does not pay.

[13] The analysis for these results, not included in the Table, is available with the authors.

credit risk state, $c_h$, drop so that the 'do nothing' strategy earns a higher return than selling the loan? If $q_l$, the probability of $c_h$ with low monitoring, drops to 0.756 or less, holding the unhedged loan in portfolio earns a higher expected rate of return than selling the loan. The large drop in the required probability reflects both reduced capital requirements and the bank's obtaining the full benefit of the tax shield in all four states.

Thus, it is possible in the model to find a derivative with basis risk that reduces the variability of returns but also lowers the mean value of returns. The current standards do not fully incorporate interest rate risk and many types of credit risk hedging. Besides, the process of determining the required basis risk is based on the assumption that capital requirements are based on an accurate measure of losses in the worst state. Bank supervisors are developing revised standards that better recognize better the risk reduction benefits of hedging.[14]

**Possible empirical tests**

In principle, the implications of the model for hedging and loan sales could be tested using three types of data: (1) across different

[14] See Paper number 50 from the Basel Committee on Banking Supervision. An index of the Basel Committee's paper may be found at the URL: http://www.bis.org/publ/index.htm

loan types, (2) across different loan originators for the same type of loan, and (3) time series data for a single type of loan.[15]

***Empirical implications through time***

One implication of the model for the time series properties of loan sales is that riskier loans should make loan sales more valuable relative to hedging. Thus, a recession may be associated with an increase loan sales. Also, an increase in interest rates should increase the value of selling loans relative to hedging.

An increase in the returns to monitoring should increase the use of hedging relative to loan sales. We cannot observe directly the benefits of hedging so any test will be a joint test of two hypotheses: i) the benefits of monitoring are correlated with some observable variable, and ii) changes in the return to monitoring result in additional hedging. Testable hypotheses emerge: the benefits of monitoring may vary over the business cycle, the returns to monitoring increase over time as monitoring technology improves, and bankruptcy law changes may induce net benefit changes to monitoring.

The numerical results suggest that basis risk may be an important determinant of the hedging decision. The development of new hedging instruments that reduce basis risk should increase the proportion of loans hedged relative to the loans held in portfolio unhedged. The incentive to hedge should also increase if banks'

[15] . One limitation of extending our model to the loan sales and securitization literature is that the cost of obtaining funding in our model is solely a function of the distribution of cash flows from the loan. Thus, while our model has implications for empirical analysis of loan sales and securitization, our model omits some potentially important influences. Our model also does not include any analysis of diversification.

capital requirements more accurately reflect the risk reduction associated with hedging.

***Empirical implications across loan originators***

The model has several implications for credit derivative use across different loan originators for the same type of loan. Firstly, banks that are better monitors are more likely to hedge than sell the credit risk of a loan. This superiority in monitoring could take the form of lower costs for the same increase in probability of receiving the "high" return on the loan or a greater probability of the high state for the same cost of monitoring. Secondly, banks operating in areas that are likely to have low default rates would be more likely to sell the loan. Lastly, banks with higher tax rates are more likely to sell credit risk than to hedge credit risk using derivatives.

The results in this paper suggest that banks that are better monitors would sell and securitize fewer loans. As yet this relationship has not been tested. Neither the cost nor the quality of monitoring is directly observable. However, a proxy for the cost of monitoring is the cost efficiency of the bank, and a proxy for the quality of monitoring is the recent performance of the bank's loan portfolio.[16]

The second implication of the model is that banks operating in areas with low default rates should be more likely to sell loans. Demsetz (1999) finds that banks in states with low unemployment

---

[16] Demsetz (1999) finds that banks with higher charge-off ratios are less likely to sell commercial and industrial (C&I) loans. Although Demsetz interprets charge-offs as an observable measure of the quality of the loan screening process, that interpretation is not inconsistent with the variable being a measure of monitoring ability.

rates are more likely to sell loans. She interprets this as consistent with the comparative advantage hypothesis: selling banks tend to have access to more good lending opportunities than they can fund at low marginal cost while banks that buy loans have few lending opportunities and a surplus of low marginal cost funds. These two interpretations of Demsetz' findings are not mutually exclusive.

A third implication of the model's results for loan sales and securitization is that banks with higher tax rates should engage in more sales. The implications of the model are not supported by loan sales analysis. Corporate tax rates in the U.S. are progressive at low levels of net income but are flat at higher levels, thus, the model predicts that there should be more sales by smaller banks. An offsetting benefit of hedging for larger banks is that their portfolios may be better diversified and thus, hedging with existing derivatives may result in less basis risk with the banks' portfolios. In practice, the evidence suggests that these two effects may be offsetting. Demsetz (1999) finds that the small banks are approximately equally likely to be selling loans as larger banks.

***Empirical implications across loan types***

The model suggests that hedging with derivatives should be preferred to selling as the net returns to monitoring increase. That is, hedging becomes more desirable as the cost of additional monitoring decreases and as the probability of the good state increases with additional monitoring. When applied to the existing market for loan sales and securitization, this implies that banks should sell those loans where the gains from monitoring are low relative to the cost. Another implication of the results across loan types is that more hedging should be done where hedging derivatives for loans are available without large amounts of basis risk. These implications are generally consistent with the findings in the academic and practitioner literature.

## 5. *Conclusion*

The increasing availability of risk management tools is providing banks with the ability to select which of the risks embedded in a loan they will retain and which they will sell. This paper analyzes three options for managing risk: holding the loan in portfolio and not engaging in any hedge, hedging with a derivative, and selling the loan.

The results suggest that if the hedge is not subject to basis risk then hedging dominates the 'do nothing' strategy. Whether hedging without basis risk dominates selling the loan depends upon the specific parameters of the model. If the hedge is subject to basis risk then a 'do nothing' strategy might dominate the hedging and loan sales strategy for risk neutral banks. A theoretical implication of the model is that an increase in the tax rate and an increase in the interest rate favor selling the loan relative to hedging the loan. An empirical implication is that better hedging contracts that have less basis risk should lead banks to hedge more of the loans that they hold in portfolio.

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**Table 1**
**Numerical Example of the "Sell, Hedge or Do Nothing" alternatives**

Assumed common parameters

$L$=100 $c_h = 0.7$ $c_l = 0.5$ $o_h = 0.5$ $o_l = 0.$ $r = 1.1$ $t = 0.3$ $mc_m$=0.1

| | | Alternative risk management strategies | | | |
|---|---|---|---|---|---|
| | | No hedging (full monitoring) | Hedged (no basis risk) | Hedged (basis risk) | Sell loan (reduced monitoring) |
| **Assumed parameters by strategies** | | | | | |
| Probabilities of various states | $q_h$ (Prob $c_h$) | 0.99 | 0.99 | 0.99 | 0.756 |
| | $p$ (Prob $o_h$) | 0.50 | 0.50 | 0.50 | 0.50 |
| Hedge payments in each of the four states | $c_l$ and $o_l$ | 0.0 | 5.0 | 0.0 | Loan sold, no hedge payments |
| | $c_l$ and $o_h$ | 0.0 | -5.0 | -10.0 | |
| | $c_h$ and $o_l$ | 0.0 | 5.0 | 5.0 | |
| | $c_h$ and $o_h$ | 0.0 | -5.0 | -4.899 | |
| **Results calculated from the model** | | | | | |
| Capital structure | Capital | 9.524 | 7.143 | 9.524 | 0 |
| | Deposits | 90.476 | 92.857 | 90.476 | 100 |
| Excess return after taxes, cost of equity, and monitoring cost | Mean | 0.103 | 0.881 | 0.089 | 0.084 |
| | Variance | 14.727 | 2.580 | 4.118 | 0.0 |

# Appendix

**Result 1 [from footnote # 7]: Given a predetermined monitoring policy, *j*, a policy of hedging increases firm value relative to a policy of not hedging by,**

$$(U^{j,hg} - U^{j,no}) = [E(C^{j,hg}) - RK^{hg} r - mc_j] - [E(C^{j,no}) - RK^{no} r - mc_j]$$
$$= (1-p)q_j[L(1+o_b+c_g) - (L - RK^{hg})r](-t) + pq_j(RK^{no} - RK^{hg})rt$$

**Proof of Result 1**

The result follows immediately from using equations (3), (4), (5), and (6) for the capital requirements (in the no hedging and hedging cases respectively) and the expected cashflows if the banker engages in monitoring level *j* (in the no hedging and hedging cases respectively). Thus,

$$
\begin{aligned}
(U^{j,hg} - U^{j,no}) &= [E(C^{j,hg}) - RK^{hg}r - mc_j] - [E(C^{j,no}) - RK^{no}r - mc_j] \\
&= q_j(1-t)\{[L(h+c_g) - (L - RK^{hg})r] - p[L(o_g + c_g) - (L - RK^{no})r]\} \\
&+ (1-q_j)\{[L(h+c_b) - (L - RK^{hg})r] - p[L(o_g + c_b) - (L - RK^{no})r]\} \\
&- (1-p)q_j\{[L(o_b + c_g) - (L - RK^{no})r] - [L(o_b + c_b) - (L - RK^{no})r]\} \\
&- (1-p)[L(o_b + c_b) - (L - RK^{no})r] + (RK^{no} - RK^{hg})r
\end{aligned}
$$

$$
\begin{aligned}
&= q_j(-t)\{[L(h+c_g) - (L - RK^{hg})r] - p[L(o_g + c_g) - (L - RK^{no})r]\} \\
&+ \{[L(h+c_b) - (L - RK^{hg})r] - p[L(o_g + c_b) - (L - RK^{no})r]\} \\
&+ [L(o_b + c_b) - (L - RK^{no})r]\} + (RK^{no} - RK^{hg})r
\end{aligned}
$$

$$
\begin{aligned}
&= q_j(-t)(1-p)[L(c_g - r)] \\
&+ q_j(-t)\{Lpo_g + L(1-p)o_b + RK^{hg}r - pLo_g - pRK^{no}r\} \\
&+ [L(o_b + c_b) - (L - RK^{no})r] + (RK^{no} - RK^{hg})r \\
&+ \{[L\{po_g + (1-p)o_b\} + Lc_b - (L - RK^{hg})r] - p[L(o_g + c_b) - (L - RK^{no})r]\}
\end{aligned}
$$

$$
= (1-p)q_j[L(1+o_b + c_g) - (L - RK^{hg})r](-t) + pq_j(RK^{no} - RK^{hg})rt
$$

The first term after the equality sign is the tax gain from hedging in the presence of convex income taxes (The term in square brackets is negative under the assumption that the bank has negative income in the states with the low return on the observable risk). The second term is the tax gain associated with reduced capital requirements for the bank using the credit derivative in the state of a high return on both the observable and the credit risk.

**Proof of Proposition**

To show this result, first substitute for $E(C^{m,hg})$.

So, $$E(C^{m,hg}) = E(C^{l,hg}) + [E(C^{m,hg}) - E(C^{l,hg})].$$

Then, $$(E(C^{l,s}) - mc_l) - (E(C^{m,hg}) - RK^{hg}r - mc_m)$$

$$= (E(C^{l,s}) - mc_l) - [E(C^{l,hg}) + \{E(C^{m,hg}) - E(C^{l,hg})\} - RK^{hg}r - mc_m]$$

Now, given a predetermined monitoring policy, *j*, a policy of insurance increases the

value of the firm to the banker relative to a policy of hedging by:

$$[E(C^{j,s}) - mc_j] - [E(C^{j,hg}) - RK^{hg}r - mc_j] = (1-q_j)[L(h+c_b) - Lr](-t) + tq_j RK^{hg}r$$

Since $mc_l = 0$, by assumption, using the above equation, we can write this as,

$$= (1-q_l)[(L(s^{hg} + c_b) - Lr)(-t)] + tq_l RK^{hg}r$$

$$-[E(C^{m,hg}) - E(C^{l,hg}) - mc_m]$$

$$= \quad (1-q_l)[L(h+c_b)-Lr)(-t)]+tq_l RK^{hg} r$$

$$-[\{q_m[L(h+c_g)-(L-RK^{hg})r](1-t)$$

$$+(1-q_m)[L(h+c_b)-(L-RK^{hg})r]\}$$

$$-\{q_l[L(h+c_g)-(L-RK^{hg})r](1-t)$$

$$+(1-q_l)[L(s^{hg}+c_b)-(L-RK^{hg})r]\}-mc_m]$$

$$= (1-q_l)[(L(h+c_b)-Lr)(-t)]+tq_l RK^{hg} r$$

$$-[(q_m-q_l)(L(h+c_g)-(L-RK^{hg})r)(1-t)$$
$$+(q_l-q_m)(L(h+c_b)-(L-RK^{hg})r)-mc_m]>0$$

**Corollaries**

The text defines $k=\dfrac{RK^{hg}}{L}$. From equation (4),

Therefore, $RK^{hg} = -L(h+c_b-r)/(1+r)$

$k = RK^{hg} / L = -(h + c_b - r)/(1 + r)$

**Corollary 1** **An increase in the probability of the good state both if the bank undertakes the maximum amount of monitoring and if it undertakes the minimum amount of monitoring reduces the benefit of insuring relative to hedging.**

**Proof:** To prove this corollary we need to prove two lemmas.

**Lemma 1: An increase in the probability of the good state if the bank undertakes the maximum amount of monitoring effort causes an increase in the benefit of insuring relative to hedging.**

**Proof:** Taking a partial derivative of *X* with respect to $q_m$, we have

$$\frac{\partial X}{\partial q_m} = -[-\{h + c_b - (1-k)r\} + \{h + c_g - (1-k)r\}(1-t)] < 0$$

since the first term in {} is negative given our assumption on page 8 on taxes while the second term in {} is positive. Thus, the overall result is that the partial derivative is negative.

**Lemma 2 An increase in the probability of the good state if the bank undertakes the low amount of monitoring effort causes an increase in the benefit of insuring relative to partially hedging.**

**Proof:** Taking a partial derivative of *X* with respect to $q_l$, we have

$$\frac{\partial X}{\partial q_l} = (h + c_b - r)t + tkr - \{h + c_b - (1-k)r\} + \{h + c_g - (1-k)r\}(1-t)$$

$$= (h + c_b - r)t\{1 - \frac{r}{(1+r)}\} - \{h + c_b - (1-k)r\} + \{h + c_g - (1-k)r\}(1-t)$$

$$= (h + c_b - r)[\frac{t}{1+r} - 1] + \{h + c_g - r\}(1-t) - krt$$

$$= (h + c_b - r)[\frac{t}{1+r} - 1 + \frac{tr}{(1+r)}] + \{h + c_g - r\}(1-t)$$

$$= (h + c_b - r)[\frac{t(1+r)}{1+r} - 1] + \{h + c_g - r\}(1-t)$$

$$= (h + c_b - r)(t-1) + \{h + c_g - r\}(1-t)$$

The simplification is achieved by substituting $k = -(h + c_b - r)/(1+r)$
in the third

step above. From the last step above it is easy to see that $\frac{\partial X}{\partial q_l} > 0$ .

From Lemmas 1 and 2 above, we can get the sum of the partial derivatives of
*X* with respect to $q_m$ and $q_l$ as,

$$\frac{\partial X}{\partial q_m}+\frac{\partial X}{\partial q_l}=-[-\{h+c_b-(1-k)r\}+\{h+c_g-(1-k)r\}(1-t)]$$
$$+(h+c_b-r)(t-1)+\{h+c_g-r\}(1-t)$$
$$=krt+(h+c_b-r)t=krt-k(1+r)t=-kt<0$$

This implies that an increase in $q_m$ and $q_l$ reduces the benefit of insuring.

**Corollary 2** **In the bad state, an increase in the cash flows, $c_b$, increases the value of insuring relative to hedging if**

$$(1-q_l)(-t)+(q_m-q_l)-\frac{q_m rt}{(1+r)}>0$$

**Proof:** Taking the partial derivative of $X$ with respect to $c_b$ we have,

$$\frac{\partial X}{\partial c_b}=(1-q_l)(-t)+tq_l r\frac{\partial k}{\partial c_b}-[(q_l-q_m)(1+\frac{\partial k}{\partial c_b}r)+(q_m-q_l)\frac{\partial k}{\partial c_b}r(1-t)]$$

Simplifying,

$$\frac{\partial X}{\partial c_b}=(1-q_l)(-t)+tq_l r\frac{\partial k}{\partial c_b}-[(q_l-q_m)+\{(q_l-q_m)\frac{\partial k}{\partial c_b}r(1-(1-t))\}]$$

which can be written as,

$$\frac{\partial X}{\partial c_b}=(1-q_l)(-t)+tq_l r\frac{\partial k}{\partial c_b}-[(q_l-q_m)(1+\frac{\partial k}{\partial c_b}rt)]$$

Substituting $\frac{\partial k}{\partial c_b}=-\frac{1}{(1+r)}$, we have

$$\frac{\partial X}{\partial c_b} = (1-q_l)(-t) + tq_l r\{-\frac{1}{(1+r)}\} + [(q_m - q_l)(1+\{-\frac{rt}{(1+r)}\})]$$

which can be written as,

$$\frac{\partial X}{\partial c_b} = -(1-q_l)(t) - \frac{tq_l r}{(1+r)} + [(q_m - q_l)(1 - \frac{rt}{(1+r)})]$$

$$\frac{\partial X}{\partial c_b} = -t + tq_l + (q_m - q_l) - \frac{q_m rt}{(1+r)}$$

Under the assumption stated in the proposition, it follows that

$$\frac{\partial X}{\partial c_b} > 0$$

Whether this condition holds or not, depends on the values of $q_l$, $q_m$, $t$ *and* $r$.

Comparative static results from analyzing Proposition 1 are facilitated by noting that

i) the condition given by equation (10) can be written by observing equation (11) as
X > 0.

ii) X > 0 iff

$$f=[(q(m)-q(l))(c(g)-(1-\{-\frac{(c(b)-r)}{(1+r)}\})r)(1-t)+(q(l)-q(m))(c(b)-(1-\{-\frac{(c(b)-r)}{(1+r)}\})r)]$$
$$-\{(1-q(l))[(c(b)-r)(-t)]+tq(l)[-\frac{(c(b)-r)}{(1+r)}]r\}$$

satisfies the condition:

$$0<f<(q(m)-q(l))(c(g)-c(b)).$$

**Corollary 2A: The incremental monitoring cost incurred equals the incremental expected cashflow benefit, for feasible values of the tax rate, t,** iff

$$0<f<(q(m)-q(l))(c(g)-c(b))\Leftrightarrow 0<t<\frac{(q_m-q_l)}{(q_m-q_l)+(1-\frac{q_m}{(1+r)})(\frac{c_b-r}{c_g-c_b})}$$

**Proof:**

A practical question illustrating this corollary is: "For what values of c(g), c(b), q(m), q(l), r, and t such that parameter values are in the following ranges (as observed in real-world data):

| | |
|---|---|
| A1 | $c(g) = 1$, $0 < c(b) < 1$; |
| B1 | $0 < q(l) < q(m) < 1$; |
| C1 | $0 < r \leq 1.2$ ; |
| D1 | $0 < t \leq 0.5$ ; |

does

$$\varphi = [(q(m) - q(l))(c(g) - (1 - \{-\frac{(c(b) - r)}{(1 + r)}\})r)(1 - t) + (q(l) - q(m))(c(b) - (1 - \{-\frac{(c(b) - r)}{(1 + r)}\})r)]$$
$$- \{(1 - q(l))[(c(b) - r)(-t)] + tq(l)[-\frac{(c(b) - r)}{(1 + r)}]r\}$$

satisfy the condition:

$0 < \varphi < (q(m) - q(l))(c(g) - c(b))$. ?"

First note that $\varphi$ is a linear function of t.

Then, the y-intercept (i.e., the value of $\varphi$ when t = 0) is given by

$(q(m) - q(l))(c(g) - c(b))$.

Next note that $\varphi$ is a decreasing function of t.

The x-intercept (i.e., the value of t when $\varphi$ = 0) is given by

$$t_0 = \frac{1}{1 + \frac{(c_b - r)(1 - \frac{q_m}{(1 + r)})}{(q_m - q_l)(c_g - c_b)}}$$

Feasible solutions to the problem requires that the x-intercept be positive, since tax rates are positive. This is so if c(b) > r. It is so for c(b) < r, iff

$$\left|\frac{(c_b - r)(1-\frac{q_m}{(1+r)})}{(q_m - q_l)(c_g - c_b)}\right| < 1$$

***Thus,***

$$0 < f < (q(m) - q(l))(c(g) - c(b)) \Leftrightarrow 0 < t < \frac{(q_m - q_l)}{(q_m - q_l) + (1-\frac{q_m}{(1+r)})(\frac{c_b - r}{c_g - c_b})}$$

**Corollary 3** **A simultaneous increase in $c_g$ and $c_b$ decreases the value of insuring relative to partial hedging.**

**Proof:** Taking the sum of the partial derivatives of $X$ with respect to $c_g$ and $c_b$, we have,

$$\frac{\partial X}{\partial c_g} = -(q_m - q_l)(1-t) < 0 \text{ and}$$

$$\frac{\partial X}{\partial c_b} = (1-q_l)(-t) + (q_m - q_l) - \frac{q_m rt}{(1+r)} > 0 \text{ .}$$

Therefore,

$$\frac{\partial X}{\partial c_g}+\frac{\partial X}{\partial c_b}=-(q_m-q_l)(1-t)+(q_m-q_l)+(1-q_l)(-t)-\frac{q_m rt}{(1+r)}$$

$$=t(q_m-q_l)+(1-q_l)(-t)-\frac{q_m rt}{(1+r)}$$

$$=(1-q_l)(-t)+t(q_m-q_l)-\frac{q_m rt}{(1+r)}<0$$

which implies that a simultaneous increase in the rates of return in good and bad states results in a decrease in the value of insurance relative to the value of hedging.

**Corollary 4** **An increase in $c_g$ and a simultaneous mean-preserving reduction in $c_b$ increases the value of insuring relative to hedging** if

*For j = l,*

$$\frac{\partial X}{\partial c_g}-w^l\frac{\partial X}{\partial c_b}=-\frac{(q_m-q_l)}{(1-q_l)}+t(q_m-q_l)-\frac{q_l}{(1-q_l)}[(1-q_l)(-t)-\frac{q_m rt}{(1+r)}]$$

For $j=m$,

$$\frac{\partial X}{\partial c_g}-w^m\frac{\partial X}{\partial c_b}=-\frac{(q_m-q_l)}{(1-q_m)}+t(q_m-q_l)-\frac{q_m}{(1-q_m)}[(1-q_l)(-t)-\frac{q_m rt}{(1+r)}]$$

**are each greater than zero.**

**Proof:** If the value of $c_g$ is increased by $\Delta$ then in order to hold the mean of the distribution constant, the value of $c_b$ must decrease by $w\Delta$ where $w$ adjusts for the differences in probability of the good and bad states. That is:

$$q_j c_g + (1-q_j)c_b = q_j(c_g + \Delta) + (1-q_j)(c_b - w^j\Delta)$$

Then, it follows that,

$$q_j\Delta - w^j\Delta(1-q_j) = 0$$

This implies that,

$$w^j = \frac{q_j}{(1-q_j)},\ j = l, m$$

Given

$$\frac{\partial X}{\partial c_g} = -(q_m - q_l)(1-t) < 0 \text{ and}$$

$$\frac{\partial X}{\partial c_b} = (1-q_l)(-t) + (q_m - q_l) - \frac{q_m rt}{(1+r)} > 0\ ,$$

we have, for j = l,

$$\frac{\partial X}{\partial c_g} - w^l\frac{\partial X}{\partial c_b} = -(q_m - q_l)(1-t) - \frac{q_l}{(1-q_l)}[(1-q_l)(-t) + (q_m - q_l) - \frac{q_m rt}{(1+r)}]$$

$$= -(q_m - q_l) + t(q_m - q_l) - \frac{q_l}{(1-q_l)}(q_m - q_l) - \frac{q_l}{(1-q_l)}[(1-q_l)(-t) - \frac{q_m rt}{(1+r)}]$$

$$= (q_m - q_l)[-1 - \frac{q_l}{(1-q_l)}] + t(q_m - q_l) - \frac{q_l}{(1-q_l)}[(1-q_l)(-t) - \frac{q_m rt}{(1+r)}]$$

$$= -\frac{(q_m - q_l)}{(1-q_l)} + t(q_m - q_l) - \frac{q_l}{(1-q_l)}[(1-q_l)(-t) - \frac{q_m rt}{(1+r)}]$$

S

similarly, for j = m, we have,

$$\frac{\partial X}{\partial c_g} - w^m \frac{\partial X}{\partial c_b}$$

$$-(q_m - q_l)(1-t) - \frac{q_m}{1-q_m}\left\{(1-q)(-t) + (q_m - q) - \frac{q_m rt}{1+r}\right\}$$

$$= -(q_m - q_l)(1-t) - \frac{q_m}{(1-q_m)}(q_m - q_l) - \frac{q_m}{(1-q_m)}\{(1-q_l)(-t) - \frac{q_m rt}{(1+r)}\}$$

$$= (q_m - q_l)[t - 1 - \frac{q_m}{(1-q_m)}] - \frac{q_m}{(1-q_m)}[(1-q_l)(-t) - \frac{q_m rt}{(1+r)}]$$

$$= -\frac{(q_m - q_l)}{(1-q_m)} + t(q_m - q_l) - \frac{q_m}{(1-q_m)}[(1-q_l)(-t) - \frac{q_m rt}{(1+r)}]$$

**Corollary 4A: If the incremental monitoring cost incurred equals the incremental expected cashflow benefit, for feasible values of the tax rate, *t*, and**

$$\frac{(c_b - r)}{(c_g - c_b)} > 1,$$

i. $\frac{\partial X}{\partial c_b} > 0 \Rightarrow [\frac{\partial X}{\partial c_g} - w^j \frac{\partial X}{\partial c_b}] < 0,\ j = l, m$

**Proof:** The statement of the corollary can be interpreted as:

"*Given* $0 < f < (q(m) - q(l))(c(g) - c(b))$**,** ***and*** $\frac{(c_b - r)}{(c_g - c_b)} > 1$

*show that* $\frac{\partial X}{\partial c_b} > 0 \Rightarrow [\frac{\partial X}{\partial c_g} - w^j \frac{\partial X}{\partial c_b}] < 0,\ j = l, m$ **"**

*From Corollary 2A,*

$$0 < f < (q(m) - q(l))(c(g) - c(b)) \Leftrightarrow 0 < t < \frac{(q_m - q_l)}{(q_m - q_l) + (1 - \frac{q_m}{(1+r)})(\frac{c_b - r}{c_g - c_b})}$$

***Since*** $\frac{(c_b - r)}{(c_g - c_b)} > 1$ **,** $0 < t < \frac{(q_m - q_l)}{(q_m - q_l) + (1 - \frac{q_m}{(1+r)})} = \frac{1}{[1 + \frac{(1 - \frac{q_m}{(1+r)})}{(q_m - q_l)}]}$

Now

$$\frac{\partial X}{\partial c_b} = (1 - q_l)(-t) + (q_m - q_l) - \frac{q_m rt}{(1+r)} = (q_m - q_l)[(1 - t) - t\frac{1 - \frac{q_m}{(1+r)}}{(q_m - q_l)}]$$

Since $t < \frac{1}{[1 + \frac{(1 - \frac{q_m}{(1+r)})}{(q_m - q_l)}]}$,

$$\frac{\partial X}{\partial c_b} > 0 \cdot$$

Now

$$\frac{\partial X}{\partial c_g} - w^j \frac{\partial X}{\partial c_b}, \; j = l, m$$

$$= -\frac{(q_m - q_l)}{(1 - q_l)} + t(q_m - q_l) - \frac{q_j}{(1 - q_j)}[(1 - q_l)(-t) - \frac{q_m rt}{(1 + r)}]$$

Therefore, we can write on simplification,

$$\frac{\partial X}{\partial c_g} - w^l \frac{\partial X}{\partial c_b} = -(q_m - q_l)[(1 - t) + \frac{q_l}{(1 - q_l)}(1 + \frac{(1 - q_l)(-t) - \frac{q_m rt}{(1 + r)}}{(q_m - q_l)})] < 0$$

$$\frac{\partial X}{\partial c_g} - w^m \frac{\partial X}{\partial c_b} = -(q_m - q_l)[(1-t) + \frac{q_m}{(1-q_m)}(1 + \frac{(1-q_l)(-t) - \frac{q_m rt}{(1+r)}}{(q_m - q_l)})] < 0$$

**Corollary 4B:Given values of parameters that validate Corollary 2A, the subset of these values that satisfy the following condition sets are:**

**q(m) <** $\frac{(c_b - r)}{(c_g - c_b)} < 1$**,** **q(l) <** $\frac{(c_b - r)}{(c_g - c_b)} < q(m)$**,** ***0*** **<** $\frac{(c_b - r)}{(c_g - c_b)} < q(l)$**,**

**respectively.**

*Condition Set A***:**

$$\frac{\partial X}{\partial c_b} < 0,$$

$$[\frac{\partial X}{\partial c_g} - w^j \frac{\partial X}{\partial c_b}] < 0, \; j = l, m$$

*Condition Set B:*

$$\frac{\partial X}{\partial c_b} < 0,$$
$$[\frac{\partial X}{\partial c_g} - w^j \frac{\partial X}{\partial c_b}] < 0, j = l$$
$$[\frac{\partial X}{\partial c_g} - w^j \frac{\partial X}{\partial c_b}] > 0, \ j = m$$

*Condition Set C:*

$$\frac{\partial X}{\partial c_b} < 0,$$
$$[\frac{\partial X}{\partial c_g} - w^j \frac{\partial X}{\partial c_b}] > 0, j = l, m$$

**Proof:** The incremental monitoring cost incurred equals the incremental expected cashflow benefit, for feasible values of the tax rate, t, iff

$$0 < f < (q(m) - q(l))(c(g) - c(b)) \Leftrightarrow 0 < t < \frac{1}{1 + \frac{(1 - \frac{q_m}{(1+r)})}{(q_m - q_l)}[\frac{c_b - r}{c_g - c_b}]}$$

Lemmas A, B, and C below are needed for this proof.

**Lemma A**

$$\frac{\partial X}{\partial c_b} > 0 \text{ iff } t < \frac{1}{1 + \frac{(1 - \frac{q_m}{(1+r)})}{(q_m - q_l)}}$$

**Proof:** Note that $\frac{\partial X}{\partial c_b} = (1-t) - t\frac{1 - \frac{q_m}{(1+r)}}{(q_m - q_l)}$.

It follows that

$$\frac{\partial X}{\partial c_b} > 0 \Leftrightarrow t < \frac{1}{1+\frac{(1-\frac{q_m}{(1+r)})}{(q_m - q_l)}}.$$

**Lemma B**

$$\frac{\partial X}{\partial c_g} - w^m \frac{\partial X}{\partial c_b} > 0 \ \text{ iff } \ t > \frac{1}{1+\frac{(1-\frac{q_m}{(1+r)})}{(q_m - q_l)} q_m}$$

**Proof:** Note that

$$\frac{dX}{dc_g} - w^m \frac{dX}{dc_b} = (1-t) - tq_m \frac{1-\frac{q_m}{(1+r)}}{(q_m - q_l)}$$

It follows that

$$\frac{dX}{dc_g} - w^m \frac{dX}{dc_b} > 0 \Leftrightarrow t > \frac{1}{1 + \frac{\left(1 - \frac{q_m}{(1+r)}\right)}{q_m - q_l} q_m}$$

**Lemma C**

$$\frac{\partial X}{\partial c_g} - w^l \frac{\partial X}{\partial c_b} > 0 \text{ iff } t < \frac{1}{1 + \frac{(1 - \frac{q_m}{(1+r)})}{(q_m - q_l)} q_l}$$

Proof: Note that:

$$\frac{dX}{dc_g} - w^l \frac{dX}{dc_b} = (1-t) - tq_l \frac{1 - \frac{q_m}{(1+r)}}{(q_m - q_l)}$$

It follows that

$$\frac{\partial X}{\partial c_g} - w^m \frac{\partial X}{\partial c_b} > 0 \Leftrightarrow t > \frac{1}{1 + \frac{(1 - \frac{q_m}{(1+r)})}{(q_m - q_l)} q_l}.$$

Given the condition on parameters that satisfy Corollary 2A, and Lemmas A, B, and C,

Condition set A is satisfied iff $q(m) < \frac{(c_b - r)}{(c_g - c_b)} < 1$,

Condition set B is satisfied iff $q(l) < \frac{(c_b - r)}{(c_g - c_b)} < q(m)$, and

Condition set C is satisfied iff $0 < \frac{(c_b - r)}{(c_g - c_b)} < q(l)$,

A set of parameter values that satisfy these condition sets are provided in Table 1.

Overall Condtition sets A, B, and C together provide the case when

$0 < f < (q(m) - q(l))(c(g) - c(b))$, and $0 \leq \frac{(c_b - r)}{(c_g - c_b)} \leq 1.$

**Corollary 5** **An increase in the interest rate increases he value of insuring the loans relative to the value of partially hedging.**

**Proof:** Given the definition of *k*, we have,

$$\frac{\partial k}{\partial r} = \frac{1}{1+r} + \frac{-(h+c_b-r)(-1)}{(1+r)^2} = \frac{(1-k)}{(1+r)}$$

So, $(h+c_b-r)<0 \Rightarrow \frac{\partial k}{\partial r} > 0$ .

Now, taking the partial derivative of *X* with respect to *r*, we get,

$$\frac{\partial X}{\partial r} = -(1-q_l)(-t) + tq_l k + tq_l r\frac{\partial k}{\partial r}$$
$$-[-(q_l-q_m)(1-k) + (q_l-q_m)\frac{\partial k}{\partial r}r - (q_m-q_l)(1-k)(1-t) +$$
$$+(q_m-q_l)\frac{\partial k}{\partial r}r(1-t)]$$

Therefore,

$$\frac{\partial X}{\partial r} = -(1-q_l)(-t) + tq_l k + tq_l r\frac{\partial k}{\partial r} - [(q_m-q_l)t\{(1-k) - \frac{\partial k}{\partial r}r\}]$$

This equation can be rewritten as,

$$\frac{\partial X}{\partial r} = (1-q_l)t + tq_l k + tq_l r\frac{\partial k}{\partial r} - (q_m - q_l)t(1-k) + (q_m - q_l)t\frac{\partial k}{\partial r}r$$

,

which can be further simplified as,

$$\frac{\partial X}{\partial r} = t[(1-q_l) - (q_m - q_l)(1-k)] + tq_l k + tq_l r\frac{\partial k}{\partial r} + (q_m - q_l)t\frac{\partial k}{\partial r}r$$

On the right hand side of the equation above, everything is positive except the square-bracketed section. As long as $q_m < 1$, and $0 < k < 1$, this is positive too. Thus, $q_m < 1$, and $0 < k < 1$, is a sufficient condition for ,

$$\frac{\partial X}{\partial r} > 0$$

i.e., an increase in *r* will lead to more insurance relative to hedging.

**Corollary 6** **An increase in the tax rate, *t*, increases the value of insuring the loans relative to the value of partial hedging.**

**Proof:** Taking the partial derivative of *X* with respect to *t*, we have,

$$\frac{\partial X}{\partial t} = -(1-q_l)(h+c_b-r)+q_l kr+[(q_m-q_l)\{h+c_g-(1-k)r\}]$$
$$= -(h+c_b-r)+q_l(h+c_b-r)+q_l kr+q_m\{h+c_g-(1-k)r\}-q_l\{h+c_g-(1-k)r\}$$
$$= -(h+c_b-r)-q_l(c_b-c_g)+q_m\{h+c_g-(1-k)r\}$$

From the last step it follows that $\frac{\partial X}{\partial t} > 0$.

i.e., an increase in taxes results in an increase in the value of insurance.