On the Effects of Regulating Price Discrimination by a Price Capped Firm[§]

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1. Introduction

This paper analyses how the regulation of price discrimination by a price capped firm can affect its pricing decision and the entry decision by potential competitors, thereby influencing social welfare. Given the existence of a standard tariff basket price cap, we focus on the effects on entry and welfare of two regulatory schemes that embed a different additional constraint meant to reduce potential anti-competitive effects of price cap regulation.

In order to elucidate the issue tackled in this paper, consider the following stylised description of a rather common condition in regulated markets. A standard price cap is imposed on a monopolistic firm that operates in two markets. The price cap places an upper limit to the weighted average of the prices set by the regulated firm. Suppose now that in market 1 the regulated firm operates as the unique supplier, while in market 2 entry could be profitable for a potential rival firm. The regulated firm, even in complying with the regulatory rule, may exploit its freedom to vary prices within the regulated basket with possible anti-competitive consequences. Indeed, by allowing price discrimination, price cap can lead the incumbent monopolist to

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price aggressively in market 2 while rising the price in market 1 where it can exploit its monopoly power (Armstrong and Vickers, 1993).

A solution worked out in the practice of regulation to mitigate this problem is to impose additional caps over the prices in the captive markets in order to limit the regulated firm's ability to obtain extra revenues (Oftel, 1995). We define as *Absolute* the regulatory regime that adhere to this approach and is given by the combination of a standard price cap and an additional constraint on the absolute level of price in the monopolistic market. We hypothesise that the regulator can use an alternative approach to restrict the incumbent's freedom to price discriminate. This alternative approach entails, along with the traditional price cap constraint, a constraint on relative prices. Henceforth in this paper, we refer to this regime as the *Relative* regime.

This paper aims at comparing the alternative regimes - *Relative* and *Absolute* - in terms of fostering competition and maximising social welfare. In particular, we want to analyse whether *Relative* regulation facilitates entry with respect to the case of *Absolute* regime and which regime delivers higher social welfare.

The role of public policy towards price discrimination has been already analysed in economic literature that has focused mainly on welfare consequences. It is well known that such welfare consequences are of ambiguous sign (Phlips, 1983; Tirole, 1988; Varian, 1989). In general, a negative implication of third degree price discrimination by an unregulated monopolist is that it causes marginal rates of substitution to differ among consumers. As to the overall change in welfare, price discrimination increases welfare only if it increases total output. As a matter of fact, total output is unlikely to remain unchanged unless one focuses on the case of linear demand curves. In this last case, the result is more clear-cut, since welfare is lower under price discrimination (Schmalensee, 1981; Varian, 1985). The first attempt to investigate the welfare effects of price discrimination by a monopolist subject to different average price constraints is due to Armstrong and Vickers (1991) who however do not consider the possible implications upon competition. To our knowledge, this issue is first tackled in Armstrong and Vickers (1993). In their set-up, a competitive threat is hypothesised in one market, while the other one is captive. By endogenising the choice on entry, Armstrong and Vickers show that public policy towards price discrimination does not make any difference to the entry decision if the sunk cost of entry is low enough or high enough. On the other hand, for intermediate values of the entry costs, entry will be feasible if and only if price discrimination is banned. Even in this analysis, the welfare consequences are of ambiguous sign. A theoretical argument in favour of the use of relative price regulation is set forth in Ireland (1992) who proposes it in those situations where asymmetric information prevents regulation upon the absolute price level. In a more recent paper, Armstrong and Vickers (2000) analyse the general issue of whether a regulated monopolist should have some degree of discretion over its pricing policy. While recognising different possible motivations for setting limits to the ability to price discriminate by a regulated firm, they study the effects of these limits only with respect to the objective of allocative efficiency in a monopolistic context. They show that the answer to this question (and to the question of the optimal degree of pricing discretion) crucially depends on the nature of regulator's uncertainty.[1]

Our analysis partly follows the reasoning set forth in Armstrong and Vickers (1993), but differs in that it examines how competition and welfare are affected by the adoption of the different policies that limit the ability to price discriminate by a price capped firm rather than by the adoption of different forms of price caps. Also, our analysis is complementary to that in Armstrong and Vickers (2000) in that we analyse the effects that

[1] In accordance with the above cited literature, we do not explicitly address here dynamic aspects of price regulation which are analysed, for instance, in Iozzi (2001).

these constrains on price discrimination may have on the development of competition and social welfare rather than on allocative efficiency in a monopolistic market.

In this paper we focus on a simple set-up where a monopolist serves two markets with identical linear demand functions, differing only for the viability of competition. In the potentially competitive market, a price-taker firm may enter at different scales of entry. A regulatory authority, whose objective is the maximisation of social welfare defined as aggregate consumer surplus, may choose between the two regulatory regimes. At a first stage, we analyse the solutions of the game in the case of *exogenous* scale of entry. In other words, the new entrant's choice is, in this case, simply whether or not to enter. Under this assumption, we also explore the effects of the two regulatory regimes upon social welfare. In the last part of the paper, we allow the potential entrant to optimally choose its scale of entry; we then investigate the effects on welfare of this optimal entry decision under the two different regulatory regimes.

The main results of the paper are as follows. Under the hypothesis that entry may occur only at a fixed scale and provided that this scale is sufficiently large, equilibrium prices in the potentially competitive market are weakly higher under the *Relative* regime than under the *Absolute* one. Hence, the *Relative* regime is more able to foster competition than the *Absolute* regime. Given that this higher likelihood of entry comes at a cost of higher prices in the competitive market and at the advantage of (weakly) lower prices in the captive market, we also assess the overall effect of the two regimes in terms of social welfare, which we take to be given only by aggregate consumers' surplus. We find that social welfare is higher under the *Relative* regime whenever entry occurs only under this regime (that is, when entry would not occur under the other regime). However, when entry occurs under both regimes, consumer surplus is found to be higher under the *Absolute* regime.

This welfare ranking across the two regimes in case of entry is overturned when we allow the potential entrant to optimally choose its level of scale. Assuming a linear cost of entry, we show that entry occurs always at a (weakly) larger scale under the *Relative* regime than under the *Absolute* one. This larger scale of entry positively affects prices and consumers surplus making consumers better off under the *Relative* regime. Hence, not only is the *Relative* regime able to guarantee entry in cases where this would not occur under the other regulatory regime, but, when the scale of entry is endogenously determined by a rival firm, it grants entry at a larger scale and, by way of the more competitive environment, it is able to raise social welfare.

The paper is organised as follows. Section 2 sketches the basic model and describes the features of the two regulatory regimes under analysis. In section 3 the implications for prices and entry stemming from the *Absolute* and *Relative* schemes are studied under the hypothesis of an exogenously given scale of entry. In the same section we investigate the welfare consequences of adopting the two schemes. Finally, section 4 extends the analysis to the case of an endogenously determined scale of entry and carries out a welfare comparison of the two regulatory regimes. All the proofs are relegated to the Appendix

2. *The model*

We employ a very simple two-market model based on Armstrong and Vickers (1993). Price in each market is denoted by p_i (i = 1, 2). Demand is given by $x(p_i)$ and is assumed to be independent and symmetric across markets, that is demand in each market does not depend on the price set in the other market and both markets have the same demand function. While independence is assumed for the sake of simplicity, we impose symmetry in order to leave out any differences across markets that are not due to the regulatory rules and the different possibility of entry.

Consumers have quasi-linear utility functions, therefore demands are independent of income. Roy's identity implies that $x(p_i) =$

$= -v'(p_i)$, where $v(p_i)$ denotes the aggregate consumer surplus in each market.

An incumbent profit maximising monopolist, firm M, operates in both markets. Firm M has constant unit cost c in each market. A potential new entrant, firm E, may enter market 2, but not market 1 where sunk costs are so high that entry is not profitable. If firm E enters, it operates as a price taker to maximise profits. We denote its supply function with $k \cdot s(p_2)$, where k is the scale of entry and $s(p_2)$ is the net supply function per unit of capital. By Hotelling's lemma we obtain that $s(p_2)=e'(p_2)$ where $e(p_2)$ is an increasing convex function giving E's profit per unit of capital. We denote by $f(k)$ the cost of entry at scale k and assume that $f(0)= 0$, $f'(k) > 0$; $f''k) \geq 0$.

We hypothesise that there exists a benevolent industry regulator. To pursue its objective, the regulator chooses between two regulatory regimes that constrain the prices set by firm M. The two alternative set of constraints are as follows:

$$wp_1 + (1-w)\,p_2 \leq \overline{p} \qquad [1]$$

$$p_1 \leq \overline{p}_1 \qquad [1.a]$$

and

$$wp_1 + (1-w)\,p_2 \leq \overline{p} \qquad [1]$$

$$p_1 \leq \beta\, p_2 \qquad [1.b]$$

where $w \in [0, 1]$ and $\overline{p}$ is the upper limit set by the regulator to the weighted average of prices chosen by firm M. Moreover, $\overline{p}_1$ is the upper limit to p_1, that is, to the price that firm M can charge in the captive market, while β is the upper limit to the ratio of prices set by the regulated incumbent firm. On the basis of the nature of these second constraints operating in each regime, we define the regulatory regime given by constraints [1] and

[1.a] as the *Absolute* regime, while the other one, given by constraints [1] and

[1.b] is referred to as the *Relative* regime.

We suppose hereafter that w and $\overline{p}$ are identical across regimes and exogenously given. This hypothesis is at odds with the practice of regulation where the management of the parameters in the price cap formula represents an important instrument in the hands of the regulatory authority. However, this allows us to focus on the consequences of the additional constraints characterising the *Absolute* and the *Relative* regimes.

We deal with a finite game of perfect information with the following order of moves:

- *stage* 1: the regulator chooses either the *Absolute* or the *Relative* regulatory regime;
- *stage* 2.*a*: firm E chooses the scale of entry $k \in \{0, K\}$;
- *stage* 3: firm M chooses p_1, $p_2 \in [0, p_{max}]$ subject to the regulatory regime selected by the regulator;
- *stage* 4: firm E chooses the optimal quantity.

The structure of the game is also described in Figure 1. The levels of the variables with subscript *max* or *min* will be derived later. We will also analyse the case in which stage 2 takes the following form:

- *stage* 2.*b*: firm E chooses the scale of entry $k \in [0, K_{\max}]$, that is the actual scale of entry is optimally chosen by the firm;

Insert Figure 1 about here.

In other words, first the regulator chooses the regulatory regime. Then, the potential entrant chooses whether or not to enter by paying the sunk cost of entry. Entry may occur either at a given scale K (stage 2.a) or at a scale which is endogenously chosen by the firm (stage 2.b). In the subsequent stage, the regulated incumbent chooses its optimal prices subject to the regulatory regime. Finally, if entry has occurred, the entrant selects its optimal quantity. Given the nature of the game, we solve it by backward induction.

Given the scale of entry k chosen by firm E, the incumbent's total profits are given by

$$\Pi(p_1, p_2, k) = \pi(p_1) + \pi(p_2) - k\ (p_2 - c)\ s(p_2) \qquad [2]$$

where $\pi(p_1) = x(p_1)\,(p_1 - c)$. Profits of firm E are given by

$$\theta(p_2, k) = k\ e(p_2) - f(k) \qquad [3]$$

Finally, the regulator's payoff is given by the social welfare function

$$W = v(p_1) + v(p_2) \qquad [4]$$

2.1 Some further assumptions

This section presents and discusses some assumptions on the functions and the parameters set forth in the previous section to be employed in the rest of the analysis. In particular, we make the following assumptions:

Assumption 1: $x(p_i) = a - p_i$, for $i = 1, 2$. [A1]

This implies linear and identical demand functions in each market. It also entails that firm M's strategy space at stage 3 of the game can be refined as $p_1, p_2 \in [0, a]$.

Assumption 2*:* $s(p_2) = 1$. [A2]

This implies that E's supply function per unit of capital is completely inelastic and normalised to 1. By Hotelling's lemma, it also implies that the profit function per unit of capital $e(p_2)$ is linear and equal to p_2.

Assumption 3: $w = 1 - w = ½$. [A3]

In other words, the two prices set by firm M have the same influence on the price cap formula. This symmetrical treatment of the two markets allows to focus only on the effects on entry and welfare of the additional constraint entailed in any of the two alternative regulatory regimes under analysis. Indeed, given this assumption, the only regulatory instrument meant to influence the structure of prices set by the regulated firm is given by the constraints [1.a] and [1.b].

Assumption 4: $\overline{p} = ½\, a$. [A4]

By this Assumption, we fix the level of the price cap, that is the maximum allowed average price for the regulated firm, so that it will never permit a regulated firm with strictly positive marginal costs to set unconstrained monopoly prices (and will just allow to set unconstrained monopoly prices whenever the firm has zero marginal cost). This assumption is just a normalisation and is very useful insofar as, together with ½. [A3], it allows to re-write the price cap formula [1] as follows:

$$p_1 + p_2 \leq a \qquad [5]$$

Assumption 5: $2\,c \leq a$. [A5]

The porpuse of the assumption is twofold. First, if combined with [A4], it grants that $\overline{p} \geq c$, that is the level of the average price cap is always above firm M's marginal cost in both markets. This is necessary to ensure that the regulated firm always makes nonnegative profits. Secondly, if combined with the conditions on the entry scale of the entrant, it is necessary to grant that the incumbent firm always faces a non-negative residual demand curve at equilibrium prices[2]

Assumption 6: $\overline{p}_1 = \dfrac{a\beta}{1+\beta}$ [A6]

This assumption is introduced to make the two alternative regulatory regimes more readily comparable. By introducing it, we basically make the simplifying hypothesis that the same maximum level for the monopoly price is allowed under both schemes. To illustrate this point, we plot in Figure 2 in the price space the constraints for both regulatory regimes. The average price cap [1] which is included in both regimes is given by CF. The constraint [1.a] on the absolute level of the monopoly price set forth in the *Absolute* regime is given by BG. Finally, the constraint [1.b] on the price ratio which makes part of the *Relative* regime is given by AD. Hence, the *Absolute* regulatory regime obliges the regulated firm to choose any combination of prices in the area ABDF, while under the *Relative* regime prices may be chosen in the area ADF. Assumption 6 simply implies that BG and AD cross CF at the same point. Analytically, this involves that it is always possible to write $\overline{p}_1$ in terms of β and vice versa.

Insert Figure 2 about here

Assumption 7: If the regulator chooses the *Relative* regime, its choice of β is restricted so that:

$$\beta \in \left[1, \frac{a+c}{a-c}\right]; \qquad \text{[A7]}$$

or, equivalently,

[2] Notice that this condition on *c* is more restrictive than necessary for the existence of the equilibrium under the *Relative* regime, for which it would suffice that $c \leq 1/3a$.

Assumption 7': If the regulator chooses the *Absolute* regime, its choice of $\bar{p}_1$ is restricted so that:

$$\bar{p}_1 \in \left[\frac{a}{2}, \frac{a+c}{2}\right]. \qquad \text{[A7']}$$

First, note that the equivalence between the two assumptions follows immediately from [A6]. The rationale behind these assumptions is more easily understood if one looks at Assumption 7'. The condition on the maximum level of $\bar{p}_1$ basically says that we restrict our analysis to those cases where the maximum level of price in the captive market is at most equal to the unconstrained monopoly price. Moreover, since we want to focus on cases where revenues foregone in the competitive market can be recouped in the captive one, through the condition on the minimum level of $\bar{p}_1$ we restrict our analysis to those situations where the regulatory regime allows the firm to set in the captive market a higher price than in the competitive market. Notice that this is equivalent to assuming that the minimum allowed value of β is equal to 1 (that is the regulator will always permit the firm to set a higher price in the captive market). Given that Ramsey prices in this model necessarily lye above the 45° degree line (see, for instance, Vickers 1997), this Assumption makes our static regulatory framework consistent with long run allocative properties of price cap.

3. The case of exogenous scale of entry

This section analyses the solution of the game in the case of an exogenous scale of entry, that is when the potential entrant may simply choose whether or not to enter at a fixed scale *K*.

3.1 Effects on prices and entry

In this subsection we first characterise the optimal choice of the regulated firm at stage 3 under different regulatory regimes. The equilibrium prices set by the incumbent under the different regulatory regimes are necessary to ascertain the distinct effects on the entry decision taken by the potential entrant due to the different regimes under analysis. Then, we proceed to characterise the optimal choice on entry of the potential entrant. We recall that, through an explicit choice of the form of the entrant's profits function, we are already taking into account its optimal behaviour in the last stage of the game.

Consider now the problem faced by the incumbent under the *Absolute* regime. This is given by

$$\begin{aligned} &\max_{p_1, p_2} (a - p_1)(p_1 - c) + (a - p_2)(p_2 - c) - (p_2 - c)k \\ &s.t.\ p_1 + p_2 \le a \qquad p_1 \le \overline{p}_1 \end{aligned} \qquad [6]$$

Note that this is the problem faced by the incumbent both with and without entry. It is indeed sufficient to set k equal to zero to have the problem faced by the regulated firm when operating as a monopolist.

Assume now that entry has occurred, so that $k = K$. Let now p_1^A and p_2^A be the optimal prices set by firm M under the *Absolute* regime in the captive and competitive market respectively when entry has occurred. Using standard constrained maximisation techniques, it turns out that:

$$p_1^A = \frac{1}{2}a + \frac{1}{4}K;\ p_2^A = \frac{1}{2}a - \frac{1}{4}K \qquad \text{for } 0 < K \le K' \ [7]$$

$$p_1^A = \frac{ab}{1+b};\ p_2^A = \frac{a}{1+b} \qquad \text{for } K' \le K \le K'' \qquad [8]$$

$$p_1^A = \frac{ab}{1+b};\ p_2^A = \frac{a+c-K}{2} \qquad \text{for } K'' \le K \le K^A_{max} \qquad [9]$$

where $K' \equiv \dfrac{2a(b-1)}{b+1}$, $K'' \equiv (a+c) - \dfrac{2a}{1+b}$ and $K^A_{max} \equiv a - c$.[3]

Looking at K' [7], it is also immediate to find out that, if one denotes with p_1' and p_2' the optimal incumbent's prices in the absence of entry, by continuity one can easily conclude that[4]

$$p_1' = p_2' = \frac{1}{2}a \qquad [10]$$

Consider now the problem faced by the incumbent under the *Relative* regime. In both cases, that is with and without entry, this can be formally stated as

$$\begin{aligned} &\max_{p_1, p_2} \ (a - p_1)(p_1 - c) + (a - p_2)(p_2 - c) - (p_2 - c)k \\ &\text{s.t. } p_1 + p_2 \leq a \quad\ p_1 \leq \beta p_2 \end{aligned} \qquad [11]$$

Let now p_1^R and p_2^R be the optimal prices set by firm M under the *Relative* regime in the captive and competitive market respectively when entry has occurred. Using standard constrained maximisation techniques, it turns out that:

$$p_1^R = \frac{1}{2}a + \frac{1}{4}K;\ p_2^R = \frac{1}{2}a - \frac{1}{4}K \text{ for } 0 < K \leq K' \qquad [12]$$

$$p_1^R = \frac{ab}{1+b};\ p_2^R = \frac{a}{1+b} \qquad \text{for } K' \leq K \leq K'' \qquad [13]$$

$$p_1^R = \frac{b\left[(a+c)(b+1) - K\right]}{2(b^2+1)};\ p_2^R = \frac{(a+c)(b+1) - K}{2(b^2+1)}$$

$$\text{for } K'' \leq K \leq K^R_{max} \qquad [14]$$

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[3] Details are available from the authors upon request.

[4] This result could also be easily derived by solving [6] by standard constrained maximisation techniques for a *K* equal to zero.

where K' takes on the same value as before,

$$K'' = (a+c)(1+b) - \frac{2a(1+b^2)}{1+b} \text{ and}$$

$$K^R{}_{max} = \frac{2ab^2 + a - c - b(a+c)}{1+2b^2}. \text{ Notice also that, since}$$

$K^R{}_{max} < K^A{}_{max}$, an equilibrium exists under the *Relative* regime for a range of values of K wider than under the *Absolute* regime.[5] Finally, it is easy to show that optimal prices in the absence of entry are identical than under the *Absolute* regime and then equation [10] applies.

Our main findings on the incumbent prices are summarised in the following Proposition:

> **Proposition 1:** When $K \leq K''$, optimal prices are independent of the regulatory regime.
>
> When $K > K''$, $p_1^A \geq p_1^R$ and $p_2^A \leq p_2^R$.

The first immediate observation regarding this Proposition is that the two regimes do not have differential effects on prices when the scale of entry K is sufficiently small neither in the absence of entry nor if entry actually occurs. Hence in the rest of the analysis, we will disregard this case and concentrate on the case of $K > K''$.

Optimal incumbent's prices under the two alternative regimes are plotted in Figure 3 for the whole admissible range of values for K. Comparing the optimal prices when the scale of entry is sufficiently large, it is easy to ascertain that the regulatory regime affects the level of prices in each market. For intermediate values of K (i.e. $K'' \leq K \leq K'''$), while the level of the monopoly price is

[5] As in the previous case, details of the procedure are available from the authors upon request.

identical under both regimes, the competitive price is higher under the *Relative* regime. When K is sufficiently large (i.e. $K''' \leq K \leq K^{A}_{max}$) the higher competitive price under this regime is associated with a lower monopoly price.

Insert Figure 3 about here

We can now move on to characterise the optimal choice of entry by firm E at stage 2. At this stage, firm E chooses to enter and pays the entry cost $f(K)$ only if it anticipates non-negative profits in the subsequent stages of the game, that is only if $\theta(p_2^i, K) = K\, p_2^i - f(K) \geq 0$, where $i = A, R$. The effect of the regulatory regime on this choice is illustrated in the following Proposition, which follows immediately from Proposition 1:

> **Proposition 2:** Let $K > K''$. If $f(K) \quad K \quad p_2^A$ $\left(f(K) > K \cdot p_2^R\right)$, then firm E always chooses to (not to) enter. If $K \; p_2^A < f(K) \; K \; p_2^R$, then firm E chooses to enter only under the *Relative* regime.

This Proposition illustrates that, provided that the scale of entry is large enough, the *Relative* regulatory regime is able to foster competition for a larger range of values of the model's parameters. The key reason for this result is that, for any value of the potential entry scale K, the equilibrium price in the competitive market is always higher under the *Relative* regime than under the *Absolute* regime. Hence, the entrant's profits in case of entry are higher under the *Relative* regime. This may lead the potential entrant to choose to enter under conditions that make entry unprofitable under the alternative regime.

However, since this higher likelihood of entry is obtained by trading off higher prices in the competitive market with (weakly) lower prices in the captive market, the overall effect of the two regulatory regimes is not clear. A more detailed analysis of the welfare consequences of the two regulatory regimes is carried out in the next section.

3.2. Equilibrium choice of the regulatory regime

In this section we characterise the optimal choice of the regulator at stage 1 of the game. Recall that its objective function, already set forth in [4], is given by $W = v(p_1) + v(p_2)$. The following Proposition deals with the optimal choice of the regulatory regime:

> **Proposition 3:** Let $K > K''$. Whenever entry occurs only under the *Relative* regime, this regime represents the optimal choice of the regulator. When entry occurs under both regimes, the *Absolute* regime is weakly welfare-superior to the *Relative* regime.

To illustrate this Proposition, consider Figure 4. In this Figure, we plot against K the aggregate consumers' surplus evaluated at the different equilibrium price for a given value of β such that $\beta \in (\beta_{\min}, \beta_{\max})$. In particular, we denote by V^A and V^R the equilibrium social welfare in case of entry under the *Absolute* and *Relative* regime respectively, that is the social welfare evaluated at those price given by K' [7]-[9] or by K' [12]-[14]. We also plot in the Figure the function V^0, which is the aggregate consumers' surplus when entry does not occur. This is a horizontal line, being independent of K.

Insert Figure 4 about here

The Figure illustrates that social welfare is always higher when entry occurs. Hence, whenever the choice of the regulatory regime affects the entry decision, the regime that fosters entry is socially superior. Since Proposition 2 states that there are conditions under which entry occurs only under the *Relative* regime, under the same very conditions, this regime has to be preferred. However, the situation is reversed when entry occurs under both regimes. Comparing V^A and V^R in the Figure, it is straightforward to see that social welfare is higher under the *Absolute* regime for any $K > K''$.[6]

4. The optimal choice of the scale of entry

In the previous sections we assumed that the new entrant choice was simply whether or not to enter at a given level of scale

$k \in [0, K]$. However, in order to examine the effects of different regulatory policies upon competition, it is desirable to make the scale of entry endogenous. In this section, we analyse the case of a potential entrant that optimally sets its scale of entry, as already formally described in Section 2. This generalisation of the model turns out to be useful to assess the consequences on the scale of entry stemming from the different regulatory regimes under analysis. It highlights how equilibrium welfare depends not only from the potential entrant's choice whether or not to enter, but also on the scale optimally chosen by the new entrant firm.

For the remaining part of this section, we make the following assumption:

Assumption 8: $f(k) = t\,k$ [A8]

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[6] Two special cases of the result given in the Proposition 3 come out when β takes on the extreme values of the interval $[\beta_{min}, \beta_{max}]$. First, when β is at its minimum allowed value, K' is equal to zero. On the other hand, when β equals β_{max}, there exists only one threshold value (that is $K' = K'' = K'''$). For K smaller than this threshold value, prices are identical under both regimes. As K gets larger, both prices decrease under the *Relative* regime while only the competitive price decreases under the *Absolute* regime. This pattern of prices implies that the range of values for K where social welfare functions overlap is wider for $\beta = \beta_{max}$ than in the case depicted in Figure 3, thus raising the level of consumer surplus under the *Relative* regime. Our analysis therefore suggests that social welfare is sensitive to the choice of the parameter β representing the allowed degree of price differentiation. However the level of this parameter does not affect the optimal choice of the regulatory regime by a regulatory authority.

This hypothesis simply implies that the cost of entry linearly increases with capacity. Notice that [A8] is perfectly consistent with previous assumptions on the nature of $f(k)$.

This section is first devoted to the analysis of the optimal choice on the scale of entry by the rival firm at stage 2.b of the game under the hypothesis that entry is profitable. Then, given that the scale of entry is optimally set and entry occurs, we compare social welfare functions. Clearly, this is not sufficient to characterise the equilibrium of the whole game when the scale of entry is endogenously chosen by the potential entrant. However, we have previously shown how the *Relative* regime is able to foster entry for a larger intervals of the parameters of the model and to deliver higher social welfare when entry occurs only under this regime. Now, we want to focus the analysis on the optimal choice of the scale of entry when entry occurs under both regimes to ascertain which regulatory regime is able to grant entry at larger scale and what are the effects on welfare of the different levels of scale coming out under the two regimes.

Given this, we first focus on the choice of $k \in (0, K_{\max}]$ at stage 2.b of the game. This choice is made by the new entrant firm to maximise its profits in the final stage of the game that, because of [A8], are given by $(k \cdot p^i_2 - t\,k)$ for $i = A, R$.. In other words, in making its choice, firm E anticipates the optimal prices set by firm M in the subsequent stage of the game, which in turn depend on the regulatory regime in force.

The main result obtained in this section is illustrated in the following Proposition:

Proposition 4: Let $t \leq t'$,

where $t' \equiv \dfrac{a(3-\beta)-c(1+\beta)}{2(1+\beta)}$. The scale of entry optimally chosen by the new entrant firm is greater under the *Relative* constraint than under the *Absolute* constraint.

The first thing to notice about this Proposition is the role of the initial assumption on the cost parameter t. Assuming that t is smaller than t' is necessary to grant, on one side, that entry is profitable under both regulatory regimes, and, on the other side, that a comparison between the choices on the scale of entry is feasible. Notice that, when t falls within this interval, not only entry occurs under both regimes both also the equilibrium prices in the following stage of the game differ depending on the regulatory regime. Given that our analysis focuses on this case, the Proposition concludes that when the entry cost is sufficiently low to make entry profitable under both regimes, the scale of entry chosen by the potential entrant is larger under the *Relative* regime. Combined with the results of the previous sections, this implies that not only the *Relative* regimes is able to grant that entry occurs for parametric conditions under which entry would not occur under the *Absolute* regime, but also, when entry is profitable under both regime, the market becomes more competitive under the *Relative* regime than in the alternative case.

Now, it is intuitive that, when the decision about the scale of entry is made endogenous, this also affects the equilibrium values of the social welfare function under the different regimes. We recall here that, similarly to section 3.2, we take social welfare as given by the sum of the consumer surplus in the two markets.

The implications on consumer welfare stemming from the two regulatory regimes under analysis when entry always occurs are summarised in the following proposition:

Proposition 5: Let $t \leq t'$. Welfare under the *Relative* regime is weakly greater than under the *Absolute* regime.

This proposition suggests that the *Relative* regulatory regime is welfare enhancing with respect to the *Absolute* one when the parameter *t*, expressing the cost of entry per unit of capital, is low enough to make entry profitable independently of the regulatory regime in place. In the set-up described in this section, that is under the hypothesis that the rival firm always chooses to enter, the *Relative* scheme is able to positively affect the scale of entry optimally chosen by the new entrant. Throughout this causal chain, entry at a higher scale brings about a downward pressure on prices increasing consumer welfare and thus leading to a reverse welfare ranking of the regulatory regimes with respect to the case of a fixed scale of entry.

5. Conclusions

The purpose of this paper has been to compare two regulatory policies towards price discrimination by a price capped firm in an environment where there is a competitive threat. The issue at hand is of both theoretical and practical importance.

First, from a theoretical point of view, it is well known that the overall welfare consequences of price discrimination are of ambiguous sign, even in the simplest case of a monopolist without regulation. Therefore it might be helpful to extend the analysis to environments where the dominant firm faces potential or effective competition and at the same time some kind of public policy towards price discrimination is in place.

Secondly, the idea of assessing different regulatory schemes towards price discrimination is of great practical importance. As a matter of fact, it is nowadays rather common in regulated markets that an incumbent monopolist subject to price cap regulation might exploit its freedom to vary prices within the regulated basket in order to counterbalance a competitive threat. This response by the dominant firm, which may prevent socially beneficial entry from occurring, thus calls for appropriate public intervention. A solution implemented in the practice of regulation is to impose additional caps over prices in the captive markets (what we defined as the *Absolute* regulatory regime). On the other hand, we hypothesised that the regulator may choose another approach to limit the regulated firm ability to obtain extra revenues and possibly deter entry. This approach, which has been defined as the *Relative* regime, involves a constraint on relative prices along with a price cap constraint.

In this paper we focused on a stylised model, based mainly on Amstrong and Vickers (1993). We assumed that a price capped firm operates as the unique supplier in a certain market, while in another market entry could be profitable. Moreover a regulatory authority may opt for either the *Absolute* or the *Relative* regulatory regime. This last choice proved to be crucial in terms of fostering competition and influencing social welfare.

First it was shown that under the hypothesis that the degree of competition, i.e. the scale of entry, is exogenously given, prices in the competitive market are weakly higher if the *Relative* regulatory scheme is applied. As a consequence of this statement, the *Relative* regulatory regime is more able to foster competition than the alternative regime, as there exists a range of sunk costs values at which entry is feasible *only* under this regulatory scheme. As far as the welfare effects stemming from the application of the two different regulatory schemes are concerned, we showed that this higher likelihood of entry comes at a cost of lower social welfare in those cases where the choice of the regulatory regime does not affect entry, that is when entry always occur. On the other hand, whenever the *Relative* regime is able to foster entry that would not occur under the alternative regime, the development of a competitive market brings about higher soc ial welfare.

The analysis was finally generalised to consider the optimal choice on the scale of entry by a rival firm. When scale of entry is no longer a binary choice variable the *Relative* regulatory regime proved to be able to encourage competition, as it allows for entry at a (weakly) larger scale than the *Absolute* regime would do.

Furthermore, via the effect induced by competition upon prices, consumer surplus is higher under the *Relative* scheme.

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Appendix

This Appendix contains the proofs of all the Propositions of the paper.

Proof of Proposition 1

Trivial, by comparing K' [7]-[9] and K' [12]-[14] over the appropriate ranges of K.

Proof of Proposition 2

Trivial, by simply combining the result of Proposition 1 and the assumption that entry occurs whenever $\theta(p_2,k)\ \ 0$.

Proof of Proposition 3

Let $V^A = v(p_1^A) + v(p_2^A)$, $V^R = v(p_1^R) + v(p_2^R)$ and

$V^0 = v(p_1') + v(p_2')$, where p_i', i= A, R, are defined as in [10]. When $K > K''$, we know from Proposition 2 that the application of the two regimes may have different effects in terms of entry. Let us first consider the case where $K\ \ p_2^A < f(K)\ K\ \ p_2^R$ so that entry occurs only under the *Relative* regime. We then need to compare V^R against V^0 to establish our result.

When $K'' < K\ K'''$, $V^0 = \dfrac{(1-a)^2}{4}$ and $V^R = \dfrac{a^2 + a^2 b^2}{2(1+b)^2}$.

Then, $V^R > V^0$ if $\dfrac{a^2(1-b)^2}{4(1+b^2)} > 0$, which always holds true. When $K''' < K\ \ K^A_{max}$, the same result holds since p_1^R and p_2^R are now both lower than when $K'' < K\ \ K'''$.

Consider now the case $f(K)$ K p_2^A so that entry occurs under both regulatory schemes. We now need to compare V^A against V^R to establish our results. Obviously in the range [0, K''], we have that $V^A = V^R$. When $K'' < K < K'''$, since p_2^A is always lower than p_2^R and the price in market 1 is identical under both regimes, then $V^R < V^A$. As a matter of fact, in this interval $\frac{\partial V^A}{\partial K} > 0$ and $\frac{\partial^2 V^A}{\partial K^2} > 0$, while $\frac{\partial V^R}{\partial K} = 0$. At $K = K'''$, $V^A > V^R$ for $\beta_{min} \leq \beta < \beta_{max}$, while $V^A = V^R$ when β reaches its highest feasible value. When $K''' < K < K^A_{max}$, we may consider $[V^A - V^R]$ as a polynomial in K. In order to ascertain the behaviour of this polynomial for $K \in \left[K''', K^A_{\max}\right]$ we employ the notion of Cauchy index of a real rational function $R(x)$ between the limits a and b, where a and b are real numbers or $\pm \infty$. The Cauchy index, denoted henceforth as $I_a^b R(x)$, is the difference between the number of jumps of $R(x)$ from $-\infty$ to $+\infty$ and that of jumps from $+\infty$ to $-\infty$ as the argument varies from a to b. By means of Sturm's theorem it is possible to determine the number of distinct real roots of a polynomial $f(x)$ in the interval (a, b), being this number given by $I_a^b \frac{f'(x)}{f(x)}$.

Applying Sturm's theorem, it comes out that $I_{K'''}^{K^A_{\max}} \frac{\partial[V^A - V^R]/\partial K}{[V^A - V^R]}$ = 0, thus implying that V^A and V^R never cross in the interval under analysis. Finally, when $K = K^R_{max}$., it is easily found that V^A is higher than V^R both when β is equal to β_{min} and to β_{max}. Notice that, by means of Sturm's theorem, it is possible to assess that the above inequality holds also for any

$\beta \in [\beta_{min}, \beta_{max}]$. This analysis then allows to conclude that

$V^A \geq V^R$ for all admissible values of K, with the two measures of social welfare being equal only when $K = K'''$ and $\beta = \beta_{max}$.

Proof of Proposition 4

Assume that the *Absolute* regime is in force and that entry has occurred. Consider the case when $K'' \leq k \;\; K^A_{max}$. From [9], the potential new entrant's maximisation problem becomes $\max_k \left(\frac{k(a+c)-k^2}{2} - kt \right)$. Let k^A be the solution of this problem. Thus, $k^A = \frac{a+c-2t}{2}$; because of the initial assumption on entry, k^A must be strictly positive, which implies $t < \frac{a+c}{2} \equiv t_0^A$. Also, consistency with the initial assumption on k and p_2^A implies that $K'' \leq k^A \;\; K^A_{max.}$ Rearranging this inequality, we obtain that:

$$-\frac{a+c}{2} \leq t \leq t' \qquad \text{[A.1]}$$

where $t' = \frac{a(3-b)-c(1+b)}{2(1+b)}$. Notice also that [A.1] implies that $t < t_0^A$ is always verified, since $t' < t_0^A$.

Assume now that the *Relative* regime is in force.

When $K''' \leq k \;\; K^R_{max}$, the new entrant solves $\max_k \left[\frac{k(a+c)(b+1)}{2(b^2+1)} - \frac{k^2}{2(b^2+1)} - tk \right]$. Let k^R be the solution of this problem; then $k^R = \frac{(a+c)(1+b)}{2} - t(1+b^2)$. Again, because of the initial assumption on entry, k^R must be strictly positive, which implies $t < \frac{(a+c)(1+b)}{2(1+b^2)} \equiv t_0^R$. Also, consistency with the initial assumptions on k and p_2^A implies that $K''' \leq k$ $K^R{}_{max.}$ and requires now that

$$\frac{(a^2 - c^2)(1+b) - 4a(a+c)}{2(a-c)(1+b^2)} \leq t \leq t'' \qquad \text{[A.2]}$$

where $t'' = \frac{2a}{b+1} - \frac{(a+c)(b+1)}{2(b^2+1)}$. Notice also that [A.2] implies that $t < t_0^R$ is always verified, since $t'' < t_0^R$.

When K'' k K''', the problem faced by the potential entrant is given by $\max_k \left(\frac{ak}{1+b} - tk \right)$, which is linearly increasing in k.

Hence, provided that $t < \frac{a}{1+b}$, the new entrant will always pick the highest possible k, that is K'''. Notice that, since $t' < \frac{a}{1+b}$, choosing K''' is a possible solution of the entrant's problem also when t is particularly small. However, in such a case, it easy to show that choosing k^R is always preferred to the firm: hence, K''' is optimal only for a large enough t, that is when $t \geq t'$.

Now, notice that the constraints on the l.h.s. of inequalities [A.1][15] and [A.2][16] always hold since both lower bounds are negative for all admissible values of β. Moreover, since $t' < t''$, we can focus on values of $t \in (0, t']$.

To establish the result, it is now sufficient to note that: i) $k^R > k^A$ when $t = 0$; ii) $k^R > k^A$ when $t = t'$ provided that $\frac{1+b^2}{2b} > \frac{a-c}{a+c}$, which always holds true, and iii) k^R and k^A are both linearly decreasing functions of t.

Proof of Proposition 5

Let $V(k^R)$ and $V(k^A)$ be the social welfare functions in case of entry under the *Relative* and the *Absolute* regime respectively given that the entrant optimally chooses the scale of entry (k^R and k^A respectively). Consider now the difference between these functions, given by $\Psi = [V(k^R(\beta, t)) - V(k^A(\beta, t))]$, as a polynomial in t. In order to ascertain the behaviour of this polynomial when $t \in [0, t']$, we apply Sturm's theorem. (see section 3.2). It comes out that $I_0^{t'} \frac{\partial\Psi / \partial t}{\Psi} = 0$, thus implying that $V(k^R)$ and $V(k^A)$ never cross in the interval under analysis.

Then, throughout a comparison between $V(k^R)$ and $V(k^A)$ evaluated at $t = 0$, we conclude that $\Psi\big|_{\beta=\beta_{min}} > 0$, and $\Psi\big|_{\beta=\beta_{max}} > 0$. Moreover, reckoning that Ψ, again at $t = 0$, can be seen as a polynomial in β, we evaluate $I_{b_{min}}^{b_{max}} \frac{\partial\Psi/\partial b}{\Psi}$ obtaining the there are no real roots in the interval (β_{min}, β_{max}).
Finally, both functions $V(k^R)$ and $V(k^A)$ are evaluated at $t = t'$ getting that $\Psi\big|_{\beta=\beta_{min}} = 0$ while $\Psi\big|_{\beta=\beta_{max}} > 0$. As before, when t is held fixed at t' $I_{\beta_{min}}^{\beta_{max}} \frac{\partial\Psi/\partial\beta}{\Psi} = 0$ meaning that there is no intersection between $V(k^R)$ and $V(k^A)$ for feasible values of the parameter β.

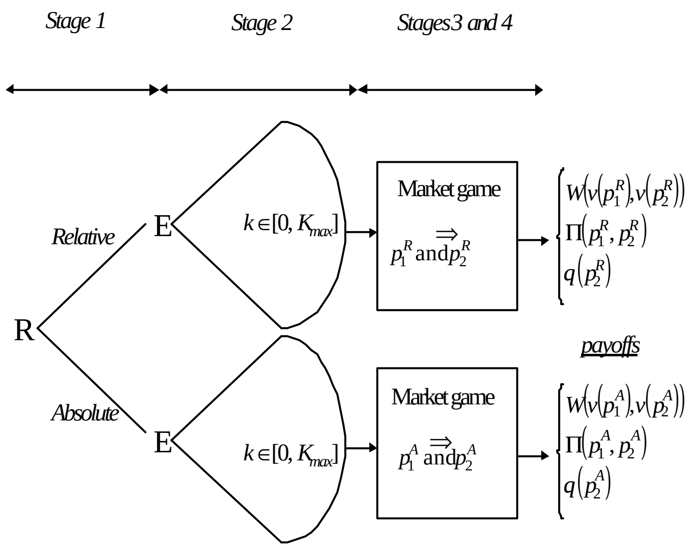

Figure 1: The game

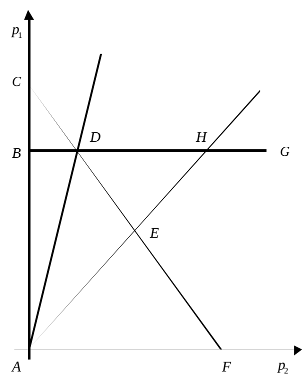

Figure 2: Prices allowed under the different regulatory regimes

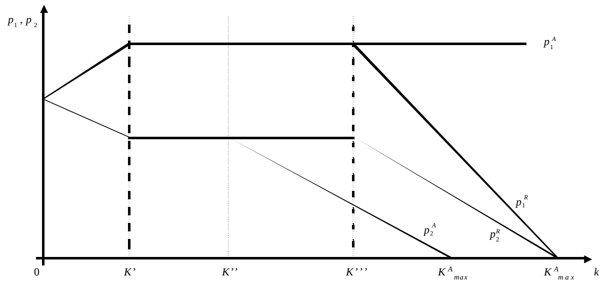

Figure 3: Equilibrium prices with K exogenous

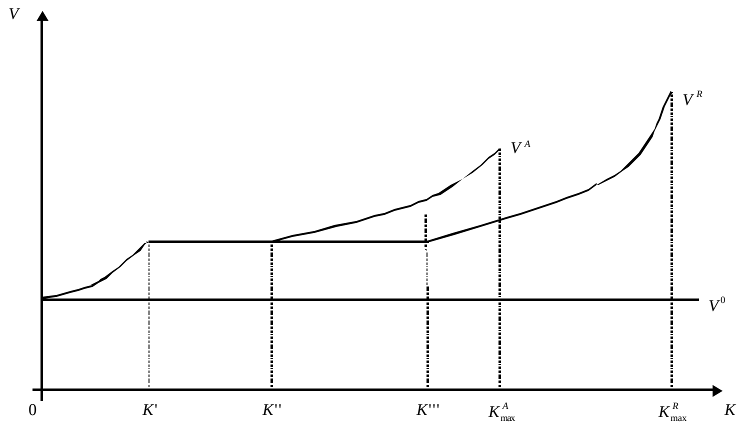

Figure 4: Welfare pattern when the scale of entry is exogenous