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Uncertainty and Endogenous Selection of Economic Equilibria

by

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Abstract

This paper presents a model of co-ordination failures based on market power and local oligopoly. The economy exhibits a multiplicity of Pareto-ranked equilibria. The introduction of uncertainty generates an endogenous equilibrium selection process, due to a strategic use of information by firms. The economy is more likely to settle on some equilibria than on others. We argue that a full understanding of these robustness criteria is needed before any policy which is intended to help co-ordinate the level of activity to a Pareto dominant outcome can be successfully implemented.

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1. Introduction

The attempt to provide rigorous microeconomic foundations for equilibrium outcomes is arguably one of the most actively pursued areas of current research. It is increasingly being recognized that a satisfactory explanation of aggregate relationships must rely on the analysis of the interactions amongst individual agents (see for instance Mankiw and Romer, 1991, and Dixon and Rankin, 1995). Within this literature, an important strand of research has focused on the investigation of the properties of non-Walrasian models, in which agents wield market power and are therefore able to influence the market in which they operate. The resulting market failures can be responsible for multiple Nash equilibria, whereby the economy could settle at more than one level of aggregate activity.

The idea that the economy may exhibit a multiplicity of equilibria, and that agents may fail to co-ordinate their actions to a high level of activity, goes back at least to Kaldor (1940), where this possibility arises due to non-linearities in the *ex ante* savings and investment functions. Recent accounts of co-ordination problems have emphasized the role of incomplete markets, increasing returns, and search costs as possible sources for the lack of co-ordination (see Silvestre, 1993, for a survey). Often, the underlying model of an economy with multiple equilibria can be described as a supermodular game, with strategic complementarity among the agents' payoffs, and its Nash equilibria can be Pareto-ranked (Cooper and John, 1988; Milgrom and Roberts, 1990).

The literature on models with multiple equilibria typically focuses on the causes of multiplicity, but does not explicitly suggest which of these equilibria is more likely (although there is a tendency to focus on the Pareto optimal equilibrium as being focal). Since an equilibrium is usually interpreted as a rest point for the economy, one approach would be to rule out some of these equilibria by imposing additional restrictions on the behavior of the agents, as in most of the refinements concepts. In the present paper we follow a new approach to the problem of co-ordination in macro models: we require the equilibrium to be robust to the introduction of a small amount of incomplete information. We exploit the notion that, in a decentralized market economy, agents mainly interact with their neighbors. When noise is introduced, and agents receive correlated stochastic signals, they will be forced endogenously to co-ordinate their decisions, even if the amount of noise is very small. Under some conditions, this could lead the economy to converge to a unique equilibrium.

The idea that some equilibria are not robust to the introduction of a small amount of incomplete information might appear to be at odds with a conventional rational expectations approach, where all self-fulfilling expectations can generate an equilibrium. The reason for the result is that noise is correlated among neighbors, and the strategic effects are so strong that only some equilibria are robust.

The main reason for the selection of equilibria is closely related to the notion of risk dominance, as analyzed by Harsanyi and Selten (1988). The intuition for our results is best understood by an example. Suppose that we start from an economy in which the fundamentals are consistent with only one strategy (*i.e.* a dominant strategy). Suppose then that the fundamentals of the economy slightly change, in such a way that additional strategies are now also possible in equilibrium. Under such conditions, it is clear that the first strategy is "*less risky*" than the latter ones. Agents will look at their own signal, which provides some information about the fundamentals, and will make inferences about the possible actions of the other players. The less risky strategy would also become the *dominant strategy* following the perturbation.

When strategic interactions only take place among neighbors, we show that locally correlated idiosyncratic signals are sufficient to generate equilibrium selection. No such endogenous co-ordination mechanism exists in a completely deterministic framework.

The results in this paper are closely related to the literature on equilibrium selection in game theory. In the latter, it is suggested that an endogenous selection of equilibrium may follow from rational behavior, in the presence of uncertainty about some features of the game. Carlsson and van Damme (1993a), in a seminal paper, consider the equilibrium selection process in a very general (2×2) game in the presence of uncertainty about payoffs. Their results have been generalized to n-person co-ordination games by Kim (1996). Other relevant contributions are Morris (1995), who analyses a work-shirk model with uncertainty about timing, and Carlsson and van Damme (1993b), who analyze an n-player stag-hunt game with payoff uncertainty. Shin's (1995) search model with idiosyncratic noise is, to our knowledge, the first implementation of this robustness criterion (or confidence, in Shin's terminology) to a macro model.

We believe however that our approach is particularly suitable for macroeconomic models on two grounds. First, uncertainty over fundamentals is a pervasive feature of macroeconomics, and therefore it is appropriate to explore the robustness of the equilibria with respect to the explicit introduction of uncertainty. Second, a common interpretation for the Nash equilibria is that they could represent the rest point for the economy. It is important therefore to analyze whether agents in the economy do indeed co-ordinate to these equilibria.

Relative to the existing game-theoretic literature, the present paper has the following innovative features. First, the equilibrium selection process takes place in a large economy with both local interactions and aggregate effects. Second, the results by Carlsson and van Damme (1993a) and Kim (1996) are generalized to an economy formed of agents with a continuum of strategies. Third, we show that the Carlsson-van Damme's findings can be applied to models with strategic complementarities. Fourth, we show that signals need only be correlated among neighbors (and not among all agents in the economy): this makes the application of our endogenous selection process to a large economy particularly appropriate.

We develop our arguments by using a simplified model of local oligopoly based on Salop's (1979) circular economy. The model exhibits strategic complementarities, and has a multiplicity of Pareto-ranked Nash equilibria. Agents only interact locally, and observe stochastic signals which are correlated across neighbors. Each firm is assumed to be exogenously located on a circle (which represents varieties of horizontally differentiated commodities), and to be exposed to competition only *vis à vis* its immediate neighbors (its potential market rivals). For simplicity, we abstract from price decisions. We assume instead that the level of activity of a firm directly affects the intensity of competition with its rivals. This captures a "business stealing" effect (see *e.g.* Mankiw and Whinston, 1986), whereby a firm producing at a higher level of activity can attract customers away from its rivals.

The firms' investment and production decisions affect the competitive conditions in their local neighborhood, but have a negligible influence on demand farther away in the preference space. However, the demand for each firms' output depends on the investment and production decisions of *all* firms in the economy. The payoff of each firm is therefore a function both of the strategies chosen by its neighbors (*via* oligopolistic competition), and of the behavior of all other firms in the economy (which affects the total demand for the firm's product). There is thus an aggregate demand externality.

Local markets may exhibit idiosyncratic features, which could result in a different strategic advantage from a more aggressive behavior over the various locations on the preference space. Furthermore, firms could also differ with respect to their information. In general, a firm's information set includes both common knowledge and private information. The common knowledge comprises past history, whereas the private information is related to the firms' perception of their local environment.

The balance between common knowledge and private information could lead to endogenous co-ordination to the most robust equilibrium. The main result of the paper is to show that even a small amount of idiosyncratic uncertainty could lead to endogenous co-ordination.

The structure of the paper is as follows. The next section presents the model and motivates our analysis. Section 3 describes the properties of the economy in the absence of idiosyncratic noise. Section 4 analyses the more general model and discusses the equilibrium selection process. Section 5 considers the case of locally correlated signals. Section 6 concludes.

2. The model

We set out a highly simplified model of local oligopoly, based on Salop (1979). Our choice of model enables us to analyze the strategic interactions amongst agents in a macro framework. A continuum of consumers is uniformly located over a circle, whose measure is normalized to unity. Their position on the circle represents their preferences. There is a large number n of firms in the economy, located at uniformly spaced points on the circle. Firms do not choose their location, *i.e.* their variety, but decide on their level of output. We assume that a higher level of production is associated with a higher market share. This is meant to capture a "business stealing" effect (Mankiw and Whinston, 1986) without explicitly modeling price decisions. Qualitatively similar results would hold under explicit pricing decisions, as in Shaked and Sutton (1987). Our assumption could be rationalized on the grounds that larger firms are associated with lower search costs for consumers, and therefore attract a larger market share.

Our remaining assumptions on the model are fairly standard. Each consumer supplies an equal share of labor to every firm in the economy, and receives an equal share of the total wage payments of every firm. The wage rate is normalized to coincide with the price of output. Workers have an infinite elasticity of labor supply at the going wage. Firms are uniformly owned by consumers, who act as their shareholders. Aggregate demand is given by the sum of labor incomes in the economy.

Total costs are $c=a$, where a is labor input and where $c \in [\underline{c}, \overline{c}] = [0, \overline{a}]$. Output increases with labor input: $y=f(c)$, $f'(\cdot)>0$. We denote $\underline{y} = f(\underline{c})$ and $\overline{y} = f(\overline{c})$. If firm i invests more than its neighbor $i+1$ then it attracts a larger share of the demand over the arc $(i, i+1)$. Formally, we let

$$\text{(1)} \qquad \theta_i(|a_i - a_{i+1}|) = \frac{1}{2} + \frac{\overline{\theta} - 1/2}{\overline{c} - \underline{c}} |a_i - a_{i+1}|$$

where $\overline{\theta} \in [1/2,\ 1)$. If $a_i \geq a_{i+1}$ then firm i receives a fraction θ_i of the demand in the arc, otherwise it receives $1-\theta_i$. If $a_i = \overline{a}$ and $a_{i+1}=0$ then $\theta_i = \overline{\theta}$ denotes the maximum *ex ante* demand on the arc.

From the above expressions it is clear that we assume linearity of both the cost function and market shares. In Section 3 we also assume linearity of the production function, $f(c)$. Linearity is not crucial for our results, but considerably simplifies computations. The exact cut-off points identified in Section 3 depend on these assumptions, but the qualitative results of Sections 4 and 5 do not.

Before investment decisions are made, firms observe $\overline{\theta}$. However, this observation can be noisy. Formally, we assume that each firm observes $\overline{\theta}_i = \overline{\theta} + \nu_i$ where ν_i is an idiosyncratic stochastic signal which is assumed to be symmetrically distributed over $[-\nu, \nu]$. One of the main purposes of this paper is to show that even when ν is arbitrarily small the structure of equilibria in the economy is drastically changed. If $\nu=0$ then all firms have the same expectations about $\overline{\theta}$. In general, these expectations could be different. $\overline{\theta}$ is a random variable whose realization could be different, *ex post*, on different arcs.

In game-theoretic terms, we have a simultaneous-move game with a continuum of strategies. The payoff of firm i is a function of the strategy profile of the other n-1 firms. However, since firms do not interact directly with firms other than their immediate neighbors, a sufficient statistic for the behavior of the remaining n-3 firms is their average behavior, which can be approximated by total output in the economy divided by the measure of the set of firms. Let $Y \equiv \sum_{j=1}^{n} y_j$ denote total output. Then the expected payoff for firm i is

(2) $$\pi_i = \frac{Y}{n}[\theta_i \cdot \mathrm{I}(a_i \geq a_{i+1}) + (1-\theta_i) \cdot \mathrm{I}(a_i < a_{i+1}) + (1-\theta_{i-1}) \cdot \mathrm{I}(a_{i-1} \geq a_i) + \theta_{i-1} \cdot \mathrm{I}(a_{i-1} < a_i)] - c_i$$

where $\mathrm{I}(\cdot)$ denotes the indicator function. Firms strategically interact at the local level, but neglect the economy-wide effects of their actions. The simultaneous move nature of the game, together with the aggregate demand externalities, will generate a co-ordination game.

3. Multiple equilibria and dominant strategies

In this section we analyze the deterministic version of the model. The competitiveness parameter $\overline{\theta}$ is constant and common knowledge to all economic agents and the idiosyncratic component of the noise is absent, *i.e.* ν=0. We identify values of the competitiveness parameter $\overline{\theta}$ for which firms have a dominant strategy. These values play a crucial role in the analysis of section 4. We calculate the symmetric equilibria of the game and show that it can have multiple Pareto-ranked equilibria. We also show that, when firms' choice is restricted to two activity levels only, the equilibrium conditions in the market do not depend on the exact configuration of firms along the circle, *i.e.* location does not matter.

Let us first consider whether there are regions of values for $\overline{\theta}$, where $\underline{c}$ is a dominant strategy. The best candidate behavioral profile for firm i to have an incentive to switch from $\underline{c}$ to a higher level of investment occurs when: (1) both neighbors are investing $\underline{c}$; and (2) all other firms invest $\overline{c}$, hence gaining an additional fraction of demand is most valuable. Firm i's profit is $\pi_i = \overline{y} - \underline{c}$ (the three firms investing $\underline{c}$ have a negligible impact on total demand). Switching to $c_i = \underline{c} + \varepsilon$, ε>0 will result in profit $\pi_i^* = 2\theta(\varepsilon)\overline{y} - (\underline{c} + \varepsilon)$, where $\theta(\varepsilon) = \theta_i(\varepsilon)$ is computed from (1). The switch is not profitable iff $\pi_i^* < \pi_i$, which is equivalent to

(3) $$\overline{\theta} < \frac{1}{2}\left(1 + \frac{\overline{c} - \underline{c}}{\overline{y}}\right) \equiv \theta^1$$

Symmetrically, the condition for $\overline{c}$ to be a dominant strategy (when both neighbors invest $\overline{c}$ and all other firms invest $\underline{c}$) is

(4) $$\overline{\theta} > \frac{1}{2}\left(1 + \frac{\overline{c} - \underline{c}}{\underline{y}}\right) \equiv \theta^2$$

From (3) and (4), $\theta^1 < \theta^2$ since $\overline{y} > \underline{y}$. Thus, for small values of $\overline{\theta}$ it is dominant to invest $\underline{c}$ and for large values it is dominant to invest $\overline{c}$ [1]. For intermediate values we will now show that there is a multiplicity of equilibria.

If all firms invest $\overline{c}$ then the payoff for firm *i* is $\overline{y} - \overline{c}$. For this to be an equilibrium no firm should have an incentive to deviate by investing a lower amount. This requires that $\overline{y} - \overline{c} > 2(1-\theta(\varepsilon))\overline{y} - (\overline{c} - \varepsilon)$, which holds when $\overline{\theta} > \theta^1$. Similarly, when all firms invest $\underline{c}$ the payoff for firm *i* is $\underline{y} - \underline{c}$. This is an equilibrium when $\underline{y} - \underline{c} > 2\theta(\varepsilon)\underline{y} - (\underline{c} + \varepsilon)$, which holds when $\overline{\theta} < \theta^2$.

Assume now that all firms invest $\widetilde{c} = \alpha\underline{c} + (1-\alpha)\overline{c}$: if the production function $y = f(c)$ is linear, *Y/n* is therefore equal to $[\alpha\underline{y} + (1-\alpha)\overline{y}]$. Firm *i* has no incentive to change its level of investment when both the following inequalities hold: (1) $Y/n - \widetilde{c} \geq 2\theta(\varepsilon)Y/n - (\widetilde{c} + \varepsilon)$, and (2) $Y/n - \widetilde{c} \geq 2(1-\theta(\varepsilon))Y/n - (\widetilde{c} - \varepsilon)$. These conditions are jointly met if and only if

(5) $$\overline{\theta} = \frac{1}{2} + \frac{\overline{c} - \underline{c}}{2Y/n} = \frac{1}{2} + \frac{\overline{c} - \underline{c}}{2[\alpha\underline{y} + (1-\alpha)\overline{y}]}$$

To summarize, there is a unique symmetric Nash equilibrium when $\overline{\theta} < \theta^1$ and when $\overline{\theta} > \theta^2$. For $\theta \in (\theta^1, \theta^2)$ there are three symmetric Nash equilibria where all firms invest $\underline{c}$, or $\overline{c}$, or $\widetilde{c}$ (which depends on $\overline{\theta}$, as one can see by solving equation (5) for α).

Next, consider what happens when in the economy (1-*α*)*n* contiguous firms invests $\overline{c}$, and the remaining *αn* firms invest $\underline{c}$ instead. The payoff for $\overline{c}$-firms in the interior region is $[\alpha\underline{y} + (1-\alpha)\overline{y}] - \overline{c}$, and $[\alpha\underline{y} + (1-\alpha)\overline{y}] - \underline{c}$ for interior $\underline{c}$-firms. Firms are better off if they operate in an environment in which their neighbors are not aggressive. For firms at the edges, the payoff is $(1/2 + \overline{\theta})[\alpha\underline{y} + (1-\alpha)\overline{y}] - \overline{c}$ if they invest $\overline{c}$ and $(1/2 + (1-\overline{\theta}))[\alpha\underline{y} + (1-\alpha)\overline{y}] - \underline{c}$ if they invest $\underline{c}$. In order for the configuration to be an equilibrium, firms at the edges must be indifferent between the two extreme strategies. This implies

[1] When the competitive advantage is a non-linear function of costs similar conditions hold, although the actual values are different.

(6) $$\bar{\theta} = \frac{1}{2}\left[1 + \frac{\bar{c} - \underline{c}}{\alpha\underline{y} + (1-\alpha)\bar{y}}\right] \in (\theta^1, \theta^2)$$

or

(7) $$\alpha = \frac{1}{\bar{y} - \underline{y}}\left(\bar{y} - \frac{\bar{c} - \underline{c}}{2\bar{\theta} - 1}\right)$$

If firms are restricted to only two levels of investment, one can prove the following lemma.

Lemma 1. If $\theta \in [\theta^1, \theta^2]$, there is a pure-strategy Nash equilibrium in which a proportion α of firms invest $\underline{c}$ and the remaining (1-α) invest $\bar{c}$, where α is given by equation (7).

Proof. See Appendix.

Lemma 1 shows that the previous result does not depend on the fraction of low-investing firms being contiguous. The above conditions are independent of the exact configuration of firms along the circle, *i.e.*, location does not matter. However *ex-post*, each firm in the economy prefers to have less competition in its environment. Expected profit to firm *i* if both neighbors invest $\underline{c}$ is $\alpha\underline{y} + (1-\alpha)\bar{y} - \underline{c}$; if both its neighbors invest $\bar{c}$ it is $\alpha\underline{y} + (1-\alpha)\bar{y} - \bar{c}$; and it is $\alpha\underline{y} + (1-\alpha)\bar{y} - (\underline{c} + \bar{c})/2$ if one neighbor invests $\underline{c}$ and the other $\bar{c}$.

When all firms invest $\alpha\underline{c}+(1-\alpha)\bar{c}$, the firms' total surplus is

(8) $$W = n[\alpha(\underline{y} - \underline{c}) + (1-\alpha)(\bar{y} - \bar{c})]$$

It follows that, when $\bar{y} - \bar{c} > \underline{y} - \underline{c}$, firms' surplus is maximized iff all firms invest $\bar{c}$. The opposite holds when $\bar{y} - \bar{c} < \underline{y} - \underline{c}$.

Note that, under constant or increasing returns to scale, the condition $\bar{y} - \bar{c} > \underline{y} - \underline{c}$ is always satisfied. Therefore the optimum is achieved when all firms invest $\bar{c}$ (and in general the surplus increases with the probability with which firms invest $\bar{c}$).

If $\theta \in (\theta^1, \theta^2)$ there could be a co-ordination failure, with firms implementing a strategy which is individually rational but socially inefficient. All firms could be made better off if it were possible to co-ordinate their activity to the high productivity equilibrium.

The social optimum is achieved when firms can co-ordinate their investment to the level of activity characterized by the highest productivity. When this happens, in equilibrium there will be no net competitive advantages among firms, because they all undertake the same level of investment.

Note that the game is supermodular in pure strategies, according to the definition of Milgrom and Roberts (1990). Supermodularity is an extension of the notion of strategic complementarities. These arise if "an increase in one player's strategy increases the *optimal strategy* of the other players" (Cooper and John, 1988, p. 442), which requires that the set of strategies be ordered. If payoffs are monotonic in the strategies of the other players and the supermodular game exhibits a multiplicity of symmetric equilibria, then these can be Pareto-ranked. Hence one has a *co-ordination game*, whereby a decentralized economy can find itself in a "bad" equilibrium. Individual, non-co-operative rationality prevents the economy from moving to a better equilibrium, even when such an equilibrium exists.

4. Uncertainty and equilibrium selection

In this section we assume that there is an idiosyncratic component in the uncertainty about the competitive advantage of each firm. Each firm *i* perceives the maximum *ex ante* demand on the arc to be $\overline{\theta}_i = \overline{\theta} + \nu_i$, where ν_i is symmetrically distributed over $[-\nu,\nu]$, $\nu>0$. There is an idiosyncratic component in the uncertainty about the competitive advantage of each firm. There are several ways to introduce uncertainty which are relevant in our model. According to a first approach, the firm has imperfect knowledge of the advantage associated with a more aggressive strategy, in terms of its ability to steal business away from its rivals. We assume that this uncertainty can be characterized by some small and symmetrically distributed noise about the true mean of the parameter. A second approach is to let firms observe a noisy signal about their local competitive advantage, where these signals are correlated between neighbors (but not necessarily farther away on the variety space). The firms' exact behavior will depend on the specific assumptions about the noise. Under both approaches an endogenous selection process will take place: in the first case firms will almost always co-ordinate on a particular equilibrium, in the second case there could still remain a region of indeterminacy.

In the presence of noise, the firm's behavior should be modeled as a function both of its own signal and of all the possible signals of the other firms. Each firm makes inferences about the possible behavior of the other firms given the possible signal they might receive, and chooses its reaction function to maximize its expected payoff. The symmetry of the noise implies that firm *i*'s best predictor for the true value of the competitive parameter is its own signal. Moreover, the signal is also the best predictor for the neighbor's signal. These properties are crucial for the results.

Firms have a dominant strategy for extreme values of the competitive advantage. Taking this into account when calculating their optimal reaction function, some strategies become dominant over a larger region of $\overline{\theta}$. In fact, if the noise is sufficiently small, then the global game is dominant solvable, *i.e.* iterated elimination of strictly dominated strategies will lead firms to co-ordinate their behavior for any possible value of the competitive advantage.

Suppose firm *i*'s signal is $\overline{\theta}_i$, where $\overline{\theta}_i - \theta^1 >> \varepsilon$. The firm knows with certainty that $\overline{\theta}$ is greater than θ^1. However, if its behavior is part of a consistent plan, it must take account of what its neighbors will do when their signal is $\overline{\theta}_i - \varepsilon$. The neighbors' behavior in turn depends on what firm *i* will do for $\overline{\theta}_i - 2\varepsilon$, *etc.* Hence, each firm must consider the optimal behavior for signals smaller than θ^1. If the signal observed by the firm is less that the average of the critical values $(\theta^1 + \theta^2)/2$, this can lead firms to co-ordinate to the low-investment equilibrium $\underline{c}$. Each firm will in fact compute the posterior probability for their neighbors' signals, and this procedure will lead to the least risky course of action. Ultimately, the strategy of the firm is critically influenced by the result that all firms are restricted to investing $\underline{c}$ when $\overline{\theta}_i < \theta^1$. By repeated elimination of strictly dominated strategies, the firm will find it optimal to choose a low level of activity whenever $\overline{\theta}_i < (\theta^1 + \theta^2)/2$. Conversely, if the firm observes a signal larger than the average $(\theta^1 + \theta^2)/2$, the process of iterated elimination of the "riskier" strategies will lead to co-ordination to the high-activity equilibrium, $\overline{c}$.

The above intuition is formalized in the following proposition.

Proposition 1. If v>0 then iterated elimination of strictly dominated strategies results in each firm investing $\underline{c}$ if $\overline{\theta}_i < (\theta^1 + \theta^2)/2$, and $\overline{c}$ if $\overline{\theta}_i > (\theta^1 + \theta^2)/2$.

The proof of Proposition 1 is in the Appendix. It is important to consider the implication of this result for the aggregate behavior of the economy. If θ is such that the support of individual firms' posteriors $[\theta \pm 2v]$ does not include the value $(\theta^1 + \theta^2)/2$, then all firms in the economy co-ordinate on the risk-dominant equilibrium. Otherwise, the proportion of firms investing $\underline{c}$ is $1/2 + [(\theta^1 + \theta^2)/2 - \theta]/2v$ if $\theta \leq (\theta^1 + \theta^2)/2$, and $[\theta - (\theta^1 + \theta^2)/2]/2v$ otherwise. In any case, if v is small, firms will almost always co-ordinate.

This case shows that, if shocks are correlated among firms, then even a very small amount of uncertainty will lead firms endogenously to co-ordinate (almost always). Note however that the selection process is not guided by Pareto optimality.

5. Locally correlated signals

In the present section, the information structure is directly related to the local interaction structure in the economy. The signal perceived by firms is correlated amongst neighboring firms and uncorrelated otherwise. A justification for this assumption is that neighboring firms compete over overlapping segments of the market, and each firm specialized in collecting and processing information related to the local market in which it operates.

In order to analyze firms' behavior in this setting, we first need to specify their expectations about the "average behavior" in the economy. We can now make use of the fact that firm *i*'s payoff depends on the behavior of its immediate neighbors and on the average output of the other *n-3* firms in the economy. Let the competitiveness parameter on arc (*i-1,i*) be $\theta_i = \theta + \eta_i$, where θ is a r.v. with expected value $\hat{\theta}$ and variance σ_θ^2, and η_i are i.i.d. r.v.s with expected value $E(\eta_i) = 0$ and variance σ_η^2, orthogonal to θ. Firm i receives a signal equal to the average of the competitiveness parameters on both sides: $\overline{\theta}_i = (1/2)(\theta_i + \theta_{i+1}) = \theta + (1/2)(\eta_i + \eta_{i+1})$.

In other words, firms have a prior expectation on the average signal in the economy, $\hat{\theta}$. This signal is related, in equilibrium, to the total level of output in the economy. In addition, each firm observes an idiosyncratic signal $\overline{\theta}_i$, whose realization depends on the local market conditions. The firm therefore has to weigh its prior expectation on the total level of activity with the individual information received, that reflects local market conditions.

The solution of the firm's problem yields the following Proposition.

Proposition 2.

- If $\hat{\theta} < \theta^1$, all firms invest $\underline{c}$ if $\overline{\theta}_i < \theta^2$ and $\bar{c}$ otherwise.
- If $\hat{\theta} > \theta^2$, all firms invest $\bar{c}$ if $\overline{\theta}_i > \theta^1$ and $\underline{c}$ otherwise.
- If $\hat{\theta} \in (\theta^1, \theta^2)$, all firms invest $\underline{c}$ if $\overline{\theta}_i < \hat{\theta}$ and $\bar{c}$ otherwise.

Proposition 2 says that, if firms' expectations about total output are very low, they will co-ordinate on the low investment equilibrium in the region of multiplicity. If expectations are very high, they will co-ordinate on the high investment equilibrium. For intermediate values of the expectations, the endogenous selection will ensure co-ordination: the critical threshold will be consistent with the economy-wide expected value of the signal, $\hat{\theta}$.

In the proof of Proposition 2 we have made use of the assumption $\sigma_\eta^2 / \sigma_\theta^2 \to 0$, that is, relative to the aggregate expectation, the local signal is uninformative about the idiosyncratic shock. This is required for firms always to co-ordinate to the risk-dominant equilibrium. As the local signal becomes more informative, there will still be some endogenous selection, but regions of indeterminacy will arise.

It is important to consider the implications of Proposition 2 for aggregate behavior. We have that aggregate behavior depends on the statistical distribution of θ. The proportion of firms investing $\underline{c}$ is given by $\text{Prob}(\overline{\theta} \le \hat{\theta})$. As in the case of Proposition 1, if v is small, firms will almost always co-ordinate.

The previous result depends on all firms sharing the same expectation regarding average output. If this assumption is removed, the result does not hold. Think, for example, of firms as receiving an additional signal about the state of the economy in the form of $\hat{y}_i$. Behavior in this multi-dimensional signal space is, in general, much more complicated than described before. It is still possible to see, using the previous calculations, that the following holds: if $\hat{\theta}_i(\hat{y}_i)$, and $\overline{\theta}_i$ are not too "distant", iterated elimination of dominated strategies will force all firms to switch from $\underline{c}$ to $\bar{c}$ at $\widetilde{\theta} = (\theta^1 + \theta^2)/2$. Hence, the relative proportion of firms investing $\underline{c}$ and $\bar{c}$ is $F(\widetilde{\theta})$ and $1 - F(\widetilde{\theta})$, where $F(\cdot)$ is the cumulative distribution function. This, however, is not true when the two observations are "distant". Assume, for example, that $\overline{\theta}_i = \theta^1 + \gamma$ and $\hat{\theta}_i = \theta^2 - \gamma$ (where γ is small). Two conflicting forces operate on the decision maker: on the one hand, $\bar{c}$ is still riskier than $\underline{c}$; on the other, the expected aggregate income in the economy is high, thus making a switch to $\bar{c}$ more profitable. Firms will invest $\underline{c}$ for values of individual signals less than θ^1 or slightly above it, and $\bar{c}$ for values greater than θ^2 or slightly smaller (the exact boundaries depending on the signal about the state of the economy). Iterated elimination of strictly dominated strategies leaves a region of indeterminacy. However, we still obtain some endogenous co-ordination over regions with a multiplicity of Nash equilibria.

6. Conclusions.

This paper analyses a simple model of co-ordination failure based on local oligopoly. The key parameter for firms is the competitive advantage they can gain over their neighbors by undertaking higher levels of investment, through a business stealing effect. Over a non-singular range of values of the competitive advantage parameter, the economy exhibits multiple equilibria. The decentralized market outcome could be socially inefficient because of the firms' failure to co-ordinate on a high-productivity equilibrium. The neighborhood structure described in the paper can be responsible for multiplicity of equilibria and market failures.

In the absence of noise, the set of possible equilibria depends on the competitiveness parameter, θ. Either firms have a dominant strategy, or there is a multiplicity of equilibria. If firms only have two investment strategies, the proportions engaging in a high or a low level of investment depend on the exact value of θ, but are independent of the exact configuration of firms in the economy. In the absence of explicit co-ordination devices, the economy could settle on any of the possible equilibria.

However, if one introduces uncertainty in the economy, and allows firms to observe imperfect signals about the competitiveness conditions in the local output market, an endogenous equilibrium selection process could take place. In particular, when firms' noisy signals are correlated among neighbors, iterated elimination of strictly dominated strategies significantly reduces the set of possible market outcomes. Firms choose a low level of investment for bad signals, and a high level for good signals. This would correspond to the adoption of the risk-dominant strategy, in the sense of Harsanyi and Selten (1988). If the firms' expectation about the state of the economy is the same for all firms, then they will all switch from low to high levels of investment at a critical value of θ. This value is the unique θ for which the expected average output of the economy is equal to its value in the intermediate investment equilibrium identified in section 3. If firms' expectations about the state of the economy depend on an additional signal, then the switch from low to high investment will occur at $(\theta^1 + \theta^2)/2$, provided this signal is not too different from their own observation of θ.

Our analysis shows that the economy is more likely settle on some equilibria than on others. A full understanding of these robustness criteria is necessary before any policy which is intended to help co-ordinate the level of activity to a Pareto dominant outcome can be successfully implemented.

Appendix

Proof of Lemma 1.

In Section 3 we have already shown that the strategy profile where all firms invest $\underline{c}$ is a Nash equilibrium if and only if $\overline{\theta} \leq \theta^2$, and that the case where all firms invest $\overline{c}$ is a Nash equilibrium if and only if $\overline{\theta} \geq \theta^1$. To complete the proof of Proposition 1, it remains to be shown that, for $\overline{\theta} \in [\theta^1,\ \theta^2]$, every configuration in which a proportion $\alpha(\overline{\theta}) = [1/(\overline{y} - \underline{y})][\overline{y} - (\overline{c} - \underline{c})/(2\overline{\theta} - 1)]$ of firms invest $\underline{c}$, and $(1 - \alpha(\overline{\theta}))$ invest $\overline{c}$ is a Nash equilibrium.

Each firm in the economy can be in exactly one of the following six configurations of investment behavior and neighborhood structure:

(1) the firm invests $\underline{c}$ and both its neighbors invest $\underline{c}$;

(2) the firm invests $\underline{c}$ and both its neighbors invest $\overline{c}$;

(3) the firm invests $\underline{c}$, one of its neighbors invests $\underline{c}$ and the other $\overline{c}$;

(4) the firm invests $\overline{c}$, one of its neighbors invests $\underline{c}$ and the other $\overline{c}$;

(5) the firm invests $\overline{c}$ and both its neighbors invest $\underline{c}$;

(6) the firm invests $\overline{c}$ and both its neighbors invest $\overline{c}$.

We next show that, if the relationship between θ and α is as in equation (7), the firm will have no incentive to change its behavior in any of the possible configurations.

Cases (1) and (2). Firm *i*'s payoff from investing $\underline{c}$ is $\pi_i = (1/n)(Y) - \underline{c}$, whereas if it invests $\overline{c}$ it will receive $\pi_i = (2\overline{\theta}/n)(Y) - \overline{c}$. The difference (the incentive to deviate) is:

$$\frac{Y.}{n}(1 - 2\overline{\theta}) - (\underline{c} - \overline{c}) \;=\; \frac{\alpha n\underline{y} + (1-\alpha)n\overline{y}}{n}\left(1 - 1 - \frac{\overline{c} - \underline{c}}{\alpha\underline{y} + (1-\alpha)\overline{y}}\right) - (\underline{c} - \overline{c}) \;=\; 0$$

Therefore the firm has no incentive to change its investment strategy in either of these two cases.

Cases (3) and (4). Firm *i*'s payoff from investing $\underline{c}$ is $\pi_i = (1/n)[(3/2) - \overline{\theta}](Y) - \underline{c}$, whereas if it invests $\overline{c}$ it will receive $\pi_i = (1/n)[(1/2) + \overline{\theta}](Y) - \overline{c}$. The difference (the incentive to deviate) is:

$$\frac{Y}{n}\left(\frac{3}{2} - \overline{\theta} - \frac{1}{2} - \overline{\theta}\right) - (\underline{c} - \overline{c}) \;=\; \frac{\alpha n\underline{y} + (1-\alpha)n\overline{y}}{n}\left(1 - 1 - \frac{\overline{c} - \underline{c}}{\alpha\underline{y} + (1-\alpha)\overline{y}}\right) - (\underline{c} - \overline{c}) \;=\; 0$$

Therefore the firm has no incentive to change its investment strategy in either of these two cases.

Cases (5) and (6). Firm i's payoff from investing $\underline{c}$ is $\pi_i = [2(1-\overline{\theta})/n](Y) - \underline{c}$, whereas if it invests $\overline{c}$ it will receive $\pi_i = (1/n)(Y) - \overline{c}$. The difference (the incentive to deviate) is:

$$\frac{Y}{n}(2-2\overline{\theta}-1)-(\underline{c}-\overline{c}) \;=\; \frac{\alpha n\underline{y}+(1-\alpha)n\overline{y}}{n}\left(1-1-\frac{\overline{c}-\underline{c}}{\alpha\underline{y}+(1-\alpha)\overline{y}}\right)-(\underline{c}-\overline{c}) \;=\; 0$$

Therefore the firm has no incentive to change its investment strategy in either of these two cases.

Proof of Proposition 1.

The investment behavior of each firm can be described by a function $f_i:[\overline{\theta}_i - v, \overline{\theta}_i + v] \to [0,\overline{a}]$. As all firms are restricted to investing $\underline{c}$ when $\overline{\theta}_i < \theta^1$ (and in particular firms *i-1* and *i+1*), then the expected payoffs to firm i when it observes $\overline{\theta}_i = \theta^1$ are:

$$\frac{1}{3}\left(\frac{Y}{n}-\underline{c}\right)+A$$

when it invests $\underline{c}$, and

$$\frac{1}{3}\left(2\theta(\varepsilon)\frac{Y}{n}-(\underline{c}+\varepsilon)\right)+B$$

when it invests $\underline{c}+\varepsilon$, where A and B are defined as:

$$A = \int_{[0,v]^2}\{[1-\theta_i(|f_{i+1}(\overline{\theta}_{i+1})|)]+[1-\theta_{i-1}(|f_{i-1}(\overline{\theta}_{i-1})|)]-\underline{c}\}h(\overline{\theta}_{i-1},\overline{\theta}_{i+1}|\overline{\theta}_i)d\overline{\theta}_{i-1}d\overline{\theta}_{i+1}$$

$$B = \int_{[0,v]^2}\{[1-\theta_i(|\varepsilon-f_{i+1}(\overline{\theta}_{i+1})|)]+[1-\theta_{i-1}(|\varepsilon-f_{i-1}(\overline{\theta}_{i-1})|)]-(\underline{c}+\varepsilon)\}\cdot$$
$$\cdot h(\overline{\theta}_{i-1},\overline{\theta}_{i+1}|\overline{\theta}_i)d\overline{\theta}_{i-1}d\overline{\theta}_{i+1}$$

where $h(\overline{\theta}_{i-1},\overline{\theta}_{i+1}|\overline{\theta}_i)$ is the joint probability density function of $(\overline{\theta}_{i-1},\overline{\theta}_{i+1})$, conditional on $\overline{\theta}_i$. The inequality $A \geq B$ follows from the definition of $\theta(\cdot)$ and from substituting $\overline{\theta}_i = \theta^1$ from equation (3). In order to show that $\underline{c}$ dominates all other investment strategies it remains to prove the following inequality:

$$\frac{Y}{n}-\underline{c} > 2\theta(\varepsilon)\frac{Y}{n}-(\underline{c}+\varepsilon)$$

Substituting $\overline{\theta}_i = \theta^1$ from (3) we obtain that the above inequality is equivalent to

$$\overline{y} > \frac{Y}{n}$$

The last inequality holds since *Y/n* (the expected average demand) is at most equal to $(\underline{y}+\overline{y})/2$ (because half of the firms are expected to receive signals smaller than θ^1, and therefore to invest $\underline{c}$), and in particular is smaller than $\bar{y}$.

Therefore iterated elimination of strictly dominated strategies forces all firms to invest $\underline{c}$ for $\bar{\theta}_i = \theta^1$. Denote by $\tilde{\theta}$ the smallest value of $\bar{\theta}_i$ for which iterated elimination of strictly dominated strategies does not force firms to invest $\underline{c}$. From the above inequalities it is clear that

(A1) $$\tilde{\theta}(y) \geq \frac{\theta^1 + \theta^2}{2}$$

Symmetrically, all firms are restricted to investing $\bar{c}$ when $\bar{\theta}_i > \theta^2$ (and in particular firms *i-1* and *i+1*). The expected payoffs to firm *i* when it observes $\bar{\theta}_i = \theta^2$ are:

$$\frac{1}{3}\left(\frac{Y}{n} - \bar{c}\right) + C$$

when it invests $\bar{c}$, and

$$\frac{1}{3}\left(2(1-\theta(\varepsilon))\frac{Y}{n} - (\underline{c} - \varepsilon)\right) + D$$

when it invests $\bar{c} - \varepsilon$, where

$$C = \int_{[-v,0]^2} \{[\theta_i(|f_{i+1}(\bar{\theta}_{i+1})|) + \theta_{i-1}(|f_{i-1}(\bar{\theta}_{i-1})|)] - \bar{c}\} h(\bar{\theta}_{i-1}, \bar{\theta}_{i+1} | \bar{\theta}_i) d\bar{\theta}_{i-1} d\bar{\theta}_{i+1}$$

$$D = \int_{[-v,0]^2} \{[\theta_i(|\varepsilon - f_{i+1}(\bar{\theta}_{i+1})|) + \theta_{i-1}(|\varepsilon - f_{i-1}(\bar{\theta}_{i-1})|)] - (\bar{c} - \varepsilon)\} \cdot$$
$$\cdot h(\bar{\theta}_{i-1}, \bar{\theta}_{i+1} | \bar{\theta}_i) d\bar{\theta}_{i-1} d\bar{\theta}_{i+1}$$

For similar considerations as above, *C≥D*. Investing $\bar{c}$ dominates all other strategies when:

$$\frac{Y}{n} - \bar{c} > 2(1-\theta(\varepsilon))\frac{Y}{n} - (\bar{c} - \varepsilon)$$

Substituting $\bar{\theta}_i = \theta^2$ from equation (4) we obtain

$$\underline{y} < \frac{Y}{n}$$

The last inequality holds since Y/n is at least equal to $(\underline{y} + \bar{y})/2$ and in particular is greater than $\underline{y}$.

Therefore iterated elimination of strictly dominated strategies forces all firms to invest $\bar{c}$ for $\bar{\theta}_i = \theta^2$. Denote by $\tilde{\tilde{\theta}}$ the largest value of $\bar{\theta}_i$ for which iterated elimination of strictly dominated strategies does not force firms to invest $\bar{c}$. From the above inequalities it is clear that

(A2) $$\tilde{\tilde{\theta}}(y) \leq \frac{\theta^1 + \theta^2}{2}$$

Additionally, $\tilde{\theta}(y) \leq \tilde{\tilde{\theta}}(y)$. Combining this with equations (A1) and (A2) we obtain $\frac{\theta^1+\theta^2}{2} \leq \tilde{\theta}(y) \leq \tilde{\tilde{\theta}}(y) \leq \frac{\theta^1+\theta^2}{2}$, *i.e.* firms' behavior is as described in Proposition 1.

QED

Proof of Proposition 2.

The solution to the signal extraction problem gives $E(\overline{\theta}_{i+1}|\overline{\theta}_i) = E(\overline{\theta}_{i-1}|\overline{\theta}_i) = \gamma \cdot \overline{\theta}_i + (1-\gamma)\cdot\hat{\theta}$ where $\gamma \equiv \frac{1/4\cdot\sigma_\eta^2}{\sigma_\theta^2 + 1/2\cdot\sigma_\eta^2} \in (0,1)$. As in the proof of Proposition 1, all firms are restricted to investing $\underline{c}$ when $\overline{\theta}_i < \theta^1$ (and in particular firms i-1 and i+1). The expected payoffs to firm i when it observes $\overline{\theta}_i = \theta^1$ are:

$$g(\gamma)\left(\frac{Y}{n} - \underline{c}\right) + [1-g(\gamma)]A'$$

when it invests $\underline{c}$, and

*

$$g(\gamma)\left[2\theta(\varepsilon)\frac{Y}{n} - (\underline{c}+\varepsilon)\right] + [1-g(\gamma)]B'$$

when it invests $\underline{c}+\varepsilon$, where $g(\gamma) \in (0,1)$ is the probability that firm i attaches to the event that its neighbors observe a signal smaller than $\overline{\theta}_i$ and where A' and B' are defined as in the proof of Proposition 1, with the difference that here we integrate over $[0,\eta]^2$ instead of over $[0,\nu]^2$:

$$A' = \int_{[0,\eta]^2} \{[1-\theta_i(|f_{i+1}(\overline{\theta}_{i+1})|)] + [1-\theta_{i-1}(|f_{i-1}(\overline{\theta}_{i-1})|)] - \underline{c}\} h(\overline{\theta}_{i-1}, \overline{\theta}_{i+1}|\overline{\theta}_i)\, d\overline{\theta}_{i-1} d\overline{\theta}_{i+1}$$

$$B' = \int_{[0,\eta]^2} \{[1-\theta_i(|\varepsilon - f_{i+1}(\overline{\theta}_{i+1})|)] + [1-\theta_{i-1}(|\varepsilon - f_{i-1}(\overline{\theta}_{i-1})|)] - (\underline{c}+\varepsilon)\} \cdot$$
$$\cdot h(\overline{\theta}_{i-1}, \overline{\theta}_{i+1}|\overline{\theta}_i)\, d\overline{\theta}_{i-1} d\overline{\theta}_{i+1}$$

The expected payoff from investing $\underline{c}$ is greater than from investing $\underline{c}+\varepsilon$ if

$$\bar{y} > \frac{Y}{n}$$

Here, unlike in Proposition 1, the expected average income Y/n is obtained by replacing $\hat{\theta}$ into equation (5). Let $\hat{y}$ denote expected average income. Iterated elimination of strictly dominated strategies will force firms to invest $\underline{c}$ until the appropriate inequalities (see the proof for Proposition 1) no longer hold. This implies that firms will invest $\underline{c}$ for $\overline{\theta}_i \leq \hat{\theta}$. Similarly, all firms will invest $\overline{c}$ for $\overline{\theta}_i \geq \hat{\theta}$. Hence the result.

QED

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