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Fiscal Policy and Exchange Rates

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Abstract

This paper examines the dynamics of the nominal exchange rate and fiscal deficits in a continuous time optimizing general equilibrium model with finite horizon. It is shown that alternative financing modes of budget deficits imply different patterns of adjustment along the transitional path towards the steady state equilibrium. In particular, the respect of public solvency without money financing is not sufficient to avoid the depreciation of the exchange rate in the long-run after a fiscal expansion.

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1 Introduction

Monetary variables and equilibrium in international financial markets typically are the crucial variables determining exchange rate dynamics. Recent contributions have challenged this view, stressing the importance of fiscal variables in determining the time pattern of exchange rates. In particular, the fiscal theory of the price level has been extended to open economy models. The effects of fiscal policies on real economic variables, on the other hand, have been analyzed in a variety of models with forward-looking agents and finite horizons. Notably, Frenkel and Razin (1986) study the effects of tax cuts on the world interest rates, on consumption and on the current account in a two country interdependent economy model. Helpman and Razin (1987) analyze the dependence of exchange rate management on the time pattern of fiscal and monetary policies. Daniel (1993) studies the effects of a tax cut in an economy where there is uncertainty regarding the time of a future tax increase to balance the budget. Kawai and Maccini (1995) examine the effects of fiscal deficits on a small open economy with flexible exchange rates and finite horizons. The fiscal deficit is assumed to be financed by selling bonds and is anticipated to be financed in the future by seignorage, tax increases or some combination of the two. The effects of a permanent change in the price of the imported materials on the current account are examined by Matsuyama (1987), while the steady state effects of monetary policy are studied by Giovannini (1988) in a small open economy model with capital accumulation. Van der Ploeg (1991) analyzes the implications of monetary policy in a two-country optimizing model with capital accumulation.

More recently, the fiscal theory of price determination has been extended to the open economy with infinite horizon agents, showing the implications for exchange rate systems and common currency areas. The fiscal theory of the price level has become increasingly popular following the contributions of

Woodford (1994, 1995) and Sims (1994) and has its roots in the paper originally presented by Sargent and Wallace (1981). Canzoneri et al. (2001) show that more severe fiscal discipline is required in common currency areas where national fiscal authorities enjoy less autonomy in pursuing their objectives. They distinguish between *Ricardian* and *non-Ricardian* regimes, following the terminology coined by Woodford (1995); in the first case the nominal anchor is provided by monetary policy and the exchange rate is determined according to standard theories, while in the second case fiscal policy serves as the nominal anchor and determines the exchange rate. In other words, in a *non-Ricardian* regime prices and not deficits adjust to satisfy the intertemporal budget constraint of the government¹. Dupor (2000) studies the determination of the exchange rate in a two-country cash-in-advance model and shows that the exchange rate cannot univocally be determined when governments are assumed to peg the nominal interest rate on domestic bonds. Daniel (2001a) extends the fiscal theory of the price level to an open economy and demonstrates how the exchange rate is determined to ensure intertemporal fiscal solvency of the public sector, showing how indeterminacy of prices and of exchange rates can be avoided. In a different contribution Daniel (2001b) provides a fiscal theory of currency crises, showing that the crisis cannot be avoided when the present discounted value of primary surpluses is less than the present value of government debt at the pegged level of exchange rate. The monetary authorities are forced to abandon the peg, in

¹The fiscal theory of the price level uses the term *Ricardian* to indicate the irrelevance of a fiscal regime for the determination of the price level, i.e. the intertemporal budget constraint of the government necessarily holds regardless of the time path of the price level. On the other hand, according to Aiyagari and Gertler (1985) a fiscal regime can be defined as *Ricardian* only when there is no money financing and the value of the public debt equalizes the present discounted value of future taxes. The validity of the fiscal theory of the price level has recently been seriously questioned by Buiter (2002).

order to extract the necessary seignorage revenues to restore public sector solvency.

The main purpose of this paper is to build a general framework describing the implications of fiscal policy in the determination of the nominal exchange rate when there are finite horizons. The fiscal authorities may close the deficits by reducing public expenditure or by raising taxes while the monetary authorities independently set the rate of money growth in order to provide the monetary anchor for the exchange rate. Alternatively, the authorities may elect to extract seignorage revenues to close the deficits and therefore monetary policy is not independent, since the rate of money growth is chosen in order to satisfy the intertemporal budget constraint of the government. The present paper emphasizes the role of fiscal policy in the determination of the nominal exchange rate.

We present a small open economy of perpetually young consumers with flexible exchange rates and forward-looking agents, in which Ricardian equivalence does not hold. The aim is to analyze the effects of a fiscal deficit, followed by later surpluses, according to a tax rule increasing in public debt, so as to always satisfy the intertemporal budget constraint of the government. In particular, this paper studies the effects of a current and unexpected lump-sum tax cut which initially implies a sequence of budget deficits. As debt accumulates the government is assumed to increase taxes, use seignorage or employ some combinations of the two to close the deficit².

The analysis shows how fiscal variables determine the dynamics of the exchange rate. We demonstrate that after a fiscal expansion the respect of public solvency without money financing is not sufficient to avoid the

²Kawai and Maccini (1995) present a model where current fiscal deficits are anticipated to be closed in the future by a tax increase, money creation or some combination of the two. Similarly, but in a different set-up with infinite horizon agents, Drazen and Helpman (1987) describe how different stabilization policies affect the dynamics of economic variables.

depreciation of the exchange rate. In particular, it is shown that for any financing mode a fiscal expansion would lead on impact to an appreciation of the domestic currency, an increase in consumption and in real money holdings and to a depreciation of the exchange rate in the long run. On the other hand, alternative financing modes determine different effects on the level of net financial wealth of households in the long run: tax financed budget deficits reduce the level of wealth, while money financed budget deficits increase it.

The paper is organized as follows. Section II presents the optimizing general equilibrium model; Section III analyzes the short and long-run effects of a once-for-all lump-sum tax cut. Conclusions are summarized in Section IV.

2 The general equilibrium optimizing model

Consider a small open economy composed by two types of agents: households and the government. To keep the presentation simple, assume that it is a one-good world so that purchasing power parity (PPP) holds, $P_t^* E_t = P_t$, where E_t is the nominal exchange rate, P_t is the domestic price level and P_t^* is the foreign price level normalized to unity. There is perfect capital mobility and domestic and foreign assets are perfect substitutes, that is uncovered interest parity (UIP) is always verified. The real interest rate r_t^* on foreign assets is exogenously given and assumed to be constant over time. The two conditions on price levels and nominal interest rates imply real interest rate parity, so that the domestic real interest rate is $r_t = r^*$.

2.1 Households

The demand side of the economy is an extended version of the Yaari (1965)-Blanchard (1985) model where forward looking agents have finite horizons,

lifetime is uncertain and there is no bequest motive. Agents are assumed to be identical and face the same probability of death δ in each period of time.

The birth and death rates are the same. For convenience the size of each period generation of agents is normalized to δ and total population is equal to unity.

Households are assumed to maximize the discounted value of their expected utility subject to the appropriate budget constraint. The instantaneous utility function is logarithmic in total consumption and real money balances³. Individuals hold their financial wealth in government bonds, real money balances and foreign real assets. Each individual born at time s at each time period $t \geq s$ receives a positive endowment, $\omega(s, t)$.

The representative agent of generation s is assumed to solve the following optimization problem

$$\max_{c(s,t), \varphi(s,t)} \int_t^\infty \log [c(s, v)^\xi \varphi(s, t)^{1-\xi}] e^{-(\beta+\delta)(v-t)} dv \quad (1)$$

subject to the individual consumer's flow budget constraint

$$\begin{aligned} \frac{da(s, t)}{dt} &= (r^* + \delta)a(s, t) \\ &\quad + \omega(s, t) - \tau(s, t) - c(s, t) - i(t)\varphi(s, t) \end{aligned} \quad (2)$$

³Entering real money balances directly into the utility function allows to account for the services provided by money holdings. The idea behind this approach is that money yields direct utility to the consumers. It can be shown that, under some regularity conditions, the procedure of introducing money in the utility function is equivalent to the maximization problem with money modelled by means of a cash-in-advance constraint *à la* Clower or by liquidity costs (see Feenstra, 1986).

The introduction of real money balances as an argument of the utility function within a Blanchard-Yaari overlapping generations framework is common to several papers including Marini and van der Ploeg (1988), van der Ploeg (1991), Daniel (1993), Kawai and Maccini (1995).

and to the transversality condition

$$\lim_{t \rightarrow \infty} [a(s, t)] e^{-(r^* + \delta)t} = 0 \quad (3)$$

where $0 < \xi < 1$, $\beta < r^*$ ⁴ is the constant rate of time preference, $i(t)$ is the nominal interest rate at time t , $\varphi(s, t) = m(s, t)/E(t)$ represents real money balances, $c(s, t)$, $\omega(s, t)$, $m(s, t)$, $a(s, t)$ and $\tau(s, t)$, denote consumption, endowment, nominal money balances, total real non-human wealth and lump-sum taxes, of the generation born at time s , respectively. Total real non-human wealth consists of real money holdings, government bonds, $b(s, t)$, and foreign assets, $f(s, t)$.

The effective discount rate of consumers is given by $(\beta + \delta)$, where $\beta + \delta > r^*$ ⁵. Each individual is assumed to receive for every period of his life an actuarial fair premium equal to $\delta a(s, t)$ from a life insurance company. At the time of his death the individual's net wealth goes to the insurance company.

The solution to the consumer's maximization problem yields the following demand functions⁶

$$c(s, t) = \frac{\beta + \delta}{1 + \eta} [a(s, t) + h(s, t)] \quad (4)$$

⁴This condition ensures that consumers are relatively patient, in order that the steady state-level of aggregate financial wealth $A(t)$ is positive (see Blanchard, 1985 and Matsuyama, 1987).

⁵This condition ensures that savings are a decreasing function of wealth and that a steady-state value of aggregate consumption $C(t)$ exists. See Blanchard (1985).

⁶Given that preferences are intertemporally separable and the utility function is homothetic, the consumer's problem can be solved by using a two-stage budgeting procedure. Defining the total consumption as the sum of consumption plus the interest foregone on money holdings, the procedure works as follows. In the first stage, one determines the optimal mix of consumption and real money holdings, conditional upon a given level of total consumption. In the second stage, one derives the optimal time path of total consumption, given the budget constraint and the solvency condition (see Marini and van der Ploeg, 1988).

$$\frac{m(s, t)}{E(t)} = \frac{\eta c(s, t)}{r^* + \pi(t)} \quad (5)$$

where, $\eta \equiv \frac{1-\xi}{\xi}$ and $\eta < 1$, by assumption; $\pi(t) = \frac{\dot{E}(t)}{E(t)}$ is the rate of depreciation of the national currency and $h(s, t)$ is human wealth defined as the present discounted value of labor income net of taxes

$$h(s, t) = \int_t^\infty [\omega(s, v) - \tau(s, v)] e^{-(r^* + \delta)(v-t)} dv \quad (6)$$

Equation (4) shows that consumption is linear in total wealth, while equation (5) is the portfolio balance condition equalizing the marginal rate of substitution between consumption and real money holdings to the nominal interest rate.

Assuming that the wage rate and taxes are independent of age and that non-human wealth for newly born agents is zero, $a(t, t) = 0$, after aggregation over all the cohorts one obtains

aggregate consumption at time t

$$C(t) = \frac{\delta + \beta}{1 + \eta} [H(t) + A(t)] \quad (7)$$

the portfolio balance condition

$$\frac{\eta C(t) E(t)}{M(t)} = \frac{\dot{E}(t)}{E(t)} + r^* \quad (8)$$

total non-human wealth

$$A(t) = F(t) + \frac{M(t)}{E(t)} + B(t) \quad (9)$$

and total human-wealth

$$H(t) = \frac{\omega}{r^* + \delta} - \int_t^\infty T(v) e^{-(r^* + \delta)(v-t)} dv \quad (10)$$

where $C(t)$, $T(t)$, $A(t)$, $F(t)$, $M(t)$ and $B(t)$ denote aggregate consumption, taxes, non-human wealth, foreign assets, real money balances and government bonds at time t , respectively. The total endowment ω received by households is assumed to be constant over time. Macroeconomic variables, $x(s, t)$, are obtained by aggregating over all the generations as follows

$$X(t) = \int_{-\infty}^t x(s, t) \delta e^{\delta(s-t)} ds \quad (11)$$

The aggregate non-human wealth accumulation equation is

$$\dot{A}(t) = r^* A(t) + \omega - T(t) - C(t) - i(t) \frac{M(t)}{E(t)} \quad (12)$$

and the dynamic equation of aggregate human wealth is

$$\dot{H}(t) = (r^* + \delta)H(t) - \omega + T(t) \quad (13)$$

2.2 Government

The government can finance public expenditures and interest payments by seignorage, lump-sum tax and bond issues. The intertemporal budget constraint is given by

$$\dot{B}(t) = r^* B(t) + G(t) - T(t) - \frac{\mu(t)M(t)}{E(t)} \quad (14)$$

where G is total government spending and μ is the rate of nominal money growth. The government must remain solvent and respect the usual condition precluding Ponzi games

$$\lim_{t \rightarrow \infty} B(t) e^{-r^* t} = 0 \quad (15)$$

Forward integration of equation (14), given the transversality condition (15), yields the intertemporal budget constraint of the government

$$B(t) = \int_t^\infty \left[T(v) - G(v) + \mu(v) \frac{M(v)}{E(v)} \right] e^{-r^*(v-t)} dv \quad (16)$$

Equation (16) states that the government debt is equal to the present discounted value of future budget surpluses.

The authorities adopt the policy rules

$$T(t) = \psi \alpha B(t) - Z(t) \quad (17)$$

$$Z(t) = \begin{cases} 0 & \text{for } t < 0 \\ Z & \text{for } t \geq 0 \end{cases} \quad (18)$$

$$\mu(t) \frac{M(t)}{E(t)} = (1 - \psi) \alpha B(t) \quad (19)$$

$$B_0 = 0 \quad (20)$$

$$G(t) = 0 \text{ for } t \geq 0 \quad (21)$$

where Z is a once-for-all lump-sum tax cut, α is a positive parameter which links the taxes and the seignorage revenues to the level of public debt, B_0 is the initial condition for the stock of public debt, while government spending is set equal to zero for simplicity⁷. Taxes and seignorage are increasing functions of the public debt. The rate of nominal money growth $\mu(t)$ is endogenously determined. The parameter ψ is the weight of tax finance in closing the deficits and is such that $0 \leq \psi \leq 1$. When $\psi = 0$ budget deficits are entirely financed by seignorage and for $\psi = 1$ only by lump-sum taxes. Under pure tax finance the fiscal authority is assumed to adjust the sequence of taxes to satisfy its intertemporal budget constraint (16). The policy experiment is the following. At time $t = 0$ there is an unexpected and once-for-all increase of Z from zero, creating a sequence of deficits and an

⁷The government is assumed to adopt a *Ricardian regime*, as defined by Woodford (1995), Canzoneri et al. (2001) and Daniel (2001a). When $\psi = 1$ the policy is *Ricardian* in the sense of Aiyagari and Gertler (1985).

increase in government debt according to equation (16), that can be rewritten as

$$\dot{B}(t) = (r^* - \alpha)B(t) + Z \quad (22)$$

The transversality condition on the debt level (15) is satisfied if and only if $\alpha \geq r^*$. For simplicity it is assumed that the condition $\alpha > r^*$ holds all the time.

Following Blanchard (1985), the effects of current and anticipated fiscal changes on wealth and consumption can be summarized by the fiscal policy index

$$d(t) = \frac{\delta + \beta}{1 + \eta} \left\{ B(t) - \int_t^\infty T(v) e^{-(\delta+r^*)(v-t)} dv \right\} \quad (23)$$

Fiscal policy affects aggregate consumption through two channels. Public debt is part of the non-human wealth and lump-sum taxes reduce disposable labor income. Money finance reduces instead the real value of money balances via the inflation tax and increases the opportunity costs of money holdings. Integrating the equation of motion (22) for the time path of B and T , and substituting the result in (23) yields

$$d(t) = \frac{\delta + \beta}{1 + \eta} \frac{1}{(\alpha - r^*)} \left[\frac{\delta + \alpha}{\delta + r^*} - \psi \frac{\alpha}{\delta + r^*} - \left(1 - \psi \frac{\alpha}{\alpha + \delta}\right) e^{-(\alpha - r^*)t} \right] Z \quad (24)$$

Differentiating equation (24) with respect to time one obtains

$$\dot{d}(t) = (r^* - \alpha)d(t) + \frac{(\delta + \beta)}{(r^* + \delta)(1 + \eta)} [\delta + \alpha(1 - \psi)] Z \quad (25)$$

and

$$d_0 = \frac{\delta + \beta}{1 + \eta} \left[\frac{1}{\delta + r^*} - \psi \frac{\alpha}{(\delta + r^*)(\alpha + \delta)} \right] Z \quad (26)$$

$$d_\infty = \tilde{d} = \frac{(\delta + \beta)}{(r^* + \delta)(1 + \eta)(\alpha - r^*)} [\delta + \alpha(1 - \psi)] Z > d_0 \quad (27)$$

The value of the fiscal index depends on the level of the initial tax cut and on the degree of future tax finance ψ . During the adjustment process, as debt accumulates, the government budget goes from deficit to surplus, but the net effect is positive; hence, the index increases from d_0 to \tilde{d} ⁸.

2.3 The Basic Model

The rate of growth of real money balances is given by

$$RM\dot{(t)} = \left(\frac{\dot{M}(t)}{E(t)} \right) = [\mu(t) - \pi(t)] \frac{M(t)}{E(t)} \quad (28)$$

which combined with the portfolio budget condition (8) yields the equation of motion of real money balance $RM(t)$

$$RM\dot{(t)} = [r^* + \mu(t)] RM(t) - \eta C(t) \quad (29)$$

Substituting (22) and (28) into the dynamic equation of non-human wealth (12), one obtains

$$F\dot{(t)} = r^* F(t) + \omega - C(t) \quad (30)$$

which is the current account of the balance of payments and denotes the dynamic evolution of the economy's net external assets.

The basic model can be described by the following set of equations

$$C(t) = \frac{\delta + \beta}{1 + \eta} \left[\frac{\omega}{r^* + \delta} + F(t) + \frac{M(t)}{E(t)} \right] + d(t) \quad (31)$$

⁸It should be noted that the fiscal index only summarizes the direct effects produced by fiscal policy on consumption through lump-sum taxation and public debt. For this reason the index is equal to zero for $\delta = 0$ and $\psi = 1$, but not for $\delta = 0$ and $\psi = 0$. In the Appendix it is shown that in the case of infinite horizon and pure money finance, long-run superneutrality of money holds.

$$\dot{E}(t) = \frac{\eta C(t)E(t)^2}{M(t)} - r^*E(t) \quad (32)$$

$$\frac{\dot{M}(t)}{E(t)} = (1 - \psi) \frac{\alpha Z}{(\alpha - r^*)} [1 - e^{-(\alpha - r^*)t}] \quad (33)$$

$$\dot{F}(t) = \omega + r^*F(t) - C(t) \quad (34)$$

$$\dot{d}(t) = (r^* - \alpha)d(t) + \frac{(\delta + \beta)}{(r^* + \delta)(1 + \eta)} [\delta + \alpha(1 - \psi)]Z \quad (35)$$

with

$$\lim_{t \rightarrow \infty} B(t)e^{-r^*t} = \lim_{t \rightarrow \infty} M(t)e^{-r^*t} = \lim_{t \rightarrow \infty} F(t)e^{-r^*t} = 0$$

$B(0) = 0$, $F(0) = 0$, $M(0) = M_0$, $d(0) = d_0$ and $E(0)$, $C(0) = free$.

The time path of real money balances is

$$RM\dot{(t)} = r^*RM(t) + (1 - \psi) \frac{\alpha Z}{(\alpha - r^*)} [1 - e^{-(\alpha - r^*)t}] - \eta C(t) \quad (36)$$

obtained using (32) and (33)⁹.

⁹Equation (32) can be rewritten as

$$\frac{\dot{E}(t)}{E(t)} RM(t) = \eta C(t) - r^*RM(t) \quad (1n)$$

which gives the inflation tax. Similarly, equation (33) can be expressed as

$$\frac{\dot{M}(t)}{M(t)} RM(t) = (1 - \psi) \frac{\alpha Z}{(\alpha - r^*)} [1 - e^{-(\alpha - r^*)t}] \quad (2n)$$

which is seignorage revenues.

3 Fiscal Deficits and Exchange Rates

In this Section we examine the short and long run effects of a current and once-for-all lump-sum tax cut. After the fiscal expansion the government may increase taxes, use seignorage or employ some combinations of the two to finance the sequence of deficits.

Firstly, we analyze the short-run comparative static properties of the model and then we consider the long-run effects of a fiscal expansion. We derive the stability properties of the system and the transitional dynamics for the three alternative financing modes of the budget deficit.

3.1 Short-Run Effects of a Fiscal Expansion

Suppose that the economy is initially in a steady state equilibrium with the steady-state values of the macrovariables denoted by C_0 , M_0 , F_0 , B_0 , d_0 , RM_0 and E_0 and that there is an unexpected and permanent lump-sum tax cut Z . The impact effect of a permanent and unanticipated tax cut on consumption and on the nominal exchange rate are obtained by combining equation (31) and the portfolio balance condition

$$RM_0^+ = \frac{M_0}{E_0^+} = \frac{\eta C_0^+}{r^*} \quad (37)$$

where the plus superscript denotes the value of the jump variables after the shock. Both consumption and the exchange rate jump on impact and, as shown in detail in the Appendix, the partial derivatives are

$$C = C(Z) \quad C_Z > 0 \quad (38)$$

$$E = E(Z) \quad E_Z < 0 \quad (39)$$

Nominal money holdings do not change and the nominal exchange rate jumps as consequence of the shock. Using equations (31) and (37) and solving for C

and RM , one obtains the partial derivatives describing the short-run behavior of real money holdings

$$RM = RM(Z) \quad RM_Z > 0 \quad (40)$$

These results can be explained as follows.

A tax cut increases human wealth and positively affects consumption¹⁰. The portfolio balance condition implies an increase in real money holdings and this excess of demand for the domestic currency requires a real appreciation of the exchange rate to clear the money market. A tax cut leads on impact to an appreciation of the exchange rate and to an increase in both consumption and in real money holdings, independently of the financing mode, as shown in the Appendix. However, the short-run effects on the jump macrovariables are shown to be larger when money is the main financing source of budget deficits.

3.2 Long-Run Effects of a Fiscal Expansion and Dynamics

Long-run macroeconomic equilibrium is described by the following set of equations

$$\tilde{C} = \frac{\delta + \beta}{1 + \eta} \left[\frac{\omega}{r^* + \delta} + \tilde{F} + \tilde{RM} \right] + \tilde{d} \quad (41)$$

$$\tilde{RM} = \frac{\eta \tilde{C}}{r^*} - (1 - \psi) \frac{\alpha Z}{r^*(\alpha - r^*)} \quad (42)$$

¹⁰In this model agents have finite horizon and the positive effect on total wealth produced by a tax cut on impact outweighs the negative effects of the anticipated future tax increase and of seignorage. All these effects are summarized by the index of fiscal policy computed at the time of the tax cut, d_0 .

$$\tilde{F} = \frac{\tilde{C} - \omega}{r^*} \quad (43)$$

$$\tilde{d} = \frac{(\delta + \beta)}{(r^* + \delta)(1 + \eta)(\alpha - r^*)} [\delta + \alpha(1 - \psi)] Z \quad (44)$$

where \tilde{C} , \tilde{RM} , \tilde{F} and \tilde{d} denote the steady state values of the macrovariables.

In the steady state the exchange rate and the nominal stock of money grow at the same rate

$$\frac{\dot{M}(t)}{M(t)} = \frac{\dot{E}(t)}{E(t)} = \frac{\eta \tilde{C}}{\tilde{RM}} - r^* = (1 - \psi) \frac{\alpha Z}{(\alpha - r^*)} \frac{1}{\tilde{RM}} \quad (45)$$

As shown in the Appendix, the long-run relationships among the variables of interest can be obtained by solving equations (41)-(44) for \tilde{C} , \tilde{RM} and \tilde{F} .

3.2.1 Mix-Financed Fiscal Expansion , $0 < \psi < 1$

Consider the case when the government is assumed to use seignorage and to levy lump-sum taxes on households to close the deficits. The steady-state effects of once-for-all tax-cut on consumption, foreign assets and real money balances are

$$\tilde{C} = \tilde{C}(Z) \quad \text{and} \quad \tilde{C}_Z = \begin{cases} > 0 & \text{for } \psi < \bar{\psi} \\ = 0 & \text{for } \psi = \bar{\psi} \\ < 0 & \text{for } \psi > \bar{\psi} \end{cases} \quad (46)$$

$$\tilde{F} = \tilde{F}(Z) \quad \text{and} \quad \tilde{F}_Z = \begin{cases} > 0 & \text{for } \psi < \bar{\psi} \\ = 0 & \text{for } \psi = \bar{\psi} \\ < 0 & \text{for } \psi > \bar{\psi} \end{cases} \quad (47)$$

$$\tilde{RM} = \tilde{RM}(Z) \quad \tilde{RM}_Z \leq 0 \quad (48)$$

and the rate of depreciation of the exchange rate is given by monetary growth, as described by equation (45).

The long-run effects of a fiscal expansion are shown to critically depend on the degree of tax finance, ψ . If seignorage is the primary source to finance budget deficits, consumption and foreign assets increase to reach a new equilibrium above the original level; conversely, if taxation is mainly used to close the deficits, consumption and foreign assets decline in the long-run. In particular, there is a threshold level of tax financing, $\bar{\psi} = \frac{\alpha - r^*}{\alpha}$, for which an increase in Z would not have any effect on the long-run values of consumption and of net external assets. After the initial jump, the economy experiments an adjustment process towards the new steady state, but foreign assets and consumption return to their original level. The overall effect on real money balances is ambiguous depending in a complex fashion on the values of the parameters.

To examine the transitional dynamics of the economy after a once-for-all tax-cut, consider the following system, linearized around the new steady state¹¹

$$\begin{pmatrix} \dot{RM}(t) \\ \dot{C}(t) \\ \dot{F}(t) \\ \dot{B}(t) \end{pmatrix} = \begin{pmatrix} r^* & -\eta & 0 & (1-\psi)\alpha \\ -\delta\Gamma & r^* - \beta & -\delta\Gamma & -\delta\Gamma \\ 0 & -1 & r^* & 0 \\ 0 & 0 & 0 & r^* - \alpha \end{pmatrix} \begin{pmatrix} RM(t) - \tilde{RM} \\ C(t) - \tilde{C} \\ F(t) - \tilde{F} \\ B(t) - \tilde{B} \end{pmatrix} \quad (49)$$

where $\Gamma \equiv \frac{\beta + \delta}{1 + \eta}$.

In order to have a unique convergent adjustment path the system must have two positive and two negative eigenvalues, since foreign assets F and

¹¹We define the system describing the economy in \dot{RM} , \dot{C} , \dot{F} and \dot{B} , so that all the coefficients are independent of time.

domestic bonds B are predetermined, while real money balances RM and consumption C jump instantaneously. The system satisfies property and the two stable roots are $\lambda_1 = r^* - \alpha$ and $\lambda_2 = r^* - \beta - \delta$, as shown in the Appendix.

The stable path towards the steady state is described by the following set of equations

$$RM(t) - \tilde{RM} = \frac{1}{v_{41}}(B_0 - \tilde{B})(e^{\lambda_1 t} - \eta v_{31} e^{\lambda_2 t}) + \eta(F_0 - \tilde{F})e^{\lambda_2 t} \quad (50)$$

$$C(t) - \tilde{C} = \frac{1}{v_{41}}(B_0 - \tilde{B})(v_{21} e^{\lambda_1 t} - \eta v_{31} e^{\lambda_2 t}) + (\beta + \delta)(F_0 - \tilde{F})e^{\lambda_2 t} \quad (51)$$

$$F(t) - \tilde{F} = \frac{v_{31}}{v_{41}}(B_0 - \tilde{B})(e^{\lambda_1 t} - e^{\lambda_2 t}) + (F_0 - \tilde{F})e^{\lambda_2 t} \quad (52)$$

$$B(t) - \tilde{B} = (B_0 - \tilde{B})e^{\lambda_1 t} \quad (53)$$

where v_{21} , v_{31} and v_{41} denote the components of the eigenvector associated to the root λ_1 and can be shown to depend on the degree of tax finance ψ .

According to equations (50), (51) and (52) the dynamics of real money balances, aggregate consumption and external assets around the steady state can be split into two different components. The first component is due to the change in domestic bonds; the second component reflects the variation in the level of external assets.

The intuition behind these results can be understood by focusing on two opposite modes of budget deficits financing: pure money-financing and pure tax-financing.

3.2.2 Pure money-financed fiscal expansion, $\psi = 0$

When deficits are entirely financed by using seignorage, the new steady state is characterized by the following partial derivatives

$$\tilde{C} = \tilde{C}(Z) \quad \tilde{C}_Z > 0 \quad (54)$$

$$\tilde{RM} = \tilde{RM}(Z) \quad \tilde{RM}_Z \begin{matrix} \leq \\ \geq \end{matrix} 0 \quad (55)$$

$$\tilde{F} = \tilde{F}(Z) \quad \tilde{F}_Z > 0 \quad (56)$$

In the long-run foreign assets and consumption are above their original level. This result can be explained by the adjustment dynamics followed by the system after the fiscal shock. There is first a decumulation of external assets due to the excess of domestic absorption and at the same time households start to decrease consumption since the inflation tax reduces the real value of their money balances. The decrease in domestic absorption however makes the economy starts to run current account surpluses at a later stage. At the end of the adjustment process the level of external assets and financial wealth are above the original level¹². The intuition behind this result is that, in the case of money finance, inflation tends to redistribute wealth from current to future generations which fully benefit from the tax cut¹³. On

¹²These results are consistent with the "unpleasant fiscal arithmetic" obtained by Kawai and Maccini (1995) in the case of pure money finance.

¹³Long-run nonsuperneutrality of money is consistent with the analysis of Weil (1991) in which agents are assumed to have infinite horizon and new generations enter the economy at each time period. In this case a faster growth of nominal money, necessary to finance a tax cut, has two effects on the wealth of current generations: it increases the present discounted value of the nominal transfers they will receive from the government and it raises the opportunity cost of money holdings. The positive probability of death and the entry of new cohorts attribute some of the increased wealth, derived from the tax cut, to new generations, while current generations suffer the burden of the inflation tax.

the other hand, the net effect on real money balances cannot be unambiguously be determined. As shown in the Appendix, the partial derivative of real money holdings depends in a complex fashion on the parameters of the model.

In the steady state the exchange rate depreciates at a constant rate proportional to the initial tax-cut and the equilibrium inflation tax corresponds exactly to the seignorage revenues.

The dynamics of the economy towards the steady state is described by the following set of equations

$$RM(t) - \tilde{R}\tilde{M} = -(B_0 - \tilde{B})e^{\lambda_1 t} + \eta(F_0 - \tilde{F})e^{\lambda_2 t} \quad (57)$$

$$C(t) - \tilde{C} = (\beta + \delta)(F_0 - \tilde{F})e^{\lambda_2 t} \quad (58)$$

$$F(t) - \tilde{F} = (F_0 - \tilde{F})e^{\lambda_2 t} \quad (59)$$

$$B(t) - \tilde{B} = (B_0 - \tilde{B})e^{\lambda_1 t} \quad (60)$$

Equations (57)-(60) show that the transitional path of real money balances depends on the changes in government bonds and in foreign assets; conversely, in the neighborhood of the new steady state, foreign assets are shown to converge to \tilde{F} independently of the time path of the domestic assets. The dynamics of consumption, on the other hand, is entirely explained by the time path of foreign assets.

3.2.3 Pure tax-financed fiscal expansion, $\psi = 1$

When deficits are financed only by taxation the stock of nominal money does not change and the long-run effects of a fiscal shock are described by

the following partial derivatives

$$\tilde{C} = \tilde{C}(Z) \quad \tilde{C}_Z < 0 \quad (61)$$

$$\tilde{E} = \tilde{E}(Z) \quad \tilde{E}_Z > 0 \quad (62)$$

$$\tilde{F} = \tilde{F}(Z) \quad \tilde{F}_Z < 0 \quad (63)$$

In the new steady state consumption, real money balances and foreign assets are below their original levels, while there is a depreciation of the nominal exchange rate. After the fiscal shock, current generations profit from the present lump sum tax cut, since they share the burden of future increase in taxation with future generations. However, during the adjustment process the economy runs current account deficits and there is a decumulation of external assets¹⁴. In the long run total non-human wealth and the level of consumption are below their original level. The decrease in the demand for domestic currency determines the depreciation of the exchange rate¹⁵.

To examine the dynamics of the economy after a once-for-all tax-cut, we linearize the system of equations (32), (34) and (35) around the new steady state. The stable solution is of the form

$$E(t) - \tilde{E} = \frac{1}{v_{31}}(e^{\lambda_1 t} - v_{21}\eta\tilde{E}^2 e^{\lambda_2 t})(d_0 - \tilde{d}) + (F_0 - \tilde{F})\eta\tilde{E}^2 e^{\lambda_2 t} \quad (64)$$

$$F(t) - \tilde{F} = \frac{v_{21}}{v_{31}}(e^{\lambda_1 t} - e^{\lambda_2 t})(d_0 - \tilde{d}) + (F_0 - \tilde{F})e^{\lambda_2 t} \quad (65)$$

¹⁴The displacement of foreign assets by government debt and the twin deficits phenomenon are typical results of finite horizons models. See Blanchard (1985), Kawai and Maccini (1995) and Piersanti (2001).

¹⁵In the Mathematical Appendix it is shown that a fiscal expansion is neutral with respect to the short and the long-run values of consumption, foreign assets and exchange rate when deficits are entirely financed by lump-sum taxation and under the hypothesis that agents have infinite life. In this case the discipline of a *Ricardian regime* and the absence of money financing guarantee the ineffectiveness of a fiscal expansion.

$$d(t) - \tilde{d} = (d_0 - \tilde{d})e^{\lambda_1 t} \quad (66)$$

where v_{21} and v_{31} denote the components of the eigenvector associated to the stable root λ_1 . Equation (64) shows that the exchange rate dynamics is explained by the time evolution of the fiscal index and of the current account. The exchange rate is shown to depreciate when d rises (since $v_{31} > 0$) and to appreciate when the economy accumulates external assets.

As the economy moves towards the new steady state and the budget goes from deficit to surplus, consumption begins to decline while the exchange rate rises. During the adjustment process the economy will experience a current account deficit, so that foreign assets end up to a lower level in the steady state.

4 Conclusions

In a optimizing general equilibrium model with finite lives, we have analyzed how a fiscal expansion affects the time path of the exchange rate and of relevant macroeconomic variables for alternative financing modes of budget deficits.

Our major finding is that when the government is assumed to satisfy the intertemporal budget constraint without resorting to seignorage revenues, a once-for-all tax-cut still leads to a depreciation of the exchange rate in the long run. After the fiscal shock, the exchange rate appreciates on impact and then starts to depreciate along the time path towards the new steady state. During the adjustment process the economy runs current account deficits and there is a decumulation of foreign assets. In the long run total financial wealth and aggregate consumption are below their initial level. On the other hand, with money finance the non-indexed financial wealth of current generations starts to reduce after the tax cut, so that consumption starts to decrease

and the economy runs current account surpluses. During the adjustment process, there is first a decumulation of external assets due to the excess of domestic absorption and a decrease in consumption as a consequence of the inflation tax which reduces the real value of money balances. The process of decumulation of foreign assets stops when domestic absorption has decreased to a point where the economy starts to run current account surpluses.

Money and tax financing generate different wealth effects and opposite intergenerational reallocation of resources. Hence, in the case of mixed finance the net effect of a fiscal expansion on financial wealth crucially depends on the degree of tax finance in closing the deficits. It is shown that there is a degree of tax finance for which a fiscal expansion is neutral with respect to the long-run values of consumption and foreign assets.

In a set-up in which Ricardian equivalence does not hold a fiscal expansion financed entirely by lump-sum taxation determines a depreciation of the exchange rate in the long-run. On the other hand, in the new long-run equilibrium after a mix-financed fiscal expansion the exchange rate depreciates steadily at a constant rate. These results further narrow the concept of fiscal discipline, in the sense that the respect of fiscal solvency without money finance is not sufficient to avoid a worsening of the fundamentals and the depreciation of the exchange rate.

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Appendix

Short-run Equilibrium

Assuming that the economy is initially in a steady state, after the fiscal shock at time $t = t_0^+$ the economy is described by the following system of equations

$$C_0^+ = \frac{\delta + \beta}{1 + \eta} \left[\frac{\omega}{r^* + \delta} + \frac{M_0}{E_0^+} \right] + \frac{\delta + \beta}{1 + \eta} \left[\frac{1}{\delta + r^*} - \psi \frac{\alpha}{(\delta + r^*)(\alpha + \delta)} \right] Z \quad (1A)$$

$$RM_0^+ = \frac{M_0}{E_0^+} = \frac{\eta C_0^+}{r^*} \quad (2A)$$

After the fiscal expansion the real values of money stock is affected by the jump of the nominal exchange rate.

The system can be solved for C and RM and by total differentiating one obtains the following partial derivatives

$$C_Z = \frac{\partial C}{\partial Z} = \frac{r^*(\beta + \delta)[\alpha(1 - \psi) + \delta]}{(\alpha + \delta)(r^* + \delta)\Delta} > 0$$

$$RM_Z = \frac{\partial RM}{\partial Z} = \frac{\eta(\beta + \delta)[\alpha(1 - \psi) + \delta]}{(\alpha + \delta)(r^* + \delta)\Delta} > 0$$

with $\Delta \equiv r^*(1 + \eta) - \eta(\beta + \delta) > 0$.

The system of equations (1A)-(2A) can be solved for C and E . Total differentiating one obtains the impact effect on the exchange rate of a change in Z

$$E_Z = \frac{\partial E}{\partial Z} = -\frac{\eta(\beta+\delta)[\alpha(1-\psi)+\delta]E^2}{(\alpha+\delta)(r^*+\delta)\Delta} < 0$$

$$C_Z = \frac{\partial C}{\partial Z} = \frac{r^*(\beta+\delta)[\alpha(1-\psi)+\delta]}{(\alpha+\delta)(r^*+\delta)\Delta} < 0$$

$$\text{with } \Delta \equiv r^*(1+\eta) - \eta(\beta+\delta) > 0$$

Remark 1 When $\psi = 0$ the short-run effects on the jump macrovariables, C and E , are larger than for $\psi > 0$.

Remark 2 When $\psi = 1$ and $\delta = 0$ the fiscal expansion is neutral with respect to the short-run values of C and E .

Long-run Equilibrium

To analyze the long-run responses to the tax cut consider the system of equations (41)-(44) in the text. The equilibrium system can be solved for \tilde{C} , \tilde{RM} and \tilde{F} . Total differentiation yields the following partial derivatives

$$\tilde{C}_Z = \frac{\partial \tilde{C}}{\partial Z} = \frac{(\beta+\delta)[(1-\psi)\alpha-r^*]\delta}{\Lambda} \begin{cases} > 0 & \text{for } \psi < \bar{\psi} \\ = 0 & \text{for } \psi = \bar{\psi} \\ < 0 & \text{for } \psi > \bar{\psi} \end{cases}$$

$$\tilde{F}_Z = \frac{\partial \tilde{F}}{\partial Z} = \frac{(\beta+\delta)[(1-\psi)\alpha-r^*]\delta}{\Lambda r^*} \begin{cases} > 0 & \text{for } \psi < \bar{\psi} \\ = 0 & \text{for } \psi = \bar{\psi} \\ < 0 & \text{for } \psi > \bar{\psi} \end{cases}$$

$$\tilde{RM}_Z = \frac{\partial \tilde{RM}}{\partial Z} = \frac{(1+\eta)(1-\psi)(1-\beta)\alpha r^* - (\beta+\delta)(\eta r^* + 1 - \psi)\delta}{\Lambda r^*} \leq 0$$

$$\text{where } \Lambda \equiv (1+\eta)(\beta+\delta-r^*)(\alpha-r^*)(\delta+r^*) > 0.$$

When $\psi = 1$ the equilibrium system (41)-(44) can be solved for \tilde{C} , \tilde{E} and \tilde{F} . Differentiating with respect to Z yields the following partial derivative

$$\tilde{E}_Z = \frac{\partial \tilde{E}}{\partial Z} = \frac{(\beta+\delta)\eta\delta\tilde{E}^2}{\Lambda} > 0$$

Remark 3 When $\psi = 1$ and $\delta = 0$ the fiscal expansion is neutral. (i.e. Ricardian equivalence holds).

Remark 4 When $\psi = 0$ and $\delta = 0$ the fiscal expansion is neutral with respect to the long-run values of C and F .

Stability and Transitional Dynamics

The Jacobian matrix of (49) is block recursive. The characteristic equation is

$$(r^* - \alpha - \lambda)(r^* - \lambda)[(r^* - \lambda)(r^* - \beta - \lambda) - \delta(\beta + \delta)] = 0 \quad (3A)$$

and the roots are $\lambda_1 = r^* - \alpha < 0$, $\lambda_2 = r^* - \beta - \delta < 0$, $\lambda_3 = r^* > 0$ and $\lambda_4 = r^* + \delta > 0$. The eigenvector $v_i = (1 \ v_{2i} \ v_{3i})'$ associated with the negative root $i = 1, 2$ satisfies

$$\begin{pmatrix} r^* - \lambda_i & -\eta & 0 & (1 - \psi)\alpha \\ -\delta\Gamma & r^* - \beta - \lambda_i & -\delta\Gamma & -\delta\Gamma \\ 0 & -1 & r^* - \lambda_i & 0 \\ 0 & 0 & 0 & r^* - \alpha - \lambda_i \end{pmatrix} \begin{pmatrix} 1 \\ v_{2i} \\ v_{3i} \\ v_{4i} \end{pmatrix} = 0 \quad (4A)$$

Solving the system one obtains

$$v_1 = \begin{pmatrix} 1 \\ v_{21} \\ v_{31} \\ v_{41} \end{pmatrix} \text{ and } v_2 = \begin{pmatrix} 1 \\ (\beta + \delta)/\eta \\ 1/\eta \\ 0 \end{pmatrix}$$

where

$$v_{21} = \begin{cases} \frac{\alpha}{\eta} & \text{if } \psi = 1 \\ \frac{\alpha\psi\delta(\beta+\delta)}{(1-\psi)[\alpha(\beta-\alpha)(1+\eta)+\delta(\beta+\delta)]+\delta\eta(\beta+\delta)} & \text{if } 0 < \psi < 1 \\ 0 & \text{if } \psi = 0 \end{cases}$$

$$v_{31} = \begin{cases} \frac{1}{\eta} & \text{if } \psi = 1 \\ \frac{\psi\delta(\beta+\delta)}{(1-\psi)[\alpha(\beta-\alpha)(1+\eta)+\delta(\beta+\delta)]+\delta\eta(\beta+\delta)} & \text{if } 0 < \psi < 1 \\ 0 & \text{if } \psi = 0 \end{cases}$$

and

$$v_{41} = \begin{cases} \frac{(\alpha+\delta)(1+\eta)(\alpha-\beta-\delta)}{\delta\eta(\beta+\delta)} & \text{if } \psi = 1 \\ \frac{(\alpha+\delta)(1+\eta)(\alpha-\beta-\delta)}{(1-\psi)[\alpha(\beta-\alpha)(1+\eta)+\delta(\beta+\delta)]+\delta\eta(\beta+\delta)} & \text{if } 0 < \psi < 1 \\ -1 & \text{if } \psi = 0 \end{cases}$$

The components of the eigenvector associated to the stable root λ_1 vary with the degree of tax finance ψ . The stable general solution is of the form

$$RM(t) - \tilde{RM} = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$$

$$C(t) - \tilde{C} = v_{21} A_1 e^{\lambda_1 t} + \frac{\beta+\delta}{\eta} A_2 e^{\lambda_2 t}$$

$$F(t) - \tilde{F} = v_{31} A_1 e^{\lambda_1 t} + \frac{1}{\eta} A_2 e^{\lambda_2 t}$$

$$B(t) - \tilde{B} = v_{41} A_1 e^{\lambda_1 t}$$

where A_1 and A_2 are the arbitrary constants which can be obtained from the initial conditions F_0 and B_0

$$A_1 = \frac{(B_0 - \tilde{B})}{v_{41}}$$

$$A_2 = \eta(F_0 - \tilde{F}) - \eta \frac{v_{31}}{v_{41}} (B_0 - \tilde{B})$$

The transition equations of the model are

$$RM(t) - \tilde{RM} = \frac{1}{v_{41}} (B_0 - \tilde{B}) e^{\lambda_1 t} + [\eta(F_0 - \tilde{F}) - \eta \frac{v_{31}}{v_{41}} (B_0 - \tilde{B})] e^{\lambda_2 t} \quad (5A)$$

$$C(t) - \tilde{C} = \frac{v_{21}}{v_{41}} (B_0 - \tilde{B}) e^{\lambda_1 t} + \frac{\beta+\delta}{\eta} [\eta(F_0 - \tilde{F}) - \eta \frac{v_{31}}{v_{41}} (B_0 - \tilde{B})] e^{\lambda_2 t} \quad (6A)$$

$$F(t) - \tilde{F} = \frac{v_{31}}{v_{41}}(B_0 - \tilde{B})(e^{\lambda_1 t} - e^{\lambda_2 t}) + (F_0 - \tilde{F})e^{\lambda_2 t} \quad (7A)$$

$$B(t) - \tilde{B} = (B_0 - \tilde{B})e^{\lambda_1 t} \quad (8A)$$

The slope of the stable path in the space F, B is obtained by differentiating with respect to time (7A) and (8A) and by combining the results

$$\frac{\partial F(t)}{\partial B(t)} = \frac{\lambda_1 \frac{v_{31}}{v_{41}}(B_0 - \tilde{B})e^{\lambda_1 t} - \lambda_2 \frac{v_{31}}{v_{41}}(B_0 - \tilde{B})e^{\lambda_2 t} + \lambda_2(F_0 - \tilde{F})e^{\lambda_2 t}}{\lambda_1(B_0 - \tilde{B})e^{\lambda_1 t}}$$

The slope of the stable trajectory changes over time. The direction in which the path approaches the new steady state is determined by the dominant eigenvector, which is associated to the larger of the stable eigenvalues (see Calvo,1987). Let $\lambda_1 > \lambda_2$, then

$$\lim_{t \rightarrow \infty} \frac{\partial F(t)}{\partial B(t)} = \frac{v_{31}}{v_{41}} < 0$$

where

$$\frac{v_{31}}{v_{41}} = \begin{cases} \frac{\delta(\beta + \delta)}{(\alpha + \delta)(1 + \eta)(\alpha - \beta - \delta)} & \text{if } \psi = 1 \\ \frac{\psi \delta(\beta + \delta)}{(\alpha + \delta)(1 + \eta)(\alpha - \beta - \delta)} & \text{if } 0 < \psi < 1 \\ 0 & \text{if } \psi = 0 \end{cases}$$

When $\psi = 1$ the transition equations are

$$RM(t) - \tilde{R}M = \frac{\delta \eta (\beta + \delta)}{(\alpha + \delta)(1 + \eta)(\alpha - \beta - \delta)} (B_0 - \tilde{B}) [e^{\lambda_1 t} - e^{\lambda_2 t}] + \eta (F_0 - \tilde{F}) e^{\lambda_2 t} \quad (9A)$$

$$C(t) - \tilde{C} = \frac{\delta(\beta + \delta)}{(\alpha + \delta)(1 + \eta)(\alpha - \beta - \delta)} (B_0 - \tilde{B}) (\alpha e^{\lambda_1 t} - \eta e^{\lambda_2 t}) + \frac{\beta + \delta}{\eta} \eta (F_0 - \tilde{F}) e^{\lambda_2 t} \quad (10A)$$

$$F(t) - \tilde{F} = \frac{\delta(\beta + \delta)}{(\alpha + \delta)(1 + \eta)(\alpha - \beta - \delta)}(B_0 - \tilde{B})(e^{\lambda_1 t} - e^{\lambda_2 t}) + (F_0 - \tilde{F})e^{\lambda_2 t} \quad (11A)$$

$$B(t) - \tilde{B} = (B_0 - \tilde{B})e^{\lambda_1 t} \quad (12A)$$

When $\psi = 0$ the transition equations are

$$RM(t) - \tilde{R}\tilde{M} = -(B_0 - \tilde{B})e^{\lambda_1 t} + \eta(F_0 - \tilde{F})e^{\lambda_2 t} \quad (13A)$$

$$C(t) - \tilde{C} = (\beta + \delta)(F_0 - \tilde{F})e^{\lambda_2 t} \quad (14A)$$

$$F(t) - \tilde{F} = (F_0 - \tilde{F})e^{\lambda_2 t} \quad (15A)$$

$$B(t) - \tilde{B} = (B_0 - \tilde{B})e^{\lambda_1 t} \quad (16A)$$

Alternatively, when $\psi = 1$ the system linearized around the new steady state is

$$\begin{pmatrix} \dot{\tilde{E}} \\ \dot{\tilde{F}} \\ \dot{\tilde{d}} \end{pmatrix} = \begin{pmatrix} r^* - \eta\Gamma & -\eta\Gamma\tilde{E}^2 & \eta\tilde{E}^2 \\ -\Gamma/\tilde{E}^2 & r^* - \Gamma & -1 \\ 0 & 0 & r^* - \alpha \end{pmatrix} \begin{pmatrix} E - \tilde{E} \\ F - \tilde{F} \\ d - \tilde{d} \end{pmatrix} \quad (17A)$$

where M_0 has been normalized to unity for simplicity.

The Jacobian matrix is block recursive and the system presents the same stable eigenvalues of system (49). The solution of the system is

$$E(t) - \tilde{E} = \frac{1}{v_{31}}(e^{\lambda_1 t} - v_{21}\eta\tilde{E}^2 e^{\lambda_2 t})(d_0 - \tilde{d}) + (F_0 - \tilde{F})\eta\tilde{E}^2 e^{\lambda_2 t} \quad (18A)$$

$$F(t) - \tilde{F} = \frac{v_{21}}{v_{31}}(e^{\lambda_1 t} - e^{\lambda_2 t})(d_0 - \tilde{d}) + (F_0 - \tilde{F})e^{\lambda_2 t} \quad (19A)$$

$$d(t) - \tilde{d} = (d_0 - \tilde{d})e^{\lambda_1 t} \quad (20A)$$

where

$$v_{21} = \frac{2\eta(\beta+\delta)-(1+\eta)\alpha}{\tilde{\eta}E^2 [2(\beta+\delta)-\alpha(1+\eta)]} \quad v_{31} = \frac{(1+\eta)(\beta+\delta-\alpha)\alpha}{\tilde{\eta}E^2 [2(\beta+\delta)-\alpha(1+\eta)]}$$

The slope of the stable trajectory in the space E, d is obtained by differentiating (18A) and (20A) with respect to time

$$\frac{\partial E(t)}{\partial d(t)} = \frac{\lambda_1 \frac{1}{v_{31}}(d_0 - \tilde{d}) - \lambda_2 \frac{v_{21}}{v_{31}}(d_0 - \tilde{d})\tilde{\eta}E^2 e^{(\lambda_2 - \lambda_1)t} + \lambda_2(F_0 - \tilde{F})\tilde{\eta}E^2 e^{(\lambda_2 - \lambda_1)t}}{\lambda_1(d_0 - \tilde{d})}$$

The slope of the stable path is time varying. The direction in which the path approaches the new steady state is determined by the dominant eigenvector, which is associated to the larger of the stable eigenvalues. When $\lambda_1 > \lambda_2$, the slope of the stable trajectory in the neighborhood of the steady state is

$$\lim_{t \rightarrow \infty} \frac{\partial E(t)}{\partial d(t)} = \frac{1}{v_{31}} > 0$$

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