PROCEEDINGS A

royalsocietypublishing.org/rspa

CC BY

Research

Cite this article: Majhi S, Mancini A, Cimini G. 2026 Modelling structure and credit risk of the economy: a multi-layer bank–firm network approach. *Proc. R. Soc. A* **482**: 20260271.
https://doi.org/10.1098/rspa.2026.0271

Received: 26 March 2026
Accepted: 15 September 2026

Subject Category:
Physics

Subject Areas:
complexity

Keywords:
network science, complexity, network reconstruction, systemic risk, financial contagion, statistical physics, multi-layer networks

Author for correspondence:
Anna Mancini
e-mail: anna.mancini@cref.it

Supplementary material is available online at https://doi.org/10.6084/m9.figshare.c.8758997.

THE ROYAL SOCIETY PUBLISHING

Modelling structure and credit risk of the economy: a multi-layer bank–firm network approach

Soumen Majhi, Anna Mancini and Giulio Cimini

Physics Department and INFN, University of Rome Tor Vergata, Rome, Lazio, Italy

SM, 0000-0003-0612-6125; AM, 0000-0002-0705-1252

Assessing economic resilience requires accounting for its intrinsic multi-layer nature, by which disruptions at the firm level, for instance, propagate through production networks and spread to the banking sector. Existing network-based stress-testing methods can quantify these cascading effects across supply chains, corporate lending relationships and interbank exposures, but they rely on detailed network data that are typically confidential and inaccessible to researchers and regulators. Here, we develop a unified framework that reconstructs the multi-layer structure of the economy from balance-sheet information of banks and firms and models shock propagation across the production and financial layers. We apply the framework to the Italian economy, identifying the most systemically important firms and industries as well as the most vulnerable banks, while recovering determinants of systemic risk consistent with the empirical literature on network contagion. Overall, the framework enables detailed network-based stress testing on a digital twin of the economy without requiring access to confidential network data.

1. Introduction

Modern economies are characterized by complex multi-layered interdependencies linking corporate entities and financial institutions. Production networks describe the web of supply relationships through which firms exchange goods and services [1–3], while financial networks capture credit exposures among banks and firms [4–6]. The structural coupling between these two domains is fundamental for the functioning of modern economies but also plays a crucial role in determining how localized disruptions evolve into systemic crises [7]. Assessing economic

© 2026 The Authors. Published by the Royal Society under the terms of the Creative Commons Attribution License http://creativecommons.org/licenses/by/4.0/, which permits unrestricted use, provided the original author and source are credited.

Downloaded from royalsocietypublishing.org/rspa/article-pdf/doi/10.1098/rspa.2026.0271/6179767/rspa.2026.0271.pdf
by guest
on 26 November 2026

resilience therefore requires understanding not only contagion within each network layer but also the feedback mechanisms that connect the economic and financial sectors.

Recent global events have highlighted the importance of such cross-layer interactions. Climate-related disasters [8–10], geopolitical conflicts [11] and the COVID-19 pandemic [12,13] have revealed how disruptions in production networks can rapidly propagate across supply chains, generating firm-level losses that spill over to creditor banks. When suppliers halt production, downstream firms experience revenue declines that may translate into liquidity shortages and loan defaults, thereby impairing bank balance sheets. Financial distress can in turn feed back into the real economy through credit contraction. These reinforcing loops lie at the core of systemic risk and economic resilience [14–16]. Developing tools capable of quantifying such multi-layer contagion dynamics has thus become a priority for regulators and policymakers.

A fundamental obstacle to this endeavour is data availability. Detailed information on interfirm transactions, bank–firm credit exposures and interbank lending relationships is typically confidential and inaccessible to researchers and stakeholders. Yet systemic risk is inherently shaped by network topology [17,18]. Being able to reconstruct hidden economic and financial networks from partial balance-sheet information has therefore become a prerequisite for realistic stress testing and policy analysis [19].

In the aftermath of the global financial crisis of 2007/2008, research efforts concentrated primarily on the interbank market. In this context, the reconstruction problem consists of inferring bilateral exposures from aggregate balance-sheet data. This challenge has been addressed using statistical physics methods that infer the least-biased probabilistic representation of the system, by maximizing entropy under the constraints given by the available information [20]. Applied to networks, this approach leads to exponential random graph (ERG) models [21,22]. Fitness-based ERG specifications [23] require no topological information yet have proven highly effective at reconstructing interbank networks, consistently outperforming alternative probabilistic methods in validation exercises [19]. More recently, attention has shifted towards production networks. Unlike interbank markets, supply chains are shaped by technological compatibility constraints: not all pairs of firms can realistically trade, as connections are determined by input requirements and production activities. Standard ERG formulations, which treat all node pairs as potentially connected, are therefore unable to capture the functional structure of production systems. To overcome this limitation, recent models explicitly incorporate sectorial compatibility constraints, thereby generating realistic synthetic production networks [24,25]. However, methods to reconstruct the entire multi-layer structure of the economy are still unexplored.

Another limitation of the literature lies in the fragmentation of contagion modelling. A substantial body of work has focused on shock propagation within financial networks [4,26,27] or within production networks [13,28], possibly including multiple transmission channels [29–31]. Yet most approaches treat the real economy and the financial system as largely separate domains. Only a few recent studies explicitly link firm-level supply chains with bank exposures. For instance, Guth *et al.* [32] investigate the impact of COVID-19 supply-chain disruptions on Austria's banking sector using aggregated input–output (IO) data. Borsos & Mero [33] analyse bidirectional amplification between firms and banks. Tabachová *et al.* [34] develop a multi-layer contagion model integrating firm-level supply chains and bank–firm credit relationships, but without incorporating the interbank market. Fialkowski *et al.* [35] study how production shocks generate correlated defaults and spillover into interbank solvency contagion. Despite these advances, a fully unified data-driven framework that simultaneously reconstructs all relevant layers and models their ordered contagion dynamics remains absent.

Here, we address this gap by developing a computational framework that reconstructs and integrates the interbank, bank–firm and interfirm networks into a single multi-layer economic-financial structure. Importantly, our approach relies exclusively on balance-sheet information, enabling the construction of a 'digital twin' of the economy without requiring access to confidential microlevel transaction data. Each layer is reconstructed using state-of-the-art techniques tailored to its structural properties. The interbank network is inferred using the density-corrected gravity model (DCGM) [23], the bank–firm credit layer via the enhanced capital asset pricing model (ECAPM) [36] and the firm–firm production network through the stripe-corrected gravity model (SCGM), using the IO gravity model (IOGM) [24,25] specification. These reconstructed layers are then combined into a coherent multi-layer network.

Downloaded from royalsocietypublishing.org/rspa/article-pdf/doi/10.1098/rspa.2026.0271/6179767/rspa.2026.0271.pdf
by guest
on 26 November 2026

On top of this structure, we design an ordered contagion pipeline linking production and financial distress. Shocks originate in the production layer, where disruptions propagate across supply chains and generate output losses measured by the economic systemic risk index (ESRI) [28], which accounts for essential and non-essential input constraints [13]. The resulting erosion of firm profit margin translates into equity losses for creditor banks, quantified through the financial systemic risk index (FSRI) [34]—which we extend to take into account non-performing loans (NPL). These losses then reverberate within the interbank market via DebtRank (DR) dynamics [37], capturing financial amplification effects of equity deterioration. Together, these coupled processes link the economic and financial sectors into a single cascade framework, allowing us to track how local failures evolve into systemic events.

We apply this unified framework to balance-sheet data of Italian firms and banks. Our analysis identifies the most systemically important firms and industries, as well as the most vulnerable banks, and provides empirical evidence on the determinants of systemic risk. Overall, our results demonstrate that integrating reconstructed production and financial networks within a single cascade framework enables detailed network-based stress testing of the economy even in the absence of granular transaction data. Figure 1 provides a conceptual representation of the full modelling pipeline underlying our firm–bank contagion analysis. Panel A illustrates the reconstruction stage, where interfirm, firm–bank and interbank layers are inferred separately and combined into a multi-layer network. Panel B depicts the ordered propagation mechanism: shocks originate at the firm level, spread across production relationships and induce bank distress through credit exposures towards firms and other banks. Panel C summarizes the resulting systemic risk measures, emphasizing how risk rankings progressively change as additional layers of contagion are incorporated.

2. Material and methods

(a) Variables definitions

We model a multi-layer weighted directed network composed of N_B banks and N_F firms. Banks are labelled with Greek letters ($\alpha = 1, \ldots, N_B$) while firms are labelled with Latin letters ($i = 1, \ldots, N_F$). Table 1 and figure 2 report the value we extract from balance sheets that are used to inform reconstruction methods (see below). Bank-level attributes for each bank α include interbank assets A_α, interbank liabilities L_α, corporate loans C_α and equity E_α. Firm-level attributes for each firm i are revenues R_i, costs S_i, bank loans B_i, shareholder funds E_i and sector of production p_i. Further, we use industrial sector attributes (IO table entries $\{s_{p\to p'}\}$ and essentiality relations $\{e_{p\to p'}\}$) to represent IO flows and critical supplier dependencies (see below). We exploit this information to reconstruct a statistically principled ensemble of multi-layer networks, which then we use as substrate for the shock propagation dynamics.

(b) Network reconstruction

The objective of the reconstruction stage is to generate plausible microscopic representations of economic interactions when transaction-level data are unavailable. Rather than attempting to recover the exact observed network, our approach uses balance-sheet information as aggregate constraints and reconstructs an ensemble of networks consistent with those observable quantities, allowing systemic risk simulations while preserving the economic scale and heterogeneity of banks and firms.

The procedures explained below are derived from constrained entropy maximization augmented by a fitness ansatz [20]. They define connection probabilities and weights for each layer of the network, which can be used to generate individual network samples of the ensemble. The only required input is the balance-sheet information described above, which by definition is preserved on average over the ensemble.

(i) Interbank layer $\mathscr{B}$

Bank–bank links define the interbank market $\mathscr{B}$. From an economic perspective, banks with larger interbank assets have greater lending capacity, while banks with larger interbank liabilities have stronger funding demand. The reconstruction model therefore assumes that bilateral interbank exposures emerge

Downloaded from royalsocietypublishing.org/rspa/article-pdf/doi/10.1098/rspa.2026.0271/6179767/rspa.2026.0271.pdf
by guest
on 26 November 2026

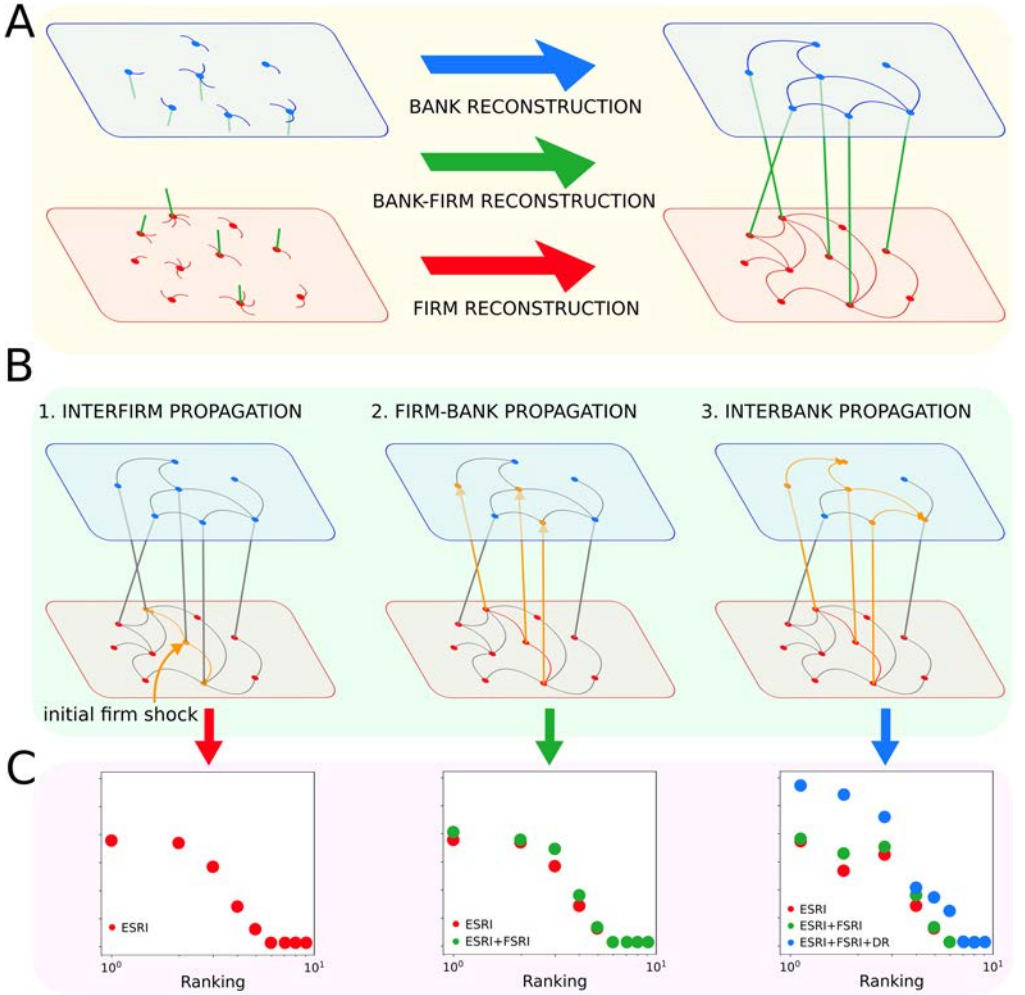

Figure 1. Schematic of the multi-layer bank–firm systemic risk framework. (A) *Network reconstruction*: starting from balance-sheet information for banks and firms in our sample, three interconnected networks are reconstructed independently: the interbank market layer (top, blue links), the bank–firm credit network (middle, green links) and the interfirm production network layer (bottom, red links). (B) *Shock propagation*: Once the multi-layer network is reconstructed, we simulate the spreading of a shock originating in the interfirm layer. The shock first diffuses through interfirm production links, according to ESRI dynamics (left), then is transmitted to the bank layer via firm exposures to banks, according to FSRI (centre) and finally amplified through interbank contagion following DR dynamics (right). (C) *Systemic risk analysis*: the cumulative effects of the shock propagation are summarized through ranked risk profiles, showing how successive layers contribute to economic losses: reduction of production (ESRI, left), additional bank credit losses (ESRI+FSRI, middle) and interbank amplification (ESRI+FSRI+DR, right).

probabilistically from the interaction between these two balance-sheet characteristics, generating sparse networks that reproduce aggregate exposures without requiring confidential transaction data.

We denote the amount of the interbank loan from lender bank α to borrower bank β as $w^{\mathscr{B}}_{\alpha\to\beta}$. This is reconstructed through the DCGM [23] as

$$w^{\mathscr{B}}_{\alpha\to\beta} = \frac{A_\alpha L_\beta}{W^{\mathscr{B}} p^{\mathscr{B}}_{\alpha\to\beta}} \quad \text{with probability} \quad p^{\mathscr{B}}_{\alpha\to\beta} = \frac{z^{\mathscr{B}} A_\alpha L_\beta}{1 + z^{\mathscr{B}} A_\alpha L_\beta}, \tag{2.1}$$

and $w^{\mathscr{B}}_{\alpha\to\beta} = 0$ otherwise. That is, the loan amount is given by the product of the interbank assets of α, A_α and interbank liabilities of β, L_β, normalized by the total weight of layer $\mathscr{B}$, $W^{\mathscr{B}}$ and the connection

Downloaded from royalsocietypublishing.org/rspa/article-pdf/doi/10.1098/rspa.2026.0271/6179767/rspa.2026.0271.pdf
by guest
on 26 November 2026

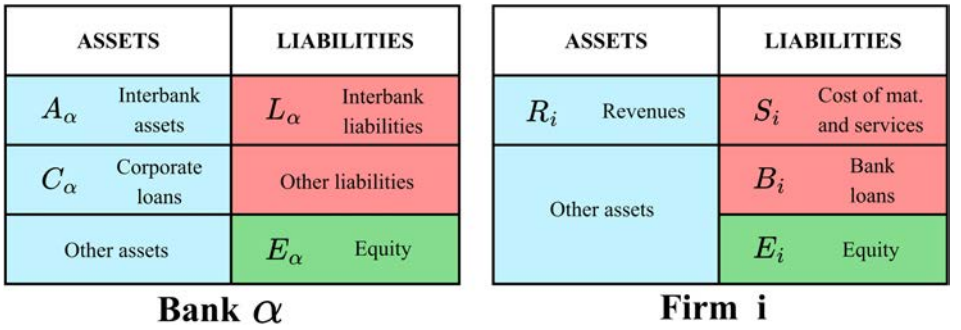

Figure 2. Balance-sheet composition for banks and firms.

Table 1. Variables used in our framework and their economic meaning, at the bank, firm and industrial sector levels.

variable	description
A_α	total interbank assets of bank α (claims on other banks)
L_α	total interbank liabilities of bank α (obligations to other banks)
C_α	total corporate loans of bank α (credit granted to firms)
E_α	total equity of bank α (net worth)
R_i	total output of production for firm i (proxied by revenues)
S_i	total input of production for firm i (proxied by cost of materials and services)
B_i	total bank loans of firm i (borrowing from banks)
E_i	total equity of firm i (shareholders'funds)
p_i	sector of production of firm i (NACE2 classification)
$s_{p\to p'}$	IO table entry: flow from sector p to sector p' (NACE2)
$e_{p\to p'}$	essentiality relation: whether sector p is an essential input for sector p' (NACE2)

probability $p^{\mathscr{B}}_{\alpha\to\beta}$ of the two banks, given by a logistic function of the same quantities (known as *fitness model* in network science). The parameter $z^{\mathscr{B}}$ sets the network density (see below), while $W^{\mathscr{B}} = \sqrt{AL}$ is the total weight of layer $\mathscr{B}$, given by the geometric mean of the two total masses $A = \sum_\alpha A_\alpha$ and $L = \sum_\alpha L_\alpha$. When the system is closed (meaning that the total masses are equal, $A \equiv L$), by construction the model preserves the total interbank assets and liabilities of each bank, as ensemble averages. For instance, $\langle A_\alpha\rangle = \sum_\beta \langle w^{\mathscr{B}}_{\alpha\to\beta}\rangle = \sum_\beta w^{\mathscr{B}}_{\alpha\to\beta} p^{\mathscr{B}}_{\alpha\to\beta} = A_\alpha(L/W^{\mathscr{B}}) = A_\alpha$. Deviations appear if the system is not closed: $\langle A_\alpha\rangle = A_\alpha\sqrt{L/A}$ and $\langle L_\alpha\rangle = L_\alpha\sqrt{A/L}$.

(ii) Bank–firm layer $\mathscr{I}$

Bank–firm links define the bipartite (interlayer) corporate credit relationships $\mathscr{I}$. This layer connects the financial sector to the real economy. We assume that banks with larger corporate loan portfolios are more likely to finance firms with greater borrowing needs, so that credit allocation emerges from matching aggregate supply and demand of credit while remaining consistent with observed balance sheets.

The amount of the loan from bank α to firm i, denoted as $w^{\mathscr{I}}_{\alpha\to i}$, is reconstructed through the bipartite version of the DCGM, also known as ECAPM [36,38]:

$$w^{\mathscr{I}}_{\alpha\to i} = \frac{C_\alpha B_i}{W^{\mathscr{I}} p^{\mathscr{I}}_{\alpha\to i}} \quad \text{with probability} \quad p^{\mathscr{I}}_{\alpha\to i} = \frac{z^{\mathscr{I}} C_\alpha B_i}{1 + z^{\mathscr{I}} C_\alpha B_i}, \tag{2.2}$$

Downloaded from royalsocietypublishing.org/rspa/article-pdf/doi/10.1098/rspa.2026.0271/6179767/rspa.2026.0271.pdf
by guest
on 26 November 2026

and $w^{\mathcal{I}}_{\alpha\to i} = 0$ otherwise. As for the DCGM, the loan amount is given by the normalized product of the corporate loans of α, C_α and bank loans of i, B_i, while the connection probability $p^{\mathcal{I}}_{\alpha\to i}$ is a logistic function of the same quantities. As before, the parameter $z^{\mathcal{I}}$ sets the network density, while $W^{\mathcal{I}} = \sqrt{CB}$ is the total weight of layer $\mathcal{I}$, given by the geometric mean of the total corporate loans $C = \sum_\alpha C_\alpha$ and total bank debt $B = \sum_i B_i$. Again, when the system is closed ($C \equiv B$), by construction the model preserves the total corporate loans of each bank and total bank debt of each firm, as ensemble averages. For instance, $\langle C_\alpha\rangle = \sum_i \langle w^{\mathcal{I}}_{\alpha\to i}\rangle = C_\alpha(B/W^{\mathcal{I}}) = C_\alpha$, with deviations occurring if the system is not closed: $\langle C_\alpha\rangle = C_\alpha\sqrt{B/C}$ and $\langle B_i\rangle = B_i\sqrt{C/B}$.

(iii) Firm–firm layer $\mathcal{F}$

Firm–firm links define the production network $\mathcal{F}$. Production networks differ from financial networks because commercial relationships are constrained by technological compatibility: firms only demand input that is relevant for their production processes. For this reason, the reconstruction combines firm-level size information with sectoral IO dependencies, generating supply links that reflect both economic scale and production requirements.

As customarily done in the literature [24,28], we assume that firms produce output only in their own industrial sector. Therefore, a link $w^{\mathcal{F}}_{i\to j}$ corresponds to the (monetary) amount of product type p_i traded by supplier firm i to customer firm j. The SCGM [24] reconstructs these links as

$$w^{\mathcal{F}}_{i\to j} = \frac{R_i S_{p_i\to j}}{W^{\mathcal{F}}_{p_i}\, p^{\mathcal{F}}_{i\to j}} \quad \text{with probability} \quad p^{\mathcal{F}}_{i\to j} = \frac{z^{\mathcal{F}} R_i S_{p_i\to j}}{1 + z^{\mathcal{F}} R_i S_{p_i\to j}}, \tag{2.3}$$

and $w^{\mathcal{F}}_{i\to j} = 0$ otherwise. As before the parameter $z^{\mathcal{F}}$ sets the total network density, while now the model uses sector-specific normalization $W^{\mathcal{F}}_{p_i} = \sqrt{R_{p_i} S_{p_i}}$. Here, $R_{p_i} = \sum_{i\in p_i} R_i$ is the total production of sector p_i and $S_{p_i} = \sum_j S_{p_i\to j}$ is the total supply of firms belonging to sector p_i. According to the SCGM formulation, outgoing connections (sales) of firm i are ruled by its total production R_i (all in sector p_i) while, differently from DCGM, incoming connections (purchases) of firm j depend on how much input its production process requires from a given sector p, indicated as $S_{p\to j}$. Then, when the system is closed ($R_p \equiv S_p$ for each sector p), the model preserves the total production of each firm and its total input by sector, as ensemble averages: $\langle R_i\rangle = \sum_j \langle w^{\mathcal{F}}_{i\to j}\rangle = R_i$ and $\langle S_{p_i\to j}\rangle = \sum_{i\in p_i}\langle w^{\mathcal{F}}_{i\to j}\rangle = S_{p_i\to j}$, therefore also $\langle S_j\rangle = S_j$ for each firm and $\langle R_p\rangle = R_p$ and $\langle S_p\rangle = S_p$ for each sector. Deviations in this case read $\langle R_i\rangle = R_i\sqrt{S_{p_i}/R_{p_i}}$ and $\langle S_{p_i\to j}\rangle = S_{p_i\to j}\sqrt{R_{p_i}/S_{p_i}}$.

However, values of the costs by sector $S_{p\to j}$ (how much j buys from suppliers belonging to sector p) are not directly available from balance-sheet information. We thus exploit IO tables [39,40] to translate aggregate production requirements into firm-level purchasing patterns, assuming that firms allocate their total input expenditures across sectors proportionally to the technological composition observed at the sector level. This is achieved through the IOGM [25]:

$$S_{p_i\to j} = S_j \frac{s_{p_i\to p_j}}{\sum_p s_{p\to p_j}}, \tag{2.4}$$

where $s_{p_i\to p_j}$ is the entry of IO tables corresponding to the total flux from sector p_i to sector p_j (how much p_j requires from p_i) and $\sum_p s_{p\to p_j}$ is the total incoming flux of sector p_j (how much p_j requires from all other sectors). In other words, we split the total costs of firm j across industrial sectors proportionally to what IO tables prescribe.

Downloaded from royalsocietypublishing.org/rspa/article-pdf/doi/10.1098/rspa.2026.0271/6179767/rspa.2026.0271.pdf
by guest
on 26 November 2026

(c) Shock propagation and systemic risk metrics

Once a sample of the multi-layer structure is reconstructed using the procedure described above, we simulate how local disruptions at the firm level spread across the economy through successive transmission channels. Production losses affect firms first, then transmit to banks through deteriorating credit quality and finally amplify through financial interconnections. Technically, this is achieved in three steps (described in detail below):

- Quantify the amount of disruption in the production network after an initial shock, using ESRI dynamics [28];
- Update the banks' equity buffers owing to NPL to disrupted firms, generalizing FSRI dynamics [34]; and
- Propagate credit shocks in the interbank market owing to NPL of affected banks, using DR dynamics [27].

(i) Shocks in the production network: the economic systemic risk index

The ESRI [28] captures how disruptions propagate through production dependencies. The underlying intuition is that firms are connected through IO requirements: when suppliers reduce production, downstream firms may be unable to substitute missing input; when customers reduce demand, upstream firms may struggle finding alternative market opportunities. The resulting output contractions then cascade across the supply chain.

Technically, ESRI quantifies the output reduction experienced by the whole production network after an initial production shock hitting one or more firms. The initial shock is described by a vector ψ, with generic component ψ_j representing the remaining production level of firm j. For instance, $\psi_j = 0$ and $\psi_i = 1$ $\forall i \neq j$ represent the initial default of firm j, while other firms remain unaffected. Shock propagation is then ruled by a generalized Leontief production function: input from essential sectors ($p \in \mathrm{Ess}_i$) sets a hard constraint on the output of firm i, while the input from non-essential sectors ($p \notin \mathrm{Ess}_i$) is treated in a linear way. This distinction reflects the idea that some production factors are absolutely required to maintain any production in the short term, while others can be partially compensated without shutting down operations. Sector essentiality relations are derived from [13,41]: for each pair of sectors p and p', their essentiality relation $e_{p\to p'} = 1$ if p is an essential input for p', while $e_{p\to p'} = 0$ otherwise. The set of essential inputs for firm i is denoted as $\mathrm{Ess}_i = \{p : e_{p\to p_i} = 1\}$.

ESRI dynamics is based on the downstream impact matrix, whose element $\Lambda^{\mathrm{d}}_{ji}$ determines the fraction of production firm i loses if firm j stops supplying to it:

$$\Lambda^{\mathrm{d}}_{ji} = \begin{cases} \dfrac{w^{\mathscr{F}}_{j\to i}}{S_{p_j\to i}}, & \text{if } p_j \in \mathrm{Ess}_i, \\ \dfrac{w^{\mathscr{F}}_{j\to i}}{S_i}, & \text{if } p_j \notin \mathrm{Ess}_i, \end{cases} \tag{2.5}$$

and the upstream impact matrix, whose element $\Lambda^{\mathrm{u}}_{ji}$ is the fraction of production firm i loses if firm j stops buying from it:

$$\Lambda^{\mathrm{u}}_{ji} = \frac{w^{\mathscr{F}}_{i\to j}}{R_i}. \tag{2.6}$$

After applying the initial shock, downstream and upstream shocks are propagated to any firm i through two iterative equations:

$$h^{\mathrm{d}}_i(t+1) = \min\left[\min_{p\in \mathrm{Ess}_i}\left(\tilde{\Pi}_{ip}(t)\right), \tilde{\Pi}_i(t), \psi_i\right] \tag{2.7}$$

Downloaded from royalsocietypublishing.org/rspa/article-pdf/doi/10.1098/rspa.2026.0271/6179767/rspa.2026.0271.pdf
by guest
on 26 November 2026

$$\text{and} \quad h_i^{\mathrm{u}}(t+1) = \min\left[\sum_j \Lambda_{ji}^{\mathrm{u}} h_j^{\mathrm{u}}(t), \psi_i\right]. \tag{2.8}$$

Here, t indicates the time step of the propagation, while $h_i^{\mathrm{d}}(t)$ and $h_i^{\mathrm{u}}(t)$ are the residual fraction of production of firm i at time t following the propagation of the downstream and upstream shock, respectively, with initial values $h_i^{\mathrm{d}}(0) = h_i^{\mathrm{u}}(0) = \psi_i$. The relative amount of essential input in sector $p \in \mathrm{Ess}_i$ available for firm i is

$$\tilde{\Pi}_{ip}(t) = 1 - \sum_{j \in p} \sigma_j(t) \Lambda_{ji}^{\mathrm{d}} \left(1 - h_j^{\mathrm{d}}(t)\right), \tag{2.9}$$

while the relative share of all non-essential inputs is

$$\tilde{\Pi}_i(t) = 1 - \sum_{p \notin \mathrm{Ess}_i} \sum_{j \in p}^{n} \sigma_j(t) \Lambda_{ji}^{\mathrm{d}} \left(1 - h_j^{\mathrm{d}}(t)\right), \tag{2.10}$$

and the firms' market share, used as a proxy for how replaceable a firm is for its buyers, is

$$\sigma_j(t) = \min\left(\frac{R_j}{\sum_{l \in p_j} R_l h_l^{\mathrm{d}}(t)}, 1\right). \tag{2.11}$$

After the two independent shocks of equations (2.7) and (2.8) have converged at $t = t^*$, we compute the residual output fraction of firm i:

$$h_i(t^*) = \min\{h_i^{\mathrm{d}}(t^*), h_i^{\mathrm{u}}(t^*)\}. \tag{2.12}$$

The ESRI value is then given by the fraction of total production lost in the system:

$$\mathrm{ESRI}(\psi) = \sum_i \frac{R_i}{\sum_j R_j} [1 - h_i(t^*)]. \tag{2.13}$$

(ii) Credit shocks to the bank layer: the financial systemic risk index

Production disruptions affect not only firms' revenues but also their financial conditions and, in turn, those of their borrowers. In our framework, lower production translates into lower profits and weaker balance sheets, reducing firms' ability to service debt and generating losses for creditor banks through loan devaluation.

Practically, the reduced production level of firm i, computed by ESRI, translates into a reduction of its revenues and material costs that affects profit π_i as [34]

$$\Delta\pi_i = \max\left\{0, \left[1 - h_i^{\mathrm{u}}(t^*)\right] R_i - \left[1 - h_i^{\mathrm{d}}(t^*)\right] S_i\right\}, \tag{2.14}$$

where revenues R_i and material costs S_i are reduced proportionally to upstream and downstream shocks, respectively.[1] This causes a direct reduction of the firm's equity as $E_i' = \max[0, E_i - \Delta\pi_i]$ that in turns affects its ability to repay bank debt—which become NPL for creditor banks. In particular, we assume that the value of the loan $w_{\alpha \to i}^{\mathcal{F}}$ that a bank α has towards firm i reduces proportionally to the equity loss of i: $\left(w_{\alpha \to i}^{\mathcal{F}}\right)' = w_{\alpha \to i}^{\mathcal{F}} \left(E_i'/E_i\right)$.[2] Hence, when i has no profit reduction ($\Delta\pi_i = 0$), then the loan keeps its original value, while when $E_i' = 0$, the firm has defaulted and the entire exposure is written off.

[1] With this formula, we link the reduction in production costs to the downstream shock, i.e. the supply shock, and the reduction in revenues to the upstream or demand shock. We also take only positive values of $\Delta\pi$. Overall, this is different from the formula $\Delta\pi_i = \left(1 - h_i(t^*)\right)(R_i - S_i)$ used in [34].

[2] The linear scaling assumption of bank corporate loans with respect to firm equity losses is in line with the linear formulation of DR, where interbank claims are revaluated with bank equity losses in order to set their market value [42] (see a more detailed discussion in §7).

Downloaded from royalsocietypublishing.org/rspa/article-pdf/doi/10.1098/rspa.2026.0271/6179767/rspa.2026.0271.pdf
by guest
on 26 November 2026

Considering potential losses from all firms, the reduced equity of bank α is calculated as $E'_\alpha = \max\left\{0, E_\alpha - \sum_i [w^{\mathcal{F}}_{\alpha\to i} - (w^{\mathcal{F}}_{\alpha\to i})']\right\}$, so the relative equity loss reads[3]

$$h_\alpha(1) = 1 - \frac{E'_\alpha}{E_\alpha} = \min\left\{1, \frac{1}{E_\alpha}\sum_i w^{\mathcal{F}}_{\alpha\to i}\left(1 - \frac{E'_i}{E_i}\right)\right\} = \min\left\{1, \sum_i \frac{w^{\mathcal{F}}_{\alpha\to i}}{E_\alpha}\min\left(1, \frac{\Delta\pi_i}{E_i}\right)\right\}. \quad (2.15)$$

Overall, direct credit losses for the banking system stemming from the initial shock ψ at the firm level are quantified by the FSRI [34]:

$$\mathrm{FSRI}(\psi) = \sum_\alpha \frac{E_\alpha}{\sum_\beta E_\beta} h_\alpha(1). \quad (2.16)$$

Here, FSRI is the equity-weighted sum of losses suffered by individual banks in the network, representing the fraction of total bank equity that is lost after the initial shock propagates through the supply-chain network, generating NPL for banks.

(iii) Credit shocks in the interbank market: the DebtRank

Bank losses generated by deteriorating corporate credit may, in turn, propagate within the financial system through interbank exposures. DR [27,37] captures this amplification mechanism by assuming that even partial deterioration of a borrower's financial condition can reduce the market value of interbank claims, allowing distress to spread before outright defaults occur. Total bank losses are then obtained by iteratively spreading the individual distress levels, weighted by the potential wealth affected. In practice, DR describes the level of financial distress of each bank α at each round t of the shock propagation dynamics, given by the relative change of equity with respect to the original value: $h_\alpha(t) = 1 - E_\alpha(t)/E_\alpha$. By definition, $h_\alpha = 0$ when no equity losses occurred for bank α, while $h_\alpha = 1$ when that bank defaults and $0 < h_\alpha < 1$ for intermediate distress levels. In our case, initial credit shocks are transmitted from the bank–firm layer, i.e. they correspond to banks' NPL to firms: $h_\alpha(1) = 1 - E'_\alpha/E_\alpha$, as of equation (2.15). These shocks then spread to the interbank layer: subsequent values of h_α are obtained according to the following iterative equation describing the evolution of the bank's equity:

$$h_\alpha(t+1) = \min\left\{1, h_\alpha(t) + \sum_{\beta\in\mathcal{A}(t)} \frac{w^{\mathcal{B}}_{\alpha\to\beta}}{E_\alpha}[H_\beta(t) - H_\beta(t-1)]\right\}. \quad (2.17)$$

Here, $\mathcal{A}(t) = \{\beta : h_\beta(t-1) < 1\}$ is the set of banks that have not defaulted up to time t (and thus can still spread their financial distress). We assume that any bank β propagates shocks as long as it keeps receiving them, i.e. provided that $H_\beta(t) > H_\beta(t-1)$, where $H_\beta(t)$ is the probability of default of bank β at time t. As in [43], we assume the form

$$H_\beta(t) = h_\beta(t) e^{\eta[h_\beta(t)-1]}, \quad (2.18)$$

where $\eta^{-1} \geq 0$ is a free parameter representing the typical relative equity loss after which banks start to propagate distress to their creditors. Here, η allows interpolating between two of the mostly widely used contagion models: $\eta = 0$ leads to the linear DR [27], where the default probability is the relative equity loss (i.e. assets valuation is a linear function of equity) [42], while $\eta \to \infty$ recovers contagion-at-default or Furfine algorithm [44], where the probability of default is unity only if the equity is depleted (and zero otherwise).

[3]Differently from [34], we do not only count banks that default but quantify the reduction of banks equity owing to NPL. However, we do not consider the effect of reduced profit on liquidity buffers, which can have larger effects on potential insolvencies than equity erosion.

Downloaded from royalsocietypublishing.org/rspa/article-pdf/doi/10.1098/rspa.2026.0271/6179767/rspa.2026.0271.pdf
by guest
on 26 November 2026

The described dynamics stops at t^* when no more banks can propagate their distress, i.e. $H_\alpha(t^*) = H_\alpha(t^* - 1)\ \forall\alpha$. The overall DR of the interbank network derived from the initial firm-level shock ψ is then obtained as

$$\mathrm{DR}(\psi) = \sum_\alpha \frac{E_\alpha}{\sum_\beta E_\beta} h_\alpha(t^*). \tag{2.19}$$

Note that while DR is a function of the initial shock, we can also compute a resilience indicator for individual banks by averaging over different realizations of the initial shock. We thus define the *vulnerability* of bank α as [45]

$$V_\alpha = \langle h_\alpha(t^*|\psi)\rangle_\psi, \tag{2.20}$$

where $h_\alpha(t^*|\psi)$ specifies the final value of h obtained by using ψ as initial condition of the dynamics, and the average is performed over a chosen set of initial shocks $\{\psi\}$ (see below).

3. Data description

(a) Dataset construction

We inform the multi-layer framework using a proprietary dataset covering Italian banks and firms for the year 2023, compiled from regulatory balance sheets.

Bank data are obtained from the BankFocus database by Bureau van Dijk. We applied four filtering steps to select the relevant banks for our analysis: (i) status: active company; (ii) world region: Italy; (iii) specialization: commercial bank, finance company, cooperative bank, savings bank, specialized governmental credit institution, private banking, investment bank; and (iv) total assets in 2023 > 2 000 000€, with the exclusion of companies with no recent financial data and public authorities. The resulting bank dataset is composed of $N_B = 109$ Italian banks.[4]

Firm data are obtained through AIDA, Bureau van Dijk's database of Italian firms. We applied the following filters: (i) companies who are ultimate owners of their groups; (ii) companies whose NACE2 code is not 64, 65, 66 (financial companies are removed); and (iii) total assets for 2023 ≥ 10 000 000€ and total shareholder's funds for 2023 ≥ 1000€. The resulting firm data are composed of $N_F = 7109$ Italian firms.[5]

IO table entries $\{\sigma_{p\to p'}\}$ are obtained from the Italian Institute of Statistics[6] for the last available year 2022. To ensure consistency with our firm-level data and thus improve reconstruction accuracy, we rescale IO entries such that the total output of a sector matches the total revenues of firms in that sector for our data.[7] This means that the factors entering equation (2.4) are computed as

$$s_{p\to p'} = R_p \frac{\sigma_{p\to p'}}{\sum_{p''}\sigma_{p\to p''}}. \tag{3.1}$$

(b) Descriptive statistics

Section S1 in the supplementary material contains summary tables and distribution plots for the balance-sheet variables of our sample. Fat tails appear for all quantities, highlighting the wide heterogeneity of banks

[4]This sample correspond to 25.9% of all Italian banks but accounts for 96.0% of their total assets (with respect to the case of the fourth filter not implemented).

[5]This sample corresponds to 26.6% of all Italian firms but accounts for 95.2% of their total assets (with respect to the case of the third filter not implemented).

[6]http://www.istat.it/tavole-di-dati/il-sistema-di-tavole-input-output-anni-2020-2022/

[7]See §S5 in the supplementary material for further details and robustness checks.

Downloaded from royalsocietypublishing.org/rspa/article-pdf/doi/10.1098/rspa.2026.0271/6179767/rspa.2026.0271.pdf
by guest
on 26 November 2026

and firms: most institutions have intermediate balance-sheet figures while a few very large ones dominate the upper tail. This is confirmed by mean values being much higher than median ones, with maximum values being orders of magnitude larger than typical ones. These patterns may suggest that systemic importance is concentrated in a small subset of large banks and firms, with shocks to top-ranked firms potentially dominating aggregate outcomes. Therefore, in the following, we perform sensitivity analyses to examine tail dependence and robustness with respect to the upper end of the distributions.

4. Results: reconstructed networks

Before running the multi-layer network reconstruction framework, we need to specify the three free parameters, $z^{\mathcal{B}}$, $z^{\mathcal{I}}$ and $z^{\mathcal{F}}$, which determine the link density of the interbank, bank–firm and interfirm layers, respectively. We do so by using empirical measures of connectivity provided by the literature.[8] In real networks with large-enough size, the link density ρ typically scales with the inverse of the number of nodes, $\rho \sim N^{-1}$. This means that we cannot directly use density values taken from empirical studies about networks of different sizes; in addition, our network only represents a subset of the Italian economy.[9] A more reliable quantity is the average node degree $\langle k \rangle \sim \rho N$, which in principle does not vary with the size of the network as it represents the intrinsic capacity of a bank or firm to connect with others (which should not depend on the size of the economy). We thus proceed as follows.

In the interbank layer, according to equation (2.1) each bank α has an average out-degree (number of borrower banks) equal to $\langle k_\alpha^{\text{out},\mathcal{B}} \rangle = \sum_\beta p^{\mathcal{B}}_{\alpha\to\beta}$ and average in-degree (number of lending banks) equal to $\langle k_\alpha^{\text{in},\mathcal{B}} \rangle = \sum_\beta p^{\mathcal{B}}_{\beta\to\alpha}$. We thus set $z^{\mathcal{B}}$ so that the mean values of these quantities over banks equal 20 [46,47], corresponding to a density value of $\rho_{\mathcal{B}} = 0.17$:

$$z^{\mathcal{B}} \text{ such that } \frac{1}{N_B}\sum_\alpha \langle k_\alpha^{\text{out},\mathcal{B}} \rangle \equiv \frac{1}{N_B}\sum_\alpha \langle k_\alpha^{\text{in},\mathcal{B}} \rangle = \frac{1}{N_B}\sum_{\alpha\neq\beta} p^{\mathcal{B}}_{\alpha\to\beta} = 20. \tag{4.1}$$

Similarly, in the interfirm layer, according to equation (2.3) each firm i has an average out-degree (number of customer firms) equal to $\langle k_i^{\text{out},\mathcal{F}} \rangle = \sum_j p^{\mathcal{F}}_{i\to j}$ and average in-degree (number of supplier firms) equal to $\langle k_i^{\text{in},\mathcal{F}} \rangle = \sum_j p^{\mathcal{F}}_{j\to i}$. We set $z^{\mathcal{F}}$ so that the mean values of these quantities over firms equal 40 [48], corresponding to a density value of $\rho_{\mathcal{F}} = 0.0054$:

$$z^{\mathcal{F}} \text{ such that } \frac{1}{N_F}\sum_i \langle k_i^{\text{out},\mathcal{F}} \rangle \equiv \frac{1}{N_F}\sum_i \langle k_i^{\text{in},\mathcal{F}} \rangle = \frac{1}{N_F}\sum_{i\neq j} p^{\mathcal{F}}_{i\to j} = 40. \tag{4.2}$$

For the bank–firm layer, according to equation (2.2) each bank α has an average out-degree (number of borrower firms) equal to $\langle k_\alpha^{out,\mathcal{I}} \rangle = \sum_i p^{\mathcal{I}}_{\alpha\to i}$, while each firm has an average in-degree (number of lending banks) equal to $\langle k_i^{in,\mathcal{I}} \rangle = \sum_\alpha p^{\mathcal{I}}_{\alpha\to i}$. We set $z^{\mathcal{I}}$ so that the average firm degree equals 1.8 [49], corresponding to an average bank degree of 117.5 and a density value of $\rho_{\mathcal{I}} = 0.016$:

$$z^{\mathcal{I}} \text{ such that } \frac{1}{N_F}\sum_i \langle k_i^{\text{in},\mathcal{I}} \rangle = \frac{1}{N_F}\sum_{i,\alpha} p^{\mathcal{I}}_{i\to j} = 1.8 \Longrightarrow \frac{1}{N_B}\sum_\alpha \langle k_\alpha^{\text{out},\mathcal{I}} \rangle = 117.5. \tag{4.3}$$

Using these values, we can generate independent samples of the multi-layer network. Results presented below are computed as average values over an ensemble of 100 networks.

Section S2 in the supplementary material illustrates how the reconstruction procedure determines node degrees, while §S3 in the supplementary material shows that the reconstruction process is able (as intended) to reproduce balance-sheet variables of banks and firms—which we generically refer to as *strengths*. This confirms that reconstructed networks retain the key balance-sheet figures necessary for subsequent contagion and systemic risk dynamics.

[8] See §S6 in the supplementary material for robustness checks against the network density.

[9] See §S7 in the supplementary material for robustness checks against the sample size.

Downloaded from royalsocietypublishing.org/rspa/article-pdf/doi/10.1098/rspa.2026.0271/6179767/rspa.2026.0271.pdf
by guest
on 26 November 2026

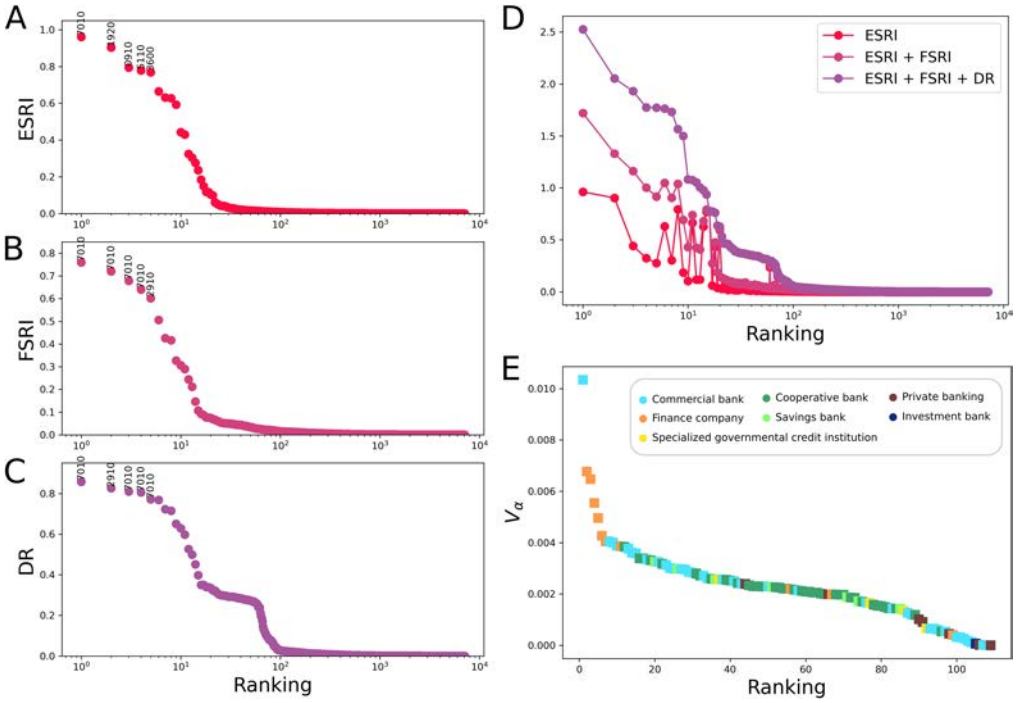

Figure 3. Ranking plots in terms of systemic risk metrics, with nodes placed in descending order of the corresponding scores. (A) Ranking based on ESRI scores of firms, where for each firm *i* the initial condition is given by its complete production shutdown ($\psi_i = 0.0$), while all other firms remain active. The ESRI of *i* thus measures the fraction of total production lost in the interfirm layer through supply-chain contagion, owing to the default of *i*. (B) Ranking based on FSRI scores of firms: FSRI of *i* measures the fraction of total bank equity lost in the bank–firm layer owing to the results of the ESRI dynamics with the same initial condition as before. (C) Ranking based on DR scores of firms: DR of *i* measures the fraction of total bank equity lost in the interbank layer owing to the results of the FSRI dynamics with the same initial condition as before. (D) Ranking based on total systemic risk scores of firms (ESRI+FSRI+DR), highlighting the total impact of firms' default on the overall economy. Total impact of *i* thus measures the fraction of total firm production and bank equity lost owing to the default of *i*, subsequent supply-chain contagion, bank loans devaluation and re-evaluation of interbank claims. (E) Ranking based on vulnerability scores of banks, obtained by averaging over shutdowns of all individual firms as initial conditions. Vulnerability of α thus measures how much the bank can be affected by economic losses, in terms of equity losses. Marker colours denote bank specialization.

5. Results: systemic risk of individual firms

We now discuss simulation results of the shock propagation dynamics on the reconstructed multi-layer network, studying the impact of individual firms failures. We thus represent the default of firm j with the initial condition $\psi_j = 0$ and $\psi_i = 1\ \forall i \neq j$ (all other firms are unaffected). Given such initial shock, we run ESRI dynamics of equations (2.7) and (2.8) up to convergence at t^*, then do FSRI dynamics of equation (2.15) and finally run DR dynamics equation (2.17) up to convergence at t^*, using the nonlinear formulation with $\eta = 2$ (we avoid using linear propagation that leads to overwhelming disruption for the banking layer). With this scenario, we measure the overall losses that the failure of a firm would cause to the whole economy, without any mitigation action taken on the system.

(a) Ranking plots

Figure 3A shows the ranking of individual firms in terms of their ESRI values (the fraction of total production lost in the interfirm network after their initial default). Similar to that found in related studies on empirical production networks [25,28,50], the ranking curve features an initial region of a handful

Downloaded from royalsocietypublishing.org/rspa/article-pdf/doi/10.1098/rspa.2026.0271/6179767/rspa.2026.0271.pdf
by guest
on 26 November 2026

high-impact firms, with ESRI values close to unity, meaning that their initial collapse can ultimately destroy the entire production of the economy.[10] The top five firms belong to NACE4 sectors 7010 (business management), 1920 (manufacture of refined petroleum products), 0910 (petroleum and natural gas extraction services), 5110 (air passenger transport) and 3600 (water production, treatment and supply). The ranking curve then declines quickly: only about 0.3% of the firms have ESRI larger than 0.1, while the rest of the firms have smaller or negligible impact, indicating a strong heterogeneity of the ESRI distribution.

Figure 3B displays firms' ranking in terms of their FSRI values (the fraction of total bank equity lost in the bank–firm network after their initial default). Again, we observe an initial region of high-impact firms, with FSRI up to 0.8. The top four firms in this case belong to NACE4 sector 7010 (business management), while the fifth belongs to NACE4 2910 (manufacture of motor vehicles). FSRI then quickly declines to negligible values: again, only about 0.3% of firms have FSRI larger than 0.1. Figure 3C finally shows firms' ranking in terms of their DR values (the fraction of total bank equity lost in the interbank network after their initial default). The initial region of high-impact firms, with DR around 0.8, basically overlaps with that of FSRI. However, in this case, we see a slower decline towards negligible values, characterized by an additional flat region of medium-impact firms (DR around 0.3). Indeed, in this case, more than 1% of firms have DR larger than 0.1. This signals the presence of many firms whose failure has little impact on the economy but a considerable one on the financial system.

As the three rankings just discussed do not necessarily coincide (for instance a top-rank firm according to ESRI may be in the DR rank tail), in order to assess the overall impact of firms on the multi-layer system, we rank them according to the sum of their ESRI, FSRI and DR scores. As figure 3D shows, the ranking for total systemic risk differs from that of ESRI in a few important firms, mainly relating to the ESRI plateau. Indeed, top-ranked ESRI firms can have very different effects on the banking sector. While FSRI is close to zero for firms with NACE4 category 3600 and 5110, the other systemically risky firms achieve top overall impact, along with those firms having intermediate ESRI values but large corporate loans, which allow their shock to travel and propagate within the banking sector.

Finally, concerning banks' vulnerability, the rank plot of figure 3E shows that banks are hardly immune to shocks originating in the interfirm layer. The distribution is not particularly heterogeneous, with most values lying in the range 0.1–0.4%. Owing to their specialization, finance companies typically have the largest values, while the most vulnerable are commercial banks (some of which were placed under extraordinary administration in 2023 after inspections by the Bank of Italy). Other commercial banks as well as cooperative and savings banks tend to populate the middle of the ranking, while investment and specialized institutions are more prevalent in the lower tail.

(b) Empirical analysis of determinants

After assessing the outcomes of systemic risk dynamics, we now aim at understanding the drivers of the various impact metrics.

ESRI. Important quantities that enter in the computation of the ESRI of firm i are its revenues R_i, production costs S_i, market share σ_i of equation (2.11), industrial sector p_i and essentiality relations with other sectors $\{e_{p_i \to p}\}$, as well as the topology of the production network around it. We thus estimate a minimal

[10] We do not interpret these estimates as predictions of actual aggregate output collapse following the failure of a single firm, but rather as upper-bound estimates of systemic exposure under the assumptions of the model (in particular, unlimited spreading of shocks up to convergence). In addition, we believe the extreme values we observe arise because our firm-level dataset is constructed at the level of ultimate owners using consolidated balance-sheet information. As a result, economic activity that in other datasets may appear fragmented across multiple legal entities is represented here as belonging to a single corporate node. Relative to studies relying on unconsolidated firm data, this aggregation can generate firms that are effectively larger, more connected and more difficult to replace within the production network, thereby increasing the potential for large cascading effects. This is especially the case of sector 69–70: ultimate owner firms are often classified as 'legal and accounting activities; activities of head office', while subsidiary activities can fall in other categories. Moreover, the firms appearing in the upper tail of the ESRI distribution are characterized not only by large size but also by high market concentration and high essentiality scores (see the quantile regression results below). These firms supply inputs that are difficult to replace and are connected to a large fraction of downstream economic activity, making them structurally capable of generating large cascades. Finally, because balance sheets are consolidated at the ultimate-owner level, all bank liabilities of affiliated subsidiaries are aggregated into a single node. This increases the exposure of the ultimate owner towards the bank layer and, consequently, amplifies its potential financial systemic impact.

Downloaded from royalsocietypublishing.org/rspa/article-pdf/doi/10.1098/rspa.2026.0271/6179767/rspa.2026.0271.pdf
by guest
on 26 November 2026

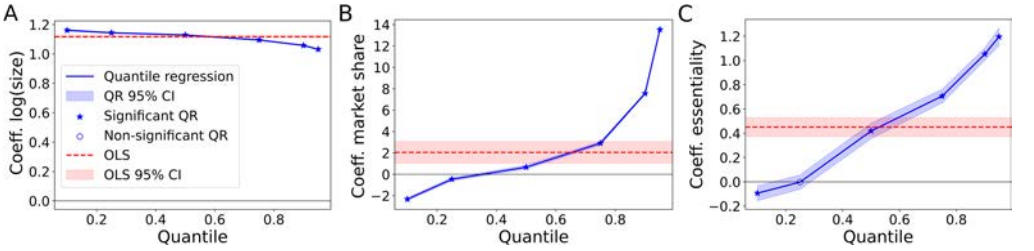

Figure 4. Determinants of ESRI for firm-level shocks: size effects and significance of firm size (A), market share (B) and essentiality score (C) in OLS and quantile regressions of ESRI values.

cross-sectional specification for ESRI:

$$\log(\mathrm{ESRI}_i) = \gamma + \beta_1 \log(\Theta_i) + \beta_2 \sigma_i + \beta_3 \mathcal{E}_i + \varepsilon_i, \tag{5.1}$$

where $\Theta_i = \max[R_i, S_i]$ measures firm size, while $\mathcal{E}_i$ is the 'essentiality' score of the firm in the economy, defined as

$$\mathcal{E}_i = \frac{\sum_j w^{\mathcal{F}}_{i\to j} R_j e_{p_i \to p_j}}{\sum_j w^{\mathcal{F}}_{i\to j} R_j}. \tag{5.2}$$

This index measures the share of downstream economic activity supplied by firm i that relies on the essential input the firm provides. Alternatively, it can be seen as a weighted average of essentiality indicators across customers, where weights reflect the economic importance of the relationship. Hence, the essentiality index is closely related to Bonacich centrality in production networks [51].

We estimate model equation (5.1) both by ordinary least squares (OLS) with heteroskedasticity-robust standard errors and by quantile regression to explore heterogeneity across the ESRI distribution. Results, reported in figure 4, indicate a significant association of ESRI with these variables—the OLS estimates show an extremely high explanatory power ($R^2 = 0.979$). Firm size has a strong and stable positive effect: the coefficient slightly above unity implies that systemic risk increases more than proportionally with firm size. Essentiality and market share also have a positive and highly significant effect on the OLS specification, indicating that firms providing essential input and occupying a larger position in their market can affect a greater portion of economic activity. Quantile regression reveals substantial heterogeneity across the ESRI distribution. While the effect of firm size remains positive and relatively stable across all quantiles, the impact of essentiality and market share increases markedly towards the upper tail. In particular, essentiality is negative or insignificant in the lower quantiles but becomes strongly positive from the median upward, reaching large and highly significant values in the top deciles. A similar pattern is observed for market share, the effect of which grows significantly at higher quantiles of ESRI. In other words, while size is a pervasive driver of systemic exposure, the combination of essential input and large market positions acts as a powerful nonlinear amplifier in the upper tail, shaping the concentration of extreme systemic risk.

FSRI. Relevant variables involved in the computation of the FSRI of firm i are its size Θ_i, bank loans B_i and the initial shock—proportional to ESRI_i and depending also on profit margin $\mu_i = \max(R_i - S_i, 0)$. Again, we expect firm size to be an important determinant of systemic risk. According to the previous analysis, we take ESRI as a proxy of size and observe a very high correlation between FSRI and ESRI values (figure 5A); however, firms with large ESRI can differ by orders of magnitude in FSRI, indicating that financial losses transmitted to banks do depend also on firms' exposures. We therefore estimate the following minimal cross-sectional specification:

$$\log(\mathrm{FSRI}_i) = \gamma + \beta_1 \log(\mathrm{ESRI}_i) + \beta_2 (B_i/\Theta_i) + \beta_3 (\mu_i/\Theta_i) + \varepsilon_i, \tag{5.3}$$

Downloaded from royalsocietypublishing.org/rspa/article-pdf/doi/10.1098/rspa.2026.0271/6179767/rspa.2026.0271.pdf
by guest
on 26 November 2026

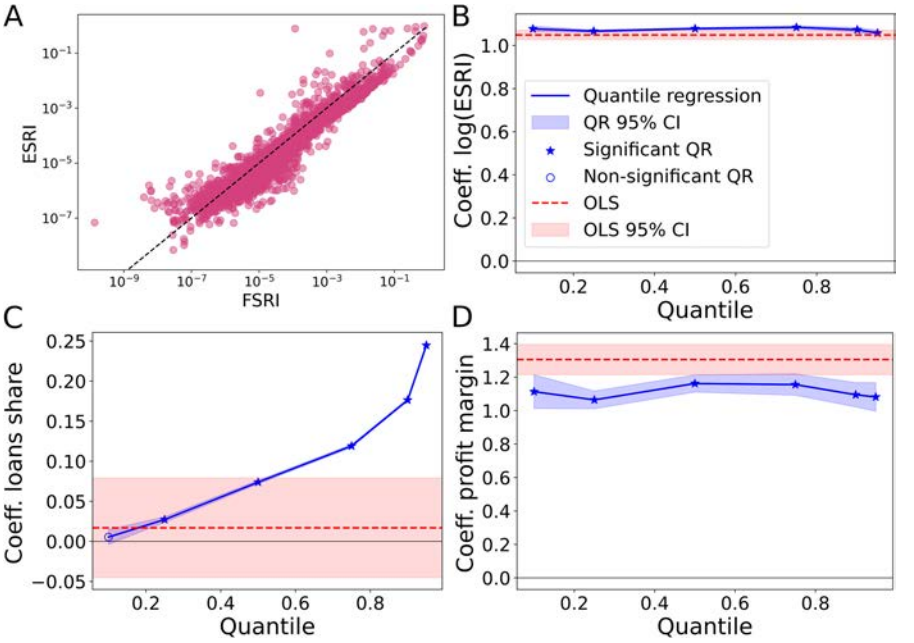

Figure 5. Determinants of FSRI for firm-level shocks. Correlation between ESRI and FSRI (A). Size effects and significance of ESRI (B), bank loans (C) and profit margin (D) in OLS and quantile regressions of FSRI values.

where we take ESRI as a proxy of both size and initial shock, while the other two variables are normalized to avoid collinearity with size. Results, reported in figure 5, reveal a strong relationship between FSRI and ESRI. In the OLS specification, with $R^2 = 0.935$, log(ESRI) exhibits a coefficient close to unity (1.048) and is highly statistically significant, indicating an almost proportional relationship between the two systemic risk measures: firms that are more systemically relevant in the production network also tend to be more relevant from a financial perspective. The coefficient of profit margin is positive and strongly significant: more profitable firms are more sensitive to the initial shock and tend to exhibit higher financial systemic relevance. By contrast, loans share is not statistically significant in the mean regression.

The quantile regression results provide a more nuanced picture by allowing the relationship to vary across the distribution of FSRI. The coefficients of log(ESRI) and profit margin remain remarkably stable, while the effect of loans share increases sharply across quantiles: it is insignificant at the 10th percentile, modestly positive around the median and becomes very large and highly significant in the upper tail of the FSRI distribution. Therefore, bank credit exposure becomes particularly relevant in determining the firms that are highly systemically important.

DR and vulnerability. The situation for the DR case is more complicated, as the dynamics is tied to properties of the interbank layer, while here we are considering how firm characteristics trigger and drive DR. However, as figure 6 shows, DR is directly driven by FSRI, with saturation effects as DR approaches unity. Results of the OLS regression of log(DR) versus log(FSRI) show an almost one-to-one relationship between FSRI and DR ($R^2 = 0.997$): FSRI alone explains virtually all cross-sectional variation in DR.

Vulnerability instead can be more easily mapped to bank variables. However, as figure 6 shows, there is almost perfect correlation between V_α and C_α/E_α, that is, the proportion of corporate loans over the bank's equity. This confirms that corporate loan share alone almost entirely explains the cross-sectional variation in vulnerability ($R^2 = 0.986$).

(c) Summary

The analysis of individual firm defaults reveals a highly heterogeneous systemic risk landscape. A tiny fraction of firms—predominantly in business management, petroleum and essential utilities—can trigger

Downloaded from royalsocietypublishing.org/rspa/article-pdf/doi/10.1098/rspa.2026.0271/6179767/rspa.2026.0271.pdf
by guest
on 26 November 2026

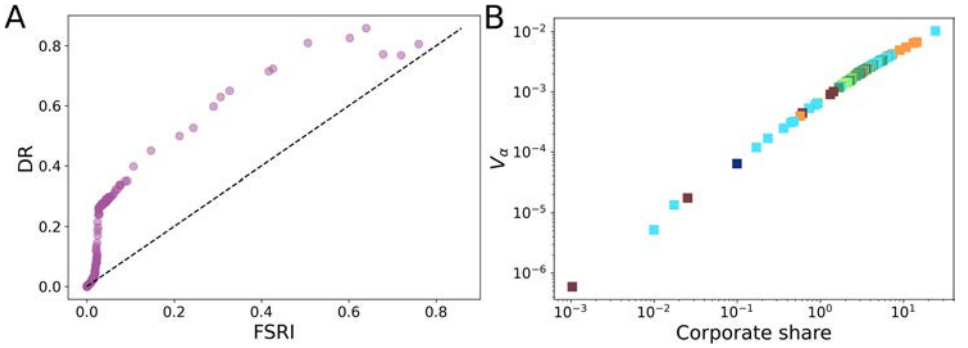

Figure 6. Determinants of DR and bank vulnerability for firm-level shocks. Correlation between DR and FSRI (A) and between vulnerability and corporate loans share (B).

cascading losses that propagate across the entire multi-layer economy. Firm size is the primary and most stable driver of systemic impact at all layers, while essentiality and market share act as powerful amplifiers, specifically in the upper tail of the risk distribution. Crucially, a firm's economic and financial systemic relevance do not always coincide: firms critical for production are not necessarily the most dangerous for the banking sector, underscoring the need for a full multi-layer perspective. Bank vulnerability is almost entirely explained by the share of corporate loans relative to equity, making balance-sheet composition rather than size the key determinant of exposure to real-economy shocks.

6. Results: systemic risk of industrial sectors

We now study the impact of losses stemming from all firms within an industrial sector. Therefore, we use as initial condition a 10% production loss for each firm belonging to sector p, identified at the NACE2 level: $\psi_j = 0.9$ if $p_j = p$ (firm j belongs to sector p) and $\psi_i = 1$ otherwise (all other firms are unaffected). Differently from the previous scenario, here we consider a one-step propagation dynamics: we run ESRI for one step ($t = 1$), then do FSRI contagion and finally run DR for one step ($t = 1$), using the linear formulation $\eta = 0$. Hence, in this scenario, we study the short-term impact of the initial shock, i.e. the losses that occur before any mitigation action can possibly take place.

Section S4 in the supplementary material reports the basic balance-sheet characteristics of each sector p: total size, defined by summing the sizes of firms belonging to it: $\Theta_p = \sum_{i \in p} \Theta_i$; essentiality score, defined by averaging the essentiality index of its firms: $\mathcal{E}_p = \langle \mathcal{E}_i \rangle_{i \in p}$; total bank loans $B_p = \sum_{i \in p} B_i$ and total margin $\mu_p = \sum_{i \in p} \mu_i$.

(a) Ranking plots

Figure 7 shows the sector rankings in terms of systemic risk scores, with sectors identified by their NACE Rev. 2 two-digit codes. Since in each sector we now aggregate firms of different sizes, rankings show less heterogeneity than in the case of individual firms. The top ESRI value of 0.04 belongs to sectors 69–70 (legal, accounting, management consultancy and head offices), which is the biggest and most essential overall. Other top sectors, whose 10% decrease in productivity would cause at least a 1% contraction of the whole economy, are: 61 (telecommunications), 19 (manufacture of coke and refined petroleum products), 46 (wholesale trade, except of motor vehicles and motorcycles), 36 (water collection, treatment and supply), 05–09 (mining and quarrying support and extraction activities) and 35 (electricity, gas, steam and air conditioning supply). At the lower tail of the ranking, we find sectors 94 (activities of membership organizations), 03 (fishing and aquaculture) and 53 (postal and courier activities—we only have one firm belonging to this sector in our data).

The FSRI and DR rankings reveal that sector 69–70 has an even higher impact on the financial system, owing to its large bank loans and profit margin. Indeed, a 10% production contraction in this sector would

Downloaded from royalsocietypublishing.org/rspa/article-pdf/doi/10.1098/rspa.2026.0271/6179767/rspa.2026.0271.pdf
by guest
on 26 November 2026

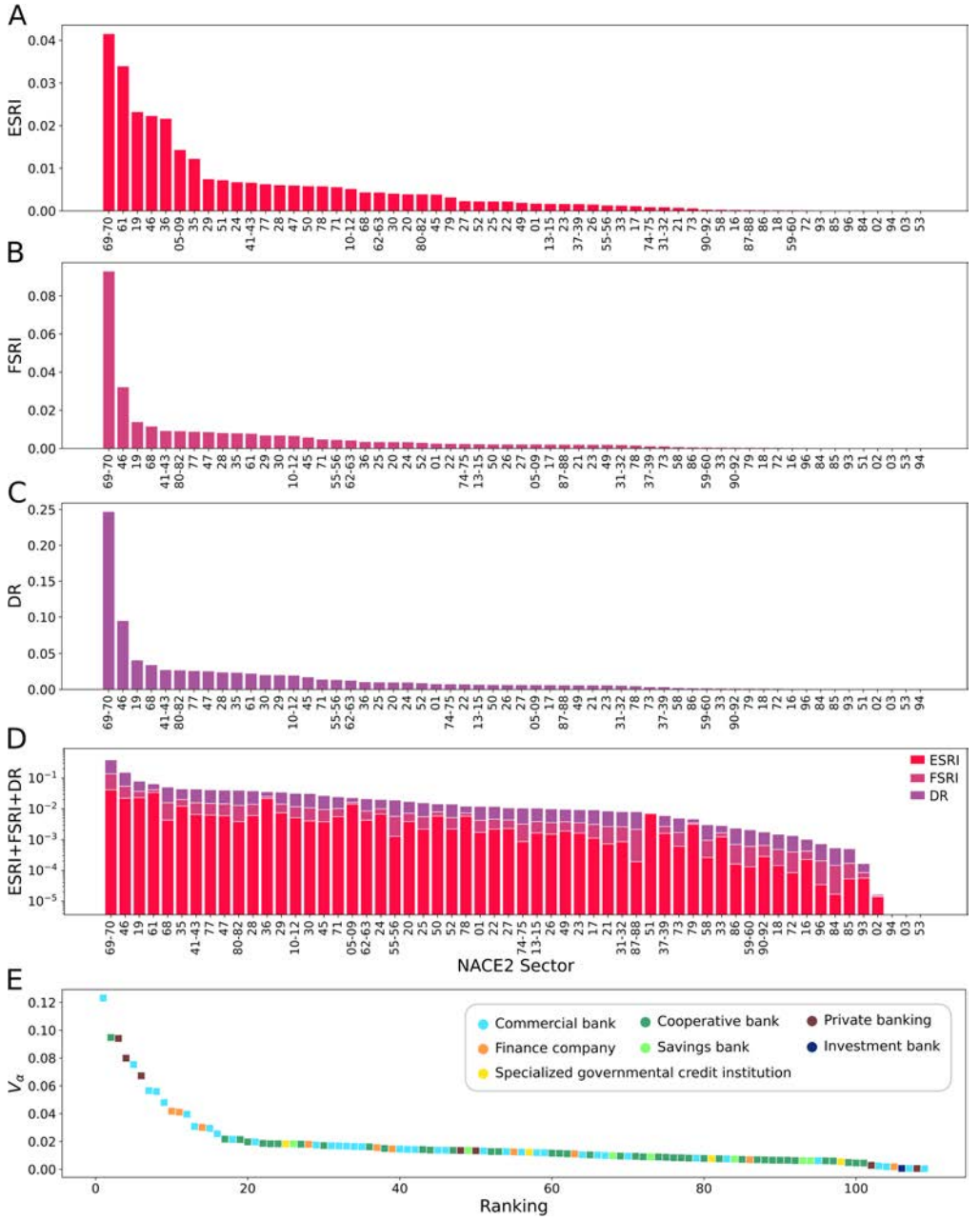

Figure 7. Ranking plots in terms of systemic risk metrics, with nodes placed in descending order of the corresponding scores. (A) Ranking based on ESRI scores of sectors, where for each sector the initial condition is given by a 10% reduction of production, while all other sectors remain fully active. (B) Ranking based on FSRI scores of sectors. (C) Ranking based on DR scores of sectors. (D) Ranking based on total systemic risk scores of sectors (ESRI+FSRI+DR), quantifying the total impact of 10% production reduction of the sector on the overall economy. Each bar is decomposed into its economic (ESRI), bank-level (FSRI) and interbank (DR) components, highlighting the relative contribution of each layer to aggregate systemic importance. (E) Ranking based on vulnerability scores of banks, obtained by averaging over 10% shocks of each sectors firms as initial conditions. Marker colours denote bank specialization.

Downloaded from royalsocietypublishing.org/rspa/article-pdf/doi/10.1098/rspa.2026.0271/6179767/rspa.2026.0271.pdf
by guest
on 26 November 2026

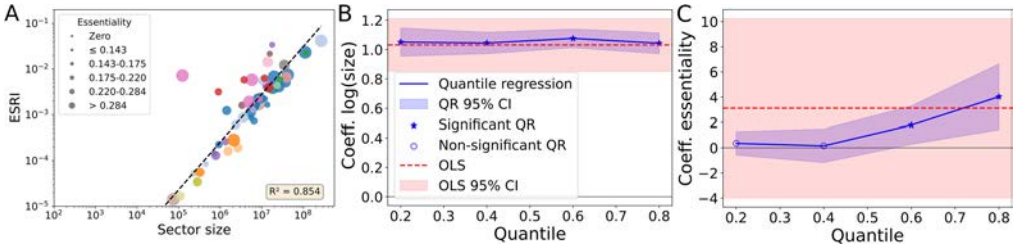

Figure 8. Determinants of ESRI for sector-level shocks. Correlation between ESRI and sector size (A). Size effects and significance of sector size (B) and sector essentiality (C) in OLS and quantile regressions of ESRI values.

cause a loss of 25% of the total bank equity. Other high-impact sectors are 46 (wholesale trade, except of motor vehicles and motorcycles), 19 (manufacture of coke and refined petroleum products), 68 (real estate activities) and 41 (construction of buildings), but roughly 25% of sectors have DR larger than 0.02.

The stacked ranking plot shows the relative contribution of ESRI, FSRI and DR for each sector (we use a logarithmic scale here to highlight the contribution of the different metrics also for all sectors and not just the top-impact ones). With respect to the previous scenario of individual firm defaults, now the initial shock is more broadly distributed, causing network amplification effects especially in the interbank layer (where propagation is now linear). As a result, bank vulnerability values are about 10 times higher, with most banks above 0.01 and the most vulnerable ones around 0.1 (they lose on average 10% of equity for a 10% production reduction in any NACE2 sector).

(b) Empirical analysis of determinants

ESRI. Repeating the same exercise as above, we estimate a minimal cross-sectional specification for the ESRI of sectors:[11]

$$\log(\mathrm{ESRI}_p) = \gamma + \beta_1 \log(\Theta_p) + \beta_2 \mathcal{E}_p + \varepsilon_p. \tag{6.1}$$

Regression results (shown in figure 8) indicate that sector size is again the primary determinant of ESRI, while the role of sector essentiality appears heterogeneous across the distribution of systemic risk. In the OLS specification, the model explains a large share of the variation in ESRI ($R^2 = 0.87$); the coefficient of $\log(\Theta)$ is close to unity and highly significant, while the essentiality index is not: being an essential sector does not systematically increase ESRI once size is controlled for. In the quantile regressions, the elasticity of ESRI with respect to size remains remarkably stable and strongly significant, while the coefficient of essentiality becomes positive and statistically significant in the upper quantiles (0.6 and especially 0.8). Therefore, essential sectors do not necessarily have high impact, except when they are very large.

FSRI. Figure 9A shows the positive correlation of FSRI with ESRI: sectors that generate large economic losses tend to be more financially damaging, but the mapping is far from one-to-one—owing to differences in bank credit exposures. We therefore estimate the following minimal cross-sectional specification:

$$\log(\mathrm{FSRI}_p) = \gamma + \beta_1 \log(\mathrm{ESRI}_p) + \beta_2 (B_p/\Theta_p) + \beta_3 (\mu_p/\Theta_p) + \varepsilon_i. \tag{6.2}$$

Regression results (figure 9) indicate that financial systemic risk is strongly driven by economic systemic relevance, with balance-sheet characteristics playing a more limited, heterogeneous role. In the OLS specification, the model explains a substantial fraction of the variation in FSRI ($R^2 = 0.73$). The coefficient of ESRI is large, positive and highly significant, with an elasticity close to unity. By contrast, loan share (B/Θ) and profit margin (μ/Θ) are not statistically significant, meaning that, on average, financial exposure and profitability do not systematically affect FSRI once real systemic importance is accounted for.

[11] Now, we cannot consider the market share of a sector, which is unity by definition.

Downloaded from royalsocietypublishing.org/rspa/article-pdf/doi/10.1098/rspa.2026.0271/6179767/rspa.2026.0271.pdf
by guest
on 26 November 2026

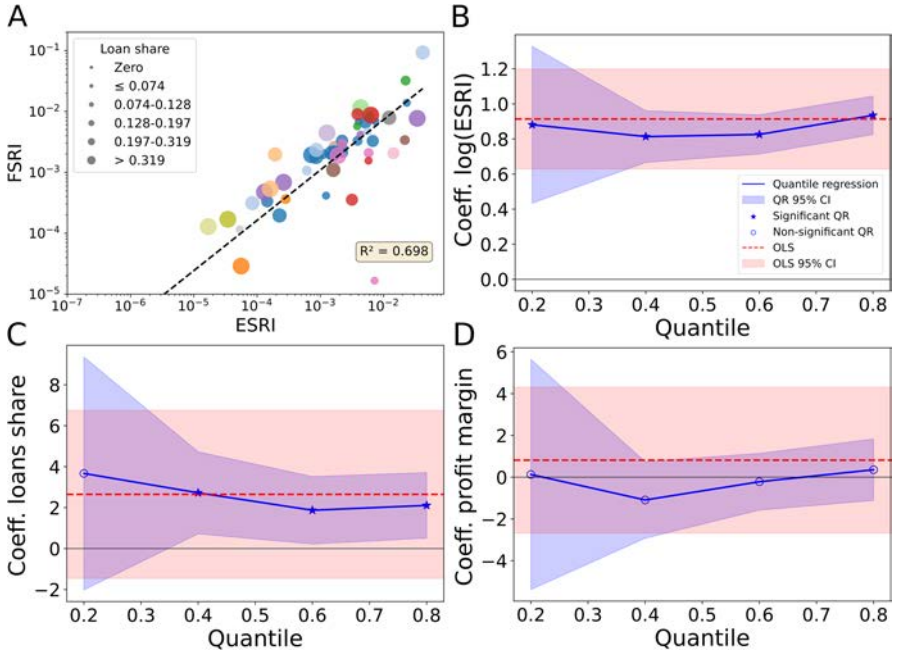

Figure 9. Determinants of FSRI for sector-level shocks. Correlation between FSRI and ESRI (A). Size effects and significance of ESRI (B), bank loans share (C) and profit margin (D) in OLS and quantile regressions of FSRI values.

In the quantile regressions, the coefficient of ESRI remains highly significant and remarkably stable across all quantiles. By contrast, loan share becomes positive and statistically significant from the median quantile onward (0.4, 0.6 and 0.8), indicating that leverage through bank financing contributes to systemic exposure mainly for firms that are already structurally relevant in the economy. Finally, profit margin is never statistically significant, and its coefficients fluctuate in sign across quantiles, providing little evidence that firm profitability systematically affects financial systemic importance. Overall, these findings suggest that financial systemic risk largely reflects the economic counterpart, with credit exposures acting as a reinforcing mechanism for firms that are already systemically important.

DR and vulnerability. Figure 10A shows that DR is directly driven by FSRI ($R^2 = 1.000$), implying that sectoral DR is nearly a deterministic, proportional amplification of the initial bank-equity losses captured by FSRI. Indeed, FSRI represents the initial condition of the DR dynamics, which in the considered scenario propagates for just one step.

A more interesting picture is provided by bank vulnerability values V_α. As figure 10 shows, vulnerability is now little driven by corporate loans shares (C_α/E_α), but correlates more with interbank loans shares (A_α/E_α). Indeed, the one-step shock propagation in the interbank layer, driven by A_α, is much larger than the initial shock, determined (among other things) by C_α. We therefore estimate the following minimal cross-sectional specification:

$$\log(V_\alpha) = \gamma + \beta_1 \log(E_\alpha) + \beta_2(A_\alpha/E_\alpha) + \beta_3(C_\alpha/E_\alpha) + \varepsilon_i, \tag{6.3}$$

where we used bank equity E_α as a proxy of size, while interbank assets A_α and corporate loans C_α are normalized to avoid collinearity and to represent leverage values. Regression results (figure 10) suggest that bank vulnerability is primarily driven by leverage rather than size. In the OLS specification, the model explains a substantial share of the variation in vulnerability ($R^2 = 0.63$), with both interbank exposure and corporate loan exposure being positive and statistically significant. By contrast, bank size, proxied by equity, is not statistically significant: balance-sheet scale alone does not systematically determine vulnerability, once leverage ratios are accounted for.

Downloaded from royalsocietypublishing.org/rspa/article-pdf/doi/10.1098/rspa.2026.0271/6179767/rspa.2026.0271.pdf
by guest
on 26 November 2026

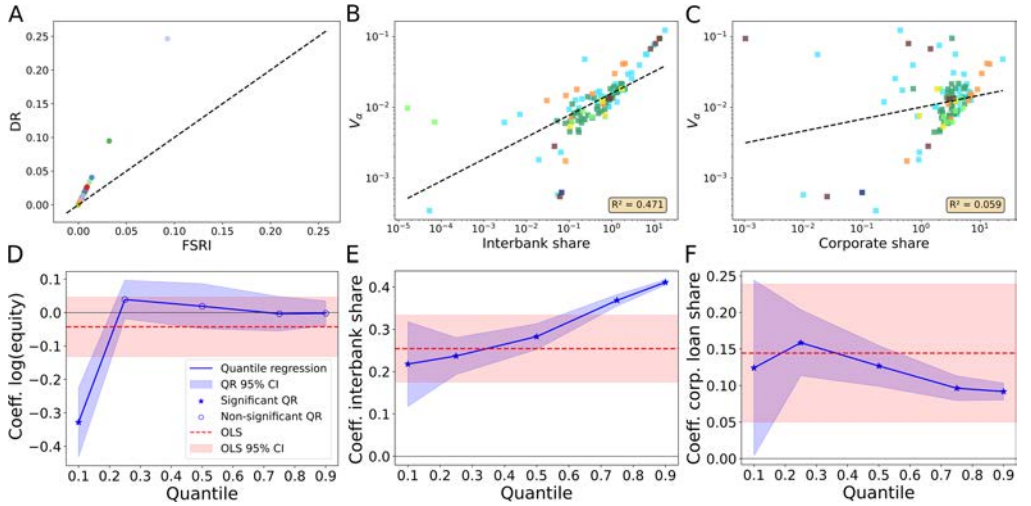

Figure 10. Determinants of DR and bank vulnerability for sector-level shocks. Correlation between DR and FSRI (A), between vulnerability and interbank share (B) and between vulnerability and corporate loans share (C). Size effects and significance of equity (D), interbank loans share (E) and corporate loans share (F) in OLS and quantile regressions of vulnerability values.

The quantile regressions reveal important heterogeneity across the distribution of vulnerability. The coefficient of interbank share rises steadily from about 0.22 at the 10th percentile to more than 0.41 at the 90th percentile, indicating that interbank exposures play an increasingly dominant role among the most vulnerable banks. Corporate loan exposure also contributes positively across quantiles, though with a smaller and slightly declining effect at higher percentiles. By contrast, equity size shows significance only in the lowest quantile, where it enters with a negative coefficient, indicating that larger capital buffers may reduce vulnerability among the least fragile institutions, but this effect disappears for more vulnerable banks.

(c) Summary

Sector-level shocks paint a broadly consistent but more nuanced picture. The legal, accounting and management sector (NACE 69–70) dominates systemic risk rankings across all three layers, with a 10% production contraction there translating into a loss of roughly 25% of total bank equity. As with individual firms, sector size is the dominant and most robust driver of economic systemic impact, while essentiality only becomes a significant amplifier for the largest sectors. Financial systemic risk tracks economic risk closely, with bank credit exposure reinforcing—rather than independently generating—sectoral importance. At the interbank level, vulnerability is now more strongly governed by interbank exposure than by corporate loan shares, reflecting the amplifying dynamics of even a single propagation step through the interbank network.

7. Conclusions

Understanding how shocks propagate between the real economy and the financial system is crucial for assessing systemic risk. Production networks and financial networks are deeply intertwined: disruptions affecting firms propagate through supply chains, reduce revenues, impair loan repayments and ultimately transmit distress to banks—which are amplified within the interbank market. However, empirical analyses of these mechanisms are typically constrained by the scarcity of detailed network data, since firm-to-firm transactions, credit exposures and interbank relationships are largely confidential.

This work addresses this limitation by proposing a framework to reconstruct the multi-layer structure linking firms and banks using only balance-sheet information. The approach combines reconstruction methods for production, bank–firm credit and interbank networks into a unified multi-layer representation of the economy. On this reconstructed structure, we simulate an ordered contagion mechanism in which shocks

Downloaded from royalsocietypublishing.org/rspa/article-pdf/doi/10.1098/rspa.2026.0271/6179767/rspa.2026.0271.pdf
by guest
on 26 November 2026

propagate from firms through supply chains, generate output losses measured by the ESRI, translate into credit losses for banks captured by the FSRI and subsequently spread within the interbank market through equity deterioration computed by DR. The resulting computational pipeline can then be used to simulate different initial shocks scenarios and quantify their short- and long-term impact.

The illustrative application to the Italian economy highlights several important insights, which are in line with the empirical findings of the literature on network-based contagion. First, systemic importance is highly heterogeneous: only a very small fraction of firms generate large economic losses when they fail, while the vast majority have negligible systemic impact. Second, the firms that are most critical for production are not necessarily the same as those posing the greatest risk to the banking system, indicating that economic and financial systemic importance overlap only partially. This highlights the importance of considering the full multi-layer structure of the economy rather than focusing on a single network layer. Finally, the empirical analysis of determinants of systemic impact provides a descriptive decomposition of the simulated systemic outcomes.[12] Firm size acts as the dominant and most stable determinant of systemic impact, while variables such as essentiality, market share and financial exposure become increasingly relevant in the upper tail, indicating that these characteristics act as nonlinear amplifiers rather than uniform drivers of systemic risk.

Of course, our works comes with limitations. First, we have considered only a sample of the Italian economy, mainly because of missing balance-sheet entries for the bank–firm layer: bank loan values are reported mainly for large firms and at the consolidated level. Furthermore, we have applied IO sectorial dependencies to our firm sample and used network density values taken from the literature. Sections S5, S6 and S7 in the supplementary material report more information and robustness checks on these issues. Second, the considered systemic risk dynamics is based on strong assumptions that abstract away from several real-world adjustment mechanisms. In ESRI, firms are not allowed to reorganize supplier–customer relationships [28]. FSRI linearly maps production losses to firm equity erosion and subsequently to bank credit deterioration, while firms facing revenue shortfalls may indeed draw on liquidity reserves, adjust inventories or access alternative financing before experiencing equity impairment. Similarly for DR, interbank claims are linearly revaluated with bank equity losses in order to set their market value [42]. Therefore, our framework provides an upper bound on systemic risk, assuming immediate and full pass-through of shocks to credit quality, without any buffering or mitigation. This modelling choice is consistent with the stress-testing tradition of estimating worst-case contagion dynamics in the absence of policy intervention or firm-level adaptation. Incorporating dynamic firm behaviour would require substantially richer microdata and a more granular modelling framework and represents an important avenue for future research.[13] Third, we do not have ground-truth data against which to assess the network reconstruction task. However, the models we employ have been extensively validated against empirical network data in prior work: the DCGM has been shown to outperform alternative probabilistic recipes across a broad range of interbank networks from different countries and time periods [19,52–54], while the SCGM was successfully tested on production networks from the Netherlands, Hungary and Ecuador [24,25,50]. These validation exercises provide strong indirect evidence that our reconstruction procedure captures the relevant structural features of the underlying networks—with deviations mainly occurring in correspondence of systemic events [50,55]. Of course, other reconstruction methods exist (see recent reviews [56,57]), but comparison with these is out of the scope of the paper.

From a policy perspective, our framework enables network-based stress testing even in the absence of detailed microdata. This opens the possibility of constructing data-driven ‘digital twins’ of economic systems

[12] Since the systemic risk indicators are computed from the reconstructed network and the shock-propagation dynamics, and these dynamics ultimately depend on firms’ balance sheet and network characteristics, part of the explanatory power is necessarily structural rather than independent. Hence, the regressions are not intended as predictive exercises, nor as evidence of causal relationships. Their purpose is to disentangle which observable firm and bank characteristics are most strongly associated with the final systemic outcomes generated by the contagion dynamics. Indeed, the mapping from explanatory variables to systemic risk metrics is not mechanical: firm size, market share, essentiality and leverage enter the model only indirectly through network reconstruction and iterative propagation equations, while systemic risk measures emerge from a nonlinear dynamic process on the multi-layer network. Therefore, *ex ante*, it is not obvious how much variation in the final outcomes should be attributable to each characteristic, nor whether their effects should remain stable across the distribution of systemic risk.

[13] Our primary goal is not to produce point forecasts of losses, but rather to provide a comparative ranking of systemic importance across firms, sectors and banks. As long as the simplifying assumptions affect all entities in a broadly symmetric way, the relative rankings we obtain are expected to remain robust.

Downloaded from royalsocietypublishing.org/rspa/article-pdf/doi/10.1098/rspa.2026.0271/6179767/rspa.2026.0271.pdf
by guest
on 26 November 2026

that allow regulators to perform integrated stress tests across production and financial networks. Such tools could help identify firms whose disruption would generate disproportionate economic losses, detect banks whose balance-sheet composition makes them particularly exposed to real-economy shocks and evaluate the systemic consequences of sectoral disruptions. At the same time, there are several directions for future research. Improving network reconstruction techniques, incorporating dynamic firm behaviour and adaptive financial responses and extending the framework to cross-border production and financial networks would allow a more realistic representation of shock propagation. Ultimately, by bridging production and financial networks within a unified multi-layer representation, this work highlights how systemic risk emerges not only from individual institutions but from the architecture of the economic network itself, emphasizing the need for analytical tools capable of capturing the interconnected nature of modern economies.

Data accessibility. The data used in this manuscript come from two proprietary datasets. The bank data come from Bureau van Dijk's BankFocus, and firm data come from AIDA, Bureau van Dijk's database of Italian firms. Therefore, we cannot publicly release our dataset. Codes to reconstruct the multi-layer network and run the shock propagation dynamics using sample data are available at [58].

Supplementary material is available online [59].

Declaration of AI use. We have not used AI-assisted technologies in creating this article.

Authors' contributions. S.M.: data curation, formal analysis, methodology, software, validation, visualization, writing—original draft, writing—review and editing; A.M.: data curation, formal analysis, methodology, software, validation, visualization, writing—original draft, writing—review and editing; G.C.: conceptualization, funding acquisition, methodology, project administration, supervision, writing—original draft, writing—review and editing.

All authors gave final approval for publication and agreed to be held accountable for the work performed therein.

Conflict of interest declaration. We declare we have no competing interests.

Funding. We acknowledge financial support from the National Recovery and Resilience Plan (NRRP), Mission 4 Component 2 Investment 1.1, funded by the European Union—NextGenerationEU: call for tender no. 1409 of 14 September 2022 by the Italian Ministry of University and Research (MUR), project title: *C2T—From Crises to Theory: towards a science of resilience and recovery for economic and financial systems*, Concession Decree no. 1381 of 1 September 2023, project code P2022E93B8—CUP E53D23018320001. Call for tender no. 104 of 2 February 2022 by the Italian Ministry of University and Research (MUR), project title: *RENet—Reconstructing economic networks: from physics to machine learning and back*, Concession Decree No. 957 of 30 June 2023, project code 2022MTBB22—CUP E53D23001770006.

References

1. Schweitzer F, Fagiolo G, Sornette D, Vega-Redondo F, Vespignani A, White DR. 2009 Economic networks: the new challenges. *Science* **325**, 422–425. (doi:10.1126/science.1173644)
2. Atalay E, Hortaçsu A, Roberts J, Syverson C. 2011 Network structure of production. *Proc. Natl Acad. Sci. USA* **108**, 5199–5202. (doi:10.1073/pnas.1015564108)
3. Pichler A *et al*. 2023 Building an alliance to map global supply networks. *Science* **382**, 270–272. (doi:10.1126/science.adi7521)
4. Elliott M, Golub B, Jackson MO. 2014 Financial networks and contagion. *Am. Econ. Rev.* **104**, 3115–3153. (doi:10.1257/aer.104.10.3115)
5. Acemoglu D, Ozdaglar A, Tahbaz-Salehi A. 2015 Systemic risk and stability in financial networks. *Am. Econ. Rev.* **105**, 564–608. (doi:10.1257/aer.20130456)
6. Battiston S *et al*. 2016 Complexity theory and financial regulation. *Science* **351**, 818–819. (doi:10.1126/science.aad0299)
7. Poledna S, Molina-Borboa JL, Martínez-Jaramillo S, van der Leij M, Thurner S. 2015 The multi-layer network nature of systemic risk and its implications for the costs of financial crises. *J. Financ. Stab.* **20**, 70–81. (doi:10.1016/j.jfs.2015.08.001)
8. Carleton TA, Hsiang SM. 2016 Social and economic impacts of climate. *Science* **353**, aad9837. (doi:10.1126/science.aad9837)
9. Carvalho VM, Nirei M, Saito YU, Tahbaz-Salehi A. 2021 Supply chain disruptions: evidence from the Great East Japan Earthquake. *Q. J. Econ.* **136**, 1255–1321. (doi:10.1093/qje/qjaa044)
10. Battiston S, Mandel A, Monasterolo I, Schütze F, Visentin G. 2017 A climate stress-test of the financial system. *Nat. Clim. Change* **7**, 283–288. (doi:10.1038/nclimate3255)

Downloaded from royalsocietypublishing.org/rspa/article-pdf/doi/10.1098/rspa.2026.0271/6179767/rspa.2026.0271.pdf
by guest
on 26 November 2026

11. Góes C, Bekkers E. 2022 *The impact of geopolitical conflicts on trade, growth, and innovation.* WTO Staff Working Paper ERSD 2022-09. Geneva, Switzerland: World Trade Organization.
12. Nehrebecka N. 2021 COVID-19: stress-testing non-financial companies: a macroprudential perspective. The experience of Poland. *Eurasian Econ. Rev.* **11**, 283–319. (doi:10.1007/s40822-020-00163-0)
13. Pichler A, Pangallo M, del Rio-Chanona RM, Lafond F, Farmer JD. 2022 Forecasting the propagation of pandemic shocks with a dynamic input-output model. *J. Econ. Dyn. Control* **144**, 104527. (doi:10.1016/j.jedc.2022.104527)
14. Fouque JP, Langsam JA. 2013 *Handbook on systemic risk*. Cambridge, UK: Cambridge University Press.
15. Hallegatte S. 2014 *Economic resilience: definition and measurement.* World Bank Policy Research Working Paper 6852. Washington, DC: World Bank Group.
16. Amini H, Cont R, Minca A. 2016 Resilience to contagion in financial networks. *Math. Finance* **26**, 329–365.(doi:10.1111/mafi.12051)
17. Bardoscia M, Battiston S, Caccioli F, Caldarelli G. 2017 Pathways towards instability in financial networks. *Nat. Commun.* **8**, 14416. (doi:10.1038/ncomms14416)
18. Ramadiah A, Di Gangi D, Lo Sardo DR, Macchiati V, Pham Minh T, Pinotti F, Wilinski M, Barucca P, Cimini G. 2020 Network sensitivity of systemic risk. *J. Netw. Theory Finance* **5**, 53–72. (doi:10.21314/JNTF.2019.056)
19. Anand K *et al*. 2018 The missing links: a global study on uncovering financial network structures from partial data. *J. Financ. Stab.* **35**, 107–119. (doi:10.1016/j.jfs.2017.05.012)
20. Cimini G, Squartini T, Saracco F, Garlaschelli D, Gabrielli A, Caldarelli G. 2019 The statistical physics of real-world networks. *Nat. Rev. Phys.* **1**, 58–71. (doi:10.1038/s42254-018-0002-6)
21. Park J, Newman ME. 2004 Statistical mechanics of networks. *Phys. Rev. E* **70**, 066117. (doi:10.1103/PhysRevE.70.066117)
22. Squartini T, Garlaschelli D. 2011 Analytical maximum-likelihood method to detect patterns in real networks. *New J. Phys.* **13**, 083001. (doi:10.1088/1367-2630/13/8/083001)
23. Cimini G, Squartini T, Garlaschelli D, Gabrielli A. 2015 Systemic risk analysis on reconstructed economic and financial networks. *Sci. Rep.* **5**, 15758. (doi:10.1038/srep15758)
24. Ialongo LN, de Valk C, Marchese E, Jansen F, Zmarrou H, Squartini T, Garlaschelli D. 2022 Reconstructing firm-level interactions in the dutch input–output network from production constraints. *Sci. Rep.* **12**, 11847. (doi:10.1038/s41598-022-13996-3)
25. Fessina M, Cimini G, Squartini T, Astudillo-Estévez P, Thurner S, Garlaschelli D. 2026 Inferring firm-level supply chain networks with realistic systemic risk from industry sector-level data. *Sci. Rep.* **16**, 19848. (doi:10.1038/s41598-026-47883-y)
26. Haldane AG, May RM. 2011 Systemic risk in banking ecosystems. *Nature* **469**, 351–355. (doi:10.1038/nature09659)
27. Bardoscia M, Battiston S, Caccioli F, Caldarelli G. 2015 DebtRank: a microscopic foundation for shock propagation. *PLoS ONE* **10**, e0130406. (doi:10.1371/journal.pone.0130406)
28. Diem C, Borsos A, Reisch T, Kertész J, Thurner S. 2022 Quantifying firm-level economic systemic risk from nation-wide supply networks. *Sci. Rep.* **12**, 7719. (doi:10.1038/s41598-022-11522-z)
29. Montagna M, Kok C. 2016 *Multi-layered interbank model for assessing systemic risk*. ECB Working Paper 1944. Frankfurt am Main, Hesse: European Central Bank.
30. del Rio-Chanona RM, Korniyenko Y, Patnam M, Porter MA. 2020 The multiplex nature of global financial contagions. *Appl Netw. Sci.* **5**, 74. (doi:10.1007/s41109-020-00301-2)
31. Wang G-J, Yi S, Xie C, Stanley HE. 2021 Multilayer information spillover networks: measuring interconnectedness of financial institutions. *Quant. Finance* **21**, 1163–1185. (doi:10.1080/14697688.2020.1831047)
32. Guth M, Lipp C, Puhr C, Schneider M. 2020 Modeling the COVID-19 effects on the Austrian economy and banking system. Financial Stability Report 40. Vienna, Austria: Oesterreichische National bank.
33. Borsos A, Mero B. 2020 *Shock propagation in the banking system with real economy feedback*. MNB Working Paper 2020/6. Budapest, Hungary: Magyar Nemzeti Bank.
34. Tabachová Z, Diem C, Borsos A, Burger C, Thurner S. 2024 Estimating the impact of supply chain network contagion on financial stability. *J. Financ. Stab.* **75**, 101336. (doi:10.1016/j.jfs.2024.101336)
35. Fialkowski J, Diem C, Borsos A, Thurner S. 2026 A data-driven econo-financial stress-testing framework to estimate the effect of supply chain networks on financial systemic risk. *J. Econ. Dyn. Control* **188**, 105333. (doi:10.1016/j.jedc.2026.105333)

Downloaded from royalsocietypublishing.org/rspa/article-pdf/doi/10.1098/rspa.2026.0271/6179767/rspa.2026.0271.pdf
by guest
on 26 November 2026

36. Squartini T, Almog A, Caldarelli G, van Lelyveld I, Garlaschelli D, Cimini G. 2017 Enhanced capital-asset pricing model for the reconstruction of bipartite financial networks. *Phys. Rev. E* **96**, 032315. (doi:10.1103/PhysRevE.96.032315)
37. Battiston S, Puliga M, Kaushik R, Tasca P, Caldarelli G. 2012 DebtRank: too central to fail? Financial networks, the FED and systemic risk. *Sci. Rep.* **2**, 541. (doi:10.1038/srep00541)
38. Di Gangi D, Lillo F, Pirino D. 2018 Assessing systemic risk due to fire sales spillover through maximum entropy network reconstruction. *J. Econ. Dyn. Control* **94**, 117–141. (doi:10.1016/j.jedc.2018.07.001)
39. Miller RE, Blair PD. 2009 *Input-output analysis: foundations and extensions*. Cambridge, UK: Cambridge University Press.
40. Leontief W. 1986 *Input-output economics*. Oxford, UK: Oxford University Press.
41. Pichler A, Pangallo M, del Rio-Chanona RM, Lafond F, Farmer JD. 2020 Production networks and epidemic spreading: How to restart the UK economy? (doi:10.48550/arXiv.2005.10585)
42. Barucca P, Bardoscia M, Caccioli F, D'Errico M, Visentin G, Caldarelli G, Battiston S. 2020 Network valuation in financial systems. *Math. Finance* **30**, 1181–1204. (doi:10.1111/mafi.12272)
43. Bardoscia M, Caccioli F, Perotti JI, Vivaldo G, Caldarelli G. 2016 Distress propagation in complex networks: the case of non-linear DebtRank. *PLoS ONE* **11**, e0163825. (doi:10.1371/journal.pone.0163825)
44. Furfine CH. 2003 Interbank exposures: quantifying the risk of contagion. *J. Money, Credit Bank.* **35**, 111–128. (doi:10.1353/mcb.2003.0004)
45. Cimini G, Serri M. 2016 Entangling credit and funding shocks in interbank markets. *PLoS ONE* **11**, e0161642. (doi:10.1371/journal.pone.0161642)
46. Iazzetta C, Manna M. 2009 The topology of the interbank market: developments in Italy since 1990. Temi di discussione (Economic working papers, Bank of Italy) 711. Rome, Italy: Economic Research and International Relations Area.
47. Finger K, Fricke D, Lux T. 2013 Network analysis of the e-MID overnight money market: the informational value of different aggregation levels for intrinsic dynamic processes. *Comput. Manag. Sci.* **10**, 187–211. (doi:10.1007/s10287-013-0171-9)
48. Bacilieri A, Borsos A, Astudillo-Estévez P, Hoefer M, Lafond F. 2026 Firm-level production networks: what do we (really) know? *J. Econ. Dyn. Control* **187**, 105313. (doi:10.1016/j.jedc.2026.105313)
49. De Masi G, Gallegati M. 2012 Bank–firms topology in italy. *Empir. Econ.* **43**, 851–866. (doi:10.1007/s00181-011-0512-x)
50. Mancini A, Lengyel B, Di Clemente R, Cimini G. 2026 Evolution and determinants of firm-level systemic risk in local production networks. *PNAS Nexus* **5**, pgag233. (doi:10.1093/pnasnexus/pgag233)
51. Bonacich P. 1987 Power and centrality: a family of measures. *Am. J. Sociol.* **92**, 1170–1182. (doi:10.1086/228631)
52. Lebacher M, Cook S, Klein N, Kauermann G. 2019 In search of lost edges: a case study on reconstructing financial networks. *J. Netw. Theory Finance* **5**, 29–61. (doi:10.21314/JNTF.2019.058)
53. Mazzarisi P, Lillo F. 2017 Methods for reconstructing interbank networks from limited information: a comparison. In *Econophysics and sociophysics: recent progress and future directions* (eds F Abergel, H Aoyama, BK Chakrabarti, A Chakraborti, N Deo, I Vodenska), pp. 201–215. Cham, Switzerland: Springer.
54. Ramadiah A, Caccioli F, Fricke D. 2020 Reconstructing and stress testing credit networks. *J. Econ. Dyn. Control* **111**, 103817. (doi:10.1016/j.jedc.2019.103817)
55. Ferracci A, Cimini G. 2022 Systemic risk in interbank networks: disentangling balance sheets and network effects. (doi:10.48550/arXiv.2109.14360)
56. Squartini T, Caldarelli G, Cimini G, Gabrielli A, Garlaschelli D. 2018 Reconstruction methods for networks: the case of economic and financial systems. *Phys. Rep.* **757**, 1–47. (doi:10.1016/j.physrep.2018.06.008)
57. Mungo L, Brintrup A, Garlaschelli D, Lafond F. 2024 Reconstructing supply networks. *J. Phys.: Complex.* **5**, 012001. (doi:10.1088/2632-072X/ad30bf)
58. mnlknt. 2026 mnlknt/bank-firm_multilayer_shocks: initial release (Version v1.0.0) [Computer software]. Zenodo. (doi:10.5281/zenodo.21382240)
59. Majhi S, Mancini A, Cimini G. 2026 Supplementary material from: Modelling structure and credit risk of the economy: a multi-layer bank–firm network approach. Figshare. (doi:10.6084/m9.figshare.c.8758997)

Downloaded from royalsocietypublishing.org/rspa/article-pdf/doi/10.1098/rspa.2026.0271/6179767/rspa.2026.0271.pdf
by guest
on 26 November 2026