CEIS Tor Vergata
RESEARCH PAPER SERIES

Working Paper No. 82 May 2006

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CEIS Tor Vergata - Research Paper Series, Vol. 28, No. 82, May 2006

Testing for Parameter Stability in Dynamic Models across Frequencies*

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March 30, 2006

Abstract

This paper contributes to the econometric literature on structural breaks by proposing a test for parameter stability in VAR models at a particular frequency ω, where $\omega \in [0, \pi]$. When a dynamic model is affected by a structural break, the new tests allow for detecting which frequencies of the data are responsible for parameter instability. If the model is locally stable at the frequencies of interest, the whole sample size can be then exploited despite the presence of a break. Two empirical examples illustrate that local stability can concern only the lower frequencies (change in the U.S. monetary policy in the early 80's) or higher frequencies (decrease in the postwar U.S. productivity).

Keywords: Structural breaks, spectral analysis, productivity slowdown, yield curve.
JEL: C32, E43

*Previous drafts of this paper were presented at the ESF-EMM 2004 meeting in Alghero, the First Italian Congress of Econometrics and Empirical Economics in Venice, the Frontier in Time Series Congress in Olbia, and the First Meeting of the Netherlands Econometric Study Group in Amsterdam. We are grateful to Marco Centoni, as well as three anonymous referees, for useful comments and corrections. Gianluca Cubadda gratefully acknowledges financial support from MIUR.

Tests for structural changes are important tools in the statistical analysis of economic time series. In this respect, the well-known Chow (1960) test still constitutes a standard reference. It consists of splitting the sample into two sub-periods, before and after the break, and testing the equality of the parameters between the two sub-samples, using an asymptotic χ^2 distribution. Because of its simplicity of implementation, it is still used in many empirical studies. Nonetheless, this test was extended in several directions.[1]

Firstly, instead of considering the date of the break as known, the testing procedure should treat it as an unknown parameter to be estimated. Following the seminal paper of Quandt (1960), a recursive sequence of Chow tests is performed, dating the break at the point where the test statistic takes the largest value. Andrews (1993) delivers the most important contribution for this extension by defining the asymptotic distribution of the sup-Chow test, which is no longer a χ^2 distribution. A further extension in this direction is developed by Bai and Perron (1998, 2003), who consider the case of multiple structural breaks with unknown dates. These authors propose several iterative methods to test for the number of breaks, and derive the asymptotic distributions of the relevant test statistics. All these procedures are valid for single equation models with no trending regressors, such as deterministic trends or I(1) processes.

The above approaches have recently been extended to multivariate regression models. Bai *et al.* (1998) generalized the single break framework in Andrews (1993) to multiple time series that are either stationary or cointegrated in the regimes of parameter stability. They showed that statistical inference is more precise when series having a common break are jointly analyzed. Bai (2000) considered the issue of multiple breaks in a segmented stationary VAR model and proved that the number of change points can be consistently estimated via information criteria, whereas Qu and Perron (2004) proposed a quasi maximum likelihood approach to analyze multiple breaks in multivariate regression models.

Secondly, some papers were devoted to the application of the standard Chow test to the Vector Error Correction Model (VECM). Hansen (2003), *inter alia*, provided tests for a break in the coefficients of the VECM, though his results are restricted to the case of known break dates. In particular, a partial structural change can be present in the cointegration parameters or in the adjustment coefficients. Such an extension has interesting economic implications, as it is possible to interpret a structural break as affecting the long-run (partial change in the cointegration relationships) or the short-run (partial change in the adjustment coefficients).

The present paper generalizes the previous idea by proposing a test for parameter stability in a segmented stationary or cointegrated VAR model at a particular frequency ω, where $\omega \in [0, \pi]$. Hence, if a VAR model is affected by a structural break, it is then possible to detect which frequencies of the data are responsible for parameter instability. Moreover, if a researcher wishes to concentrate on a subset of the frequencies of the data, the proposed test allows one to check whether the whole sample size can be exploited for the analysis, despite the presence of a break. Although the null hypothesis of local stability at frequency ω implies that the spectral density

[1] See Hansen (2001) for a detailed survey of the current state of the art.

matrix at frequency ω is stable over time, the testing procedure is easily implemented in the time domain,[2] as it is based on a set of linear hypotheses on the autoregressive parameters. The test statistic for local stability at a given frequency has a limit χ^2 distribution when the break date is either known or estimated by means of the sup-Chow test for a full structural change. For the latter case, a bootstrap procedure is also offered. We evaluate the finite-sample behavior of our testing procedure through a Monte Carlo study.

The test procedure is applied to two well-known examples. The first one copes with the predictive power of the yield curve on future output growth in the U.S. It turns out that a break is detected around July of 1980. Based on Estrella *et al.* (2003), we interpret this break as the consequence of a change in monetary policy associated with the nomination of Paul Volcker as the head of the FED. Our local stability tests confirm this hypothesis by stressing that the relationship between the spread and future output growth remains stable at low frequencies.

The second example addresses the slowdown of the post-war United States output growth. Following King *et al.* (1991), we consider a trivariate system with consumption, investment and output to get a clearer view on this issue. Similarly to Bai *et al.* (1998), a structural break is detected in the late sixties, and our local stability tests reveal that the system is stable only at high frequencies. This evidence is consistent with the view that a negative productivity shock is at the origin of the break.

The paper is organized as follows. In section 2, the concept of local stability at frequency ω is developed for segmented stationary VAR systems and known break dates. The extensions for cointegrated systems and unknown break dates are proposed in Section 3. In section 4, a simulation study is performed to investigate the properties of the tests. Section 5 presents both the empirical applications, and Section 6 concludes.

1 Local stability in stationary VAR models

Let us consider an n-vector time series $\{X_t, t = 1, \ldots, T\}$ that is generated by the following stationary linear stochastic process

$$X_t = \Theta D_t + C(L)\varepsilon_t, \tag{1}$$

where $C(L) = I_n + \sum_{i=1}^{\infty} C_i L^i$ is such that $\sum_{j=1}^{\infty} j\,|C_j| < \infty$, ε_t are i.i.d. $N_n(0, \Sigma)$ innovations, and D_t is an m-vector of deterministic terms that may contain a constant and various trigonometric functions of time.

We assume that series X_t admits the following VAR(p) representation:

$$A(L)X_t = \Phi D_t + \varepsilon_t, \quad t = 1, \ldots, T, \tag{2}$$

[2] Similar approaches can be found in Breitung and Candelon (2006) for Granger-causality tests, Centoni and Cubadda (2003) for measuring the cyclical effects of permanent-transitory shocks, and Christiano and Vigfusson (2003) for maximum likelihood analysis of business cycle models.

where $A(L) = I_n - \sum_{i=1}^{p} A_i L^i$ is such that $\det[A(c)] = 0$ implies that $|c| > 1$, and $\Phi D_t = A(L)\Theta D_t$.[3]

By expanding the polynomial matrix $A(L)$ on the complex conjugate points z and z^{-1}, where $z = \exp(-i\omega)$ and $\omega \in [0, \pi]$, we obtain

$$A(L) = \Delta_\omega(L) - \Pi_\omega(L)L - \Gamma_\omega(L)\Delta_\omega(L)L, \tag{3}$$

where $\Gamma_\omega(L)$ is a $n \times n$ polynomial matrix of order $(p-3)$ if $\omega \in (0, \pi)$, $(p-2)$ if $\omega = 0$ or $\omega = \pi$,[4] and

$$\Delta_\omega(L) = \begin{cases} 1 - 2\cos(\omega)L + L^2 & \text{if } \omega \in (0, \pi) \\ (1 - zL) & \text{if } \omega = 0 \text{ or } \omega = \pi \end{cases}.$$

Comparing both sides of equation (3) for $L = z$ yields

$$A(z) = -\Pi_\omega(z)z, \tag{4}$$

and, by equating real and imaginary parts of (4), we find

$$\Pi_\omega(L) = \begin{cases} -\operatorname{Im}[A(z)]/\sin(\omega) + (\operatorname{Re}[A(z)] + \operatorname{Im}[A(z)]\cos(\omega)/\sin(\omega))L & \text{if } \omega \in (0, \pi) \\ -zA(z) & \text{if } \omega = 0 \text{ or } \omega = \pi \end{cases}.$$

Finally, by inserting equation (3) into equation (2), we rewrite the VAR model as follows:

$$\Delta_\omega(L)X_t = \Phi D_t + \Pi_\omega(L)X_{t-1} + \Gamma_\omega(L)\Delta_\omega(L)X_{t-1} + \varepsilon_t, \tag{5}$$

Since the filter $\Delta_\omega(L)$ annihilates at $L = z$, the filtered series $\Delta_\omega(L)(X_t - \Theta D_t)$ have null spectra at frequency ω. Hence the parameters $\Pi_\omega(L)$ fully characterize the stochastic behavior of series X_t at frequency ω. Indeed, the spectral density matrix of the stochastic process $(X_t - \Theta D_t)$ at frequency ω is given by $C(z)\Sigma C(z^{-1})'$, where $C(z) = -(\Pi_\omega(z)z)^{-1}$. [5]

The frequency domain properties of model (2) are also determined by the nature of the deterministic vector D_t. Indeed, a linear combination of the trigonometric functions $[\cos(\omega t), \sin(\omega t)]$ has its spectral mass entirely concentrated at frequency ω. Hence, let us write

$$\Phi D_t = \Phi_1 D_{1,t} + \Phi_2 D_{2,t},$$

[3]The reason we assume stationarity is twofold. First, the spectral density matrix is well-defined only for stationary VAR processes. Second, the asymptotic theory of structural break tests does not generally allow for unit or explosive roots (see, *inter alia*, Andrews 1993; Bai and Perron; 1998, 2003; Bai, 2000).

[4]For reasons that will be clarified later, we are assuming that $p > 2$.

[5]Notice that similar reparametrizations of the VAR model are widely used in the context of seasonal cointegration analysis (see e.g. Cubadda, 2001).

where $D_{1,t}$ and $D_{2,t}$ are respectively composed of m_1 and m_2 elements such that

$$\Delta_\omega(L)D_{1,t} = 0,$$

$$\Delta_\omega(L)D_{2,t} \neq 0.$$

It is clear that the parameters Φ_1 fully characterize the deterministic behavior of series X_t at frequency ω.

We now allow for a possible structural break at time $T_b = bT$, where $b \in (0,1)$. Let us assume, for the moment, that there is only one single break and its date T_b is known. Model (2) is then generalized by the following sub-sample models:

$$A^-(L)X_t = \Phi^- D_t + \varepsilon_t, \quad t = 1, \ldots, T_b, \tag{6}$$

$$A^+(L)X_t = \Phi^+ D_t + \varepsilon_t, \quad t = T_b + 1, \ldots, T, \tag{7}$$

where $A^-(L) = I_n - \sum_{i=1}^p A_i^- L^i$, and $A^+(L) = I_n - \sum_{i=1}^p A_i^+ L^i$.

Notice that we can expand both the polynomial matrices $A^-(L)$ and $A^+(L)$ on 0 and the complex conjugate points z and z^{-1}, thus obtaining the sub-sample analogs of model (5). Hence, let us consider the following particular cases of the sub-sample models (6) and (7):

$$\begin{aligned} \Delta_\omega(L)X_t &= \Phi_1 D_{1,t} + \Phi_2^- D_{2,t} + \Pi_\omega(L)X_{t-1} + \Gamma_\omega^-(L)\Delta_\omega(L)X_{t-1} + \varepsilon_t, \\ t &= 1, \ldots, T_b, \end{aligned} \tag{8}$$

$$\begin{aligned} \Delta_\omega(L)X_t &= \Phi_1 D_{1,t} + \Phi_2^+ D_{2,t} + \Pi_\omega(L)X_{t-1} + \Gamma_\omega^+(L)\Delta_\omega(L)X_{t-1} + \varepsilon_t, \\ t &= T_b + 1, \ldots, T, \end{aligned} \tag{9}$$

where $\Phi_1^- = \Phi_1^+ \equiv \Phi_1$, and $\Pi_\omega^-(L) = \Pi_\omega^+(L) \equiv \Pi_\omega(L)$.

Since the structural break does not affect the components of series X_t that are associated with fluctuations at frequency ω, the sub-sample models (8) and (9) are said to be locally stable at that frequency. Notice that local stability is possible only if the polynomial parameter matrices $\Gamma_\omega^-(L)$ and $\Gamma^+(L)$ can freely vary from $\Pi_\omega(L)$. Therefore, a necessary condition for local stability at frequency ω is that $p > 2$ if $\omega \in (0, \pi)$ and $p > 1$ if $\omega = 0$ or $\omega = \pi$.

The statistical problem consists of testing for each of the following null hypotheses:

$$\begin{aligned} &\text{H}_0 \ (\textit{global stability}): && [A^-(L) = A^+(L)] \cap [\Phi^- = \Phi^+], \\ &\text{H}_* \ (\textit{local stability}): && [\Pi_\omega^-(L) = \Pi_\omega^+(L)] \cap [\Phi_1^- = \Phi_1^+], \end{aligned}$$

versus the alternative hypothesis:

$$\text{H}_1 \ (\textit{global instability}): \quad [A^-(L) \neq A^+(L)] \cup [\Phi^- \neq \Phi^+].$$

In particular, the sample-split Chow test statistics (see e.g. Doornik and Hendry, 1997) for the systems of hypotheses H_0 versus H_1 and H_* versus H_1 are, respectively, the following:

$$\Xi_{0|1}(b) = (T-2p-q_0)\frac{\det\left(\sum_{t=p+1}^{T}\widehat{\varepsilon}_t'\widehat{\varepsilon}_t\right) - \det\left(\sum_{t=p+1}^{T_b}\widehat{\varepsilon_t^-}'\widehat{\varepsilon_t^-} + \sum_{t=T_b+1}^{T}\widehat{\varepsilon_t^+}'\widehat{\varepsilon_t^+}\right)}{\det\left(\sum_{t=p+1}^{T_b}\widehat{\varepsilon_t^-}'\widehat{\varepsilon_t^-} + \sum_{t=T_b+1}^{T}\widehat{\varepsilon_t^+}'\widehat{\varepsilon_t^+}\right)} \xrightarrow{d} \varkappa^2(q_0), \quad (10)$$

$$\Xi_{*|1}(b,\omega) = (T-2p-q_*)\frac{\det\left(\sum_{t=p+1}^{T_b}\widetilde{\varepsilon_t^-}'\widetilde{\varepsilon_t^-} + \sum_{t=T_b+1}^{T}\widetilde{\varepsilon_t^+}'\widetilde{\varepsilon_t^+}\right) - \det\left(\sum_{t=p+1}^{T_b}\widehat{\varepsilon_t^-}'\widehat{\varepsilon_t^-} + \sum_{t=T_b+1}^{T}\widehat{\varepsilon_t^+}'\widehat{\varepsilon_t^+}\right)}{\det\left(\sum_{t=p+1}^{T_b}\widehat{\varepsilon_t^-}'\widehat{\varepsilon_t^-} + \sum_{t=T_b+1}^{T}\widehat{\varepsilon_t^+}'\widehat{\varepsilon_t^+}\right)} \xrightarrow{d} \varkappa^2(q_*), \quad (11)$$

where $\{\widehat{\varepsilon}_t, t = 1, \ldots, T\}$ are the residuals resulting from OLS estimation of the fixed parameter model (2), $\{\widehat{\varepsilon_t^-}, t = 1, \ldots, T_b\}$, $\{\widehat{\varepsilon_t^+}, t = T_b+1, \ldots, T\}$, $\{\widetilde{\varepsilon_t^-}, t = 1, \ldots, T_b\}$, $\{\widetilde{\varepsilon_t^+}, t = T_b+1, \ldots, T\}$ are, respectively, the residuals resulting from OLS estimation of the sub-sample models (6), (7), (8), (9), and

$$q_0 = n^2p + mn,$$

$$q_* = \begin{cases} 2n^2 + m_1n & \text{if } \omega \in (0,\pi) \\ n^2 + m_1n & \text{if } \omega = 0 \text{ or } \omega = \pi. \end{cases}$$

The statistic (10) is the usual Chow test statistic for global stability, whereas (11) is the suggested test statistic for local stability at frequency ω. These statistics may be used in a sequential fashion; starting with running the test based on the statistic $\Xi_{0|1}(b)$. If the null hypothesis of global stability is not rejected, the sequence stops. Otherwise, one can test for local stability at frequencies $\omega_j = \omega_0(\frac{k-j}{k}) + \omega_k(\frac{j}{k})$, for $0 \leq \omega_0 < \omega_k \leq \pi$ and $j = 0, 1, ..., k$, by means of the test statistics $\Xi_{*|1}(\omega_j, b)$.[6]

Remark 1 *As correctly pointed out by a referee, the definition of the polynomial matrix* $\Pi_\omega(L)$ *depends on the parameterization that is considered. We can, for instance, use an alternative representation to model* (5) *such as the following*

$$\Delta_\omega(L)X_t = \Phi D_t + \widetilde{\Pi}_\omega(L)X_{t-p+1} + \widetilde{\Gamma}_\omega(L)\Delta_\omega(L)X_{t-1} + \varepsilon_t, \quad (12)$$

in which the parameters of interest for local stability are the coefficients of $[X_{t-p-1}, X_{t-p}]$ *and not those of* $[X_{t-1}, X_{t-2}]$*. However, in the appendix we show that constancy of* $\Pi_\omega(L)$ *is equivalent to that of* $\widetilde{\Pi}_\omega(L)$*. Hence, tests for local stability are invariant to isomorphic representations of the VAR.*

[6] Notice that the choice of the interval $[\omega_0, \omega_k]$ reflects the researcher's *a priori* knowledge of the frequencies at which local stability can occur. An agnostic option is to fix $\omega_0 = 0$ and $\omega_k = \pi$.

Remark 2 *We must notice that local stability can only occur at a finite set of frequencies. Indeed, local stability at frequency* ω *requires that both the following conditions hold*

$$A^{-}(L) \neq A^{+}(L), \tag{13}$$
$$A^{-}(z) = A^{+}(z). \tag{14}$$

Given that $A^{-}(L)$ *and* $A^{-}(L)$ *are polynomial matrices of order* p*, it is clear that there can exist, at most,* p *different points on the complex unit circle that satisfy* (14) *without violating* (13)*. Since we are considering real-valued processes, this implies that local stability may occur, at most, at* $(\lfloor p/2 \rfloor + 1)$ *frequencies in* $[0, \pi]$*.*

Remark 3 *It may be of interest to test for the stability of a subset of parameters only. In this case, let us write the polynomial matrix* $A(L)$ *in* (2) *as* $A(L) = A_1(L) + A_2(L)$*. If we assume that the break may solely affect the parameters in* $A_2(L)$*, the model* (2) *can be generalized as:*

$$\begin{aligned}[A_1(L) + A_2^{-}(L)]X_t &= \Phi^{-} D_t + \varepsilon_t, \quad t = 1, \ldots, T_b, \\ [A_1(L) + A_2^{+}(L)]X_t &= \Phi^{+} D_t + \varepsilon_t, \quad t = T_b + 1, \ldots, T,\end{aligned}$$

where $A_2^{-}(L) = I_n - \sum_{i=1}^{p} A_2^{-} L^i$*, and* $A_2^{+}(L) = I_n - \sum_{i=1}^{p} A_2^{+} L^i$*. We can then expand both the polynomial matrices* $A_2^{-}(L)$ *and* $A_2^{+}(L)$ *on* 0 *and the complex conjugate points* z *and* z^{-1} *and perform tests for both global and local stability of the parameters of interest.*

2 Various extensions

This section extends the above framework in various directions. In particular, we consider the cases of the cointegrated VAR and unknown break dates.

2.1 Cointegrated time series

Let us now consider an n-vector of cointegrated time series $\{Y_t, t = 1, \ldots, T\}$ of order (1,1) that is generated by the following VAR(p) model:

$$B(L)Y_t = \Phi D_t + \varepsilon_t, \tag{15}$$

where $B(L) = I_n - \sum_{i=1}^{p} A_i L^i$ is such that $\det[B(c)] = 0$ implies that $|c| > 1$ or $c = 1$, $B(1) = -\alpha\beta'$, α and β are $n \times r$-matrices with rank equal to r, and the matrix $\alpha'_\perp \Gamma \beta_\perp$ has full rank, where $\beta_\perp$ are $n \times (n - r)$-matrices with rank equal to $(n - r)$ such that $\alpha'_\perp \alpha = \beta'_\perp \beta = 0$, $\Gamma = I_n - \sum_{i=1}^{p-1} \Gamma_i$, and $\Gamma_i = -\sum_{j=i+1}^{p} A_j$ for $i = 1, 2, ..., p - 1$.

Series Y_t also admits the following Wold representation:

$$\Delta Y_t = \Theta D_t + F(L)\varepsilon_t, \tag{16}$$

where $F(L) = I_n + \sum_{i=1}^{\infty} F_i L^i$ is such that $\sum_{j=1}^{\infty} j\,|F_j| < \infty$, and $\Theta D_t = F(L)\Phi D_t$.

In this case, a difficulty emerges in testing for local stability at the zero frequency. Since $F(1) = \beta_\perp \left(\alpha'_\perp \Gamma \beta_\perp\right)^{-1} \alpha'_\perp$ (see e.g. Johansen, 1996), the spectral density matrix of series ΔY_t is singular at $\omega = 0$. Thus, the coefficient matrix $B(1)$ does not fully characterize the long-run behavior of series Y_t. However, we can reparameterize model (15) in order to avoid such singularity.

Suppose that the cointegration matrix β is fixed over time and known. Then we can transform series Y_t such that $X_t = T(L)Y_t$, where $T(L) = (\beta', \Delta\beta'_\perp)$. Model (15) can thus be written as in equation (2), where $A(L) = B(L)T(L)^{-1}$. Notice that if a super-consistent estimate of the cointegration matrix is available, one can simply use the estimate of β instead of the unknown population values without affecting the asymptotic distributions of the test statistics (10) and (11).

However, the cointegration matrix may be affected by the structural break at time T_b as well. In this case, series Y_t can be transformed as $X_t = T(L,t)'Y_t$, where

$$T(L,t) = \begin{cases} (\beta^-, \Delta\beta_\perp^-)', & t = 1, \ldots, T_b \\ (\beta^+, \Delta\beta_\perp^+)', & t = T_b + 1, \ldots, T \end{cases} .$$

Again, one can substitute the matricies β^- and β^+ with their super-consistent estimates. Inference on time-varying cointegration relationships is discussed, *inter alia*, by Hansen (2003), and Andrade *et al.* (2005).

2.2 Unknown break date

In the previous sections of the paper, the date of the break was considered as known beforehand. However, it is especially relevant to extend our procedure to the case where the break date is determined by means of the data itself. In such a case, Quandt (1960) proposed performing the Chow (1960) test recursively, using the supremum of the statistics. It is possible to apply this approach to the test based on (10) by considering the following statistic:

$$\Xi_{0|1}(\widehat{b}) = \sup[\Xi_{0|1}(b)],$$

where $t = [bT]$, and $b \in (0,1)$. Based on Andrews (1993), Bai *et al.* (1998) provided the asymptotic distribution of the above test statistic in the multivariate case.

We recommend testing for local stability at the various Fourier frequencies fixing $b = \widehat{b}$. A rationale for this procedure lies in the fact that the limit distribution of the break date estimator is unaffected by the imposition of valid restrictions on the other parameters of the model, see Qu and Perron (2004). This implies that imposing local stability at a given frequency provides no efficiency gains for the break date estimation in large samples. Formally, we then propose using the test statistics:

$$\Xi_{*|1}(\widehat{b}, \omega_j) \tag{17}$$

where $\omega_j = \omega_0(\frac{k-j}{k}) + \omega_k(\frac{j}{k})$ for $0 \leq \omega_0 < \omega_k \leq \pi$ and $j = 0, 1, ..., k$.

Since Bai *et al.* (1998) proved that the estimators of the segmented VAR parameters have the same asymptotic distribution when the break date is either known or estimated, the test statistic (17) converges in distribution to the same as that of (11) under the null hypothesis. Nevertheless, the χ^2 distribution is sometimes a poor approximation of the exact distribution even when the break date is known, see, e.g., Candelon and Lütkepohl (2001). Hence, we propose the following bootstrap procedure:

1. Compute the usual Chow test statistic $\Xi_{0|1}(b)$ and find $\widehat{b} = \arg\{\sup[\Xi_{0|1}(b)]\}$ for $b \in [0.15, 0.85]$.

2. Save the unrestricted residuals of the sub-sample models under H_1 conditional on $b = \widehat{b}$. Then obtain one matrix of residuals $\widehat{\varepsilon}$.

3. Save the estimated parameters of the full-sample model under H_0.

4. Save the estimated parameters of the sub-sample models under H_* conditional on $b = \widehat{b}$.

5. Sample from $\widehat{\varepsilon}$ h times. Then, take the estimated parameters in (3) to rebuild the data that are used to bootstrap $\Xi_{0|1}(\widehat{b})$, and use the estimated parameters in (4) to rebuild the data that are used to bootstrap $\Xi_{*|1}(\widehat{b}, \omega_j)$.

6. Obtain the bootstrap distributions of $\Xi_{0|1}(\widehat{b})$, and $\Xi_{*|1}(\widehat{b}, \omega_j)$ for $j = 1, ..., k$.

The testing procedure for local stability can be extended to the case of multiple breaks with unknown dates. As shown by Bai and Perron (2003), a dynamic programming algorithm can be used to search for an optimal partition that globally maximizes the likelihood function for any given number of breaks. The number of breaks can then be determined by means of either information criteria, see Bai (2000) or testing procedures, see Qu and Perron (2004). After fixing the number and dates of the breaks to their estimated values, the tests for local stability can be applied to any pair of adjacent regimes. In principle, a bootstrap procedure could also be used for the case of multiple breaks. However, the combined use of dynamic programming algorithms and resampling techniques is, admittedly, rather time consuming.

3 Simulation study

In this section, a Monte Carlo experiment is conducted to evaluate the finite-sample performances of the proposed testing procedure. In particular, we examine in a simple univariate framework, the size and power of both the asymptotic and bootstrap tests for local stability, at frequency ω (H_*) versus global instability (H_1).[7]

[7] For a detailed analysis of the bootstrapped version of the global stability test (H_0 vs H_1), the reader can refer to Diebold and Chen (1996) or Candelon and Lütkepohl (2001).

To this aim, we start by considering the following simple stationary AR(3) model:

$$X_t = a_1 X_{t-1} + a_2 X_{t-2} + a_3 X_{t-3} + \varepsilon_t \ ,$$

where $\varepsilon_t \sim N(0, \sigma^2)$. We assume that the DGP under the hypothesis H_1 has the following form:

$$X_t = a_1 X_{t-1} + a_2 X_{t-2} + a_3 X_{t-3} + \varepsilon_t, \quad t = 1, \ldots, T_b \tag{18}$$

$$X_t = \tau(\mu + a_1 X_{t-1} + a_2 X_{t-2} + a_3 X_{t-3}) + \varepsilon_t, \quad t = T_b + 1, \ldots, T \tag{19}$$

While the DGP under the null hypothesis H_* of local stability at frequency ω is of the form:

$$\Delta_\omega(L) X_t = \Pi_\omega(L) X_{t-1} + a_3 \Delta_\omega(L) X_{t-1} + \varepsilon_t, \quad t = 1, \ldots, T_b, \tag{20}$$

$$\Delta_\omega(L) X_t = \Pi_\omega(L) X_{t-1} + \tau_* a_3 \Delta_\omega(L) X_{t-1} + \varepsilon_t, \quad t = T_b + 1, \ldots, T \tag{21}$$

where $\Pi_\omega(L) = a_1 - a_3 - 2cos(\omega) + [1 + a_2 + 2a_3 \cos(\omega)]L$.

The design parameters are set at the following values: $a_1 = 0.15$, $a_2 = -0.05$, $a_3 = 0.1$, $\sigma = 1$, $\mu = 0.15$, $T = 200, 500$, $b \equiv T_b/T = 0.25, 0.50, 0.75$, $\tau = 2, 3, 4$, $\tau_* = 2, 3$, and $\omega = \pi/4, \pi/2, 3\pi/4$.

Some comments on the choices of the parameter values are in order. We let the breaks occur at three different fractions of the sample and take three different sizes. Indeed, previous results in the literature suggest that the parameters b and τ are the most important in determining the performances of structural change tests, see, *inter alia,* Candelon and Lütkepohl (2001), and Bai and Perron (2004). Notice that the break fraction b is treated as an unknown parameter to be estimated, and we use a trimming parameter equal to 15%. The AR parameters are chosen such that the process is stationary in both of the regimes and for all the considered sizes of the break. We let the constant term vary across the regimes because both the models, (18-19) and (20-21), are locally unstable at the zero frequency.

The rejection rates of the tests for local stability are based on both the asymptotic and bootstrap critical values at the 5% level. In each experiment, 500 series of length $T + 50$ are generated with initial values set to zero. The first 50 observations are discarded to eliminate dependence resulting from the starting conditions. For the bootstrap tests, 500 bootstrap draws are performed in each of the 500 replications.

Table 1 shows the rejection frequencies of the tests at the 5% level when the DGP is given by the processes (20-21). We see that the rejection rates of the bootstrap test are always quite close to the nominal size, while we also see that the asymptotic test tends to be oversized, especially when $T = 200$, $\tau_* = 2$, and T_b/T differs from 0.50. With $T = 200$, the bootstrap test is better sized than the asymptotic one for all the eighteen experiments, and fifteen differences between the rejection rates are indeed significant.[8] Even with $T = 500$ the bootstrap test is less

[8] We consider a difference between the rejection frequencies as insignificant when it is less large than twice the Monte Carlo standard error at the nominal 5% level, i.e., 0.02.

size-distorted in seventeen experiments and twelve differences between the rejection rates are significant. Interestingly, the empirical sizes of the two tests are more similar when the model is locally stable at frequency $3\pi/4$.

Table 1: Rejection rates of 5% level tests under the null hypothesis of local stability at frequency ω

		Asymptotic test			Bootstrap test		
	$T_b/T =$	0.25	0.50	0.75	0.25	0.50	0.75
$T = 200$	$\omega = \pi/4$	0.192	0.164	0.186	0.052	0.050	0.054
$\tau_* = 2$	$\omega = \pi/2$	0.172	0.160	0.184	0.056	0.078	0.076
	$\omega = 3\pi/4$	0.134	0.134	0.160	0.068	0.064	0.068
$T = 200$	$\omega = \pi/4$	0.136	0.100	0.108	0.056	0.060	0.052
$\tau_* = 3$	$\omega = \pi/2$	0.130	0.082	0.102	0.060	0.058	0.062
	$\omega = 3\pi/4$	0.070	0.056	0.070	0.068	0.054	0.062
$T = 500$	$\omega = \pi/4$	0.166	0.120	0.140	0.064	0.054	0.058
$\tau_* = 2$	$\omega = \pi/2$	0.132	0.136	0.124	0.054	0.074	0.062
	$\omega = 3\pi/4$	0.092	0.086	0.090	0.046	0.054	0.054
$T = 500$	$\omega = \pi/4$	0.088	0.076	0.074	0.058	0.056	0.056
$\tau_* = 3$	$\omega = \pi/2$	0.072	0.076	0.072	0.062	0.056	0.062
	$\omega = 3\pi/4$	0.073	0.060	0.062	0.062	0.064	0.060

In order to evaluate the effects of the break at frequencies $\pi/4$, $\pi/2$, and $3\pi/4$ under the alternative hypothesis H_1, Table 2 reports the spectra of the processes (18-19) at those frequencies. It emerges that the effect of the break, as measured by the relative change in the spectrum at the frequency of interest, is the strongest at frequency $3\pi/4$.

Table 2: Spectrum of model (19)

	Frequencies		
Break size	$\pi/4$	$\pi/2$	$3\pi/4$
$\tau = 1$	0.992	1.105	0.696
$\tau = 2$	0.972	1.220	0.494
$\tau = 3$	0.942	1.342	0.362
$\tau = 4$	0.905	1.471	0.274

We report in Table 3 the rejection rates of both the asymptotic and bootstrap tests at the 5% level when the DGP is given by the processes (18-19).

Table 3: Rejection rates of 5% level tests under the alternative hypothesis of global instability

		Asymptotic test			Bootstrap test		
	$T_b/T =$	0.25	0.50	0.75	0.25	0.50	0.75
$T = 200$	$\omega = \pi/4$	0.222	0.206	0.200	0.068	0.064	0.068
$\tau = 2$	$\omega = \pi/2$	0.170	0.176	0.198	0.066	0.070	0.078
	$\omega = 3\pi/4$	0.282	0.256	0.234	0.144	0.136	0.112
$T = 200$	$\omega = \pi/4$	0.278	0.286	0.206	0.114	0.184	0.146
$\tau = 3$	$\omega = \pi/2$	0.154	0.158	0.174	0.084	0.104	0.116
	$\omega = 3\pi/4$	0.360	0.400	0.296	0.274	0.336	0.230
$T = 200$	$\omega = \pi/4$	0.432	0.538	0.404	0.272	0.512	0.372
$\tau = 4$	$\omega = \pi/2$	0.134	0.198	0.180	0.120	0.198	0.170
	$\omega = 3\pi/4$	0.636	0.672	0.456	0.608	0.690	0.456
$T = 500$	$\omega = \pi/4$	0.264	0.236	0.192	0.126	0.136	0.088
$\tau = 2$	$\omega = \pi/2$	0.168	0.160	0.188	0.082	0.090	0.074
	$\omega = 3\pi/4$	0.312	0.348	0.286	0.172	0.226	0.158
$T = 500$	$\omega = \pi/4$	0.502	0.614	0.438	0.416	0.596	0.412
$\tau = 3$	$\omega = \pi/2$	0.158	0.216	0.206	0.134	0.196	0.174
	$\omega = 3\pi/4$	0.692	0.800	0.600	0.638	0.778	0.572
$T = 500$	$\omega = \pi/4$	0.888	0.968	0.863	0.878	0.968	0.851
$\tau = 4$	$\omega = \pi/2$	0.306	0.432	0.335	0.296	0.442	0.297
	$\omega = 3\pi/4$	0.954	0.966	0.942	0.944	0.972	0.934

Given the size distortions of the asymptotic test, caution is needed in comparing the empirical power of the two tests. However, the asymptotic test rejects more often in almost all the experiments. The two tests tend to have similar power as the parameters τ and T increase, as well as when the null hypothesis is local stability at frequency $\pi/4$. For both the tests, it appears that for a shock of 2 or 3 times the standard deviation of the residuals, the power is relatively low even if a large sample is considered. This result indicates that the size of the shock should be large enough to make the distinction between local and global stability. As in Candelon and Lütkepohl (2001), it is observed that the rejection frequency is generally lower when the break is located at the borders of the sample (i.e. $T_b/T = 0.25, 0.75$). The results reveal to us that the frequency at which the break occurs is also important for the empirical power. As expected in view of Table 2, the power is the highest when the break occurs at frequency $3\pi/4$ since the relative change in the spectrum under H_1 is the strongest at that frequency.

In empirical applications, the order of the AR process is unknown. It is thus of interest to investigate the robustness of the local stability test at frequency ω when the model dynamics are misspecified.[9] The test would clearly be inconsistent if the true order is underestimated.

[9] We thank an anonymous referee for pointing out this issue.

Hence, it is of interest to examine the implications of choosing the AR order in a liberal fashion. However, simulations would become too time-consuming if we allow for estimating the AR order within the bootstrap procedure. Hence, we analyze the effects of using an AR order which has one lag more than the true one, on the size and power of our stability tests. The experiments are performed for the case $T = 200$ and $\tau = 3$, which is quite representative of the other DGP's, and the results on the size and power of the test are reported in Table 4 and 5 respectively. It turns out that the effect of over-parametrization on size is rather limited. Indeed, only few rejection rates in Table 4 are significantly larger than the corresponding ones in Table 1. It also appears that when the model is over-parametrized, the bootstrap version has lower size distortion than the asymptotic one.

Table 4: Rejection rates of 5% level tests under the null hypothesis of local stability at frequency ω based on an AR(4) model

		Asymptotic test			Bootstrap test		
	$T_b/T =$	0.25	0.50	0.75	0.25	0.50	0.75
$T = 200$	$\omega = \pi/4$	0.156	0.116	0.136	0.072	0.072	0.064
$\tau_* = 3$	$\omega = \pi/2$	0.102	0.070	0.082	0.066	0.056	0.050
	$\omega = 3\pi/4$	0.088	0.072	0.060	0.074	0.080	0.052

We also notice that power is slightly affected by the over-parametrization of the AR model, in particular for the case of the bootstrap test. Overall, these results suggest that local stability tests are quite robust to a liberal choice of the AR order.

Table 5: Rejection rates of 5% level tests under the alternative hypothesis of global instability based on an AR(4) model

		Asymptotic test			Bootstrap test		
	$T_b/T =$	0.25	0.50	0.75	0.25	0.50	0.75
$T = 200$	$\omega = \pi/4$	0.248	0.258	0.206	0.126	0.172	0.142
$\tau = 3$	$\omega = \pi/2$	0.144	0.132	0.240	0.086	0.108	0.098
	$\omega = 3\pi/4$	0.414	0.412	0.274	0.314	0.344	0.206

4 Empirical applications

4.1 The predictive power of the yield curve on output growth

There is an extensive literature documenting the predictive power of the yield curve slope for future economic activity. Several factors can explain this stylized fact. As the yield curve describes the relationship between yields and maturities, it is determined by financial markets' expectation of future interest rate and the term premium. In front of a recession, the central bank may take the decision to stimulate activity, using the traditional instrument of monetary policy, i.e. by seeking to lower short-run interest rates. Consequently, the long-run interest rate will also decrease, but to a lesser degree, because of the expectations hypothesis of the term structure, leading to an increase in the yield curve. As the monetary expansion is expected to increase future activity, the correlation between the yield curve and future output growth is positive. The same relationship can be found using a Consumption Capital Asset Pricing model (Campbell and Cochrane, 1999), a Real Business Cycle model, or even a simple IS-LM model , see Estrella *et al.* (2003).

At an empirical level, several papers have provided evidence in favor of a positive correlation between the yield curve slope and future output growth in the United States.[10] Nevertheless, Estrella *et al.* (2003) have questioned the stability of this relationship, finding evidence of a break around September 1983. They justify the presence of this rupture by pointing to the effects of the change in the US monetary policy. It became more oriented towards inflation control since the nomination in 1979 of Paul Volcker as the head of the Federal Reserve Bank.

For this example, we propose using the local stability test to investigate the stability of the predictive power of the yield curve slope on output growth in the US. We use monthly data of industrial production (IP), 10-year treasury rate ($10TT$), and the Federal fund rate (FFR) for the US economy, as extracted from the Saint-Louis Federal Reserve Bank. The sample covers the period 1955M1-2003M12. As IP presents unit roots, we compute the future growth rate of industrial production at a forecast horizon k ($\Delta IP_{t,k}$) as:

$$\Delta IP_{t,k} = (1200/k)\ln(IP_{t+k}/IP_t)$$

Since Estrella *et al.* (2003) showed that the predictive power of the spread is at its maximum maximum at a horizon of one year, we also consider this horizon ($k = 12$) in the rest of the subsection. The interest rate spreads are given as the difference between the long-run and the short-run interest rate, i.e. $Spread = (10TT - FFR)$.[11] We then build a bivariate VAR of the following form:

$$A_3(L)\begin{pmatrix}\Delta IP_{t,12}\\ Spread_t\end{pmatrix} = \Phi_3 + \varepsilon_{3,t},$$

where $A_3(L)$ is a polynomial matrix of order 4,[12] Φ_3 is a vector of constant terms and $\varepsilon_{3,t}$ are $N_2(0, \Sigma_3)$ innovations.

The parameter stability of the above model will thus been investigated using the bootstrap procedure for unknown breaks that was discussed in subsection 3.2. Following Andrews (1993), the trimming region is $[0.15, 0.85]$. In view of Table 6, we observe that a break is detected in 1980M7.

[10] See, *inter alia*, Estrella and Hardouvelis (1991), Plosser and Rouwenhorst (1994), Bonser-Neal and Morley (1997), Kozicki (1997), Dotsey (1998), Breitung and Candelon (2006).

[11] It has been checked that the results are robust if we consider the 3-month treasury rate instead of the FFR. Results are available from authors upon request.

[12] The lag length is chosen according to the Akaike information criterion. Other choices of the lag length do not qualitatively modify the results.

Table 6: Stability Test for H_0 vs H_1

break date	statistic	bootstrap p-value	asymptotic p-value
1980.07	80.221	$< 0.02\%$	$< 0.01\%$

Note: The bootstrapped p-value is obtained after 5000 replications.

This result indicates that the polynomial matrix $A_3(L)$ taken as a whole is unstable. Nevertheless, it does not imply that the same conclusion necessarily holds for a subset of coefficients. In particular, it does not clearly indicate that the break has altered the relation between the spread and future output growth.[13] Hence, we tested for the stability of the polynomial $a_{12}(L)$ in the segmented VAR model, where $a_{ij}(L)$, for $i, j = 1, 2$, denotes the generic element of matrix $A_3(L)$. The LR test rejects the stability of the coefficients of $a_{12}(L)$ with a p-value of 3.77%, which suggests that the relationship spread-future output growth changed in 1980M7.

This break date is earlier than the one found by Estrella *et al.* (2003) in 1983M9. Nevertheless, our dating of the break seems more closely associated with the change in monetary policy regime associated with the appointment of Volcker as the head of the FED in late 1979. A possible explanation for the differences between our result and that of Estrella *et al.* (2003) is that we consider a bivariate system, whereas Estrella *et al.* (2003) investigate the stability of a single-equation model where future output growth is the endogenous variable.

The study of global instability is then performed, making it possible to investigate the local stability around the break date that was previously obtained. Local stability tests are computed for the frequencies $\omega_j = (\frac{j}{100})\pi$, for $j = 1, 2, ..., 99$. In Figure 1, we plot the test statistics along with their bootstrap and asymptotic 95% critical values.

It appears that the system is locally stable for frequencies lower than 0.3, i.e. for a wavelength longer than 21 months. This evidence is consistent with the view that this break has a "nominal" origin. Indeed, the new orientation of US monetary policy, being more concerned by inflation control, has apparently affected the stability of the spread-future output growth relationship at the higher frequencies while leaving unchanged the low-frequency components of the data.

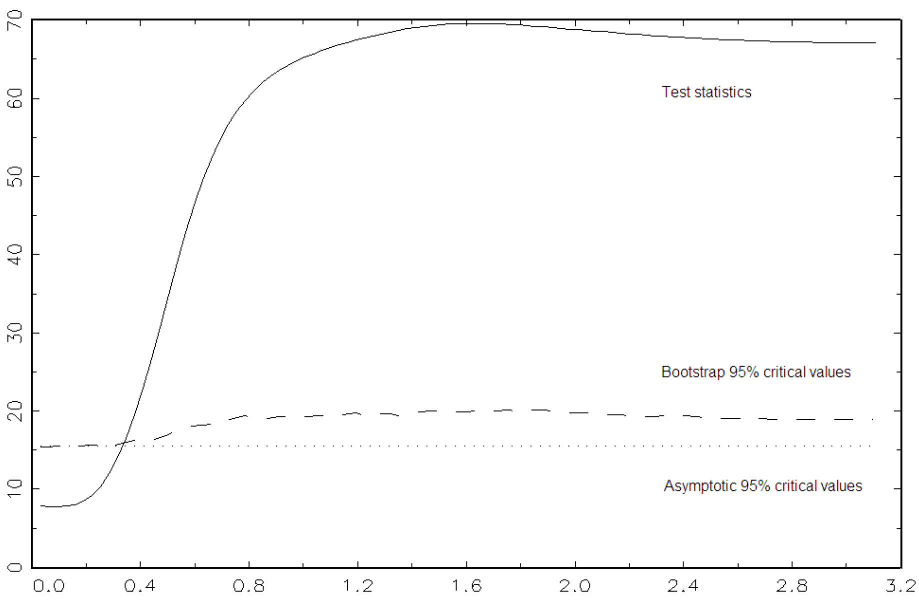
It must be noticed that the empirical results seem to indicate that local stability holds for an interval of frequencies, which is contradicted by Remark 2. We think that this phenomenon is due to a sort of leakage problem of local stability tests, namely power is likely to be low for frequencies that are close to the ones for which the model is stable. Breitung and Candelon (2006) documented a similar problem for their causality test via a local power analysis.

4.2 Output, consumption, and investment.

Several studies have been devoted to the analysis of the productivity slowdown in postwar U.S. output. As the univariate analysis of the output series by Bai *et al.* (1998) lead to inconclusive results, these authors considered a trivariate system composed of consumption (C), investment

[13] We thank an anonymous referee for pointing out this issue.

Figure 1: Tests for local stability - The yield curve



(I), and output (Y). Following King *et al.* (1991), the rationale behind this idea is that a break in the productivity process should also be present in variables possessing strong long-run links with output, in particular consumption and investment. Indeed, Bai *et al.* (1998) proved that if the stochastic growth model by King *et al.* (1998) is augmented with a break in the average growth rate of productivity, such a break will affect the three variables $c = \ln(C)$, $i = \ln(I)$, and $y = \ln(Y)$, but not the "great ratios" $(c - y)$ and $(i - y)$.

We thus investigate local and global stability in the following dynamic model:

$$A_4(L) \begin{pmatrix} c_t - y_t \\ i_t - y_t \\ \Delta(c_t + y_t + i_t) \end{pmatrix} = \Phi_4 + \varepsilon_{4,t},$$

where $A_4(L)$ is a polynomial matrix of order 4, Φ_4 is a vector of constant terms and $\varepsilon_{4,t}$ are $N_4(0, \Sigma_4)$ innovations. Quarterly data are obtained from the Saint-Louis Federal Reserve Bank and cover the period 1954Q1-2004Q4. Y_t is the private GDP per capita, C_t the real personal consumption expenditures per capita, and I_t the private fixed investment per capita. The variables are seasonally adjusted and divided by the civilian non-institutional population aged 16 and over.

As in the previous example, the global stability test is implemented using a 15% trimming, and the outcome is reported in Table 7. Similar to Bai *et al.* (1998), a break is detected in 1968Q3. Interestingly, the break is dated earlier than the first oil shock.

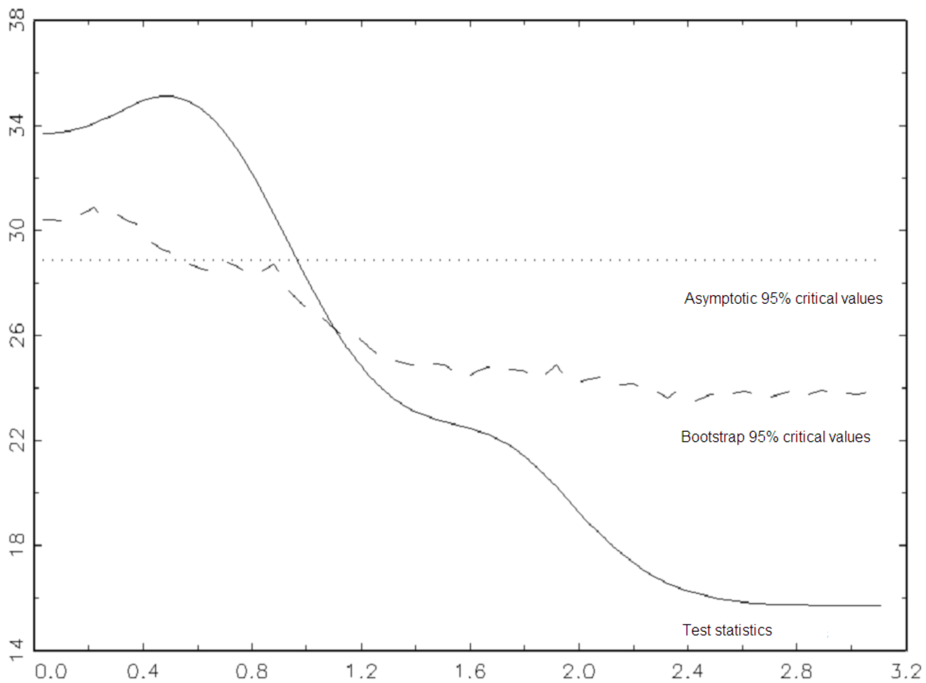
Table 7: Stability Test H_0 vs H_1

break date	statistic	bootstrap p-value	asymptotic p-value
1968q3	51.576	3.92%	$< 1.00\%$

Note: The bootstrap p-value is obtained after 5000 replications.

In order to gain a deeper insight into the origin of the break in 1968Q3, the local stability tests are performed for the frequencies $\omega_j = (\frac{j}{100})\pi$, for $j = 1, 2, ..., 99$. The tests statistics, along with their bootstrap 95% and asymptotic critical values, are plotted in Figure 2.

Figure 2: Tests for local stability - The "great ratios" system



It turns out that this system is locally stable for frequencies higher than 1, i.e. with a wavelength lower than 6 quarters, but it becomes unstable at lower frequencies, in particular when ω approaches zero.

The empirical evidence suggests that the break in 1968Q3 can be labelled as "real" as it affects the long-run properties of the variables. However, unlike the prediction of the theoretical model by Bai *et al.* (1998), the stochastic components of the data are also unstable at low frequencies. Therefore, a simple break in the productivity average growth rate is not, apparently, the only origin of this break.

5 Conclusions

In this paper, we develop a new testing procedure for parameter stability in a segmented stationary or cointegrated VAR model at a particular frequency. By doing so, it is possible to determine the range of frequencies which are responsible for the parameter instability in a dynamic model. The local stability tests can provide a deeper insight into the origin of a structural break. The two examples presented in this study highlight the practical value of this procedure in empirical studies. A structural break is detected at 1980M7 for the spread-future output growth relationship, and around 1968Q3 in the case of an output-consumption-investment system. The application of the local stability tests reveals that in the first case, the model is only stable at lower frequencies, whereas in the second case the stability exclusively concerns the high frequency components of the data. From these characteristics, it follows that a nominal shock, that being the change in the U.S. monetary policy in the early 80's, has led to the modification of the predictive power of the interest rate spread for future output growth, whereas a real productivity shock is likely to be at the origin of the instability of the output-consumption-investment system.

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6 Appendix

In thix appendix we show that tests for local stability are invariant to reparametrizations of the VAR such as that in equation (12). Indeed, the considered alternative representation is based on the following expansion

$$A(L) = \Delta_\omega(L) - \widetilde{\Pi}_\omega(L)L^{p-1} - \widetilde{\Gamma}_\omega(L)\Delta_\omega(L)L,$$

which yields at $L = z$

$$A(z) = -\widetilde{\Pi}_\omega(z)z^{p-1}.$$

By comparing the above equation with equation (4), we obtain the following relation

$$\widetilde{\Pi}_\omega(z) = z^{2-p}\Pi_\omega(z), \qquad (22)$$

which immediately reveals that $\widetilde{\Pi}_\omega(L) = \Pi_\omega(L)$ when $\omega = 0$ or $\omega = \pi$ and p is even, and $\widetilde{\Pi}_\omega(L) = -\Pi_\omega(L)$ when $\omega = \pi$ and p is odd. Hence, in the following we concentrate on the case $\omega \in (0, \pi)$.

By equating real and imaginary parts of both sides of equation (22) we find

$$\begin{aligned} \mathrm{Re}\{\widetilde{\Pi}_\omega(z)\} &= \mathrm{Re}\{\Pi_\omega(z)\}\cos[(2-p)\omega] + \mathrm{Im}\{\Pi_\omega(z)\}\sin[(2-p)\omega], && (23) \\ \mathrm{Im}\{\widetilde{\Pi}_\omega(z)\} &= \mathrm{Im}\{\Pi_\omega(z)\}\cos[(2-p)\omega] - \mathrm{Re}\{\Pi_\omega(z)\}\sin[(2-p)\omega]. && (24) \end{aligned}$$

Moreover, by writing $\Pi_\omega(L) = \Pi_{\omega,0} + \Pi_{\omega,1}L$, we obtain for $L = z$

$$\begin{aligned} \mathrm{Re}\{\Pi_\omega(z)\} &= \Pi_{\omega,0} + \Pi_{\omega,1}\cos(\omega), \\ \mathrm{Im}\{\Pi_\omega(z)\} &= -\Pi_{\omega,1}\sin(\omega). \end{aligned}$$

Substituting the above equations into (23) and (24) yields

$$\begin{aligned} \mathrm{Re}\{\widetilde{\Pi}_\omega(z)\} &= [\Pi_{\omega,0} + \Pi_{\omega,1}\cos(\omega)]\cos[(2-p)\omega] - \Pi_{\omega,1}\sin(\omega)\sin[(2-p)\omega], \quad (25) \\ \mathrm{Im}\{\widetilde{\Pi}_\omega(z)\} &= -\Pi_{\omega,1}\sin(\omega)\cos[(2-p)\omega] - [\Pi_{\omega,0} + \Pi_{\omega,1}\cos(\omega)]\sin[(2-p)\omega]. \quad (26) \end{aligned}$$

Similarly, by writing $\widetilde{\Pi}_\omega(L) = \widetilde{\Pi}_{\omega,0} + \widetilde{\Pi}_{\omega,1}L$, we obtain for $L = z$

$$\begin{aligned} \mathrm{Re}\{\widetilde{\Pi}_\omega(z)\} &= \widetilde{\Pi}_{\omega,0} + \widetilde{\Pi}_{\omega,1}\cos(\omega), \\ \mathrm{Im}\{\widetilde{\Pi}_\omega(z)\} &= -\widetilde{\Pi}_{\omega,1}\sin(\omega). \end{aligned}$$

Substituting the above equations into (26) and (25) yields the following linear system

$$\begin{bmatrix} \widetilde{\Pi}_{\omega,0} \\ \widetilde{\Pi}_{\omega,1} \end{bmatrix} = \Upsilon \begin{bmatrix} \Pi_{\omega,0} \\ \Pi_{\omega,1} \end{bmatrix},$$

where

$$\Upsilon = \begin{bmatrix} \begin{array}{c} \cos[(2-p)\omega] + \\ \cos(\omega)\sin[(2-p)\omega]/\sin(\omega) \end{array} & \begin{array}{c} \sin(\omega)\sin[(2-p)\omega] + \\ [\cos(\omega)]^2\sin[(2-p)\omega]/\sin(\omega) \end{array} \\ -\sin[(2-p)\omega]/\sin(\omega) & \begin{array}{c} \cos[(2-p)\omega] - \\ \cos(\omega)\sin[(2-p)\omega]/\sin(\omega) \end{array} \end{bmatrix}.$$

Since $|\Upsilon| = 1$, we conclude that coefficients of $\widetilde{\Pi}_\omega(L)$ are a non-singular linear transformation of those of $\Pi_\omega(L)$. Hence, constancy of $\Pi_\omega(L)$ is equivalent to that of $\widetilde{\Pi}_\omega(L)$. ■