Networks and Spatial Economics
https://doi.org/10.1007/s11067-026-09753-y

RESEARCH

Resilience of Multilayer Networks Through Community Detection

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Received: 14 November 2025 / Accepted: 11 May 2026

Abstract

This paper introduces a novel conceptualization of the resilience of a multilayer network based on a community detection procedure over the individual layers. Communities in the layers are assumed to drive the creation of interlayer links between pairs of nodes. We hypothesize that the shocks can be of two different natures. On one side, a shock appears when the clustering exercise is perturbed by imposing that one node forms a singleton community (type 1). On the other side, the shock is the removal of one of the interlayer links (type 2). The resilience measure is defined as a variation in the stability of the topological structure of the interlayer links (for shocks of type 1) and the community detection exercise (for shocks of type 2). We also provide a resilience-related instrument for ranking individual nodes and arcs in terms of their relevance when shocked. We discuss how the methodology can be efficiently exploited in the relevant empirical instance of the European Circular Economy indicators.

Keywords Networks · Resilience · Multilayer networks · Cluster analysis

1 Introduction

The resilience of networks is highly debated for its relevance and meaningfulness. This concept is quite intuitive: a network is resilient when it is able to absorb an external shock. This simple definition leads to many issues to be faced, from the statement of the problem to the conceptualization of the shocks and the consequent identification of a criterion for measuring resilience (e.g., see Cerqueti et al. 2025a; Fraccascia et al. 2018; Müllerklein and Fontaine 2025; Reggiani et al. 2002; Reggiani 2022; Scazzieri 2022). This paper enters this debate. We deal with a novel definition of resilience measures for multilayer networks based on a community detection procedure within the individual layers. Multilayer networks are particularly suitable

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Published online: 20 May 2026

for modeling settings where the same actors present different connecting attributes. Indeed, many scholars across various fields have exploited their versatility; we can cite, e.g., Lei and Cheong (2024) as well as Boccaletti et al. (2014); De Domenico (2023); Kivelä et al. (2014) explore theoretical aspects. Recent empirical studies have also employed multilayer representations to analyze the interdependencies of socio-economic systems (e.g. see Cerqueti et al. 2025b, c; Peng et al. 2024; Pitombeira-Neto et al. 2020). Therefore, multilayer networks are appropriate for exploring the reaction of a complex system to external solicitations.

More specifically, we consider a set of nodes having links with multidimensional quantitative attributes. This multidimensionality is captured by a set of layers, so that each layer is associated with a specific attribute. We assume that the weights of the intralayer links are driven by the similarity among the related attributes' value. Differently, the interlayer links are created based on a community detection algorithm. On this, we assume that two nodes in different layers are connected if they belong to the same cluster in both layers (e.g., see Cerqueti et al. 2025b, c). A similar multilayer structure has been explored in network design problems, such as in Knippel and Lardeux (2007), where interlayer connections are determined by optimization constraints rather than community structure.

Several possible methods and criteria can be adopted for implementing a community detection procedure (for a review, see Javed et al. 2018). Previous research suggests that many real-world networks exhibit hierarchical structures (e.g., see Ahn et al. 2010; Clauset et al. 2008; Sales-Pardo et al. 2007). Examples can be found in different fields, such as biology (Ravasz et al. 2002; Yu and Gerstein 2006), transportation networks (Zhang et al. 2025), and finance (Mantegna 1999; Tabak et al. 2010; Tumminello et al. 2010). When the nodes of a network show a hierarchical structure, hierarchical clustering algorithms are adopted to retrieve the community structure. The proposed multilayer network model presents a hierarchical structure, so that we are in line with this large strand of literature. In our case, we use the Ward (1963) algorithm.

The resilience of multilayer networks has been extensively studied through various approaches. For example, Caschili et al. (2015) explored the resilience of interdependent multilayer networks, focusing on how disruptions in one layer can propagate and affect other layers. Similarly, Buldyrev et al. (2010) investigated cascading failures in interdependent networks, showing how a failure in one layer can lead to widespread disruptions across interconnected layers. In social networks, Su et al. (2022) demonstrated that coupling between layers can promote prosocial behavior, where small changes in one domain catalyze positive changes in another. The resilience of multilayer networks has also been examined in the context of infrastructure systems, such as Du et al. (2016), who studied the Chinese Airline Network and found that resilience increases when low-degree nodes or high-flight flow links are removed. Furthermore, Fisher and Pinter-Wollman (2021) explored the temporal dynamics of collective behavior in multilayer networks, emphasizing the role of evolving interactions in determining network stability. In the domain of satellite communication, Gao and Fang (2020) proposed an optimization-based approach to enhance resilience in multilayer satellite networks, balancing application needs and resource constraints. Boccaletti et al. (2014) provided a comprehensive analysis of multilayer networks, highlighting the importance of temporal and contextual factors in assessing resilience. Lastly, Chen and Lu (2018) introduced a framework for evaluating layer centrality, offering insights into the contribution of each layer to the overall resilience of multilayer networks. These studies collectively contribute to the understanding of how different layers interact and influence the overall resilience of complex multilayer systems.

Differently, we introduce two types of shock. On the one hand, we modify the configuration of the communities in one layer by isolating a node in a singleton cluster. Substantially, this action removes the shocked node from the clustering exercise. In so doing, we modify the interlayer links between the layer with the removed node and the others. In this case, the shock is labelled as of type 1. On the other hand, we arbitrarily remove an interlayer link. This implies also the removal of the companion link of the removed one. To fix ideas, by removing the link between node i in layer k and node j in layer h, one has also the removal of the link between node i in layer h and node j in layer k. As a consequence, we are taking out of the community detection outcomes two nodes in the involved layers, which are forced to form a singleton cluster. Even if the community detection algorithm is not redone after the shock, setting nodes as singleton clusters modifies the outcome of the community detection procedures implemented before the shock in the considered layers. The shock is in this case of type 2. The variation of the configuration of the multilayer network in terms of interlayer links (for type 1) and community structure (for type 2) is the basis for measuring the resilience of the network. A brief discussion on the motivations for taking this form of shocks and the consequent definition of resilience can be found in Section 3.

In this paper, we develop two indicators, which are relatively simple to compute, for measuring the resilience of the multilayer network to shocks of type 1 and type 2, respectively. In the case of type 1-shock, the resilience is measured by a ratio based on the number of interlayer links before and after the shock. The larger the related proportion, the more resilient the multilayer is to shocks of type 1. Indeed, in cases where the shock has no effect, we have that this ratio is equal to one. In the case of type 2-shock, resilience is measured by evaluating the similarity of the community structure between the shocked multilayer network and the shocked one. To this aim, we develop a measure based on the Rand Index (RI, Rand 1971), which is used to assess the similarity across community structures.

We adapt our concepts of resilience measures to a global and a local perspective. Indeed, we provide a definition of local resilience for detecting the relevance of a given component of the network in the overall context of its stability. In doing so, we discuss the role of the individual components of the network and, more in general, of the topological structure of the system, in light of the resilience measure of the network (Reggiani 2022; Scazzieri 2022). Furthermore, we measure the stability of the entire system by aggregating the local resilience measures, hence deriving a global resilience setting.

To illustrate how the proposed resilience measure can be computed in practice, we develop an application to European Circular Economy (CE) indicators. Data is open and available on the Eurostat website. Studying the resilience of the CE network is important due to its evident social, economic, and environmental impact (Mies and

Gold 2021; Donaghy 2022). In this context, resilience reflects the capacity of European countries to preserve the overall structure of similarities in their circular economy performance when subject to shocks. This source of resilience is meaningful from a European-level policy perspective, as it signals whether convergence towards shared sustainability objectives is robust to external perturbations, or whether such events risk fragmenting the network and widening disparities among countries.

The framework builds upon recent studies that model multilayer networks through community detection (Cerqueti et al. 2025b, c), linking nodes across layers according to their structural similarity. Our resilience definition is naturally compatible with this representation, as it directly quantifies the stability of the community structures under perturbations. Yet, it remains general and applicable to any multilayer system, regardless of how interdependencies are defined. In the empirical analysis, we construct a multilayer network based on community detection, where each layer corresponds to a specific Circular Economy dimension, and countries are connected according to the similarity of their community memberships. This formulation enables us to demonstrate the practical implementation of the proposed measure in a real-world setting. Using data on five Circular Economy dimensions for European countries over the period 2016–2020, we find that the system is generally resilient, although certain indicators and countries exhibit temporary vulnerabilities–particularly in the material use and investment layers.

The remaining part of the paper is organized as follows. Section 2 presents in more detail the development of the multilayer network model. Section 3 contains the definition of the exogenous shocks and the methodologies adopted for measuring the resilience. Section 4 shows how the proposed methodological framework can be applied in the context of European Circular Economy indicators. Section 5 offers a discussion on the scientific content of the paper, provides final remarks and illustrates future research directions.

2 The Multilayer Network Model

This section contains the key ingredients of the employed network model. First, we introduce the main ingredients of the network model. Then, we describe how links are formed between the nodes in different layers.

2.1 Overview of the Network

We define a multilayer network $\mathcal{N}$ with K layers as follows

$$\mathcal{N} = (\mathcal{G}, \mathcal{L}), \tag{1}$$

where $\mathcal{G} = \{G_k : k = 1, \ldots, K\}$ collects K distinct graphs sharing the same set of nodes $V = \{1, \ldots, N\}$. For an easy notation, we refer hereafter to $\mathrm{K} = \{1, \ldots, K\}$ as the set of the layers. For each layer $k \in \mathrm{K}$, we define the weighted adjacency matrix $W_k = (w_k(i,j) : i, j \in V)$, where $w_k(i,j) \in [0, 1]$ describes the entity of the connection between nodes i and j at layer k and $w_k(i,j) = 0$ means no connection.

The graph at layer *k* is $G_k = (V, W_k)$. The set $\mathcal{L}$ contains the interlayer links between pairs of nodes. We assume that such links are not weighted, and we label the generic link between *i* at layer *h* and *j* at layer *k* by $L_{hk}(i, j)$.

2.2 Interlayer Links Formation

The interlayer links are built through a community detection procedure within the individual layers. For each layer $k \in$ K, one can find $H_k \in \{1, \ldots, N\}$ – which represents the number of clusters – such that the selected community detection algorithm provides a clustering of *V* in H_k non-overlapping communities. The family of clusters of the *k*-th layer is denoted by $\mathcal{C}_k$. We identify the cluster in $\mathcal{C}_k$ containing node $i \in V$ by $C_k(i)$. The community detection procedures over the layers allow us to identify the interlayer links, so that an interlayer link exists only when the pair shares identical community labels in both layers. Substantially, nodes *i* and *j* in different layers *h* and *k* are linked if and only if *i* and *j* are in the same cluster in layer *h* and in the same cluster in layer *k*, simultaneously. This condition states explicitly that two nodes evaluated according to different attributes – i.e., belonging to different layers – are connected when they present the same behavior – i.e., they belong to the same cluster – according to both attributes. This procedure for building interlayer links meets reasonability requirements, by assuming that similarity drives not only the links at an intralayer level, but also the connections between nodes at different layers.

Following previous research suggesting that many networks exhibit hierarchical structures (Ahn et al. 2010; Clauset et al. 2008; Sales-Pardo et al. 2007), we consider the hierarchical approach of Ward (1963) to find communities of nodes.

The basic tool for hierarchical clustering is the definition of pairwise similarities, *d*(*i*, *j*). The lower *d*(*i*, *j*), the more similar two nodes *i* and *j* are. Let us call $d_k(i, j)$ the pairwise dissimilarity between two nodes *i* and *j* in the *k*-th layer. Given a dissimilarity measure, the hierarchical algorithm merges, at each step, two communities to minimize the total inertia, that is,

$$W(\mathcal{C}_k) = \sum_{l=1}^{H_k} I_{lk} \tag{2}$$

where I_{lk} is the within-community inertia of the *l*-th cluster in the *k*-th layer – say, C_{lk} – and it is given by

$$I_{lk} = \sum_{i \in C_{lk}} \sum_{j \in C_{lk}} \frac{\lambda_i \lambda_j}{2 \sum_{r \in C_{lk}} \lambda_r} d_k(i, j), \tag{3}$$

where λ_i and λ_j are the weights associated to the *i*-th and *j*-th units, respectively. As we will see, in the empirical application we will assume these weights to be equal for all *N* nodes and unitary, so that $\lambda_1 = \cdots = \lambda_N = 1$. Reasonably, this assumption leads to a fair treatment of all the units when clustering. Indeed, we do not have any specific reason to include heterogeneity and overweight or underweight some of the statistical units with respect to the others. Thus, the hierarchical clustering algorithm starts with N singleton clusters, where each unit belongs to a separate cluster. At each iteration step, the two most similar clusters are merged based on the adopted dissimilarity measure. The process continues until the desired number of communities is achieved. On this, we select the number of H_k communities in each layer k by means of an automatic elbow criterion (e.g., see Onumanyi et al. 2022).

We assume that the interlayer connection appears when the considered nodes belong to the same community in the considered layers. In particular, for each $h, k \in \mathrm{K}$ and $i, j \in V$, we say that

$$L_{hk}(i,j) = \begin{cases} 1 & \text{if } C_k(i) \equiv C_k(j) \text{ and } C_h(i) \equiv C_h(j), \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

Evidently, there is a symmetry in the interlayer links. Indeed, when node *i* in *k* and node *j* in layer *h* are connected, then node *i* in layer *h* and *j* in *k* are also connected.

3 Shock Definition and Resilience Measures

The rationale behind the definition of the shock can be expressed as follows. Imagine that policies and strategies are built based on similar values of a set of elements with respect to a given attribute. If one of such elements departs from the supposed value and takes a different one, then the implemented policies and strategies might no longer be valid. The assessment of the reaction of the considered system in such a circumstance leads to the formal definition of shocks of type 1 in Definition 1 below. From a different perspective, one can imagine that policies and strategies are built based on similar values of two units in different attributes. Assume that one of such units modifies its value in one of the attributes. Also in this case, policies and strategies might no longer be valid. The measurement of the reaction of such a system in this case leads to the definition of shocks of type 2 in Definition 2 below. Therefore, in this paper, we adopt the following type of shock for the network.

Definition 1 We say that the multilayer network $\mathcal{N}$ has a shock of type 1 when there is a perturbation of the community detection procedure in one layer according to the following rule: one node *i* in a layer *h* is removed from *V* and forced to form a singleton cluster. Then, the community detection procedure is applied to the set $V \setminus \{i\}$.

Any shock of type 1 leads to a reshaping of the multilayer network, in that the novel community detection procedure implies a novel set of interlayer links. We denote the shocked network with the shock of type 1 at node *i* in layer *h* by $\mathcal{N}^{(1)}_{i;h}$.

Definition 2 We say that the multilayer network $\mathcal{N}$ has a shock of type 2 when one interlayer link is removed from the network. According to the definition of $\mathcal{N}$, the removal of the link from *i* at layer *h* to *j* at layer *k* implies that *i* and *j* are removed from their clusters in layers *h* and *k* and forced to form singleton clusters.

Shocks of type 2 modify the clustering of the involved layers. We denote the shocked network with the shock of type 2 at node $i, j \in V$ in layers $h, k \in \mathrm{K}$ by $\mathcal{N}^{(2)}_{ij;hk}$. Globally, the resilience of $\mathcal{N}$ can be obtained by suitably aggregating its reactions to the shocks over the nodes. Locally, resilience depends on the selected nodes and layers for the considered shocks in Definitions 1 and 2. From a local point of view, one can effectively assess the relevance of nodes and layers for maintaining the stability of the overall network. In all the cases, resilience is measured by comparing the unshocked network with the shocked one. To proceed, according to (4), we denote the number of interlayer links between layers h and k of network $\mathcal{N}$ by

$$L_{hk}(\mathcal{N}) = \sum_{x,y \in V} L_{hk}(x,y). \tag{5}$$

The total number of interlayer links is

$$L(\mathcal{N}) = \sum_{h,k \in \mathrm{K} \,:\, h<k} L_{hk}(\mathcal{N}). \tag{6}$$

By construction, $L_{hk}(\mathcal{N})$ is an integer in $\{0, 1, \dots, N(N-1)\}$ while $L(\mathcal{N})$ is an integer in $\{0, 1, \dots, KN(N-1)\}$.

Definition 3 The local type 1-resilience of $\mathcal{N}$ for a shock of type 1 in node $i \in V$ at layer $h \in \mathrm{K}$ is

$$\mu^{(1,loc)}_{i;h}(\mathcal{N}) = \frac{\min\left\{L\left(\mathcal{N}^{(1)}_{i;h}\right), L(\mathcal{N})\right\}}{\max\left\{L\left(\mathcal{N}^{(1)}_{i;h}\right), L(\mathcal{N})\right\}}. \tag{7}$$

The global resilience of $\mathcal{N}$ is

$$\mu^{(1,glob)}(\mathcal{N}) = \frac{1}{K \times N} \sum_{i \in V,\, h \in \mathrm{K}} \mu^{(1,loc)}_{i;h}(\mathcal{N}). \tag{8}$$

The local resilience in (7) allows us to rank the nodes in the individual layers based on the impact on the network of a shock. Therefore, such a local measure can efficiently identify the more destabilizing nodes of the multilayer network. Differently, the global measure in (8) provides a clear view of the general resilience of the network by including all the possible shocks of type 1. For the resilience measure in (7), it is worth noticing that we will have $L\left(\mathcal{N}^{(1)}_{i;h}\right) < L(\mathcal{N})$ in all our empirical instances.

For the network's resilience when shocks of type 2 occur, we notice that we do not re-run the community detection algorithm after the removal of the interlayer link; we measure the resilience by comparing the clustering outcome before the shock and that after the shock, when nodes *i* at layer *h* to *j* at layer *k* are artificially set as forming a singleton cluster. Thus, we need to exploit a measure of the similarity between different clustering outcomes. To this end, we take $i, j \in V$ and $h, k \in \mathrm{K}$ and consider the Adjusted Rand Index (ARI Hubert and Arabie 1985; Rand 1971), which provides a measure of the agreement between two community structures and it is preferred to Rand Index since it corrects for agreement expected by chance. We denote hereafter $R(\bullet_1, \bullet_2) = ARI(\bullet_1, \bullet_2)$ as the measure – ARI, in our case – of the deviation between clustering outcomes $\bullet_1$ and $\bullet_2$. In particular, given that the type 2-shock involves a couple of layers *h* and *k*, we compute two different indices, $R(\mathcal{C}_h, \mathcal{C}^{(2,h)}_{ij;hk})$ and $R(\mathcal{C}_k, \mathcal{C}^{(2,k)}_{ij;hk})$, where $\mathcal{C}^{(2,\star)}_{ij;hk}$ is the family of clusters of layer $\star \in \{h, k\}$ after the shock of type 2 on the arc connecting *i* in layer *h* and *j* in layer *k*, while $R(\mathcal{C}_\star, \mathcal{C}^{(2,\star)}_{ij;hk})$ represents the ARI computed considering the similarity of the community structure between the $\star$-th layer before and after the type 2-shock. We then obtain the local type 2-shock measure, that we call $R(\mathcal{N}, \mathcal{N}^{(2)}_{ij;hk})$, by taking the average of these two measures, that is

$$R(\mathcal{N}, \mathcal{N}^{(2)}_{ij;hk}) = [R(\mathcal{C}_h, \mathcal{C}^{(2,h)}_{ij;hk}) + R(\mathcal{C}_k, \mathcal{C}^{(2,k)}_{ij;hk})]/2. \tag{9}$$

Also in this case, we can easily define a global resilience measure as the aggregation of the local measures.

Definition 4 The local type 2-resilience of $\mathcal{N}$ for a shock of type 2 in nodes $i, j \in V$ at layers $h, k \in \mathrm{K}$ is

$$\gamma^{(2,loc)}_{ij;hk}(\mathcal{N}) = R(\mathcal{N}, \mathcal{N}^{(2)}_{ij;hk}), \tag{10}$$

where *R* is defined as in (9).

The global type 2-resilience of $\mathcal{N}$ is given by the average of the corresponding local concept of resilience as follows

$$\gamma^{(2,glob)}(\mathcal{N}) = \frac{1}{K(K-1) \times N(N-1)} \sum_{i,j \in V \,:\, i \neq j} \; \sum_{h,k \in \mathrm{K} \,:\, h \neq k} \gamma^{(2,loc)}_{ij;hk}(\mathcal{N}). \tag{11}$$

Also in this case, the local resilience in (10) leads to a ranking of interlayer links – i.e., of couples of nodes in couples of layers – accordingly the severity of the effects of a shock of type 2. The global resilience measure in (11) describes the network's resilience by considering all the shocks of type 2.

From an economic perspective, forcing nodes into singleton clusters can be interpreted as a situation in which countries become structurally incomparable with all others in a given dimension, for example, due to severe political divergences, institutional collapse or extreme macroeconomic shocks. Although such configurations may be rare, they provide a useful upper bound to assess the system's sensitivity. Examples include major geopolitical shifts such as Brexit, which can lead to a gradual divergence from previously comparable systems. In such contexts, the 'singleton' configurations should be interpreted as extreme cases representing the upper limit of structural divergence.

On this, removing a node from one of the layers is the only way to grasp immediately the systemic role of such a node in the context of the given layer. This leads to a clear view of the local properties of the resilience of the system.

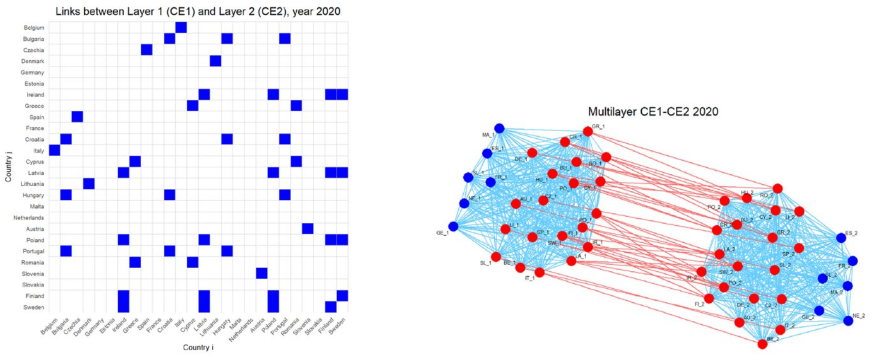
4 Application to European Circular Economy Indicators

To illustrate the proposed framework, we provide an application to a multilayer system describing the European countries' performance on the five Circular Economy (CE) indicators released by Eurostat. Each layer represents a distinct dimension of the circular economy: (i) the recycling rate of municipal waste (CE1), (ii) the circular material use rate (CE2), (iii) private investment and gross value added in CE sectors (CE3), (iv) employment in CE-related sectors (CE4), and (v) patents related to recycling and secondary raw materials (CE5). The layers are modelled as weighted networks, where nodes represent countries and links represent the degree of similarity between their achievements in a given CE dimension. In detail, edge weights reflect the inverse distance between countries according to each indicator. Differently, interlayer links capture the similarity in community memberships across the CE dimensions. More specifically, each layer of the multilayer system is constructed from a distance matrix capturing the dissimilarity among European countries with respect to a specific CE indicator. For a given indicator CEk, with $k = 1, 2, 3, 4, 5$, let $x_{i,k}$ denote the value of CEk for country *i*; the pairwise distance between countries *i* and *j* are computed as

$$d_k(i, j) = |x_{i,k} - x_{j,k}|.$$

The adjacency matrix of layer *k* is then defined as $w_k(i, j) = 1/(1 + d_k(i, j))$. This transformation converts distances into similarity scores in the range (0, 1], ensuring that countries with similar CE profiles are more strongly connected. The networks built using the CEk indices are clustered using Ward's hierarchical algorithm (see, Section 2.2), which is suitable for capturing structural similarities among EU countries. Starting with singleton clusters for each country, the algorithm aggregates countries with similar CE performance until an optimal number of communities is reached using the elbow criterion. In this context, the hierarchical structure is the ground for defining interlayer connections. In this framework, interlayer links identify countries that maintain similar cluster memberships across different CE dimensions, providing a natural interpretation of systemic alignment among circular economy policies.

Figure 1 shows, for the year 2020, the pattern of inter-layer links between the first two dimensions (CE1 and CE2) together with the corresponding multilayer network representation. The left panel displays the existence of cross-layer connections. In contrast, the right panel highlights how the two layers interact structurally: countries belonging to the same clusters in both CE1 and CE2 are connected across layers, revealing a cohesive subset of nations that exhibit similar "behaviors" in terms of both recycling efficiency and circular material use. Indeed, the existence of a link means that two countries belong to the same community in both layers for that year. The right panel focuses on the structure of the connections between these two layers



(a) Inter-layer links (2020) (b) Multilayer network CE1–CE2 (2020)

Fig. 1 Visualisation of inter-layer connections and the corresponding multilayer network for 2020

within the full CE multilayer network, where red arrows show intralayer connections. Similar figures can be derived considering other layers and years. Our aim is to investigate the resilience of such a multilayer network, as defined in Section 2.

Table 1 reports the local resilience scores under type 1-shocks, which quantify the stability of the community structure of each layer when single nodes are perturbed. High values (close to 1) indicate robust configurations, suggesting that most layers display a stable community structure. However, in 2020 the second dimension (CE2, circular material use rate) appears to be the least resilient, with relatively lower values for several countries such as Belgium (0.843) and Estonia (0.838). This indicates a high sensitivity of such an indicator to local shocks. These differences highlight that the resilience of the system is not uniform across the circular economy dimensions (Fig. 2).

As the analysis refers to a single year, a broader temporal perspective is provided by the heatmaps in Fig. 3, which illustrate the evolution of local resilience across years. In the heatmaps, darker shades correspond to higher resilience (values close to 1), while lighter shades identify layers or countries that are more vulnerable to perturbations. Overall, local resilience values are high across all years, countries, and layers, rarely dropping below 0.75–0.80. Therefore, the European multilayer system is generally resilient to type 1-shocks. At the same time, the heatmaps in Fig. 3 reveal non-negligible heterogeneity: both the least-resilient layer and the set of most affected countries vary over time. In 2020 the second dimension (CE2, circular material use rate) shows the lowest value, especially for countries such as Belgium (0.843) and Estonia (0.838). In 2016, by contrast, the least resilient values emerge in CE3 (private investment and gross value added), with countries such as Malta and Denmark showing lower scores; CE3 remains relatively vulnerable in 2017 (with Ireland among the most robust cases). In 2018, the pattern shifts back to CE2, although the countries with lower resilience differ from 2020 (e.g., the Netherlands and Finland, with Belgium again among the weaker cases). In 2019, pockets of lower resilience are visible in both CE1 and CE2.

Taken together, these results suggest that while the system is consistently robust at the aggregate level, the identity of the countries driving the weakest responses

Table 1 Local type 1-shock for the year 2020: results

Country	Layer				
	CE1	CE2	CE3	CE4	CE5
Belgium	0.979	0.843	0.979	0.984	0.984
Bulgaria	0.995	0.921	0.953	0.950	0.948
Czechia	1.000	0.969	0.969	0.979	0.984
Denmark	0.979	0.942	0.859	0.979	0.958
Germany	1.000	0.979	0.990	0.984	0.974
Estonia	1.000	0.838	0.869	0.974	0.958
Ireland	0.990	0.963	1.000	0.936	0.995
Greece	0.974	0.953	1.000	0.984	0.963
Spain	0.990	0.974	0.958	0.974	0.963
France	0.979	0.990	0.969	0.990	0.984
Croatia	0.969	0.942	0.995	0.974	0.953
Italy	0.984	0.843	0.984	0.984	0.974
Cyprus	0.969	0.937	0.963	0.950	0.948
Latvia	0.979	0.932	0.942	0.963	0.979
Lithuania	0.979	0.927	0.948	0.927	0.953
Hungary	0.985	0.932	0.942	0.974	0.937
Malta	0.984	0.838	0.985	0.918	0.979
Netherlands	0.990	1.000	1.000	0.984	0.984
Austria	0.880	0.984	0.990	0.984	0.984
Poland	0.974	0.921	0.932	0.963	0.958
Portugal	0.974	0.937	0.942	0.990	0.969
Romania	0.979	0.953	0.969	0.979	0.995
Slovenia	1.000	0.969	0.963	0.932	0.974
Slovakia	0.995	0.927	0.974	0.963	0.974
Finland	0.958	0.942	0.953	0.937	1.000
Sweden	0.974	0.890	0.948	0.948	0.958

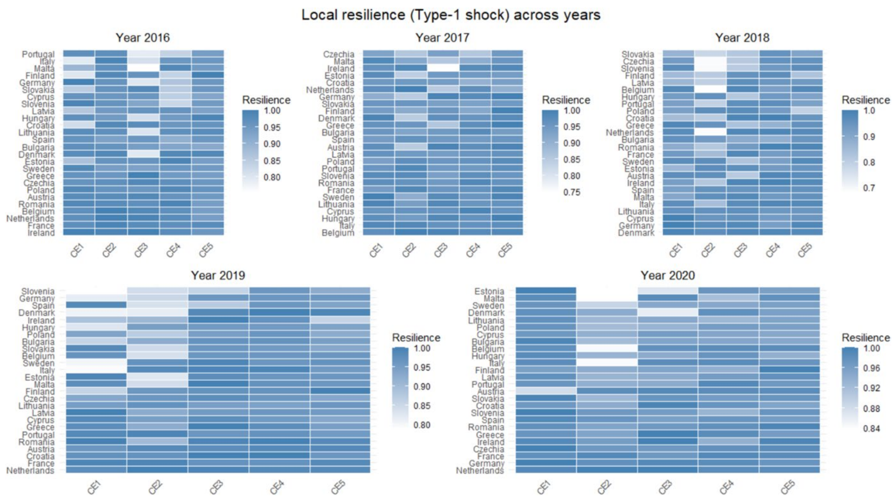


Fig. 2 Local type 1-shocks of the Circular Economy multilayer network for different years

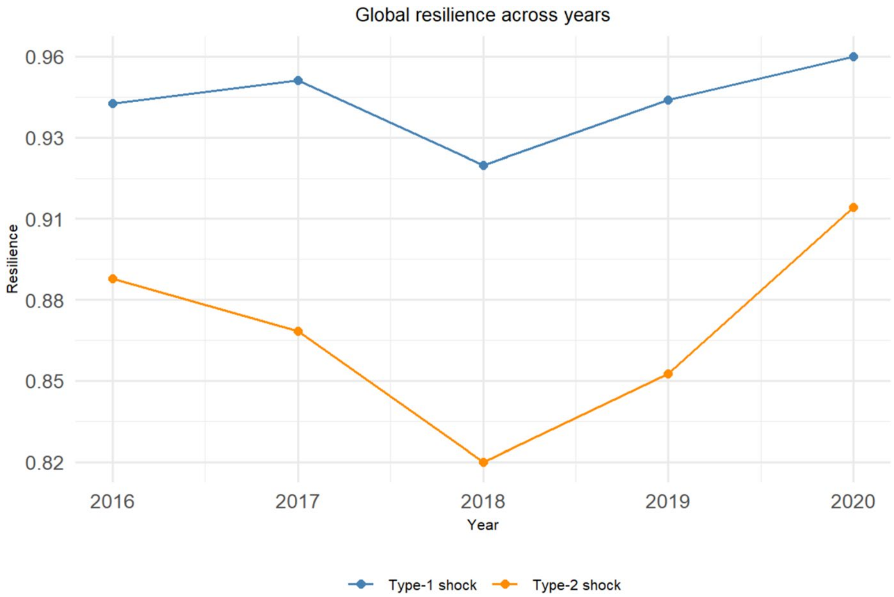


Fig. 3 Temporal evolution of global type 1- and type 2-shocks of Circular Economy multilayer network

and the dimensions most exposed to local perturbations are not time-invariant. Economically, this means that altering the community membership of a single country reshapes the overall structure, so the groups we detect are substantively robust and reflect persistent similarities in circular-economy performance at the European level.

We then compute type 2-shocks, involving the removal of interlayer links between pairs of countries and layers. Table 2 reports a sample of the possible outcomes of the type 2-shock analysis for the year 2020. Clearly, it is impractical to summarize all combinations in a single table, as each shock involves removing a pair of nodes (i, j) from a pair of layers (h, k), resulting in a large number of possible configurations.

About the interpretation of the results shown in Table 2, we have that removing the interlayer connection between Belgium and Italy (layers CE1 and CE2) yields a local resilience value of 0.830, whereas removing the link between Cyprus and Malta (layers CE4 and CE5) produces a value of 0.862. In general, the resilience values remain relatively high across all the configurations, confirming the robustness of the multilayer network to interlayer perturbations. This indicates that perturbing specific cross-layer relationships among countries does not substantially alter the overall community structure, implying that the detected interdependencies across circular economy dimensions are stable and persistent.

Although these results refer to a single year (that is, 2020), the same analysis can be replicated for each period, providing a dynamic view of how resilience evolves across time. However, unlike type 1-shocks, where heatmaps can intuitively visualize the spatial and temporal variability of local resilience, representing type 2-shocks graphically is less straightforward due to the combinatorial nature of interlayer interactions. For this reason, global resilience measures – obtained by aggregating

Table 2 Local type 2-shock for the year 2020: results

Country		Layer		Resilience
i	j	h	k	
Belgium	Italy	CE1	CE2	0.830
Bulgaria	Croatia	CE1	CE2	0.918
Bulgaria	Hungary	CE1	CE2	0.918
Bulgaria	Portugal	CE1	CE2	0.918
Czechia	Spain	CE1	CE2	0.928
Denmark	Lithuania	CE1	CE2	0.933
Croatia	Lithuania	CE4	CE5	0.942
Italy	Latvia	CE4	CE5	0.965
Cyprus	Malta	CE4	CE5	0.862
Hungary	Slovakia	CE4	CE5	0.942
Netherlands	Austria	CE4	CE5	0.959
Slovenia	Sweden	CE4	CE5	0.847

local responses across all possible shocks – offer a more concise and interpretable summary.

Figure 3, therefore, presents the temporal evolution of the global resilience indices for both type 1- and type 2-shocks, illustrating how the stability of the European multilayer system changes over the 2016–2020 period.

Both indicators follow a similar trend, with a temporary decrease around 2018 followed by recovery in 2019–2020. This pattern indicates that the overall system tends to self-stabilise, with interactions between CE dimensions becoming increasingly coherent over time.

Interestingly, the temporary decline in global resilience observed around 2018 appears to reflect a period of relative instability in the structure of similarities between countries.

One plausible explanation relies to the evolution of European policies on the circular economy during that period. In particular, the adoption of the updated Circular Economy Package (see https://commission.europa.eu/publications/documents-circular-economy-package_en) by the European Commission in 2018 introduced new goals and strategic guidelines, which may have affected countries asymmetrically. Such heterogeneous adjustments can lead to changes in the relative positioning of countries across the various dimensions, thereby altering the structure of the community and reducing the stability of the multilayer network.

More generally, the observed decline in resilience could reflect a transitional phase in which countries temporarily diverge before converging again in subsequent years, as suggested by the recovery in 2019–2020. This interpretation is consistent with the idea that resilience reflects the stability of similarity patterns rather than their absolute level.

At the same time, we cannot rule out that part of this variation may be related to data-specific factors, such as updates to measurements or revisions to the underlying Eurostat indicators. For this reason, the interpretation of the 2018 low level of resilience should be considered with care, perhaps in a devoted study.

5 Discussion and Conclusive Remarks

In this paper, we introduced a novel resilience measure for multilayer networks based on a community detection approach applied to individual layers. By defining two types of shocks–one affecting the clustering process and the other disrupting interlayer links–we assessed how perturbations influence the stability of network structures.

A way to contextualise our contribution is to compare it with existing measures of resilience for multilayer networks. Percolation-based robustness indicators typically quantify a network's ability to preserve global connectivity when nodes or links are removed, often through the size of the giant component or related connectivity thresholds. Cascade-based measures, on the other hand, focus on the propagation of failures across interdependent layers and the extent of systemic collapse triggered by local disruptions. Other topology-based indices typically evaluate structural properties such as centrality, redundancy, or layer importance. Our proposal complements these approaches. Rather than measuring only the persistence of connectivity or the propagation of failures, our measures quantify the stability of the community structure and the induced cross-layer similarity structure in the presence of disturbances. Thus, our proposed resilience is particularly well-suited to contexts where the relevant systemic characteristic is not simply whether the network remains connected, but whether the clustering structure of nodes across levels remains sufficiently stable to preserve the multilayer organisation. Therefore, our framework captures a mesoscopic dimension of resilience that standard robustness measures do not directly target. This peculiarity is significant in contexts such as the circular economy. Indeed, a disruption at the country level can leave the network largely connected while altering the structure of the community that identifies groups of countries with similar performance in the circular economy. Consequently, our measure is able to highlight instability in the architecture of interlayer similarities even when more traditional robustness indicators, based on connectivity, would suggest limited damage.

In general, the introduction of a multilayer structure in complex networks prevents a straightforward adaptation to this case of the standard definitions of resilience measures for unilayer networks for practical applications. In the present study, we advanced a reasonable criterion for building resilience measures in the challenging context of multilayer networks. The followed approach is based on community detection algorithms and interlayer links density. Thus, the proposed methodology moves from a rather simple view of the attitude of the considered networks to absorb a shock, and it becomes complicated only due to its applicability to a multilayer context. This said, we do not have a standard robustness measure of a multilayer network to be compared with the present one from the side of the applications. We have in mind some different conceptualizations of multilayer network resilience, but their formalizations require devoted papers and are left for future studies.

Thus, the proposed framework builds upon the recent literature that represents multilayer networks through community detection, linking nodes across layers according to their structural similarity. Our resilience measure is naturally compatible with this formulation, as it directly quantifies the stability of the community configuration under perturbations. Nevertheless, the approach is general and can be applied to any kind of multilayer system, regardless of how interdependencies are defined. Indeed, the procedure for assessing the resilience of the network with respect to shocks of type 1 and 2 satisfies a sort of "universality feature", in the sense that it does not depend on the specific clustering outcomes or on the number of layers and nodes. Therefore, our methodology provides a deeper understanding of how multilayer networks respond to disturbances, offering valuable insights in different applicative domains.

To show how the developed resilience measure can be computed in practice, we develop an application for European Circular Economy indicators. The results revealed that the European multilayer system is, on average, highly resilient to both types of shocks, with limited structural variation under perturbation. Nevertheless, resilience levels vary across years and indicators, suggesting that certain dimensions–such as material use and investment–are more exposed to instability than others.

We point out that the proposed definitions of resilience move from local perturbations to the overall structure of the system. This procedure is in agreement with the standard definition of *resilience*, which is the ability of the network to absorb an exogenous and impulsive shock occurring in one of its components. On this, large-scale system-wide shocks – like COVID-19 pandemic, for instance – cannot be conceptualized as shocks of type 1 or type 2. COVID-19 is included in the considered time range and deserves a specific comment.

Indeed, it is important to clarify that the measures of resilience proposed in this paper reflect the stability of the relative structure of similarities between countries, rather than changes in the absolute levels of the underlying indicators. In this regard, the impact of more systemic events like COVID-19 may not necessarily translate into a decline in resilience if countries are affected in a largely symmetrical manner. When shocks affect most units simultaneously and in a comparable way, the relative positioning of countries –- and, thus, the structure of the community -– may remain largely unchanged.

This interpretation is consistent with the data shown in Fig. 3, where no substantial deterioration in global resilience is observed in 2020. Rather than indicating the absence of disruptions, this result suggests that the pandemic may have acted as a common shock, preserving the overall structure of similarities between countries despite affecting their absolute performance.

For these reasons, although COVID-19 represents a highly significant disruptive event in the real world, its effects are not directly comparable to the stylized shocks introduced in our model.

Future developments include the design of alternative types of shocks to capture different sources of perturbation within and across layers, as well as the implementation of enhanced visualization tools to better represent the complex interactions and resilience patterns in multilayer systems. Moreover, less extreme perturbations than removing nodes might be considered. For example, one can consider shocks associated with a weakening of the intralayer connections or the possibility that some nodes move from one community to another one. These shocks would lead to a less clear measure of resilience of the network, but with the positive aspect of a more intuitive interpretation when making empirical applications. This is left for future research. Finally, further applications could also explore the use of the proposed framework in other classes of multilayer networks, such as those arising from financial markets, innovation systems, or production networks, where interdependencies among units are crucial for assessing systemic robustness.

Author Contributions All authors (R.C., G.F., R.M. and S.S.) have contributed equally to this work.

Funding Open access funding provided by Università degli Studi della Campania Luigi Vanvitelli within the CRUI-CARE Agreement. No funding was received for conducting this research.

Data Availability No datasets were generated or analysed during the current study.

Declarations

Competing interests The authors declare no competing interests.

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