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ON POLITICAL AMBIGUITY AND ANTI-MEDIAN PLATFORMS

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Abstract

This paper constructs a new, information-based explanation for political ambiguity and the success of anti-median platforms. It argues that voters' and candidates' correlated preferences about the appropriate policy combined with ambiguous platforms can help candidates with non-median preferences increase their support and even win against a median candidate. I show how ambiguity can arise in a standard citizen-candidate setting where voters have different preferences, in its extension with primaries, and even in a Condorcet jury model where disagreement arises only from differences in voters' information. The paper also offers a formal framework that allows for dog whistle politics. The model illustrates how ambiguity can have important negative welfare implications. Specifically, I show that despite having ex-ante identical preferences with voters, politicians may choose ambiguous platforms even if voters would be keen on banning them.

JEL Classification: D72, D83

Keywords: Political ambiguity, citizen-candidate, Condorcet voting, dog whistle

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politics.

1. Introduction

A common characteristic of political statements is their deliberate ambiguity. However, in basic electoral models ambiguous messages tend to generally hurt rather than help politicians. Specifically, Shepsle (1972) showed that, if ambiguous platforms are interpreted as lotteries and voters are risk averse, then a majority of them will always prefer a certain median platform to any risky ambiguous platform. This paper describes a new reason why the result can be overturned even when all agents are fully rational and risk averse as long as citizens' and candidates' policy preferences are correlated. In particular, I will show how either self-contradicting promises or promises that specify what the candidate is *not* going to do without specifying what they *are* going to do can unite two disagreeing ends of the voter spectrum against the natural median.

The mechanism in this paper rationalizes the success of a rhetoric strategy employed especially by outsider candidates or parties that break the traditional political divisions by appealing to both conservative or extreme right-wing voters as well as disenchanted blue-collar workers and the unemployed.[2] Large part of their electoral success comes undoubtedly from focusing their platforms on issues, such as immigration, where the two groups of voters have *similar* preferences. However, one would expect these two groups to have very divergent preferences on many traditional issues like redistribution, social welfare and public health care. I describe a simple, *information-based* strategy how politicians, by being ambiguous, can position themselves against the median voter, generate

[2] Recent examples include Donald Trump in the United States, the Brexit party in the UK, the Five Star Movement in Italy, FPÖ in Austria, the Party for Freedom and FvD in the Netherlands, the Sweden Democrats in Sweden, the Finns in Finland, the AfD in Germany , Javier Milei in Argentina etc.

an air of perceived preference-similarity with voters of diverse backgrounds and not alienate any of the conflicting groups within their base. The logic works also for established parties who, for example, may be clear on questions related to redistribution but whose immigration platforms are often fairly ambiguous (Han, 2020). More generally, this paper rationalizes the empirical findings in Han (2020) who shows that the more divided a party's supporters are on an issue, the more blurred the party's platform on that issue tends to be.

I first discuss how correlation in preferences interacts with ambiguous platforms in a parsimonious citizen-candidate model (Osborne and Slivinski, 1996; Besley and Coate, 1997) with uncertainty about citizens' and candidates' policy preferences. My main result shows that as long as these preferences are correlated, non-median candidates can beat a median voter's favorite platform by simply committing to a non-median platform without specifying whether that platform is to the "left" or "right" from the median.[3] Intuitively, a left-wing voter who knows that a candidate's preferences are correlated with hers, believes that an ambiguous non-median candidate is more likely to be from the left than the right. Symmetrically, a right-wing voter believes the opposite. Hence, both tails of the voter distribution are willing to vote for a candidate with an ambiguous *anti-median* platform, even when all agents are moderately risk-averse. My model hence rationalizes how very different voter groups can perceive a candidate as a champion of their cause as long as the candidate remains sufficiently ambiguous about their exact position.

[3]I use labels right and left only as a convenient modeling short-hand with no intended ideological content. The key insight is that a politician can expand their support by taking only a partial stand on issues that are divisive within their intended audience. As is shown later in Section 7, the mechanism works within smaller groups, in an extension with primaries and even in a Condorcet model where everybody has identical preferences. For example, in the extension with primaries, ambiguous platforms unite conflicting extremes *within* the party against the party median. This allows the most "left-wing" of the supporters of an ambiguous "conservative party" candidate to be more conservative than any of the supporters of the "liberal" party. This extension allows ambiguity to arise in the within-party competition in environments where voters have a strong ex-ante party identification.

The model offers a novel reason to rationalize why outsider, anti-median or anti-establishment politicians can unite voters who traditionally disagree on many issues. For example, Donald Trump's anti-immigration stance was clearly stated from the beginning. However, during the campaign both the conservative and liberal press criticized his ambiguity on a wealth of other important policy issues.[4]. For example, Pierson (2017) gives an in-depth description of how Trump's ambiguous rhetoric on many policy issues appealed to both the wealthy, conservative interest groups and his blue-collar base, and how the wealthy interest groups discounted Trump's messaging to the blue-collar base and vice versa. One indicator of the resulting divide in beliefs are the polling results before the 2016 election where close to 20% of the voters perceived Trump as having liberal political views, a number that is double relative to all previous republican candidates at least since George H. W. Bush.[5] Almost half of these voters changed their view about Trump's political ideology during his presidential term.[6] When in 2016 less than half of the polled voters thought Trump was a conservative or an extreme conservative, in 2020 that number had climbed to 68%.

I show that the equilibrium where anti-establishment candidates choose ambiguous platforms is more likely when the opposing candidate is a known centrist.[7] This finding is confirmed in an experiment by Tolvanen et al. (2022)

[4]For instance, POLITICO described his political position as "ecletic, improvisational and often contradictory" (Timothy Noah, "Will the real Donald Trump please stand up?", July 26, 2015) while still in May 2016 a headline in the conservative Washington Times read: "Donald Trump's agenda a mystery as interviews contradict position papers" (May 12, 2016).

[5]See Gallup, October 4, 2016. `https://news.gallup.com/poll/196064/trump-seen-less-conservative-prior-gop-candidates.aspx`

[6]See Alan I. Abramowitz, "How Donald Trump Turned Off Swing Voters in 2020", The Center for Politics, August 25, 2021. `https://centerforpolitics.org/crystalball/how-donald-trump-turned-off-swing-voters-in-2020/`

[7]A notable example of this was the Brexit referendum where a vote for Remain was a clear vote for the status quo while a vote for Leave was presented as a way to "take back control". Depending on the campaigner and the audience, this could mean anything from allowing for more room for state intervention in industrial and trade policies that benefit employees to cutting down worker protections and existing regulations. See, for example, Anand Menon,

who take a simplified version of my model to the laboratory and show that non-centrist candidates indeed play ambiguous platforms more often when they know they are facing a centrist.

Furthermore, the existence of ambiguous platforms depends on how appealing the centrist's favorite position is to non-centrists. The more the extremes dislike the centrist's platform relative to their own favorite, the more appealing is an ambiguous anti-median candidate. Recent surveys show that the policy preferences of the right and the left wing in the US have diverged during recent years (see e.g. Pew Research Center, 2014; Iyengar and Westwood, 2015). Similarly, income growth rates at the top and bottom deciles of the income distribution have diverged from the growth at the middle of the distribution at an exceptional rate.[8] Both trends are likely to make the median voters' preferred platform less appealing to both extremes which in my model increases the support for ambiguous candidates.

Ambiguous platforms in my model leverage uncertainty about voter and candidate preferences. Political scientists have shown that the volatility in citizens' voting behavior has increased considerably since the sixties (see e.g. Mair, 2008; Bischoff, 2013; Dassonneville and Hooghe, 2017). This trend is likely to increase the uncertainty that the voters have about each others' and candidates' preferences. My model generates a formal causal link between this increased volatility

"We still don't know what or who Brexit is actually for", *Independent*, June 3, 2021. In the case of Brexit the voters' beliefs of what it meant was likely to be shaped by their beliefs about future elections. These beliefs, in turn, were likely to be influence by the voter's lived experience. The differences in beliefs became visible even within the British parliament when Leave proponents were strongly split when voting on the terms at which Britain was to leave the EU.

[8] For example, Piketty et al. (2018) show that in the 1946-1980 period the bottom 20% of the American income distribution experienced much higher income growth rates than the percentiles from 50 to 90, who in turn experienced higher growth rates than the top 10%. This mean-reverting pattern can be argued to make both extremes more amenable to the median voter's favorite policy. The pattern is completely reversed in the 1980-2014 period where the incomes of the bottom 20% grew only by 4%, the middle 40% experienced a growth of 49% and the top tenth percentile's incomes grew by 113%. This divergent pattern is likely to distance the preferences of both the top and the bottom from the median.

and the popularity of anti-establishment candidates and parties. The model also requires voters to believe their preferences are correlated with candidates' preferences. Such perceived correlation can be induced by socio-economic factors such as economic recessions and booms, cultural phenomena, and political crisis that create unpredictable shifts in voter preferences and that may motivate political outsiders to run for office. Additionally, strategic campaigning in social media and its echo chambers may reinforce the perception of similarity that voters assign to a new candidate.

Importantly, ambiguity arises in equilibrium even in a *unidimensional* policy space where the extremes have different policy preferences. Consequently, adding a dimension where the extremes have identical preferences will only add to the appeal of the ambiguous candidate. If voters who like either low or high taxes agree on the optimal level of immigration, then being clear on this dimension makes the anti-establishment candidate's position only stronger. Conversely, an anti-immigration candidate can win by choosing not to articulate her stance on taxation, whereas she would have lost to a median candidate, were she forced to clearly declare her preferences. Hence, my model is not meant as an exhaustive model of the rise of populist or outsider candidates but a rationalization of a strategy that they as well as the existing parties use when their base disagrees on a focal topic.

If an ambiguous candidate gets elected, she will eventually have to choose which policies to enact. Given the voters' diverging expectations, a large fraction of them will eventually be disappointed. This pattern can be seen in some recent elections. For example, the Austrian populist FPÖ had a landslide victory in the 1999 elections. When in government, the party had to commit to positions on issues outside of its anti-immigration core and ended up alienating a large fraction of its blue-collar support with largely neo-liberal economic reforms (see e.g. Luther, 2003, 2008). The party eventually split and lost nearly two-thirds of its support falling from almost 27% in the 1999 election to mere 10% in the 2002 elections.[9] On a more general level, the model suggests a way for single-issue movements to survive by not articulating a stance on issues that might be divisive. However, when forced to decide on those issues they risk dissolving or splitting.

The rest of the paper is organized as follows: The next section discusses the relationship of my model to recent literature. Section 3 introduces the model and its equilibria and section 3.3 contains the main result of the paper. Section 4 discusses equilibrium selection. Section 5 derives important comparative statics. Section 6 shows that informational ambiguity can have non-trivial, negative welfare implications for voters. Section 7 discusses the generality of the mechanism. In particular, it first discusses how the model is easily extended to encompass primaries where an ambiguous left-wing candidate can first beat the left-wing party median and then go on to beat the right-wing party median in a general election. Furthermore, I show that the mechanism is able to generate ambiguity even in a Condorcet jury model where all agents have identical preferences but differ only in their information. The section also explores an asymmetric extension of the model where one of the target groups is small, much like in many of the known examples on dog whistle politics. The last section concludes. The proofs are relegated to the Appendix.

[9] A very similar story is true about the Finns party in Finland that entered a right-wing coalition government after winning almost 18% in the 2015 parliamentary elections only to experience a historical fall in support bottoming at 8% in the July 2016 poll ("Yle poll: Social Democrats hot on heels of Center Party", *Yle Uutiset*, September 8, 2016). Most of these lost voters moved back to the left-wing Social Democrats. The Finns also split into two parts in 2017.

2. Related Literature

The mechanism suggested in this paper is related to the seminal work on ambiguous political platforms by Zeckhauser (1969), Shepsle (1970) and Shepsle (1972). These papers model ambiguous political platforms as politicians committing to a probability distribution over the available policy space. They show that there exists single peaked preference profiles such that a well chosen lottery can beat a politician committing to the median voter's position. Aragones and Postlewaite (2002) generalizes the earlier work allowing politicians to have a favorite policy and to commit to only subsets of all probability distributions over the policy space. However, Shepsle (1972) shows that ambiguous platforms can be winning only if voters are risk-loving.

These models differ considerably from my model where politicians have preferences that are correlated with voter preferences and can hence make *credible* commitments only to subsets of the policy space. Compared to these papers my model highlights a new way how voters' private information can contribute to candidates' ambiguity even when voters are risk averse. I am also able to derive comparative statics in terms of political marginalization and polarization of the extremes as well as derive ex-ante welfare comparisons between equilibria where politicians are non-ambiguous and ones where they do not commit to singleton platforms. Furthermore, I show that the mechanism can generate political ambiguity in equilibrium even when everybody has identical preferences but different information about a policy relevant state.

Alesina and Cukierman (1990) show that politicians who care both about the implemented policy and re-election may also choose non-zero levels of randomness in their first-period policies. This corresponds strongly with the interpretation of ambiguity as politicians committing to lotteries over policy outcomes. Shepsle's result is true here in the sense that if voters are risk-averse, any randomness in the first period policies that is not resolved during the first period will hurt the politician's re-election chances. The increased noise in the implemented policy, on the other hand, helps the politicians to hide their first period policy choice and thus not reveal their preference type.

The recent work by Janssen and Teteryatnikova (2017) considers a version of ambiguity where politicians make non-committing announcements about their policy preferences and where the candidates have better information about each others' preferences than the voters do. They show that the resulting equilibrium outcome of the game is guaranteed to be the same as with truthful announcements if and only if politicians provide information about their own and their opponent's position and are required to include their own and their opponent's true favorite position in their announcements. In my model candidates have only information about their own type and hence there is not any additional information they can convey about their opponent and they can commit to whatever platform they like. Consequently, I can support equilibria that differ considerably from the equilibria when only truthful announcements are allowed.

Multiple authors have pointed out that ambiguity in platforms offers candidates flexibility over policies choices at a later date. Meirowitz (2005) and Alesina and Holden (2008) consider models where candidates have a strategic incentive to be ambiguous in primary elections either because of uncertainty about the electorate's preferences or because it allows them to retain an option to change their platform after observing their competitor's platform in the primary. Somewhat similarly, in Kartik et al. (2017) ambiguity allows candidates adapt to policy relevant information revealed at a later date. The source of ambiguity in these models is quite different compared to mine and could be a complementary explanation for why politicians choose not to commit in elections. In my model, ambiguous platforms can arise even when politicians are not expected to gain policy relevant information after the election. Hence, it is able to explain ambiguity on issues that are more value- or preference-based. The models where demand for flexibility generates ambiguity also rely heavily on the median voter theorem and cannot explain the appeal of ambiguity to anti-median candidates.

There are also some papers where ambiguous platforms become optimal because voters are not fully rational. For example, in Callander and Wilson (2008) voters have context-dependent preferences and in Jensen (2009) they are susceptible to projection effects, i.e. thinking that their favorite candidate is closer to them than they really are while their least favorite candidate must be even further than their true position. My suggested mechanism does not require any deviations from the standard model with rational voters and politicians.

Last, my paper complements the work by Buisseret and Van Weelden (2019). It offers a mechanism through which the anti-establishment candidate of their model can use ambiguous platforms in the divisive dimension of either the primary or the general election to unify extreme views and win with even a higher probability than predicted by their model.

3. The Voting Game and Its Equilibria

This section constructs a model of elections where correlation between the candidates' and voters' preferences can give rise to ambiguous political platforms. I chose to keep the model as simple as possible to highlight how correlation between voters' beliefs and tastes feeds into the support for ambiguous candidates. The model builds on the citizen-candidate models pioneered by Osborne and Slivinski (1996) and Besley and Coate (1997). Formally, the model resembles the one in Goeree and Großer (2007). The two key building blocks of the model are the ability of candidates to commit to non-singleton platforms and the assumed correlation between voters' and candidates' preferences. For example, changes in economic outcomes, social norms and past political decisions can all induce correlated changes in the distribution of political preferences. For example, recessions are often associated with a higher demand for redistribution than booms and could be interpreted as generating uncertainty about the distribution of voter preferences.

The first subsection describes the structure of the model, the second considers the existence of non-ambiguous perfect Bayesian equilibria. The main result that constructs the ambiguous perfect Bayesian equilibrium is in the third subsection.

3.1. The Voting Game

Consider a population of $N+2 \in \mathbb{N}$ citizen-candidates where N is a large odd number. Assume that there are three available policies $a \in \{-1, 0, 1\}$ and two equally likely states of the world $s \in \{-1, 1\}$. The state can be anything from day-to-day events to economic or cultural phenomena that shock the policy preferences in the population. In other words, the state is probabilistically correlated with the preferences of the citizens. I assume that there are three types of citizens. I denote the type of a citizen $v \in N$ by $\tau_v \in \{-1, 0, 1\}$ and I often refer to the type -1 (or type 1) as *leftists* (or *rightists*, respectively) and together I refer to them as *extremists*, while type 0 is often called *centrists*. Let $u_\tau : \{-1, 0, 1\} \to \{0, 1, u_0\}$ be the citizens' von Neumann-Morgernstern utility index where $u_0 \in [0, 1)$ is some constant. Assume that $u_\tau(a) = 1$ if $a = \tau$ and $u_\tau(a) = 0$ if $0 \neq a \neq \tau$. For the types $\tau = -1, 1$ assume that $u_\tau(0) = u_0$. In other words, everybody has single-peaked preferences and wants the policy to match their type. On the other hand, the policy 0 guarantees some intermediate payoff independent of their type for everybody. The utility u_0 can be interpreted as a measure of marginalization of the extremes, as it measures how appealing the extremists find the centrists' preferred policy.[10] I often call policy 1 (-1) rightist (leftist, respectively) and policy 0 centrist. As a robustness exercise, I also consider the case where the centrists strictly prefer one of the extremists over the other. If most centrists prefer -1 to 1, I call the leftists *favored*[11] and vice versa.

The key assumption driving the results is that there are more leftists in expectation when the state is -1 and more rightists when the state is 1. Formally, let

$$p = \mathbb{P}(\tau = 1 \mid s = 1) = \mathbb{P}(\tau = -1 \mid s = -1)$$

and

$$q = \mathbb{P}(\tau = 1 \mid s = -1) = \mathbb{P}(\tau = -1 \mid s = 1)$$

and assume that $p > q$. Conditional on the state, the type of each citizen is drawn independently from other citizens and candidates. Furthermore, I assume that in expectation neither type $\tau = 1$ nor $\tau = -1$ are unlikely to form a majority alone but together they are always likely to comprise more than half of the population: $p + q > \frac{1}{2} > p$.

The voting game proceeds as follows

1. Nature picks the state with equal probability on each state.
2. Conditional on the state, nature picks a type for each one of the citizens using the probability distribution from above and informs the citizen about her type but not about anything else.

[10] It can be also used as an incomplete measure of risk preferences of the extremists. Suppose, for example, that when the chosen policy is $a \neq 0$, type $\tau = a$ gets a monetary income H and types $\tau \neq a$ get income $L < H$ and when $a = 0$ all types get income $M = \frac{H+L}{2}$. Given that the vNM utilities are unique up to affine transformations, it is without loss of generality to set $u_1(H) = 1$ and $u_1(L) = 0$. Then given the equal prior, the extremists can be risk-averse only if $u_0 := u_1(\frac{H+L}{2}) > \frac{1}{2}u_1(H) + \frac{1}{2}u_1(L) > \frac{1}{2}$. It is also trivial to verify that for each $u_0 \in (\frac{1}{2}, 1)$ there exists a strictly concave (and hence risk-averse) utility function that maps H to 1, L to 0 and M to u_0.

[11] I.e. if $u_0(0) > u_0(-1) > u_0(1)$.

3. Nature picks randomly 2 citizens as candidates indexed by $i \in \{1, 2\}$.
4. Each candidate i simultaneously chooses a platform $A_i \in \mathcal{P}(\{-1, 0, 1\}) \setminus \emptyset$ where $\mathcal{P}(\cdot)$ denotes the power set. In other words, a platform can be any non-empty subset of the set of available policies.
5. Each voter votes for either of the candidates. For simplicity, the candidates are not allowed to vote. This assumption has no qualitative effects on the results when N is large. The winner of the election is decided by simple majority.
6. The winning candidate then implements a policy $a \in A_i$ and utilities are realized.

Candidates gain no additional benefit from holding office. As long as the benefit from holding the office is small relative to the benefit from matching the policy with the politician's type, the analysis remains unchanged.[12] I analyze the *perfect Bayesian equilibria* of this game with weakly undominated strategies. I allow voters to vote strategically and generally assume that N is sufficiently large. In other words, when forming beliefs voters are allowed to condition on events where they are pivotal. I argue later that the predictions of the paper do not depend on this assumption.

As I mentioned above the two key assumptions that generate ambiguous or non-committing equilibrium platforms are the ability of the candidates to commit to platforms with more than one elements and the correlation between the voters' and the politicians' types through the state of the world s. This state can be interpreted as anything that generates uncertainty about the preferences of the people who vote and candidates' true policy preferences. For example,

[12] The main result in Proposition 4 remains qualitatively the same even with large office motivation, as long as candidates get a small benefit from enacting their favorite policy. In that case the equilibrium may have centrists and extremist voters mixing but the reasoning stays otherwise the same.

changes in the political culture, ideological trends or even political scandals are likely to cause correlated changes in the distribution of voter and candidate preferences. Similarly, economic shocks are likely to perturb the distribution of preferences in the voter population as well as affect the relative turnout of each of the voter types. Furthermore, those same shocks can have a significant effect on who finds it meaningful to become a politician.[13]

3.2. Equilibria with Singleton Platforms in Large Populations

I now discuss when perfect Bayesian equilibria (henceforth equilibria) with singleton platforms exist. Notice first that if politicians are forced to use singleton platforms, then conditional on there being a candidate who commits to 0, a candidate committing to 0 is going to win with a probability that converges to 1 as $N \to \infty$. The result follows from two observations. First, by the law of large numbers, the fraction of voters of type τ converges to $\mathbb{P}(\tau \mid s)$ as $N \to \infty$. Second, if candidate 1 commits to $a = 0$ and candidate 2 commits to something else, say 1, then both types $\tau = 0$ and $\tau = -1$ find it weakly dominant to vote for candidate 1. The only ones who find it weakly dominant to vote for candidate 2 are the type $\tau = 1$ voters. Consequently, as $N \to \infty$, the share of voters voting for candidate 1 converges in probability to $\mathbb{P}(\tau = 1 \mid s) + \mathbb{P}(\tau = 0 \mid s) > \frac{1}{2}$ for all s. Hence, for a large N, policy 0 gets implemented with a probability arbitrarily close to 1. In this sense, with singleton platforms, the centrists are the median voters of the model.[14]

[13] In the interest of keeping the model simple and parsimonious, the choice of becoming a candidate or allowing voters to abstain is left outside of the model but could be fairly easily incorporated.

[14] Notice that in my model candidates are issue motivated and face uncertainty about their winning probabilities if both run on an extreme platform. Hence, the results from Calvert (1985) do not guarantee that the median voter's preferred action is always implemented in equilibrium. However, the reasoning above shows that when constrained to singleton platforms, if a candidate runs with the median voter's favorite platform, that platform will be implemented in any equilibrium where voters never play weakly dominated actions. However, Section 3.3 shows that this version of the median voter theorem is *not* robust to allowing candidates play *ambiguous*, multi-valued platforms.

The power of the median voter does not mean that other platforms never get proposed in equilibrium. Even truthful policy agendas can be an equilibrium as long as the likelihood that the opposing candidate is from one of the tails of the population is high enough. Even though a centrist candidate will always win conditional on running, the extremists can win if the opposing candidate is also from the extreme. The next proposition characterizes this result.

Proposition 1. *Suppose that* $\frac{p^2+q^2}{(p+q)^2} > u_0$. *Then there exists* $\hat{N} \in \mathbb{N}$ *such that for each candidate* i *of type* τ *setting platform* $A_i = \{\tau\}$ *is part of an equilibrium when there are* $N > \hat{N}$ *voters.*

Notice that here the extreme candidates are running with extreme platforms just because they hope their opponent is not a centrist. For example, with a high enough a cost of running, there will only be centrist candidates in equilibrium. The term $\frac{p^2+q^2}{(p+q)^2}$ is the probability with which two extreme candidates have the same type. Consequently, the inequality guarantees the existence of the truthful equilibrium even in the case where all centrists have a strict preference over the two extremes. In this case, the extremist type who is ranked the lowest by the centrists will not deviate to the centrists platform only if it is highly probable that, conditional on the other candidate being an extremist, they will both run on the same platform. In other words, the inequality works outside of the knife-edge case where all centrists are indifferent between the two extremes and resolve the indifference by randomizing equally between the two. If all centrists are indifferent between the two extremes, then the equilibrium will exist for a slightly larger set of parameter values.

The next result shows that everybody playing the centrist platform is never an equilibrium

Lemma 1. *All types of all candidates committing to platform* $\{0\}$ *is never an equilibrium.*

This result follows rather trivially, since if everyone is expected to commit to $\{0\}$, there is no down-side for an extreme candidate to commit to her favorite position. With a vanishing probability the majority of the voters share her type and she gets elected. If she loses, the opponent implements 0. This is, however, exactly what she would have gotten had she committed to $\{0\}$. Consequently, all extremists have a tiny but positive incentive to deviate.

The only two remaining pure strategy equilibria that are symmetric (or Markov-perfect) in the sense that the candidates' and citizens' strategies depend only on their type and where candidates commit to singleton platforms are characterized by the following proposition.

Proposition 2. *Suppose that* $\frac{p^2+q^2}{(p+q)^2} < u_0$. *Then there exists* $\hat{N} \in \mathbb{N}$ *such that all candidates of type* 1 *committing to a platform* $\{1\}$ *and all other types committing to* $\{0\}$ *is part of an equilibrium when there are* $N > \hat{N}$ *voters. Similarly, all candidates of type* -1 *committing to a platform* $\{-1\}$ *and all other types committing to* $\{0\}$ *is part of an equilibrium with* $N > \hat{N}$ *voters.*

The proof of this proposition follows almost directly from the proof of Proposition 1. The key part of supporting the equilibrium is to assume that if in the first case someone runs with platform $\{-1\}$ and the other candidate runs with platform $\{1\}$, then all of the centrist votes go to the rightist candidate. This discourages leftist candidates from deviating to their preferred platform. The immediate corollary of this is that if the centrists have a preference for one of the extreme platforms, then only one of these equilibria survives, as all of the centrists will flock to their preferred option when presented with two extremist candidates. Notice that in this case the extremist candidate wins the election only if there happens to be two extremists of the same winning type. Hence, just as above, with a large enough a cost of running, the extremists never have an incentive to run.

The assumption on u_0 implies that if one extremist is running with an extremist platform, the other extremist rather takes the certain payoff of u_0 than takes the gamble where she wins only if her type matches the state. It turns out that the equilibria from Propositions 1 and 2 cannot coexist when N is large.

This is captured by the following proposition.

Proposition 3. *Suppose that the centrist voters all strictly prefer one of the extreme options over the other. Then there exists $\hat{N} \in \mathbb{N}$ such that*

1. *if $\frac{p^2+q^2}{(p+q)^2} < u_0$ there does not exist an equilibrium where players play truthfully when $N > \hat{N}$;*
2. *if $\frac{p^2+q^2}{(p+q)^2} > u_0$, then the equilibria from Proposition 2 do not exist when $N > \hat{N}$.*

Hence, when N is large the equilibria from Propositions 1 and 2 are essentially unique among the pure strategy symmetric equilibria with singleton strategies. The proof of the Proposition is also a direct consequence of the proof for Proposition 1. In the first case, the same calculations as in that proof show that the unfavored extremists have an incentive to deviate to $\{0\}$. In the second case the direct converse of the same calculation shows that both favored and unfavored extremists have a strict incentive to deviate to their favorite platform.

It is easy to see that the only pure strategy equilibria where $\{-1, 0\}$ or $\{0, 1\}$ get played on the equilibrium path are outcome equivalent to the truthful equilibria when $u_0 < \frac{p^2+q^2}{(p+q)^2}$ and to the equilibria from Proposition 2 when the converse holds. The centrist never has an incentive to commit to these ambiguous platforms, since by deviating to 0 she can always win with certainty while choosing one of these platforms implies that she will lose half of the time to the opposing candidate choosing the same platform. Hence, in any equilibrium where, for example, $\{-1, 0\}$ gets played, that platform perfectly signals that the candidate is a leftist. Similarly, platforms $\{-1, 0, 1\}$ and $\{-1, 1\}$ can be used on the equilibrium path to fully signal the (unfavored) extremist's type. However, none of these equilibria are robust to even the smallest cost of choosing ambiguous platforms. More formally, assume $c > 0$, and suppose that any platform A with $\#A > 1$ costs c to play. Then in a fully signaling equilibrium where A is played on the equilibrium path, the type who is supposed to play this platform will always want to deviate to her preferred singleton platform as it will win with the same probability but will avoid paying c. The cost of ambiguity can be motivated by public distaste for ambiguous platforms, the added difficulty of constructing manifestos that seem to say something without really saying anything or candidate's personal moral disutility from appearing deceptive.

3.3. The Ends Against the Middle Equilibrium

This section constructs the equilibrium where candidates with extreme preferences all commit to the same non-singleton platform. The intuition behind this equilibrium is fairly simple: Both candidates of type -1 and 1 commit to the platform $\{-1,1\}$ and if elected they implement their favorite policy. Extremist voters then have to update their beliefs about the type of the candidate once faced with this platform. Each voter knows that their own type is correlated with the state of the world which in turn is correlated with the politician's preferences. Hence, a leftist voter will think that the candidate promising $\{-1,1\}$ is more likely to be type -1 than 1. The opposite is true for rightist voters. Consequently, as long as u_0 is not too high, both extremes will vote for the candidate promising $\{-1,1\}$ even over a candidate promising u_0. This result is formally stated in the following proposition

Proposition 4. 1. *Suppose the centrists are indifferent between the two extremes. If $u_0 \leq \frac{p^2+q^2}{(p+q)^2}$, then there exists $\hat{N} \in \mathbb{N}$ such that whenever $N > \hat{N}$ there exists an equilibrium where on the equilibrium path both types of extremists choose $\{-1,1\}$ as their platform, centrists choose $\{0\}$, both types of extremist voters vote for anyone running with platform $\{-1,1\}$ and centrists vote for a candidate running with $\{0\}$.*

2. *Suppose the centrists prefer one extreme over the other. Then if*

$$u_0 < \min\left\{\frac{p^2+q^2}{(p+q)^2}, 1 - \frac{pq}{(1-p-q)(p+q)}\right\},$$

there exists $\hat{N} \in \mathbb{N}$ such that, whenever $N > \hat{N}$, there is still an equilibrium where the path in part 1 is still the equilibrium path outcome.

The details about the rest of the equilibrium construction are in the proof of the proposition. When the centrists are indifferent, they can be chosen to vote against (or randomize unfavorably for) a candidate who deviates to either $\{-1\}$ or $\{1\}$. Hence, those deviations will win only when most of the voters prefer the deviation over all alternatives. This probability is vanishingly small when the population is large. Hence, the extremists do not want to commit to truthful platforms as long as all of the extremist voters are expected to vote for the non-committing extremist. The extreme voters are willing to vote for the non-committing extremist whenever $u_0 \leq \frac{p^2+q^2}{(p+q)^2}$, where the right-hand side is the posterior that a non-committing candidate shares a type with a voter conditional on the voter's own type and the fact that the candidate is from one of the extremes. Notice the stark contrast in strategic reasoning to Proposition 1 where essentially the same condition arises from the extreme candidates' calculation of whether to play truthfully and gamble against an opposing extremist or to play $\{0\}$ to secure a smaller payoff in all states of the world.

In the second case of the Proposition, the new term guarantees that the extremist candidates who are favored by the centrists do not want to deviate to truthfully committing to a platform that reveals their type. Essentially, the term comes from them weighing the probability with which the opposing candidate is from the other extreme against the probability with which the opposing candidate is a centrist. When the opponent is an extremist from the other extreme, choosing not to commit wins only with probability $\frac{1}{2}$, while when the opponent is a centrist, all extremists will vote for the non-committing candidate and she will win with a probability close to 1. On the other hand, if she deviates to a platform that reveals her type and her opponent happens to be a non-committing extremist, the deviation will win with a probability that is close to 1, since both the centrists and the extremists who are of her own type will vote for her. However, if the opponent is a centrist, the committing extremist's winning probability is close to 0 as both, the opposing extreme and the centrists, will vote against her.

The following lemma shows that the favored candidate's consideration is more binding than the voter's consideration whenever the population is more polarized in the sense that $p+q \geq \frac{2}{3}$.

Lemma 2.
$$\frac{p^2+q^2}{(p+q)^2} \geq 1 - \frac{pq}{(1-p-q)(p+q)},$$
if and only if $p+q \geq \frac{2}{3}$.

More voters in the extremes makes it more likely that the favored extremist is facing another extremist from the other side and choosing the platform $\{-1,1\}$ becomes less appealing. In this sense, when the median voter has a strict preference ranking over the two extremes, the increased competition due to polarization can even discipline the extremists to commit to clearer platforms.

Whenever voters expect the ambiguous equilibrium to be played, centrist candidates have little incentive to run, since they can win only if the other candidate happens to be a centrist. Hence, in contrast to the truthful equilibrium, here the centrists are discouraged by running costs that are high enough. However, the ambiguous equilibrium depends much less on off-equilibrium beliefs, since the only off-equilibrium platform where players do not commit to singletons is the fully ambiguous $\{-1,0,1\}$.

Notice that the equilibrium where the extremists do not commit has the fundamental feature that after the election, on average a proportion of q of their voters are going to be highly disappointed. This matches the stylized fact mentioned in the introduction of how ambiguous parties and candidates have a tendency to lose a lion's share of their voters and rarely get re-elected. As long as the extremist's type is persistent enough this would also be true in a dynamic version of the model where the previous period's winning candidate can run again against either a random or endogenously chosen opponent. If an extremist gets elected with an ambiguous platform, her type is revealed by her actions during the first term. Consequently, even a centrist can beat her in the next election cycle. Hence, centrists have a large incentive to challenge an incumbent extremist. Similarly, in light of Proposition 4, knowing that the opponent is a centrist can only make non-committing platforms more appealing. We formalize this in the following direct corollary:

Corollary 1. *If one of the candidates is known to be a centrist and* $u_0 \leq \frac{p^2+q^2}{(p+q)^2}$*, then there exists* $\hat{N} \in \mathbb{N}$ *such that for all* $N > \hat{N}$ *the equilibrium from Proposition 4 always exists. The existence does not depend on whether the centrists have a strict preference over the extremes.*

Knowing that the opponent is an established centrist makes it easy for the extremists to not commit, because now they do not have to worry about losing to another extremist from the other end of the political spectrum. This has also been perhaps the more realistic scenario in many of the recent elections (for example, the US presidential elections in 2016, the German federal elections in 2017 and the Italian general election in 2018) where untested anti-establishment candidates or parties ran against an incumbent whose future preferences and policy choices can be guessed by looking at their past choices. I show in Appendix Appendix B that this basic logic is fully robust to adding an arbitrary number of states to the model as long as there are 3 available policies and voter types.

The only remaining equilibrium of the game has all candidates committing to $\{-1, 0, 1\}$. Deviations to non-singleton platforms can be discouraged with extreme off-equilibrium beliefs. Furthermore, the centrist is always going to be the one with the highest incentive to reveal her type by committing to her preferred policy. Notice that when a rightist voter v is faced with the choice between the deviant 0 from candidate $-i$ and the equilibrium path platform

$\{-1,0,1\}$ from candidate i, she will weakly prefer $\{-1,0,1\}$, if an only if

$$\begin{aligned} & u_0 \leq \mathbb{P}(\tau_i = 1 \mid \tau_v = 1) + \mathbb{P}(\tau_i = 0 \mid \tau_v = 1)u_0 \\ \Leftrightarrow \quad & u_0 \leq \frac{\mathbb{P}(\tau_i = 1 \mid \tau_v = 1)}{1 - \mathbb{P}(\tau_i = 0 \mid \tau_v = 1)} = \frac{p^2 + q^2}{(p+q)^2}. \end{aligned} \tag{1}$$

Same holds symmetrically for leftist voters and hence when the inequality holds the deviating candidate will lose with certainty. Consequently, the fully ambiguous equilibrium exists if and only if the extremes against the middle equilibrium also exists. However, in the next section I will show that a reasonable equilibrium refinement can be used to select only the ends against the middle equilibrium.

4. Equilibrium Selection

The fully revealing equilibria above require relatively asymmetric off-equilibrium beliefs to discourage extremists from deviating to $\{-1,1\}$. If one side of the political spectrum is thought to be much more likely to embrace such ambiguous platforms, such beliefs could potentially be reasonable. However, such beliefs should be part of the model rather than an equilibrium selection criterion. If there is no reason to expect such asymmetry, then these two types of equilibria start to look unlikely.

Furthermore, the following forward induction argument supports the equilibrium where $\{-1,1\}$ gets played on the equilibrium path: Suppose, voters expect the truthful equilibrium but instead observe a candidate playing $\{-1,1\}$. This can benefit the candidate only if she is extremist and she believes that the non-commitment equilibrium is being played. In that equilibrium both extremist types are ex-ante equally likely to choose the non-commitment strategy. Hence, within the model it is natural to think of voters revising their expectation about the equilibrium being played by the deviating player to the non-commitment equilibrium and attach beliefs observed in that equilibrium after $\{-1,1\}$ to the deviating player. These beliefs will result in extremists voting for the deviating candidate whenever she is running against a centrist hence generating an incentive for such candidates to deviate.

This forward induction idea is built into the perfect sequential equilibrium of Grossman and Perry (1986). It turns out that the equilibrium where extremists choose $\{-1, 1\}$ is the only perfect sequential equilibrium when centrists have a strict preference over the extremes and there is any positive cost on ambiguous platforms.

Proposition 5. *Suppose that there are favored extremists and a cost of ambiguous platforms is $c > 0$. Then the equilibrium where extremists commit to $\{-1, 1\}$, whenever it exists, is the unique pure perfect sequential equilibrium.*

The proof is a straightforward verification that when out-of-equilibrium beliefs are restricted as above, extremists will want to deviate to $\{-1, 1\}$ whenever it is not part of the equilibrium path. The proposition holds also when there are no favored extremists but centrist voters are always required to randomize equally when faced with two different extremists.

Refinements like the Intuitive Criterion of Cho and Kreps (1987) do not deliver the result, since they do not put restrictions on the off-equilibrium distribution of types for whom the deviation was not equilibrium dominated. Hence, if players expect the fully revealing equilibrium to be played but one of the politicians deviates to $\{-1, 1\}$, the Intuitive Criterion does not rule out off equilibrium beliefs that put all of the mass on type 1. However, it turns out that when either the truthful or fully non-revealing equilibrium is expected, the only *credible* updating rule in the sense of Grossman and Perry (1986) requires type 1 voter to put exactly the weight $\frac{p^2+q^2}{(p+q)^2}$ on the deviation coming from type 1 candidate. These beliefs, in turn, will lead both extreme types of candidates to deviate to $\{-1, 1\}$.

The situation is much simpler when an unknown candidate is facing an incumbent centrist. In that case both types of extremists know that by playing truthfully, they will automatically lose the election. Consequently, deviating from the expected truth to the platform $\{-1, 1\}$ either yields the same payoff as playing the truthful platform or wins the election depending on the voters' equilibrium beliefs. Furthermore, as shown above, there exists voter beliefs that are consistent with the deviating candidate winning the election. Consequently, in any equilibrium, an extremist playing $\{-1, 1\}$ does always at least as well as playing truthfully and there exists an equilibrium where she strictly benefits from this strategy. Hence, playing $\{-1, 1\}$ equilibrium dominates truthful strategies for extremist candidates when the opponent is known to be a centrist.

5. Comparative Statics and Some Remarks

In this section I will first discuss some intuitive comparative statics that can be easily derived from the model. I will also make some remarks on the effect of polling on the equilibrium.

5.1. Comparative Statics

In the public discourse the popularity of anti-establishment platforms has been often linked to the level of marginalization of the extremes and the polarization of the society. As I argued before, u_0 is a natural measure of marginalization of the extremes, as it measures how appealing the extremes find the median voter's preferred policy. Thus, in the light of the previous section, the less the centrists can offer to the extremes, the more likely are platforms without commitment.

To study how polarization affects the likelihood of non-committing platforms, it is instructive to re-parametrize the model slightly. A natural measure of polarization is the fraction of voters in the two extremes: $\pi := p + q$. This measure captures how small the center is compared to the extremes. I also define the relative share of the extreme voters whose type mismatches the state as $r = \frac{q}{p}$. This parameter captures the covariance between the preferences of an extremist and the state of the world. More specifically, it is straightforward to verify that for a citizen with type τ,

$$cov(\tau, s \mid \tau \in \{-1, 1\}) = \frac{(1-r)}{(1+r)},$$

which is decreasing in r. Hence, the larger is r the less information an extreme citizen's own type conveys about the state of the world.

With this re-parametrization the upper bound for u_0 under which the equilibrium with no commitment exists becomes:

$$\frac{p^2+q^2}{(p+q)^2} = \frac{1+r^2}{(1+r)^2}.$$

The first thing to notice is that any changes in p and q that do not change r but affect the level of polarization, π, have no effect on the minimum level of marginalization that supports the ambiguous equilibrium. Hence, when there are no favored extremists or when $\pi \leq \frac{2}{3}$, polarization matters in the model for the viability of non-committing platforms only up to the point that there must be at least half of the voters in the two extremes. Intuitively, this is very natural, since the viability of the strategy depends on whether each type of an extremist is willing to vote for non-committing extremist when the alternative is voting for a centrist. For this decision the only important dimension is the relative likelihood of electing an extremist whose type matches the voter's own type versus getting the wrong type in the office. This quantity conditions on there being an extremist candidate and hence does not depend on the likelihood of an extremist candidate running which is captured by π.

Now, $\frac{1+r^2}{(1+r)^2}$ is decreasing in r and approaches $\frac{1}{2}$ when $r \to 1$. Hence, when both extremes are almost equally likely in each state of the world, the voter's type is a worse signal about the type of the extremist and hence the voter needs to be more marginalized to be willing to take the risk and vote for the non-committing extremist. In this sense, large economic or social fluctuations that generate large, and strongly correlated variation in voter preferences require less marginalized extremes for them to be willing to support ambiguous platforms.

On the other hand, if there is a favored extremist and if $p+q \geq \frac{2}{3}$, then the relevant upper bound for u_0 can be written as

$$1 - \frac{pq}{(1-p-q)(p+q)} = 1 - \frac{r\pi}{(1-\pi)(1+r)^2}.$$

In Proposition 4 this quantity was pinned down by the trade-off that the favored extremist candidate faces when choosing between an ambiguous platform or deviating to revealing her type. The ambiguous platform wins against a centrist but loses half the time against another extremist while revealing her type wins against a non-committing extremist but loses to a centrist. Hence, it is natural that this quantity is decreasing in π; if there are fewer centrists and more extremists revealing one's type becomes relatively more attractive.

It is also decreasing in r. This is also highly intuitive. Revealing her type is attractive to the favored extremist only if, conditional on her information, there is still a substantial risk of facing a non-committing extremist from the opposing end of the political spectrum. If p is large compared to q, r is small and it is very likely that, if her opponent is an extremist, they both will be of the same type. Just as in the previous case, a low p-to-q ratio implies that the extremist's own type is a strong signal about the non-committing, opposing extremist's type and hence reduces the expected relative loss in case she loses to another non-committing extremist.

How do these comparative statics relate to recent patterns in polarization? For example, surveys from the US show that the voters who identify as democrats or republicans have drifted apart in their views and fewer voters indentify consistently with policies from the middle of the political spectrum (see, for example, Pew Research Center, 2014). Now, one direct way of interpreting this phenomenon through the lens of my model is to say that $p + q$ has increased. This means that fewer people identify with the political center while more people have preferences that used to be fringe or extreme in the past. The comparative statics above show that this will lead to a potential increase in the popularity of ambiguous platforms. However, if polarization is associated with a strong pre-defined partisan identification of voters, luring voters across party lines becomes more difficult and ambiguous platforms become less appealing. Nonetheless, ambiguity can even then help candidates to unify voters within their own party on issues that might otherwise be divisive. Similarly, many of the recent populist movements and candidates started as new entities which were able to define their anti-median platform without a clear ex-ante association to given policies in all policy dimensions. They were also successful in unifying voters from both ends of the traditional policy spectrum.

An alternative interpretation of the recent shift in voter preferences could be that not only the distribution but also the preferences have changed. As long as this change makes the centrists platform less appealing for the voters in the extremes, the discussion above shows that the ambiguous equilibirum is easier to sustain. For example, suppose that both extreme utilities for an extreme voter become multiplied by some $x > 1$ pulling the extremes further away from one another holding the utility from the median policy, u_0, fixed.[15] The extreme voter's new von Neumann-Morgernstern preferences, $(-x, u_0, x)$, are then the same as these preferences multiplied by the positive constant $1/x$,

[15] Given that VNM utilities are unique only up to increasing affine transformations, multiplying all terms, including u_0, by x would keep the preferences unchanged.

yielding $(-1, \frac{u_0}{x}, 1)$. Since only relative rankings matter, pulling the extremes apart is hence equivalent with making the payoff from the median policy worse.[16]

5.2. The Effects of Polling and Behavioral Biases on the Equilibrium

Since the mechanism I am suggesting depends crucially on the imperfect information that the voters and candidates have about each others' preferences, accurate polling can make ambiguous platforms less likely. However, precision of polling predictions has arguably become worse lately due to, among other things, widespread use of cell phones (Skibba, 2016a) and larger numbers of undecided voters and decreasing turnouts (Skibba, 2016b). Furthermore, if polls ask only about which candidate a person is going to vote for, even an accurate polling result will not unravel the uncertainty about the distribution of policy preferences in the population when there is a non-committing candidate. In my model this would correspond to accurately polling the total number of extremists $p + q$, which is already common knowledge. This is especially true, when ambiguity is issue-specific and candidates are clear on other dimensions of their platform.

Polls will also not affect the result, if voters disregard the information that is embedded in other voters' polled preferences. This is a common assumption made in the literature on how the beliefs in the population can diverge in the long run even when everyone observes the same data. For example, Piketty (1995) and Benabou and Tirole (2006) both assume that generations do not

[16] Only if the shift in preferences implies that the payoff from the opposing extreme policy becomes relatively less appealing compared to the improvement in the voter's own ideal policy, then ambiguous platforms become less appealing. But this is indistinguishable in the model from the centrist platform's appeal becoming higher. To see this, suppose the utilities for an extreme voter from the different policies become $(-1 - x, u_0, 1)$ where $x > 0$. Consider multiplying each of these utilities by the (positive) constant $\frac{2}{2+x}$ and then adding the constant $\frac{x}{2+x}$. The resulting utilities are now $(-1, \frac{2u_0+x}{2+x}, 1)$. Since u_0 is between zero and one, the worst outcome becoming worse is hence equivalent with the centrist outcome becoming better. However, we do not seem to observe a growing support for centrist policies from the extremes making this interpretation somewhat less plausible.

learn other individuals' signals from the vote counts of the election. People may also have strategic reasons to misrepresent their preferences in polls. See, for example, Tracey and Stocken (2008).

Notice also that polls undermine the strategy of an ambiguous extremist in favor of a clear centrist by revealing information correlated with the candidate's type. Hence, voters who ex-ante prefer the equilibrium where ambiguous extremists win against centrists have little benefit from revealing their private information. Similarly, centrists are highly incentivized to run polls. This asymmetry in benefits from polls can cast a long shadow on their independence and unbiasedness.

It is somewhat unrealistic to assume that all voters have symmetrically uncertain preferences. In fact, there is evidence that a large fraction of the American voter population has moved further into the tails of the policy spectrum (Pew Research Center, 2014). The consistency of this shift over policy issues suggests that these voters are less satisfied with compromise policies that are somewhere in the middle. Interestingly, the equilibrium suggested by the model remains intact if one assumes that the population has highly entrenched extremes with known preferences and $u_0 < \frac{1}{2}$, and a small mass of uncertain, more centrist non-median voters with $u_0 > \frac{1}{2}$ and preferences that are ex-ante correlated with the preferences of at least one of the candidates. For example, one can assume that due to economic or political shocks all centrists have a small probability of becoming moderate leftists or rightists and at least one of the candidates is running as a disenchanted centrist. In this version of the model, the entrenched extremists will always prefer the 50/50-gamble offered by a non-committing extremist over the committing centrist. Thus, the moderate, uncertain leftists and rightists become the decisive voters and the model plays out just as above. The results above offers a clear connection between the growing number of undecided voters in the polls to the recent popularity of anti-establishment platforms.

Last, a number of documented biases in how people process available information can affect the prevalence of my suggested channel. For example, confirmation bias can lead to individuals interpreting the candidates' ambiguous promises more in favor of their existing beliefs and push them into believing that the candidate must be on "their side". Motivated cognition and projection effects (see, for example, Benabou and Tirole, 2016; Jensen, 2009) have comparable effects. Similarly, homophily in the choice of peers as well as individuals choosing media outlets that support their existing views can create echo chambers that further bias, and polarize voter's beliefs and Bayes updating based on these strong beliefs can make it deceptively obvious that the non-committing promises must come from someone with similar beliefs.[17] On the other hand, correlation neglect (as in Levy and Razin, 2015) and failure to form consistent higher order beliefs can protect voters from ambiguous candidates. Tolvanen et al. (2022) takes a stylized version of my model to the lab and shows that most voters in the experiment behave according to the model and the likelihood of electing an ambiguous candidate is positively influenced by both the strategic and statistical sophistication of the subject pool.

6. The Effect of Non-Commitment on Voter Welfare

Suppose it would be possible to forbid or increase the cost of ambiguous or self-contradicting campaign promises. In this section I study who would benefit from a move from the non-commitment equilibrium to the commitment equilibrium. In an ex-post sense, it is trivial that all but the most numerous extremists would have weakly preferred the equilibrium with commitment to

[17] For example, there is already clear evidence that Trump and Clinton voters followed very different news outlets during the 2016 election (Pew Research Center, 2017).

the equilibrium with no commitment. If both candidates were extremists, then the losing extremists and centrists get 0 in both equilibria. However, if one was a centrist, the commitment equilibrium guarantees the losing extremists u_0 instead of 0 that they get under non-commitment. Similarly, the centrists get 1 instead of 0. Notice also that when there is an ambiguous extremist candidate, all extremists vote for her if she is running against a committing centrist. Thus, the less numerous extremists will always exhibit regret. This feature of the model can partially explain why anti-establishment candidates have often lost a sizable fraction of their support after being elected.

A more nuanced question is whether a given type of a voter would prefer the equilibrium with commitment to the non-commitment equilibrium *before* platforms are announced. Notice that even though candidates' preferences are perfectly aligned with the voters of their own type, there is a sizable information asymmetry between voters and the candidates. A given type of a candidate knows her own type and hence has to form beliefs only about the other candidate while a given type of a voter needs to form beliefs about both candidates. Therefore, the question of voter welfare is different from whether a given type of a candidate would like to deviate to committing when they are supposed to play the equilibrium without commitment.

The following proposition shows that there is an open set of parameters where the non-commitment equilibrium exists and most voters generally dislike it.

Proposition 6. 1. *Suppose that*

$$u_0 > \frac{p^2+q^2}{(p+q)^2} - \frac{pq(p-q)}{2(p+q)^2(1-p-q)}$$

Assume further that the centrists are indifferent between the extremes and randomize equally between the two candidates whenever faced with the choice between -1 and 1. Then there exists $\hat{N} \in \mathbb{N}$ such that if $N > \hat{N}$, then all voters prefer the commitment equilibrium over the non-commitment equilibrium

2. *Suppose that*

$$u_0 > \frac{p^2+q^2}{(p+q)^2} - \frac{pq}{2(p+q)(1-p-q)}$$

and that the centrist voters favor one of the extremists over the other. Then there exists $\hat{N} \in \mathbb{N}$ such that if $N \geq \hat{N}$, then both the favored voters and the centrists prefer the commitment equilibrium to the non-commitment equilibrium.
The unfavored voters always prefer the non-commitment equilibrium.

The first parts of Propositions 4 and 6 together imply that when the centrists are indifferent between the two ends, there is a non-empty, open set of possible values of u_0, namely the interval

$$\left(\frac{p^2+q^2}{(p+q)^2} - \frac{pq(p-q)}{2(p+q)^2(1-p-q)}, \frac{p^2+q^2}{(p+q)^2} \right),$$

such that the non-commitment equilibrium exists and *all* of the voters would prefer banning non-commitment platforms. For lower values of u_0, the extremist voters are so marginalized in terms of the utility they get from the median policy that they prefer the uncertain, non-commitment equilibrium. Similarly, even if there is a group of extremists favored by the centrists, there still generally exists a range of values of u_0 such that the non-commitment equilibrium exists and the favored extremists and the centrists both prefer the commitment equilibrium to the non-commitment one. Notice also that in this case, the combination of favored extremists and centrists still comprises a strict majority of the population who prefer banning non-committing platforms in both states of the world. The general distaste for non-committing platforms helps to explain why ambiguous platforms are generally lambasted by most or all sides of the political spectrum while they can still garner large vote shares in actual elections.

It is also noteworthy that

$$\frac{pq(p-q)}{2(p+q)^2(1-p-q)} \leq \frac{pq}{2(p+q)(1-p-q)}.$$

Hence, when there are favored extremists, the range of parameters for which non-commitment hurts the majority of voters is larger than when centrists are indifferent.

Interestingly, when one of the candidates is known to be a centrist, all of the extremists ex-ante prefer the non-commitment equilibrium to banning non-committing platforms. To see this, remember that when there is a centrist incumbent, non-ambiguous extremist platform will almost always lose the election. However, when the non-commitment equilibrium exists, extremists prefer the uncertain platform to the centrist platform, and the non-committing platform will almost always win the election. Hence, the extremists must also prefer the non-commitment equilibrium to the equilibrium where the centrist platform always gets elected. Consequently, when there is a known centrist incumbent, only the centrists should be complaining about self-contradicting or ambiguous platforms.

7. Extensions

This section discusses three extensions of my model. First, I discuss a simple extension that shows that ambiguous platforms can unite two within-party extremes in a primary and go on to win a general election in two-party elections. Second, I show that ambiguity can be a winning strategy even when the target groups are highly asymmetric in size. This extension is important for the case of dog whistle politics where politician's coded messages usually target a small minority. Last, and perhaps most importanly, I show that ambiguity can arise in equilibrium even in a Condorcet model where all voters have *identical* preferences but where they differ only in terms of their signals. Hence, ambiguity can be a winning strategy even in courtrooms and corporate boards where voters' preferences are highly aligned.

In general, the mechanism presented here is does not depend much on the model specifics, as long as there is correlation between the voters' and candidates' preferences. For example, the existence of ambiguous equilibria is robust to adding states or citizen-candidate types. Most of these extensions generate higher multiplicity of equilibria. However, a subset of these equilibria generally have multiple candidate types attracting conflicting voter types by committing to non-singleton platforms by leaning on the correlation between the candidate and citizen types. Similarly, allowing for fully office motivated candidates will still result in equilibria with ambiguous extreme platforms.[18]

7.1. Elections with Primaries

It is straightforward to see that ambiguity can arise also in elections with primaries where it is more likely to unite extremes of a party rather than voters who vote for radically conflicting parties. Consider, for example, an electorate with two parties L and R that each consists of three preference-types of voters, $\{L_{-1}, L_0, M\}$ and $\{M, R_0, R_1\}$, respectively. Suppose these types have single-peaked preferences that have ideal points that are ordered accordingly from L_{-1} on the left to the R_1 on the right and assume for simplicity that these are symmetrically located, so that, for example, type R_0 gets the same utility from policy M and policy R_1. Suppose further that the state of the world has two realizations -1 and 1, and this state is correlated with the distribution of voters and candidates within each party so that in state -1 there are more voters and candidates of type L_{-1} and fewer of type M in party L than in state 1, and similarly that in state -1 there are more voters and candidates of type M and fewer of type R_1 in party R than in state 1. Assume that the median type in party R (L) is R_0 (L_0, respectively). In short, assume that the environment

[18] If the office motivation is high enough, even the centrist candidates may choose to run on an ambiguous anti-median platform. This will dilute the correlation between the voters and ambiguous candidates. If the correlation is too small for extremist voters when all centrist run on ambiguous platforms, the equilibrium will feature mixing from both centrist candidates and extremist voters after histories where one candidate was a centrist and the other was an ambiguous extremist.

on the party-level matches the model described in Section 3. Notice that this implies that the preference type M voters from the two parties hold opposite beliefs about the state. Finally, assume that voters of preference-type M are the de facto median in the combined population and all voters vote *non-strategically* choosing always their preferred option in each election.[19]

Each party first organizes a primary between two randomly chosen party candidates, and only members of the party are allowed to vote in a party's primary. The candidate chosen from that primary then runs with the same platform against the winner of the opposing primary in a general election. Since the model at the primary level is exactly like the model in Section 3, as long as candidates' and voters' preferences are sufficiently correlated, there exists an equilibrium where the L primary is won with the ambiguous platform $\{L_{-1}, M\}$. Just as in Section 3, both the national centrists M and the extremists L_{-1} in the party are going to vote for the ambiguous candidate over the party median L_0.

Now, consider the general election where this ambiguous candidate is running against the clear median candidate R_0 from the opposing party. All of the voters from party L are going to prefer $\{L_{-1}, M\}$ to the candidate promising R_0. If the preferences of type M in party L are symmetric or tilted towards the left end of the spectrum, the fact they preferred $\{L_{-1}, M\}$ to L_0 is going to imply that they also prefer $\{L_{-1}, M\}$ to R_0. Hence, with two states, the ambiguous left-wing candidate is going to win against the clear right-wing median if party L holds a majority.

The result becomes even stronger if there are three states, one of them generating more type M citizens than the two other extreme states. In this

[19] Non-strategic or expressive voting is a critical assumption for this extension, as discussed below. However, the assumption that type M voters in both parties have identical preferences is not critical and is made only for simplicity of exposition.

version of the model very similar arguments to above can be used to show that type M citizens from both parties may prefer the $\{L_{-1}, M\}$ to R_0, since the type M voters in both parties believe the ambiguous candidate to be more likely like them than a type L_{-1} candidate (given that their type is a strong enough a signal about the state being M).

If all voters are strategic and able to use backward induction on the whole voting game and candidates/parties expect them to do so, ambiguous platforms lose their appeal. If, for example, the right-wing primary includes a candidate running on platform M and the left primary a candidate running on the ambiguous platform $\{L_{-1}, M\}$, then strategic right-wing voters will foresee that other platforms they might choose can lose to $\{L_{-1}, M\}$ while M will guarantee a victory in the final election. Hence, strategic and non-myopic right-wing voters will vote for a candidate running with M (if able) over any competing platform (except potentially $\{R_{-1}, M\}$ if right-wingers are a majority, but then the same argument applies to left-wing voters). Hence, with primaries the appeal of ambiguous platforms depends strongly on whether a large fraction of voters vote expressively or whether voters base their choices on who they believe to win each primary and which of those candidates is going to win the general election. Furthermore, parties need to choose their candidates for the primaries believing that voters will behave strategically (as M is an extreme type in each party).

In practice, my mechanism does not require that, for example, Trump's ambiguity appealed to both, the most liberal and conservative citizens in the US population. The extension above suggests that his ambiguity on, for example, economic policy appealed to the voters within the Republican party who preferred either higher or lower taxes than the party median. Furthermore, this ambiguity can be appealing to the more conservative end of the Democratic voters.

7.2. Asymmetric Voter Distributions and Dog Whistles

The main purpose of this section is to highlight that the key for successful ambiguous messaging is the uncertainty about the politician's type and to a lesser extent about the preferences in the population. Importantly, one of the sub-populations that is being targeted by the ambiguous messaging can be tiny compared to the other. This tends to be true especially in what is generally understood as dog whistle politics, where a politician's message is understood very differently by a small interest group and the general majority of her base.

To my knowledge, this is the first formal model that can be used to study the viability these dog whistles. In typical examples, a politician sends messages that are understood completely differently by the large majority and a small target minority. This is often achieved by using coded language that has an alternative meaning to the targeted minority. For example, Albertson (2015) discusses a number of examples of dog-whistling from recent elections where the coded messages have used in racial, religious and anti-abortion messaging signaling. For example, according to her, Ronald Reagan, Bill Clinton and George W. Bush, all used veiled biblical references in their speeches that were clear to devout Christian voters but went unnoticed by the rest of the electorate. In my model, this would correspond to messages that credibly increase how correlated a religious voter thinks her preferences are with the candidate's preference while having no effect on how this correlation is perceived by the rest of the population. In the context of my model, dog whistles can be seen as attempts to asymmetrically manipulate the correlation that voters perceive between them and the candidate. The ability to send a coded message signals similarity to the target group who are able to decipher the message. The voters who do not notice the code will take the message at its face value and not alter their perception of the candidate. A dog whistle would hence increase the candidate's popularity within the receivers of the coded message while not adversely affecting the perceptions of other voters.

To formalize this idea, consider a slight generalization of the model where the voters' now have an asymmetric type distribution where

$$\mathbb{P}(\tau_v = 1 \mid s = 1) := \bar{r} > \underline{r} =: \mathbb{P}(\tau_v = 1 \mid s = -1)$$

and

$$\mathbb{P}(\tau_v = -1 \mid s = -1) := \bar{l} > \underline{l} =: \mathbb{P}(\tau_v = -1 \mid s = 1).$$

Assume also that $\min\{\bar{r} + \underline{l}, \bar{l} + \underline{r}\} > \frac{1}{2}$ to guarantee that the extremes still have a majority. Continue assuming that for the candidate

$$\begin{aligned} & \mathbb{P}(\tau_i = 1 \mid s = 1) = \mathbb{P}(\tau_i = -1 \mid s = -1) = p \\ > \quad & q = \mathbb{P}(\tau_i = 1 \mid s = -1) = \mathbb{P}(\tau_i = -1 \mid s = 1). \end{aligned}$$

Then an easy extension of the arguments presented in the proof of Proposition 4 yield that when an extremist is running against a known centrist, the extremist will win with the platform $\{-1, 1\}$ as long as

$$u_0 < \min\left\{\frac{p\bar{r} + q\underline{r}}{(p+q)(\bar{r} + \underline{r})}, \frac{p\bar{l} + q\underline{l}}{(p+q)(\bar{l} + \underline{l})}\right\}.$$

Now,

$$\frac{p\bar{r} + q\underline{r}}{(p+q)(\bar{r} + \underline{r})} > \frac{p\bar{l} + q\underline{l}}{(p+q)(\bar{l} + \underline{l})} \Leftrightarrow \frac{\bar{r}}{\underline{r}} > \frac{\bar{l}}{\underline{l}}.$$

Intuitively, the critical voter type is the one whose type is the least informative about the state and hence about the candidate's type. The model implies that the coded messages of dog whistling can be valuable only if they are able to affect the voters who initially perceive themselves to be less correlated with the candidate, while not changing the perceived correlation for the rest of the voters too dramatically. Notice further that if $\underline{l} \to 0$, then

$$\frac{p\bar{l} + q\underline{l}}{(p+q)(\bar{l} + \underline{l})} \to \frac{p}{p+q} = \mathbb{P}(\tau_i = -1 \mid \tau_i \in \{-1, 1\}, s = -1),$$

which is independent of $\bar{l}$. In other words, if the left interest group is very small in the right state, the size of the group in the left state or the average size of the group, in general, matters next to nothing. What is critical for the small interest group is the perceived likelihood of the *ambiguous* candidate matching the state.

7.3. Condorcet Voting with Identical Voters with Heterogeneous Beliefs

I will next show how the same mechanism can generate ambiguous equilibrium platforms even in a Condorcet voting model where all voters have ex-ante identical preferences but different policy relevant information. The results below show that ambiguous platforms can be played even in situations like share holder meetings of corporations or panels of experts where, conditional on the state, players' policy preferences are aligned.

I show that the availability of ambiguous platforms can completely eliminate beneficial information aggregation despite everybody agreeing on the optimal policy in each state and even when there is no extra benefit from holding an office. Furthermore, in the ambiguous equilibrium, candidates with non-centrist signals will choose the centrist platform with a positive probability further narrowing the scope for socially optimal decision making.

The formal model is almost as before. The key difference is that now everybody has the same preferences. Specifically, assume that there are three equally probable states $s \in \{-1, 0, 1\}$ and that everyone's payoffs are given by

$$u(s, a) = \begin{cases} 1 & \text{if} \quad s = a \\ 0 & \text{if} \quad s \neq a \neq 0 \\ u_0 & \text{if} \quad s \neq a = 0 \end{cases}$$

where $u_0 > 0$ is the extra benefit from choosing the "safer" centrist option. Suppose then that each voter and candidate gets a signal $\theta \in \{-1, 0, 1\}$ and that these signals are independent conditional on the state. To ease the exposition, assume that these signals are fully symmetric so that $\mathbb{P}(\theta = i \mid s = i) = p \in (\frac{1}{3}, \frac{1}{2})$ for all $i \in \{-1, 0, 1\}$ and $\mathbb{P}(\theta = j \mid s = i) = \frac{1-p}{2} =: q$ for all $j \neq i$.[20] The upper limit on p guarantees that the probability of a single group forming a majority vanishes as $N \to \infty$.

Otherwise the game proceeds just like in the model with private preferences. However, for simplicity of exposition I will assume that each politician has to choose their platform from the set $\mathcal{A} = \{\{-1\}, \{0\}, \{1\}, \{-1, 1\}\}$. Similar results hold even with the larger platform space. I will also assume that voters vote non-strategically.[21]

The following Proposition shows that this version of the game also has a truthful equilibrium and in equilibrium centrists win whenever they run.

Proposition 7. *Suppose $u_0 \leq \frac{p^2+2q(p-q)}{(p+q)^2}$. Then there exists $\hat{N} \in \mathbb{N}$ such that for all $N > \hat{N}$, each candidate with signal θ committing to platform $\{\theta\}$ is part of an equilibrium path. Whenever there is a candidate running with $\{0\}$, policy 0 will be implemented on the equilibrium path with probability converging to 1 as $N \to \infty$.*

Interestingly, in this model the existence of the "centrist" option that has a somewhat higher utility when mismatching the state strongly hinders information aggregation. Whenever one of the candidates gets the centrist signal, she

[20] It is easy to verify that similar results hold when the symmetry is relaxed or if signals come from a continuous distribution satisfying the monotone likelihood ratio property.

[21] In this version of the model this assumption has content. Strategic voting makes winning with $\{-1, 1\}$ less likely against candidates running with $\{0\}$. Conjecture that all voters who get an extreme signal vote for $\{-1, 1\}$ and the rest vote for $\{0\}$. Then, conditional on being pivotal, a voter with signal 1 is going to be almost certain that the state is 0, since in large populations having exactly half of the others obtaining signal 0 is much more likely when the state is 0 than otherwise. Hence, due to reasoning similar to the one in Feddersen and Pesendorfer (1998) strategic voters must mix. In contrast to Feddersen and Pesendorfer (1998), the results below show that allowing for ambiguous platforms can destroy information aggregation even when voters vote non-strategically.

will win independent of the state. However, whenever there are two extremists from the opposing ends, if one matches the state, she will win with near certainty in large populations.

It turns out that the ambiguous equilibrium is even worse for information aggregation.

Proposition 8. 1. *There does not exist a symmetric pure strategy equilibrium where $\{-1, 1\}$ gets played by both types -1 and 1.*

2. *Suppose $u_0 < \frac{p-q}{p+q}$. Then there exists $\hat{N} \in \mathbb{N}$ such that for all $N > \hat{N}$, candidate 1 committing to $\{-1, 1\}$ whenever $\theta_{c_1} \in \{-1, 1\}$ and otherwise committing to $\{0\}$ and candidate 2 always committing to $\{0\}$ is part of an equilibrium path. Whenever candidate 1 runs with $\{-1, 1\}$, she will win with probability converging to 1 as $N \to \infty$.*
3. *Suppose $u_0 < \frac{p-q}{p+q}$. Then there exists $\hat{N} \in \mathbb{N}$ such that for all $N > \hat{N}$ there exists a symmetric mixed strategy equilibrium where each candidate c commits to $\{-1, 1\}$ with probability $x_N \in (0, 1)$ whenever $\theta_c \in \{-1, 1\}$ and otherwise commits to $\{0\}$. The mixing probability*

$$x_N \underset{N \to \infty}{\longrightarrow} \frac{p - q - (p + q)u_0}{(p + q)(p - (p + q)u_0) - 2q^2}.$$

Whenever a candidate runs with $\{-1, 1\}$ she will win with probability converging to 1 as $N \to \infty$.

The intuition behind the first part of the proposition can be explained as follows: If both types -1 and 1 of the opposing candidate play $\{-1, 1\}$, then playing 0 means essentially "delegating" the decision to the other candidate. If the opponent's type is -1 or 1, she will win the election and when her type is 0, policy 0 will win as both committed to that platform. The opponent is correct with probability p. In addition to that, whenever she is incorrect with platform $\{0\}$, everybody gets u_0. Hence, the expected utility for type 1 from playing $\{0\}$ is strictly above p (as a mixture of u_0 and p). On the other hand, playing $\{-1, 1\}$ means that either she or her opponent wins with platform $\{-1, 1\}$. In both of these cases the winner's signal is correct with probability p and hence the expected utility from the ambiguous platform is only p. Consequently, both players' extreme types cannot always be ambiguous.[22]

There are two ways to get around this "insurance" motive: If one of the candidates always plays $\{0\}$, the decision gets fully delegated to the other candidate. The candidate who is responsive to her signal now wants to win whenever she has an extreme signal, since conditional on this signal, the non-responsive centrist candidate is very unlikely to choose the correct option. Similarly, if both candidates mix whenever they are extreme, that reduces the probability with which they choose the correct option if they win the election. This makes it less appealing for the other candidate to remain passive and choose $\{0\}$.

Notice that the information the *voters* have never gets aggregated in these two equilibria; either an extreme candidate wins and she will choose the correct option only if her type matches the state, or both candidates choose the centrist platform. Hence, at best, in the asymmetric equilibrium, the decision to implement the centrist platform is based on two centrist signals, and most of the time extreme policies are implemented because one of the candidates happened to get an extreme signal. In the mixing equilibrium, centrist policies are sometimes implemented even when both candidates got an extreme signal.

Full information aggregation in large elections can be (almost) restored by allowing the platforms $\{-1, 0\}, \{0, 1\}$ and asymmetric equilibria. Consider both candidates completely ignoring their signals and assume that candidate 1 always chooses $\{-1, 0\}$ and candidate 2 always chooses $\{0, 1\}$. Suppose that voters vote for the first candidate when their signal is -1, for the second candidate when their signal is 1, and otherwise randomize with equal probability. In that case, when the electorate is large, the winning candidate can deduce the signal

[22]Notice that this argument shows that the departure from the baseline model by including a third state where the centrists have the correct signal makes constructing an equilibrium with ambiguity substantially harder. Indeed, in a model where the state can be only either -1 or 1 and the centrists signal is uninformative about the state, the first part of the proposition holds only if u_0 (the insurance payoff) is higher than p.

distribution from the vote shares and implement the correct policy. Information aggregation requires candidates to ignore their own, mostly irrelevant signals and simply act as the statisticians putting together the information from the polls.

A straightforward special case of the model above is a principal-agent problem where both, the principal and agent, have correlated private signals about a mutually beneficial action. The principal observes first a commitment from the agent to an action or a set of actions and then decides whether to hire her or employ a "status-quo", centrist action. If the agent is allowed to make a commitment to an ambiguous "reform" (i.e. a set of non-status quo actions) rather than being specific about its exact type, my mechanism can generate equilibria where agents whose information favors a new policy to the status quo, commit to the ambiguous reform and get the job with a high probability. Hence, for example, a prospective CEO or a management consultant should often promise the owners to reform a struggling company while remaining ambiguous about the type of the reform.

8. Conclusions

This paper suggests a simple mechanism that connects increased volatility in voter preferences to the rise of ambiguous anti-establishment platforms. Importantly, the ambiguity tends to hurt the majority of voters both in an interim and ex-post sense even when candidates are ex-ante identical to voters. Campaign promises are supposed to impose checks and balances on politicians. However, the non-committing anti-establishment platforms allow the winning candidate to implement potentially highly unpopular policies and run the risk of a small extremist minority ruling over the majority. At worst my suggested mechanism poses a serious threat to the democratic process. Consequently, institutions, such as independent fact checkers and news media, that reduce uncertainty about candidates' preferences can have highly beneficial welfare implications. Conversely, when people's news consumption moves towards the echo chambers of social media, the ambiguous candidates will have an easier time targeting their contradicting promises to their intended audiences.

Interestingly, a simple dynamic version of the model can be also used to generate opposition cycles where extremists and centrists alternate between government and opposition. Consider a version of the model where the winner of an election continues to the next election as an incumbent. The incumbent has been forced to implement policies and hence reveal their type even if their platform was ambiguous. This means that an incumbent extremist's type is always known to the electorate and hence she will lose against a committing centrist. However, she will win against an ambiguous extremist, since the extremist voters who share her policy preferences will always vote for her. Thus, an incumbent extremist's rule can last only until they are opposed by a centrist challenger. On the other hand, the main proposition of the model shows that the centrist incumbent can lose to an ambiguous extremist challenger. Consequently, a cycle emerges where centrists and extremists alternate in power. Notice also that this generates the prediction that extremist challengers must always be new parties or new candidates to get the full benefit from the veil of uncertainty. However, the centrist challenger can even be the previous centrist incumbent and hence centrist parties and candidates can be more long-lived.

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Appendix A. Appendix: Proofs

Proof of Proposition 1. Suppose that every candidate is supposed to set truthful platforms. Assume also that a candidate who commits to a set-valued platform is expected to be type 1 with probability 1.[23] It is then weakly dominant for each type $\tau_v \neq 1$ to vote for candidate who commits to policy $a = \tau_v$ if such candidate exists. Furthermore, citizens of type $\tau_v = -1$ always prefer a candidate committing to 0 over all other platforms that are not equal to

[23] There are also beliefs compatible with the intuitive criterion where a candidate with committing to set A is believed to be the type $\tau_A = 1$ if $\{1\} \in A$ and $\tau_A = -1$ otherwise.

$\{-1\}$. Let $R_N = \sum_{c=1}^{N} \mathbb{1}\{\tau_v = 1\}$ be the number of citizens who are type 1 and, similarly, let $L_N = \sum_{c=1}^{N} \mathbb{1}\{\tau_v = -1\}$. Define the probability of right-wingers being a majority given the candidates' types as

$$P_{R_N}(\tau_i = x, \tau_{-i} = y) := \mathbb{P}\left(R_N > \frac{N}{2} \mid \tau_i = x, \tau_{-i} = y\right)$$

and similarly for the left-wingers:

$$P_{L_N}(\tau_i = x, \tau_{-i} = y) := \mathbb{P}\left(L_N > \frac{N}{2} \mid \tau_i = x, \tau_{-i} = y\right).$$

To consider the most adversarial case for the type 1 candidate, suppose that if there is only a leftist and a rightist candidate, the centrist voters all vote for the leftist candidate. This assumption guarantees that if centrist voters have a slight preference for either -1 or 1 over the other, my equilibrium construction will cover that case.[24] Now, the expected payoff from the suggested strategy for a candidate i of type $\tau_i = 1$ equals

$$\begin{aligned}
&\mathbb{P}(\tau_{-i} = 1 \mid \tau_i = 1) \\
&+ \mathbb{P}(\tau_{-i} = 0 \mid \tau_i = 1) \left[P_{R_N}(\tau_i = 1, \tau_{-i} = 0) + (1 - P_{R_N}(\tau_i = 1, \tau_{-i} = 0))\, u_0\right] \\
&+ \mathbb{P}(\tau_{-i} = -1 \mid \tau_i = 1) P_{R_N}(\tau_{-i} = -1, \tau_i = 1)
\end{aligned} \tag{A.1}$$

Deviating to $p_i = \{-1\}$ is clearly not beneficial for type 1. Consider a deviation to $p_i = \{0\}$. This yields in expectation:

$$\begin{aligned}
&\mathbb{P}(\tau_{-i} = 1 \mid \tau_i = 1) \left[P_{R_N}(\tau_i = 1, \tau_{-i} = 1) + (1 - P_{R_N}(\tau_i = 1, \tau_{-i} = 1))\, u_0\right] \\
&+ \mathbb{P}(\tau_{-i} = 0 \mid \tau_i = 1) u_0 \\
&+ \mathbb{P}(\tau_{-i} = -1 \mid \tau_i = 1) \left[(1 - P_{L_N}(\tau_i = 1, \tau_{-i} = 1))\, u_0\right]
\end{aligned} \tag{A.2}$$

Take any pair $x, y \in \{-1, 0, 1\}$. Now, since conditional on the state the types

[24] If the preference is for the right-wing candidate, then the relevant bound comes from by switching the labels -1 and 1 and thus considering the choices made by leftist candidates.

are independent,

$$\begin{aligned}
&\mathbb{P}\left(R_N > \frac{N}{2} \mid \tau_i = x, \tau_{-i} = y\right) \\
= \quad & \mathbb{P}(s = 1 \mid \tau_i = x, \tau_{-i} = y)\mathbb{P}\left(\frac{R_N}{N} > \frac{1}{2} \mid s = 1\right) \\
& + \mathbb{P}(s = -1 \mid \tau_i = x, \tau_{-i} = y)\mathbb{P}\left(\frac{R_N}{N} > \frac{1}{2} \mid s = -1\right) \qquad \text{(A.3)}
\end{aligned}$$

The random variable $\frac{R_N}{N}$ converges in probability to p conditional on $s = 1$ and to q conditional on $s = -1$. Furthermore, $\max\{p, q\} < \frac{1}{2}$ by assumption and hence (A.3) converges to 0 as $N \to \infty$. An identical argument for L_N implies that

$$\mathbb{P}\left(L_N > \frac{N}{2} \mid \tau_i = 1, \tau_{-i} = 1\right) \underset{N\to\infty}{\to} 0$$

Consequently, for any $\varepsilon > 0$ there exists N_ε such that for $N > N_\varepsilon$,

$$(A.1) > \mathbb{P}(\tau_{-i} = 1 \mid \tau_i = 1) + \mathbb{P}(\tau_{-i} = 0 \mid \tau_i = 1)u_0 - \varepsilon,$$

and

$$(A.2) < u_0 + \varepsilon,$$

as long as $N > N_\varepsilon$. Hence, for $N > N_\varepsilon$ a deviation is not beneficial if

$$\mathbb{P}(\tau_{-i} = 1 \mid \tau_i = 1) + \mathbb{P}(\tau_{-i} = 0 \mid \tau_i = 1)u_0 > u_0 + 2\varepsilon$$

which can be equivalently written as

$$\frac{\mathbb{P}(\tau_{-i} = 1 \mid \tau_i = 1)}{\mathbb{P}(\tau_{-i} = 1 \mid \tau_i = 1) + \mathbb{P}(\tau_{-i} = -1 \mid \tau_i = 1)} > u_0 + \alpha\varepsilon, \qquad \text{(A.4)}$$

where $\alpha = \frac{2}{\mathbb{P}(\tau_{-i}=1|\tau_i=1)+\mathbb{P}(\tau_{-i}=-1|\tau_i=1)}$. Using the law of total probability and the Bayes' rule the numerator becomes

$$\begin{aligned}
\mathbb{P}(\tau_{-i}=1 \mid \tau_i=1) &= \mathbb{P}(s=1 \mid \tau_i=1)\mathbb{P}(\tau_{-i}=1 \mid s=1) \\
&\quad + \mathbb{P}(s=-1 \mid \tau_i=1)\mathbb{P}(\tau_{-i}=1 \mid s=-1) \\
&= \frac{p\mathbb{P}(\tau_i=1 \mid s=1)\mathbb{P}(s=1)}{\mathbb{P}(\tau_i=1 \mid s=1)\mathbb{P}(s=1)+\mathbb{P}(\tau_i=1 \mid s=-1)\mathbb{P}(s=-1)} \\
&\quad + \frac{q\mathbb{P}(\tau_i=1 \mid s=-1)\mathbb{P}(s=-1)}{\mathbb{P}(\tau_i=1 \mid s=1)\mathbb{P}(s=1)+\mathbb{P}(\tau_i=1 \mid s=-1)\mathbb{P}(s=-1)} \\
&= \frac{p^2+q^2}{p+q}. \qquad \text{(A.5)}
\end{aligned}$$

Similarly,

$$\begin{aligned}
\mathbb{P}(\tau_{-i}=-1 \mid \tau_i=1) &= \frac{q\mathbb{P}(\tau_i=1 \mid s=1)\mathbb{P}(s=1)}{\mathbb{P}(\tau_i=1 \mid s=1)\mathbb{P}(s=1)+\mathbb{P}(\tau_i=1 \mid s=-1)\mathbb{P}(s=-1)} \\
&\quad + \frac{p\mathbb{P}(\tau_i=1 \mid s=-1)\mathbb{P}(s=-1)}{\mathbb{P}(\tau_i=1 \mid s=1)\mathbb{P}(s=1)+\mathbb{P}(\tau_i=1 \mid s=-1)\mathbb{P}(s=-1)} \\
&= \frac{2pq}{p+q}. \qquad \text{(A.6)}
\end{aligned}$$

Plugging these back into (A.4) yields that deviating from the truthful strategy is not beneficial as long as:

$$\frac{p^2+q^2}{(p+q)^2} > u_0 + \alpha\varepsilon.$$

Hence, if

$$\frac{p^2+q^2}{(p+q)^2} > u_0,$$

then one can find an $\varepsilon > 0$ small enough such that

$$\frac{p^2+q^2}{(p+q)^2} > u_0 + \alpha\varepsilon$$

and by the above argument there then exists an N_ε such that for $N > N_\varepsilon$ the candidate of type $\tau_i = 1$ does not want to deviate to announcing $\{0\}$. As this is the only remaining available deviation and all of the rest were inferior to the suggested equilibrium, this player does not benefit from any deviation. Furthermore, type $\tau_i = -1$ has even smaller incentives to deviate, since whenever there is only a leftist and a rightist candidate, all of the centrists vote for her and she wins the election while her payoff from deviating remains the same.

Last, notice that by (A.2) and the argument that follows it, by committing to $\{0\}$, a candidate can guarantee that policy 0 gets implemented with a probability that goes to 1 as $N \to \infty$. Hence, for high N a centrist candidate has never incentives to deviate. ■

Lemma 3. *Suppose that candidates are expected to play an equilibrium like the one in Proposition 4. Suppose candidate* 1 *commits to* $\{-1, 1\}$ *and candidate* 2 *commits to* $\{0\}$. *Then both type* -1 *and* 1 *voters vote for candidate 1, if* $\frac{p^2+q^2}{(p+q)^2} \geq u_0$.

Proof. Due to symmetry, it is enough to show that voters of type 1 are willing to vote for candidate 1. Denote by P_v a random variable that gets value 1 if voter v is pivotal and 0 otherwise. Now the expected utility, conditional on being pivotal, for a voter of type 1 from voting for candidate 1 is

$$\begin{aligned} & \mathbb{P}(\tau_1 = 1 \mid \tau_v = 1, \tau_1 \in \{-1, 1\}, P_v = 1) \\ = \; & \mathbb{P}(\tau_1 = 1 \mid s = 1, \tau_1 \in \{-1, 1\})\mathbb{P}(s = 1 \mid \tau_v = 1, \tau_1 \in \{-1, 1\}, P_v = 1) \\ & + \mathbb{P}(\tau_1 = 1 \mid s = -1, \tau_1 \in \{-1, 1\})\mathbb{P}(s = -1 \mid \tau_v = 1, \tau_1 \in \{-1, 1\}, P_v = 1) \\ = \; & \frac{p}{p+q}\mathbb{P}(s = 1 \mid \tau_v = 1, \tau_1 \in \{-1, 1\}, P_v = 1) \\ & + \frac{q}{p+q}\mathbb{P}(s = -1 \mid \tau_v = 1, \tau_1 \in \{-1, 1\}, P_v = 1) \end{aligned} \tag{A.7}$$

where the equality follows by using the law of total probability and the conditional independence of the types conditional on the state. Now using the Bayes' rule yields

$$
\begin{aligned}
& \mathbb{P}(s=1 \mid \tau_v=1, \tau_1 \in \{-1,1\}, P_v=1) \\
= & \frac{\mathbb{P}(\tau_v=1, \tau_1 \in \{-1,1\}, P_v=1 \mid s=1)\mathbb{P}(s=1)}{\sum_{k\in\{-1,1\}} \mathbb{P}(\tau_v=1, \tau_1 \in \{-1,1\}, P_v=1 \mid s=k)\mathbb{P}(s=k)} \\
= & \frac{\mathbb{P}(\tau_v=1, \tau_1 \in \{-1,1\}, P_v=1 \mid s=1)}{\sum_{k\in\{-1,1\}} \mathbb{P}(\tau_v=1, \tau_1 \in \{-1,1\}, P_v=1 \mid s=k)}, \qquad \text{(A.8)}
\end{aligned}
$$

where the last equality follows, because $\mathbb{P}(s=1) = \mathbb{P}(s=-1) = \frac{1}{2}$ by assumption. It is helpful to analyze the numerator separately. Conditional on s, τ_v, τ_1 and the pivotality event P_v are all independent, as P_v depends only on the types of the voters $n \in \{1,\ldots,N\}$, $n \neq v$. Hence the numerator can be written as

$$
\begin{aligned}
& \mathbb{P}(\tau_v=1, \tau_1 \in \{-1,1\}, P_v=1 \mid s=1) \\
= & \mathbb{P}(\tau_v=1 \mid s=1)\mathbb{P}(\tau_1 \in \{-1,1\} \mid s=1)\mathbb{P}(P_v=1 \mid s=1) \\
= & p(p+q)\mathbb{P}(P_v=1 \mid s=1). \qquad \text{(A.9)}
\end{aligned}
$$

Similarly, the term associated with $k=-1$ in the denominator can be written as

$$
\mathbb{P}(\tau_v=1, \tau_1 \in \{-1,1\}, P_v=1 \mid s=1) = q(p+q)\mathbb{P}(P_v=1 \mid s=-1).
$$

Plugging these back into (A.8) and noticing that because of the symmetry voter v is equally likely to be pivotal when $s=1$ and when $s=-1$, i.e. that $\mathbb{P}(P_v=1 \mid s=1) = \mathbb{P}(P_v=1 \mid s=-1)$ yields

$$
(A.8) = \frac{p}{p+q} \qquad \text{(A.10)}
$$

Using identical reasoning, one can show that

$$
\mathbb{P}(s=-1 \mid \tau_v=1, \tau_1 \in \{-1,1\}, P_v=1) = \frac{q}{p+q}
$$

Substituting this and (A.10) back into (A.7) yields that the expected utility for a type 1 voter from voting for $\{-1, 1\}$ when the other candidate commits to 0 is

$$\frac{p^2 + q^2}{(p+q)^2}.$$

Hence, a rightist voter will prefer candidate 1 to candidate 2 as long as

$$\frac{p^2 + q^2}{(p+q)^2} \geq u_0.$$

By symmetry, the same holds also for the leftist candidate. ■

Proof of Proposition 4. Suppose that if both candidates commit to $\{-1, 1\}$ then each voter votes for candidate 1 with probability $\frac{1}{2}$. Suppose first that whenever one candidate plays $\{-1, 1\}$ and the other commits to either $\{-1\}$, $\{1\}$ or any other non-singleton subset of $\{-1, 0, 1\}$ the centrists vote for $\{-1, 1\}$. This behavior can be supported with off-equilibrium beliefs that put full mass on one of the extremes, leaving the centrists always indifferent between the two candidates. Centrist candidates clearly have no incentive to deviate from $\{0\}$, since no other platform is more likely to win than this while still allowing the centrist to choose 0 as the final policy. Consider then the payoffs for candidate i of type $\tau_i = 1$. Given that

$$\frac{p^2 + q^2}{(p+q)^2} \geq u_0,$$

both rightists and leftists will vote for the candidate if the opponent is a centrist. If the opponent is a leftist or a rightist, both commit to the same platform and both win with probability $\frac{1}{2}$. Hence the expected payoff from following the suggested strategy is

$$
\begin{aligned}
&\mathbb{P}(\tau_{-i} = 1 \mid \tau_i = 1) \\
&+ \mathbb{P}(\tau_{-i} = 0 \mid \tau_i = 1)\Big[\mathbb{P}\left(R_N + L_N > \frac{N}{2} \mid \tau_i = 1, \tau_{-i} = 0\right) \\
&+ \left(1 - \mathbb{P}\left(R_N + L_N > \frac{N}{2} \mid \tau_i = 1, \tau_{-i} = 0\right)\right) u_0\Big] \\
&+ \frac{1}{2}\mathbb{P}(\tau_{-i} = -1 \mid \tau_i = 1)
\end{aligned} \tag{A.11}
$$

Since $p + q > \frac{1}{2}$, just as in the proof of Proposition 1,

$$
\mathbb{P}\left(R_N + L_N > \frac{N}{2} \mid \tau_i = 1, \tau_{-i} = 0\right) \underset{N\to\infty}{\to} 1
$$

Hence, for any $\varepsilon > 0$ there exists $\hat{N} \in \mathbb{N}$ such that for $N > \hat{N}$,

$$
(A.11) > 1 - \frac{1}{2}\mathbb{P}(\tau_{-i} = -1 \mid \tau_i = 1) - \varepsilon = 1 - \frac{pq}{p+q} - \varepsilon.
$$

where the last equality follows just as in the proof of Proposition 1. Deviating to $\{0\}$ yields 1 when the opposing candidate is type 1, u_0 when the opposing candidate is type 0 and 0 when the opposing candidate is type -1. This is clearly less than the outcome above, because the payoff is the same when the opponent is type 1 and strictly less in the two other cases. Deviating to $\{1\}$ or any non-singleton set including 1 that induces the voters to believe that her type is 1 yields:

$$
\begin{aligned}
&\mathbb{P}(\tau_{-i} = 1 \mid \tau_i = 1) \\
&+ \mathbb{P}(\tau_{-i} = 0 \mid \tau_i = 1)\Big[\mathbb{P}\left(R_N > \frac{N}{2} \mid \tau_i = 1, \tau_{-i} = 0\right) \\
&+ \left(1 - \mathbb{P}\left(R_N > \frac{N}{2} \mid \tau_i = 1, \tau_{-i} = 0\right)\right) u_0\Big] \\
&+ \mathbb{P}(\tau_{-i} = -1 \mid \tau_i = 1)\mathbb{P}\left(R_N > \frac{N}{2} \mid \tau_i = 1, \tau_{-i} = -1\right)
\end{aligned} \tag{A.12}
$$

Just as before, for any ε there exists $\hat{N}$ such that

$$(A.12) < \mathbb{P}(\tau_{-i} = 1 \mid \tau_i = 1) + \mathbb{P}(\tau_{-i} = 0 \mid \tau_i = 1)u_0 + \varepsilon$$

when $N > \hat{N}$. Then, as long as $2\varepsilon < \frac{pq}{p+q}$,

$$\begin{aligned} & \mathbb{P}(\tau_{-i} = 1 \mid \tau_i = 1) + \mathbb{P}(\tau_{-i} = 0 \mid \tau_i = 1)u_0 + \varepsilon \\ < \; & \mathbb{P}(\tau_{-i} = 1 \mid \tau_i = 1) + \mathbb{P}(\tau_{-i} = 0 \mid \tau_i = 1) + \varepsilon \\ = \; & 1 - \frac{2pq}{p+q} + \varepsilon < 1 - \frac{pq}{p+q} - \varepsilon \end{aligned} \tag{A.13}$$

but we already argued that the right-hand side is less than the utility from following the suggested equilibrium strategy. Hence, also this deviation is not profitable. Last, all of the remaining deviations do not include 1 in their platforms and clearly worse than the two considered here. Hence, given these strategies, type 1 candidate does not have incentives to deviate. By symmetry, the same holds also for candidate of type -1 and hence the profile must be an equilibrium.

Consider then the case where the centrists always vote for a candidate committing to 1 if the other candidate is committing to $\{-1, 1\}$, i.e. the case where all of the centrists potentially prefer 1 over -1. Suppose that the suggested strategy profiles and off-equilibrium beliefs are otherwise unchanged. Then the only relevant thing that changes is what a type 1 player gets if she deviates to 1 instead of $\{-1, 1\}$. Now, this deviation utility becomes:

$$\begin{aligned} & \mathbb{P}(\tau_{-i} = 1 \mid \tau_i = 1) \\ & + \mathbb{P}(\tau_{-i} = 0 \mid \tau_i = 1)\Big[\mathbb{P}\left(R_N > \frac{N}{2} \mid \tau_i = 1, \tau_{-i} = 0\right) \\ & + \left(1 - \mathbb{P}\left(R_N > \frac{N}{2} \mid \tau_i = 1, \tau_{-i} = 0\right)\right) u_0\Big] \\ & + \mathbb{P}(\tau_{-i} = -1 \mid \tau_i = 1)\left(1 - \mathbb{P}\left(L_N > \frac{N}{2} \mid \tau_i = 1, \tau_{-i} = -1\right)\right) \end{aligned} \tag{A.14}$$

Then for any $\varepsilon > 0$ there are N large enough such that

$$\begin{aligned}(A.14) \quad &< \quad \mathbb{P}(\tau_{-i} = 1 \mid \tau_i = 1) + \mathbb{P}(\tau_{-i} = 0 \mid \tau_i = 1)u_0 + \mathbb{P}(\tau_{-i} = -1 \mid \tau_i = 1) + \varepsilon \\ &= \quad 1 - \mathbb{P}(\tau_{-i} = 0 \mid \tau_i = 1)(1 - u_0) + \varepsilon \end{aligned} \tag{A.15}$$

In the proof of Proposition 1 we showed that $\mathbb{P}(\tau_{-i} = 1 \mid \tau_i = 1) = \frac{p^2+q^2}{p+q}$ and $\mathbb{P}(\tau_{-i} = -1 \mid \tau_i = 1) = \frac{pq}{p+q}$. These imply that $\mathbb{P}(\tau_{-i} = 0 \mid \tau_i = 1) = 1 - p - q$. Plugging this back into (A.15) and comparing with (A.13) we find that this deviation from the suggested strategies is not beneficial for large N, as long as,

$$\begin{aligned} & 1 - \frac{pq}{p+q} - \varepsilon > 1 - (1-p-q)(1-u_0) + \varepsilon \\ \Leftrightarrow \quad & u_0 < 1 - \frac{pq}{(1-p-q)(p+q)} - \beta\varepsilon \end{aligned}$$

where $\beta = \frac{2}{1-p-q}$. ■

Proof of Lemma 2. Reorganizing the inequality, I get that

$$\begin{aligned} & \frac{p^2+q^2}{(p+q)^2} \geq 1 - \frac{pq}{(1-p-q)(p+q)} \\ \Leftrightarrow \quad & [(1-p-q)(p+q) - pq]\,(p+q) \leq (p^2+q^2)(1-p-q) \\ \Leftrightarrow \quad & 2pq - 3pq(p+q) \leq 0 \\ \Leftrightarrow \quad & p + q \geq \frac{2}{3}, \end{aligned}$$

proving the lemma. ■

Proof of Proposition 6. Notice that the only outcomes where the commitment and non-commitment equilibria yield different utilities for a voter are the ones where there is one unknown extremist and one centrist or when there are two extremists of different kind as candidates. In other words, conditional on the candidates being the same type, both equilibria yield the same utility for everyone. Suppose that the centrists prefer policy 1 over policy -1 and consider first a voter v of type -1. Denote

$$A = \mathbb{P}(\tau_1 \in \{-1, 1\}, \tau_2 = 0 \mid \tau_v = -1) = \mathbb{P}(\tau_1 \in \{-1, 1\}, \tau_2 = 0 \mid \tau_v = 1)$$

and

$$B = \mathbb{P}(\tau_1 = 1, \tau_2 = -1 \mid \tau_v = -1) = \mathbb{P}(\tau_1 = 1, \tau_2 = -1 \mid \tau_v = 1).$$

Due to symmetry, the probability that one candidate is either type -1 or 1 and the other one is type 0 is $2A$. Similarly, the probability that the two candidates are extremists of different types is $2B$. Now, conditional on there being one centrist and one extremist of an unknown type, the commitment equilibrium yields a voter of type -1 an approximate payoff of u_0 when N is large.[25] On the other hand, the equilibrium with no commitment yields the same player and expected utility of approximately $\frac{p^2+q^2}{(p+q)^2}$. On the other hand, if there are two extremists of different types, the commitment equilibrium implies that the type 1 candidate gets chosen with a probability close to one and hence the type -1 gets approximately zero utility. Under no commitment, non of the voters know the type of either of the candidates and hence randomize between the two. Consequently, the type -1 voter gets a utility of $\frac{1}{2}$.

Putting these details together, the difference in expected payoffs for the type -1 voter between the equilibria is approximately

$$2A \left(u_0 - \frac{p^2 + q^2}{(p + q)^2} \right) + 2B(0 - \frac{1}{2}).$$

Since the no-commitment equilibrium exists only if the first term is negative, the whole difference must be negative when N is large enough. Consequently, the unfavored voters always prefer the no-commitment equilibrium to the com-

[25] Just as in the previous proofs one can use the Law of Large Numbers to show that for any ε there exists N large enough such that this payoff is almost surely within an ε from u_0 when there are at least N voters.

mitment equilibrium.

Consider then the type 1 voters. For them the utilities are otherwise the same except for the outcome of the commitment equilibrium when there are two extreme candidates of different types. In that case they get 1 instead of the 0 above. Therefore, for every $\varepsilon > 0$ there exists $\hat{N} \in \mathbb{N}$ such that the type 1 voter prefers the commitment equilibrium to the no-commitment equilibrium when

$$\begin{aligned} & 2A\left(u_0 - \frac{p^2+q^2}{(p+q)^2}\right) + B \geq \varepsilon \\ \Leftrightarrow \quad & u_0 \geq \frac{p^2+q^2}{(p+q)^2} - \frac{B}{2A} + \varepsilon \end{aligned} \tag{A.16}$$

Now, A can be written as

$$\begin{aligned} A &= \sum_{j\in\{-1,1\}} \mathbb{P}\left(s=j \mid \tau_v=1\right)\mathbb{P}\left(\tau_1=0,\tau_2\in\{-1,1\}\mid \tau_v=1,s=j\right) \\ &= \frac{p}{p+q}(1-p-q)(p+q) + \frac{q}{p+q}(1-p-q)(p+q) \\ &= (p+q)(1-p-q). \end{aligned}$$

Similarly, B can be written as

$$\begin{aligned} B &= \sum_{j\in\{-1,1\}} \mathbb{P}\left(s=j \mid \tau_v=1\right)\mathbb{P}\left(\tau_1=1,\tau_2=-1\mid \tau_v=1,s=j\right) \\ &= \frac{p}{p+q}pq + \frac{q}{p+q}pq = pq. \end{aligned}$$

Plugging these back into (A.16) yields that that the favored extremists prefer the commitment equilibrium to the non-commitment equilibrium, whenever

$$u_0 \geq \frac{p^2+q^2}{(p+q)^2} - \frac{pq}{2(p+q)(1-p-q)} + \varepsilon.$$

Consider then the case when the centrist voters are indifferent between the two extremes an randomize with equal probability between the two candidates if faced with the choice between -1 and 1. Now, due to symmetry the two types of extremists get the same expected utility in each case above conditional on their own type. These utilities are also the same except for the expected payoff from the commitment equilibrium when there are two extremists of different types.

To calculate this expected utility, notice that conditional on $s = 1$, the probability that there are more of type 1 voters than type -1 voters can be made arbitrarily close to 1 by choosing a population that is large enough.[26] Since, approximately 50% of the centrists vote for each candidate, the election is approximately decided by the relative frequency of each type of extremist voters. Hence, the expected utility for a type 1 voter is approximately

$$\mathbb{P}(s = 1 \mid \tau_v = 1, \tau_1 = 1, \tau_2 = -1) = \frac{p}{p+q}.$$

Due to symmetry, the expected utility for type -1 voters is approximately the same. Consequently, for all $\varepsilon > 0$ there exists $\hat{N} \in \mathbb{N}$ such that whenever $N \geq \hat{N}$ all types of voters prefer the commitment equilibrium to the non-commitment equilibrium as long as

$$\begin{aligned}
& 2A\left(u_0 - \frac{p^2+q^2}{(p+q)^2}\right) + 2B\left(\frac{p}{p+q} - \frac{1}{2}\right) \geq \varepsilon \\
\Leftrightarrow \quad & A\left(u_0 - \frac{p^2+q^2}{(p+q)^2}\right) + B\left(\frac{p-q}{2(p+q)}\right) \geq \frac{\varepsilon}{2} \\
\Leftrightarrow \quad & u_0 \geq \frac{p^2+q^2}{(p+q)^2} - \frac{B(p-q)}{2A(p+q)} + \varepsilon \\
\Leftrightarrow \quad & u_0 \geq \frac{p^2+q^2}{(p+q)^2} - \frac{pq(p-q)}{2(p+q)^2(1-p-q)} + \varepsilon, \qquad \text{(A.17)}
\end{aligned}$$

where I abuse the notation slightly by redefining ε up to a multiplicative, positive constant. ■

Proof of Proposition 7. I will check the game using backward induction.

[26] Just as before, conditional on $s = 1$, the law of large number implies that the fraction of type 1 voters approaches p almost surely and the fraction of type -1 voters approaches $q < p$.

Suppose candidates play truthful strategies. Since voters are not strategic, their behavior after observing each possible pair of candidates depends on their beliefs conditional on the candidates behavior and their own signal. Denote the vector of signals that the voter bases her decisions on $\theta = (\theta_v, \theta_{c_1}, \theta_{c_2})$, where the first element is the voter's own signal and the last two are the signals of the two candidates. Consider first voter v of type 1. If she is faced with the choice between a candidate committing to 1 and a candidate committing to -1, it is trivial to check that she will vote for the candidate committing to 1. Similarly, when faced with a candidate who commits to 0 and another committing to -1, she will trivially vote for the candidate committing to 0. The non-trivial choice happens when she faces a candidate committing to 1 and the other committing to 0. Then she will vote for the candidate committing to 1, if and only if

$$\begin{aligned} &\mathbb{P}(s = 1 \mid \theta = (1,1,0)) \geq \mathbb{P}(s \neq 0 \mid \theta = (1,1,0))u_0 + \mathbb{P}(s = 0 \mid \theta = (1,1,0)) \\ \Leftrightarrow \quad & u_0 \leq \frac{\mathbb{P}(s = 1 \mid \theta = (1,1,0)) - \mathbb{P}(s = 0 \mid \theta = (1,1,0))}{\mathbb{P}(s = 1 \mid \theta = (1,1,0)) + \mathbb{P}(s = -1 \mid \theta = (1,1,0))}. \end{aligned} \tag{A.18}$$

Now, by Bayes' law and symmetry,

$$\mathbb{P}(s = 1 \mid \theta = (1,1,0)) = \frac{p^2 q}{p^2 q + q^2 p + q^3},$$

$$\mathbb{P}(s = 0 \mid \theta = (1,1,0)) = \frac{q^2 p}{p^2 q + q^2 p + q^3}$$

and

$$\mathbb{P}(s = -1 \mid \theta = (1,1,0)) = \frac{q^3}{p^2 q + q^2 p + q^3}$$

Hence, (A.18) can be written as

$$u_0 \leq \frac{p^2 q - q^2 p}{p^2 q + q^3} = \frac{p(p-q)}{p^2 + q^2} \tag{A.19}$$

where the right-hand side is always positive, since $p > q$. By symmetry, an identical result holds for a voter type -1 choosing between a candidate committing to -1 and another committing to 0. A voter with a centrist signal always votes for a centrist when a centrist candidate is available and otherwise she is indifferent.

Since, $p < \frac{1}{2}$, this implies that a centrist candidate will almost always win when N is large enough and candidates are expected to commit truthfully, no matter what the state is and no matter how big the u_0 is. Hence, conditional on facing a centrist, a candidate's payoff does not depend on her own platform. Consequently, it is enough to consider the candidate's problem when facing an agent with an extreme signal.

If the state is 1, a candidate commits to 1 and another chooses -1, then for large N, the law of large numbers implies that, the candidate committing to 1 will win with a probability close to 1. Hence, for any ε, we can find N large enough, such that the difference between committing to 1 and 0 is at least:

$$\begin{aligned} & \mathbb{P}(\theta_2 = 1 \mid \theta_1 = 1)\mathbb{P}(s = 1 \mid \theta_1 = 1, \theta_2 = 1) \\ + \quad & \mathbb{P}(\theta_2 = -1 \mid \theta_1 = 1)\mathbb{P}(s \in \{-1, 1\} \mid \theta_1 = 1, \theta_2 = -1) \\ - \quad & \mathbb{P}(\theta_2 \in \{-1, 1\} \mid \theta_1 = 1)\mathbb{P}(s = 0 \mid \theta_1 = 1, \theta_2 \in \{-1, 1\}) \\ - \quad & \mathbb{P}(\theta_2 \in \{-1, 1\} \mid \theta_1 = 1)\mathbb{P}(s \in \{-1, 1\} \mid \theta_1 = 1, \theta_2 \in \{-1, 1\})u_0 \\ - \quad & \varepsilon. \end{aligned} \tag{A.20}$$

Now, by Bayes' rule,

$$\mathbb{P}(\theta_2 = 1 \mid \theta_1 = 1) = \frac{\frac{1}{3}p^2 + \frac{2}{3}q^2}{\frac{1}{3}p + \frac{2}{3}q} = p^2 + 2q^2,$$

$$\mathbb{P}(\theta_2 = -1 \mid \theta_1 = 1) = \frac{\frac{2}{3}pq + \frac{1}{3}q^2}{\frac{1}{3}p + \frac{2}{3}q} = 2pq + q^2$$

which implies that

$$\mathbb{P}(\theta_2 \in \{-1, 1\} \mid \theta_1 = 1) = p^2 + 2q^2 + 2pq + q^2 = 2q^2 + (p+q)^2.$$

Similarly,

$$\mathbb{P}(\theta_2 = 0 \mid \theta_1 = 1) = 2pq + q^2.$$

Just like above,

$$\mathbb{P}(s = 1 \mid \theta_1 = 1, \theta_2 = 1) = \frac{p^2}{p^2 + 2q^2}$$

and

$$\mathbb{P}(s \in \{-1, 1\} \mid \theta_1 = 1, \theta_2 = -1) = \frac{2pq}{2pq + q^2}.$$

Substituting all of these back into (A.20) yields

$$\begin{aligned}
(A.20) \quad &= \quad \left(p^2 + 2q^2\right) \frac{p^2}{p^2 + 2q^2} + \left(2pq + q^2\right) \frac{2pq}{2pq + q^2} \\
&\quad -(2q^2 + (p+q)^2) \frac{2q^2}{2q^2 + p(p+q) + q(p+q)} \\
&\quad -(2q^2 + (p+q)^2) \frac{p(p+q) + q(p+q)}{2q^2 + p(p+q) + q(p+q)} u_0 - \varepsilon \\
&= \quad p^2 + 2pq - 2q^2 - (p+q)^2 u_0 - \varepsilon, \qquad \text{(A.21)}
\end{aligned}$$

which is positive if and only if

$$\begin{aligned}
u_0 \quad &\leq \quad \frac{p^2 + 2q(p-q)}{(p+q)^2} - \frac{\varepsilon}{(p+q)^2} \\
&\xrightarrow[N \to \infty]{} \quad \frac{p^2 + 2q(p-q)}{(p+q)^2} \qquad \text{(A.22)}
\end{aligned}$$

Hence, as long as,

$$u_0 < \frac{p^2 + 2q(p-q)}{(p+q)^2},$$

there exists $\hat{N} \in \mathbb{N}$ such that, when $N \geq \hat{N}$, the candidates with signal 1 prefer committing to policy 1 rather than policy 0.[27]

[27] This constraint on u_0 is less stringent than the one that guarantees that also the voters of type 1 vote according to their signal when facing both a type 0 and a type 1 candidate. This can be seen by taking the difference between the two constraints

$$\frac{p(p-q)}{p^2 + q^2} - \frac{p^2 + 2q(p-q)}{(p+q)^2} = \frac{p(p-q)(p+q)^2 - (p^2 + q^2)(p^2 + 2q(p+q))}{(p^2 + q^2)(p+q)^2},$$

where the denominator is always positive and simplifying the numerator yields

$$p(p-q)(p+q)^2 - (p^2 + q^2)(p^2 + 2q(p-q)) = -p^3 q - 3q^3 p + 2q^4 < -p^3 q - q^3 p < 0,$$

I will next show that a type 1 candidiate never wants to deviate to committing to -1. The difference in expected utilities between those two options is given by

$$\begin{aligned}
& \mathbb{P}(\theta_2 = 1 \mid \theta_1 = 1)\left(\mathbb{P}(s = 1 \mid \theta_1 = 1, \theta_2 = 1) - \mathbb{P}(s \in \{-1, 1\} \mid \theta_1 = 1, \theta_2 = 1)\right) \\
& + \mathbb{P}(\theta_2 = -1 \mid \theta_1 = 1)\left(\mathbb{P}(s \in \{-1, 1\} \mid \theta_1 = 1, \theta_2 = -1) - \mathbb{P}(s = -1 \mid \theta_1 = 1, \theta_2 = -1)\right) \\
= \; & (p^2 + 2q^2)\left(\frac{p^2}{p^2 + 2q^2} - \frac{p^2 + q^2}{p^2 + 2q^2}\right) + (2pq + q^2)\left(\frac{2pq}{2pq + q^2} - \frac{pq}{2pq + q^2}\right) \\
= \; & -q^2 + pq > 0, \qquad \text{(A.23)}
\end{aligned}$$

where the inequality follows, since $p > q$. Similar argument can be used to show that a centrist candidate never wants to deviate to either -1 or 1. Non-singleton platforms can be deterred with extreme off-equilibrium beliefs just like in the original model. ■

Proof of Proposition 8. The same law of large number argument can be run through this proposition as all of the previous ones. In other words, as long as we require all of the inequalities to be strict, we can find N large enough that the arguments hold even when the winning or losing probabilities for candidates are only approximatively the ones stated below. For conciseness, I will not carry the ε and N in this proof.

I will again proceed with backward induction and show that there is no symmetric pure strategy equilibrium where $\{-1, 1\}$ gets played by both candidates when their type is $\theta \in \{-1, 1\}$. Suppose first that candidates with $\theta \in \{-1, 1\}$ commit to $\{-1, 1\}$ and all candidates with $\theta = 0$ commit to $\{0\}$. Then for a voters with $\theta_v = 1$, conditional on the other candidate committing to 0, the

---

where the first inequality holds, since $p > q$.

expectation from the ambiguous candidate is

$$
\begin{aligned}
& \mathbb{P}\left(\theta_{c_1}=s \mid \theta=(1,\theta_{c_1},0),\theta_{c_1}\in\{-1,1\}\right) \\
= \quad & \mathbb{P}\left(s=1 \mid \theta=(1,\theta_{c_1},0),\theta_{c_1}\in\{-1,1\}\right) \\
& \times \mathbb{P}\left(\theta_{c_1}=1 \mid \theta_{c_1}\in\{-1,1\}, s=1\right) \\
+ \quad & \mathbb{P}\left(s=-1 \mid \theta=(1,\theta_{c_1},0),\theta_{c_1}\in\{-1,1\}\right) \\
& \times \mathbb{P}\left(\theta_{c_1}=-1 \mid \theta_{c_1}\in\{-1,1\}, s=-1\right) \\
= \quad & \frac{pq(p+q)}{pq(p+q)+2pq^2+q^2(p+q)} \times \frac{p}{p+q} \\
& + \frac{q^2(p+q)}{q^2(p+q)+2pq^2+pq(p+q)} \times \frac{p}{p+q} \\
= \quad & \frac{p^2+qp}{(p+q)^2+2pq} \qquad \text{(A.24)}
\end{aligned}
$$

For the same voter the expected utility from implementing 0 is

$$
\begin{aligned}
& \mathbb{P}\left(s=0 \mid \theta=(1,\theta_{c_1},0),\theta_{c_1}\in\{-1,1\}\right) \\
& + \mathbb{P}\left(s\neq 0 \mid \theta=(1,\theta_{c_1},0),\theta_{c_1}\in\{-1,1\}\right) u_0 \\
= \quad & \frac{2pq^2}{pq(p+q)+2pq^2+q^2(p+q)} \\
& + \left(1-\frac{2pq^2}{q^2(p+q)+2pq^2+pq(p+q)}\right) u_0 \\
= \quad & \frac{2pq}{(p+q)^2+2pq} + \frac{(p+q)^2}{(p+q)^2+2pq} u_0 \qquad \text{(A.25)}
\end{aligned}
$$

Voting for the ambiguous candidate yields more than the centrist, as long as,

$$
\begin{aligned}
\frac{(p+q)^2}{(p+q)^2+2pq} u_0 & \leq \frac{p^2+qp}{(p+q)^2+2pq} - \frac{2pq}{(p+q)^2+2pq} \\
u_0 & \leq \frac{p(p-q)}{(p+q)^2} = \frac{p(p-q)}{p+q^2} \qquad \text{(A.26)}
\end{aligned}
$$

It is easy to check that when faced with a choice between {1} (or {-1}) and $\{-1,1\}$ both centrists and type 1 (type -1)voters always vote for the candidate committing to $\{1\}$ (or $\{-1\}$, respectively). Similarly, to the previous lemma, committing to an extreme platform loses (with a probability close to 1) to a centrist candidate as opposing extremists and centrists vote for the centrist.

I will next turn to the candidates' problem. Clearly, a centrist candidate cannot do better than to commit to $\{0\}$. An extremist, on the other hand, has two potential deviations: either commit to $\{0\}$ or commit to the policy corresponding to the candidate's type. Due to symmetry, it is enough to look at a candidate c_1 of type 1. I will first show that playing the ambiguous platform with probability 1 is never an equilibrium.

The expected utility from not deviating is

$$\begin{aligned}
&\mathbb{P}(s=1 \mid \theta_{c_1}=1, \theta_{c_2}=0)\mathbb{P}(\theta_{c_2}=0 \mid \theta_{c_1}=1) \\
&+ \left(\mathbb{P}(s=1 \mid \theta_{c_1}=1, \theta_{c_2} \in \{-1,1\}) + \mathbb{P}(\theta_{c_2}=s \mid \theta_{c_1}=1, \theta_{c_2} \in \{-1,1\})\right) \\
&\times \frac{1}{2}\mathbb{P}(\theta_{c_2} \in \{-1,1\} \mid \theta_{c_1}=1) \qquad \text{(A.27)}
\end{aligned}$$

Now,

$$\begin{aligned}
& \mathbb{P}(\theta_{c_2}=s \mid \theta_{c_1}=1, \theta_{c_2} \in \{-1,1\}) \\
= & \sum_{i \neq 0} \mathbb{P}(s=i \mid \theta_{c_1}=1, \theta_{c_2} \in \{-1,1\})\mathbb{P}(\theta_{c_2}=i \mid s=i, \theta_{c_2} \in \{-1,1\}) \\
= & \frac{p(p+q)}{(p+q)^2+2q^2} \times \frac{p}{p+q} + \frac{q(p+q)}{(p+q)^2+2q^2} \times \frac{p}{p+q} \\
= & \frac{p(p+q)}{(p+q)^2+2q^2} \qquad \text{(A.28)}
\end{aligned}$$

and

$$\begin{aligned}
& \mathbb{P}(\theta_{c_2} \in \{-1,1\} \mid \theta_{c_1}=1) \\
= & \sum_{i=-1}^{1} \mathbb{P}(s=i \mid \theta_{c_1}=1)\mathbb{P}(\theta_{c_2} \in \{-1,1\} \mid s=i) \\
= & q(p+q) + 2q^2 + p(p+q) = (p+q)^2 + 2q^2 \qquad \text{(A.29)}
\end{aligned}$$

Substituting (A.28) and (A.29) into (A.27), I get

$$\begin{aligned}
(A.27) \quad &= \frac{pq}{2pq+q^2} \times \frac{2pq+q^2}{p+2q} \\
&\quad + \left(\frac{p(p+q)}{(p+q)^2+2q^2} + \frac{p(p+q)}{(p+q)^2+2q^2} \right) \\
&\quad \times \frac{1}{2}\left((p+q)^2+2q^2\right) \\
&= \frac{pq}{p+2q} + p(p+q) = p, \qquad \text{(A.30)}
\end{aligned}$$

which is highly intuitive, since the winning event is not informative about the state in this equilibrium and hence only the signal precision matters.

The expected utility from deviating to $\{0\}$ for a candidate with signal $\theta_{c_1} = 1$ is:

$$\begin{aligned}
&\mathbb{P}(\theta_{c_2} = 0 \mid \theta_{c_1} = 1) \\
&\times \left(\mathbb{P}(s = 0 \mid \theta_{c_1} = 1, \theta_{c_2} = 0) + (1 - \mathbb{P}(s = 0 \mid \theta_{c_1} = 1, \theta_{c_2} = 0))u_0\right) \\
&+ \mathbb{P}(\theta_{c_2} = s \mid \theta_{c_1} = 1, \theta_{c_2} \in \{-1,1\})\mathbb{P}(\theta_{c_2} \in \{-1,1\} \mid \theta_{c_1} = 1) \\
= \quad & \left(q^2+2pq\right)\left(\frac{pq}{2pq+q^2} + \frac{pq+q^2}{2pq+q^2}u_0\right) \\
&+ \left((p+q)^2+2q^2\right)\frac{p(p+q)}{(p+q)^2+2q^2} \\
= \quad & pq + (pq+q^2)u_0 + p(p+q) = p + (pq+q^2)u_0 \qquad \text{(A.31)}
\end{aligned}$$

Now, clearly, $p + (pq+q^2)u_0 > p$ and hence the candidate prefers deviating to 0 instead.

I will first derive the asymmetric equilibrium. Suppose that candidate 2 always plays $\{0\}$ and candidate 1 plays $\{-1, 1\}$ whenever her type is $\theta_{c_1} \in \{-1, 1\}$ and $\{0\}$ otherwise.

I will again start with the voters' problem. Notice that now voter's are even more prone to vote for the ambiguous candidate 1, since the probability that the candidate 2's platform is correct is even smaller. Without further loss of generality, consider a voter of type 1 faced with the choice between an ambiguous candidate 1 and a candidate 2 committing to 0.

Now the expected utility from voting for the ambiguous candidate 1 is

$$\begin{aligned}
& \mathbb{P}(s=\theta_{c_1} \mid \theta_{c_1} \in \{-1,1\}, \theta_v = 1) \\
= & \sum_{i\neq 0} \mathbb{P}(\theta_{c_1} = i \mid s = i, \theta_{c_1} \in \{-1,1\}) \mathbb{P}(s = i \mid \theta_{c_1} \in \{-1,1\}, \theta_v = 1) \\
= & \frac{p}{p+q} \times \frac{p(p+q)}{(p+q)^2+q^2} + \frac{p}{p+q} \times \frac{q(p+q)}{(p+q)^2+q^2} \\
= & \frac{p(p+q)}{(p+q)^2+2q^2} \qquad \text{(A.32)}
\end{aligned}$$

Voting for candidate 2 who committed to $\{0\}$ yields

$$\begin{aligned}
& \mathbb{P}(s=0 \mid \theta_{c_1} \in \{-1,1\}, \theta_v = 1) + (1 - \mathbb{P}(s=0 \mid \theta_{c_1} \in \{-1,1\}, \theta_v = 1))u_0 \\
= & \frac{2q^2}{(p+q)^2+2q^2} + \frac{(p+q)^2}{(p+q)^2+2q^2} u_0 \qquad \text{(A.33)}
\end{aligned}$$

Hence, she prefers voting for the ambiguous extremist as long as

$$\begin{aligned}
\frac{p(p+q)}{(p+q)^2+2q^2} &> \frac{2q^2+(p+q)^2 u_0}{(p+q)^2+2q^2} \\
u_0 &< \frac{p(p+q)-2q^2}{(p+q)^2}. \qquad \text{(A.34)}
\end{aligned}$$

Notice that

$$\frac{p(p+q)-2q^2}{(p+q)^2} > \frac{p(p+q)-2pq}{(p+q)^2} = \frac{p(p-q)}{(p+q)^2}$$

and hence this inequality is indeed less stringent than the one in (A.26).

Consider candidate 1 of type 1. Her utility from playing $\{-1,1\}$ is

$$\mathbb{P}(s=1 \mid \theta_{c_1} = 1) = p$$

and her utility from deviating to $\{0\}$ is

$$\mathbb{P}(s=0 \mid \theta_{c_1} = 1) + (1 - \mathbb{P}(s=0 \mid \theta_{c_1} = 1))u_0 = q + (1-q)u_0.$$

Hence, this deviation does not benefit her as long as

$$u_0 < \frac{p-q}{1-q}.$$

Given our assumptions that $p = 1-2q < \frac{1}{2}$, it is easy to show that $\frac{p-q}{1-q} < \frac{p(p-q)-2q^2}{(p+q)^2}$ and hence the candidate's inequality is the more stringent one. Checking the suboptimality of singleton platforms is also straightforward (for any off-equilibrium beliefs). Consequently, the suggested strategy profile is indeed an equilibrium.

To prove the last part, consider then all candidates of type $\theta \in \{-1,1\}$ playing $\{-1,1\}$ with probability $x \in (0,1)$. Suppose again that a voter of type 1 is faced with the choice between candidate 1 committing to $\{-1,1\}$ and candidate 2 committing to $\{0\}$. Then her expected utility from voting for the ambiguous candidate is

$$\begin{aligned}
& \mathbb{P}(\theta_{c_1} = s \mid \theta_v = 1, \theta_{c_1} \in \{-1,1\}, p_2 = \{0\}) \\
= \; & \mathbb{P}(s = 1 \mid \theta_v = 1, \theta_{c_1} \in \{-1,1\}, p_2 = \{0\}) \\
& \times \mathbb{P}(\theta_{c_1} = 1 \mid \theta_{c_1} \in \{-1,1\}, s = 1) \\
+ \; & \mathbb{P}(s = -1 \mid \theta_v = 1, \theta_{c_1} \in \{-1,1\}, p_2 = \{0\}) \\
& \times \mathbb{P}(\theta_{c_1} = -1 \mid \theta_{c_1} \in \{-1,1\}, s = -1) \\
= \; & \frac{p(p+q)(q+(1-x)(p+q))}{(p+q)^2(q+(1-x)(p+q)) + q^2(p+2q(1-x))} \times \frac{p}{p+q} \\
& + \frac{q(p+q)(q+(1-x)(p+q))}{(p+q)^2(q+(1-x)(p+q)) + q^2(p+2q(1-x))} \times \frac{p}{p+q} \\
= \; & \frac{(p^2+pq)(q+(1-x)(p+q))}{(p+q)^2(q+(1-x)(p+q)) + 2q^2(p+2q(1-x))} \\
= \; & \frac{(p^2+pq)(1-x(p+q))}{(p+q)^2(1-x(p+q)) + 2q^2(1-2qx)}
\end{aligned} \tag{A.35}$$

Similarly, her utility from voting for candidate 2 is

$$\begin{aligned}
& \mathbb{P}\left(s=0 \mid \theta_v=1, \theta_{c_1} \in \{-1,1\}, p_2=\{0\}\right) \\
& \quad + \mathbb{P}\left(s \neq 0 \mid \theta_v=1, \theta_{c_1} \in \{-1,1\}, p_2=\{0\}\right) u_0 \\
= & \frac{2q^2(p+q(1-x))}{(p+q)^2(q+(1-x)(p+q))+q^2(p+2q(1-x))} \\
& + \left(1-\frac{2q^2(p+q(1-x))}{(p+q)^2(q+(1-x)(p+q))+q^2(p+2q(1-x))}\right) u_0 \\
= & \frac{2q^2(1-2qx)}{(p+q)^2(1-x(p+q))+2q^2(1-2qx)} \\
& + \frac{(p+q)^2(1-x(p+q))}{(p+q)^2(1-x(p+q))+2q^2(1-2qx)} u_0. \qquad \text{(A.36)}
\end{aligned}$$

Now, voting for the ambiguous candidate yields more than voting for candidate 2 as long as

$$\begin{aligned}
(p+q)^2(1-x(p+q))u_0 &< (p^2+pq)(1-x(p+q))-2q^2(1-2qx) \\
u_0 &< \frac{(p^2+pq)(1-x(p+q))-2q^2(1-2qx)}{(p+q)^2(1-x(p+q))} \qquad \text{(A.37)}
\end{aligned}$$

Notice that at $x=1$ the right-hand side becomes (as it should) exactly the right-hand side of (A.26).[28] To see that for $x<1$ this is less stringent, I will show that the right-hand side of (A.26) is decreasing in x. Differentiating with respect to x, one gets

$$\begin{aligned}
& \frac{\partial}{\partial x} \frac{(p^2+pq)(1-x(p+q))-2q^2(1-2qx)}{(p+q)^2(1-x(p+q))} \\
= & \frac{4q^3(p+q)^2-2q^2(p+q)^3}{(p+q)^4(1-x(p+q))^2} = \frac{2(q(p+q))^2(q-p)}{(p+q)^4(1-x(p+q))^2} < 0, \qquad \text{(A.38)}
\end{aligned}$$

since $q<p$. Hence, the more there is mixing the easier it is to get the extremist voters to vote for the ambiguous platform.

I will then turn to the candidates. Both candidates need to be indifferent between the ambiguous platform $\{-1,1\}$ and the "centrist" platform $\{0\}$ whenever

[28] To get this, remember that $1-p-q=q$ and thus $1-2q=p$.

they observe an extreme signal. By symmetry, it is enough to consider candidate 1 when she observes signal 1. Choosing the ambiguous platform yields her

$$\begin{aligned}
& \mathbb{P}(\{s=1\}\cap\{p_2=\{0\}\}\mid\theta_{c_1}=1) \\
& +\frac{1}{2}\left(\mathbb{P}(s=1\cap\{p_2=\{-1,1\}\}\mid\theta_{c_1}=1)+\mathbb{P}(\{\theta_{c_2}=s\}\cap\{p_2=\{-1,1\}\}\mid\theta_{c_1}=1)\right) \\
= \; & \frac{p(q+(p+q)(1-x))}{p+2q} \\
& +\frac{1}{2}\frac{px(p+q)}{p+2q} \\
& +\frac{1}{2}\frac{p^2x+qpx}{p+2q} \\
= \; & p(q+(p+q)(1-x))+px(p+q)=p
\end{aligned} \tag{A.39}$$

Choosing $\{0\}$ instead yields her

$$\begin{aligned}
& \mathbb{P}(\{p_2=0\}\cap\{s=0\}\mid\theta_{c_1}=1)+\mathbb{P}(\{p_2=0\}\cap\{s\neq 0\}\mid\theta_{c_1}=1)u_0 \\
& +\mathbb{P}(\{\theta_{c_2}=s\}\cap\{p_2=\{-1,1\}\}\mid\theta_{c_1}=1) \\
= \; & \frac{q(p+(1-x)2q)}{p+2q}+\frac{p(q+(p+q)(1-x))+q(q+(p+q)(1-x))}{p+2q}u_0 \\
& +\frac{p^2x+qpx}{p+2q} \\
= \; & q(p+(1-x)2q)+p^2x+qpx+(p+q)(q+(p+q)(1-x))u_0 \\
= \; & q(1-2qx)+px(p+q)+(p+q)(1-(p+q)x)u_0
\end{aligned} \tag{A.40}$$

Hence, a candidate with an extreme signal is indifferent between her options as long as

$$p=q(1-2qx)+px(p+q)+(p+q)(1-(p+q)x)u_0 \tag{A.41}$$

Notice that, as $x\to 0$, the right-hand side goes to $q+(p+q)u_0$ which is less than p as long as $u_0<\frac{p-q}{p+q}$. On the other hand, as $x\to 1$, the right-hand side goes to

$$qp+p(p+q)+(p+q)qu_0=p+(p+q)pu_0>p.$$

Hence, as long as $u_0 < \frac{p-q}{p+q}$, a mixing probability exists that makes extreme candidates indifferent between the ambiguous platform and $\{0\}$.

Solving for this mixing probability, I get

$$\begin{aligned}
& p = q(1-2qx) + px(p+q) + (p+q)(1-(p+q)x)u_0 \\
\Leftrightarrow \quad & x(p(p+q) - 2q^2 - (p+q)^2 u_0) = p - q - (p+q)u_0 \\
\Leftrightarrow \quad & x = \frac{p-q-(p+q)u_0}{(p+q)(p-(p+q)u_0) - 2q^2}
\end{aligned}$$

Now, if I substitute this and $q = \frac{1-p}{2}$ into (A.37), I get that voters are willing to vote for the ambiguous candidate as long as

$$u_0 < \frac{2p}{1+p} = \frac{p}{p+q}.$$

Again, the candidate's inequality is more stringent, since

$$\frac{p-q}{p+q} = \frac{3p-1}{1+p} = \frac{2p}{1+p} - \frac{1-p}{1+p} < \frac{2p}{1+p} = \frac{p}{p+q}.$$

Notice, that due to the finite number of voters, the mixing probability above works only at the limit when $N \to \infty$ and yields only ε-equilibria for finite N with $\varepsilon \to 0$ as $N \to \infty$.[29] However, since the inaccuracy is only a result of the uncertainty in the voter behavior while the mixing is there to make the other candidate indifferent, due to the continuity of the payoff function, it is clear that for each N there exists x_N close to the x above that makes extreme candidates indifferent between $\{-1, 1\}$ and 0. ■

Appendix B. Extension with a potentially large number of states

This section shows that an ambiguous extreme candidate can win against a clear centrist also when the state space has a continuum of possible states. In

[29] There is a vanishingly small chance that there is, for example, a strict majority of type 1 voters in any of the states.

particular, the extension highlights the fact that the result is not dependent on the centrists being always "wrong".

Assume still that there are three possible voter and candidate types -1, 0 and 1, and the preferences of those types over the actions of the enacted policy are the ones given in the main text. Instead of having just two states, suppose that the state of the world is drawn from some set $S \subset \mathbb{R}$ according to an absolutely continuous probability distribution F with an associated density given by some f.[30] With slight abuse of notation, denote the associated random variable by S. Assume further that the conditional probability of drawing a voter or candidate of type -1 conditional on the state being equal to s is given by some measurable function $p\colon S \to [0,1]$ such that $\mathbb{P}(\tau = -1 \mid S = s) = p(s)$ while the probability of drawing the polar opposite type is given by another measurable function $q\colon S \to [0,1]$ satisfying $\mathbb{P}(\tau = 1 \mid S = s) = q(s)$ and $0 \leq p(s)+q(s) \leq 1$, (and hence the probability of drawing a centrist is simply $1 - p(s) - q(s)$). The types of different players are still drawn independently conditional on the state. To simplify the analysis and keep the model parsimonious, I will assume that drawing a centrist is uninformative about the relative probabilities of extremists. More precicely, assuming that τ_1 and τ_2 independent draws from the type distribution, I assume that

$$\mathbb{P}(\tau_1 = -1 \mid \tau_2 = 0, \tau_1 \in \{-1,1\}) = \mathbb{P}(\tau_1 = -1 \mid \tau_1 \in \{-1,1\})$$

and

$$\mathbb{P}(\tau_1 = 1 \mid \tau_2 = 0, \tau_1 \in \{-1,1\}) = \mathbb{P}(\tau_1 = 1 \mid \tau_1 \in \{-1,1\}).$$

This assumption is made so that extremist voters do not update their beliefs about the types of other extremists after seeing a centrist candidate. Apart from

[30]It is straightforward to verify that the following arguments go through without requiring absolute continuity. The notation however becomes more burdensome.

knife-edge cases, voters' actions will be decided by a strict preference. Hence, by continuity, even if centrists are slightly more common in states where the left-wingers are also more common (or vice versa), the main result does not change qualitatively.

Consider now a voter whose type is $\tau_v = 1$ and who observes a candidate committing to policy 0 and another candidate who remains ambiguous between policies -1 and 1. The beliefs this voter holds about the ambiguous candidate's type τ_c is then given by

$$\mathbb{P}(\tau_c = 1 \mid \tau_v = 1, \tau_c \in \{-1, 1\}) = \frac{\mathbb{P}(\{\tau_c = 1\} \cap \{\tau_v = 1\})}{\mathbb{P}(\{\tau_v = 1\} \cap \{\tau_c \in \{-1, 1\}\})}.$$

Now, using the law total probability I can write the numerator as

$$\begin{aligned} \mathbb{P}(\{\tau_c = 1\} \cap \{\tau_v = 1\}) &= \mathbb{P}(\tau_c = 1 \mid \tau_v = 1)\mathbb{P}(\tau_v = 1) \\ &= \int_S \mathbb{P}(\tau_c = 1 \mid \tau_v = 1, s)\mathbb{P}(\tau_v = 1 \mid s) f(s)\, \mathrm{d}s \\ &= \int_S q(s)^2 f(s)\, \mathrm{d}s. \end{aligned} \tag{B.1}$$

A similar calculation for the denominator yields

$$\mathbb{P}(\{\tau_v = 1\} \cap \{\tau_c \in \{-1, 1\}\}) = \int_S q(s)(p(s) + q(s)) f(s)\, \mathrm{d}s. \tag{B.2}$$

Combining (B.1) and (B.2) then yields that the probability that the ambiguous candidate matches the voter's type is given by

$$\mathbb{P}(\tau_c = 1 \mid \tau_v = 1, \tau_c \in \{-1, 1\}) = \frac{\int_S q(s)^2 f(s)\, \mathrm{d}s}{\int_S q(s)(p(s) + q(s)) f(s)\, \mathrm{d}s}. \tag{B.3}$$

The probability of mismatching the voter's type is then given by the complement event as

$$\mathbb{P}(\tau_c = -1 \mid \tau_v = 1, \tau_c \in \{-1, 1\}) = \frac{\int_S p(s)q(s) f(s)\, \mathrm{d}s}{\int_S q(s)(p(s) + q(s)) f(s)\, \mathrm{d}s}. \tag{B.4}$$

A right-wing voter who votes non-strategically maximizing her expected utility then (weakly) prefers the ambiguous extremist to a clear centrist, if and only if

$$\begin{aligned}\frac{\int_S q(s)^2 f(s)\ \mathrm{d}s}{\int_S q(s)(p(s)+q(s))f(s)\ \mathrm{d}s}\times 1+\frac{\int_S p(s)q(s)f(s)\ \mathrm{d}s}{\int_S q(s)(p(s)+q(s))f(s)\ \mathrm{d}s}\times 0 &\geq u_0\\ \Leftrightarrow \frac{\int_S q(s)^2 f(s)\ \mathrm{d}s}{\int_S q(s)(p(s)+q(s))f(s)\ \mathrm{d}s} &\geq u_0 \qquad (B.5)\end{aligned}$$

This inequality holds, if and only if

$$u_0\int_S p(s)q(s)f(s)\ \mathrm{d}s \leq (1-u_0)\int_S q(s)^2 f(s)\ \mathrm{d}s.$$

But this condition is equivalent with saying that

$$\mathbb{P}(\{\tau_c=1\}\cap\{\tau_v=1\}) \geq \frac{u_0}{1-u_0}\mathbb{P}(\{\tau_c=-1\}\cap\{\tau_v=1\}).$$

By symmetry, a left-wing candidate is willing to vote for the ambiguous extremist over a centrist, if and only if

$$u_0\int_S p(s)q(s)f(s)\ \mathrm{d}s \leq (1-u_0)\int_S p(s)^2 f(s)\ \mathrm{d}s,$$

or in the more interpretable form

$$\mathbb{P}(\{\tau_c=-1\}\cap\{\tau_v=-1\}) \geq \frac{u_0}{1-u_0}\mathbb{P}(\{\tau_c=-1\}\cap\{\tau_v=1\}).$$

In other words, both extremist require that the probability of drawing two individuals of the the same extreme type is at least $\frac{u_0}{1-u_0}$ times the probability of drawing two individuals of the opposing extreme type. For example, when $u_0=\frac{1}{2}$, this boils down to the probability of matching the voter's type being twice as high as drawing the opposing extreme type instead.

The result extends fairly easily also to strategic voting in large populations following the steps in the main model. The only slight difference here is that the large state space allows for states where the centrists always win in large electorates. This implies that strategic extremists will put a large weight on states where the total number of extreme voters is likely to be close to the number of centrist voters, because those are the states where their vote is likely to matter. As long as the prior in those states over the different extremists is relatively equal, a direct extension of the logic above conditioned on those states implies that extremist candidates will have an incentive to run on an ambiguous platform when running agains a centrist incumbent and extremist voters will vote for the ambiguous extremist. What matters for the extremists are their beliefs about the relative frequency of different extremists conditional on their own type and being in a state where the extremists together form a majority. In short, even when in most states centrists are the most numerous, as long as there is enough symmetry across the extremes, there exists an equilibrium where the extremists to run on an ambiguous platform and win with that platform whenever they form a joint majority.

DECLARATION OF INTERESTS:
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Declarations of interest: NONE