Published for SISSA by Springer

Received: September 11, 2024
Revised: October 22, 2024
Accepted: November 30, 2024
Published: December 27, 2024

Half-BPS Janus solutions in AdS_7

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Abstract: We study half-BPS flows in gauged minimal 7d supergravity featured by an $\mathrm{AdS}_3 \times S^3$ slicing of the metric, supported by a dyonic three-form field. We first present a novel strategy for analytic integration of the BPS equations, which makes use of the integrals of motion. Subsequently, we discuss the suitable choice of integration constants that gives rise to smooth geometries. These flows are asymptotically locally AdS_7 and close off smoothly reproducing an $\mathrm{AdS}_3 \times \mathbb{R}^4$ geometry. We then discuss their uplifts to 11d and massive IIA supergravity and observe that they describe one-parameter deformations of their $\mathrm{AdS}_7 \times S^4$ and $\mathrm{AdS}_7 \times S^3$ vacua, respectively, their holographic interpretation being as conformal defect CFT_2's within the corresponding dual SCFT_6's. We conclude with the computation of the holographic central charge, by focussing on the M-theory interpretation.

Keywords: AdS-CFT Correspondence, M-Theory, Supergravity Models, Superstring Vacua

ArXiv ePrint: 2407.21619

https://doi.org/10.1007/JHEP12(2024)198

Contents

1 Introduction

Since its very birth, the AdS/CFT correspondence has produced many important achievements, which have completely changed our understanding of gravity, information theory and strongly coupled field theories. Within a supersymmetric set-up, one may have control on both sides of the correspondence and calculating certain protected quantities may therefore provide precision tests for holography. In this context, producing supersymmetric supergravity solutions involving AdS is of utmost importance. In particular, AdS black holes play a crucial role in analysing CFT's at finite temperature. On the other hand, by means of the so-called domain wall (DW)/QFT correspondence [1], AdS DW geometries provide the holographic description of RG flows.

Another holographic application of supergravity is the investigation of conformal defect theories. The insertion of a conformal defect within a mother CFT, breaks conformal invariance in the bulk and only retains a lower-dimensional conformal symmetry, corresponding with the spacetime directions in which the defect is extended. Signatures of this broken higher-dimensional conformal symmetry may be, *e.g.* the presence of non-zero one-point bulk correlators. The holographic dual picture of a conformal defect is given by a lower-dimensional AdS slicing of the metric (see for instance [2, 3]), which locally asymptotes to a higher-dimensional AdS geometry. The prototypical realisation of this is the Janus solution [4, 5] describing a one-parameter deformation of the $\mathrm{AdS}_5 \times S^5$ vacuum of type IIB supergravity. These geometries describe AdS_4 $\frac{1}{2}$-BPS foliations of AdS_5 and are interpreted as the gravity duals of conformal defects within $\mathcal{N} = 4$ SYM_4.

On the other hand, when it comes to the microscopic description of conformal defects, one may follow the general construction of [2], where these are argued to stem from branes ending on other branes. In [2], the construction is sketched in a few different stringy settings and a lower-dimensional AdS-sliced geometry is shown to arise at least within a probe analysis, i.e. in a limit where the defect branes are treated as probes within the remaining background branes engineering the mother CFT in their near-horizon limit.

Now, unifiying these two observations within a fully controlled set-up has so far proven to be extremely challenging. Indeed, a significant amount of work on BPS AdS flows has been carried out and some interesting results are nowadays available (see *e.g.* [6–18]), many of which include key holographic tests. It may be worth noticing that most holographic calculations in this context involve ambiguities related to the choice of a regularisation scheme, which need to be resolved. This is the main challenge to face before one can draw physical conclusions concerning the dual conformal defect theory. However still, it is very hard to come up with a brane picture for these *holographic conformal defects* in the way envisioned by [2], i.e. exhibit a brane system whose fully backreacted geometry yields the aforementioned AdS-sliced geometry in a certain limit.

In this work, we focus our attention on SCFT_6's, whose gravity duals are AdS_7 vacua in 11d supergravity or in massive IIA supergravity. These 6d theories are particularly interesting because they generically lack a Lagrangian description and hence are intrinsically strongly coupled. Surface conformal defects in massive IIA were previously analysed in [15, 19–21], in cases where the defect theory enjoys *small supersymmetry*. Due to this peculiarity, one can say more about the brane construction of the defect, as these turn out to be partially smeared within the background of the mother branes. This simplifies the supergravity analysis substantially and allows for an analytic treatment. For solutions with *large supersymmetry*, this remains an open issue, which serves us as a motivation towards this problem.

This paper is organised as follows. We first review the 7d minimal supergravity set-up. Secondly, we introduce a detailed procedure for integrating the 7d BPS equations, which is based on reducing the amount of independent fields by making use of the integrals of motion for the given class of flows. After testing this on previously known solutions with small supersymmetry, we derive the most general solutions with large supersymmetry and discuss smoothness constraints on the integration constants. Subsequently, we lift them to 11d supergravity and spell out their relation to the classification of [22], and also lift them to massive IIA to get novel AdS_3 flows. Finally, we focus again on the M-theory picture and discuss the holographic central charge.

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2 The supergravity set-up

Minimal $\mathcal{N}=1$ supergravity in $d=7$ is defined by just the $\mathcal{N}=1$ supergravity multiplet in seven dimensions. The global isometry group is $\mathbb{R}^+ \times \mathrm{SO}(3)$, where the non-compact part is associated to the shifts of a scalar field X and the SO(3) part defines the R-symmetry. For the 7d field content we consider the graviton $g_{\mu\nu}$ and a 3-form $\mathcal{B}_3$, in addition to the real scalar X.[1] In what follows we will gauge some symmetries in the supergravity set-up. Specifically, these are controlled by two gauge couplings, g and h, and correspond to the gauging of the R-symmetry group and a Stückelberg coupling, giving a mass to the 3-form. For simplicity g and h are chosen such that they satisfy $h=\frac{g}{2\sqrt{2}}$.

The Lagrangian has the following form

$$\mathcal{L} = R_7 - 10\,X^{-2}(\partial_\mu X)^2 - \frac{1}{2}X^4 \star_7 \mathcal{F}_4 \wedge \mathcal{F}_4 - h\mathcal{F}_4 \wedge \mathcal{B}_3 - V(X)\,, \tag{2.1}$$

where $\mathcal{F}_4 = d\mathcal{B}_3$. The scalar potential can be expressed in terms of a superpotential as

$$V = \frac{4}{5}\left(-6f(X)^2 + X^2(\partial_X f)^2\right)\,. \tag{2.2}$$

Taking for instance the supersymmetric superpotential

$$f(X) = \frac{1}{2}\left(h\,X^{-4} + \sqrt{2}\,g\,X\right)\,, \tag{2.3}$$

the scalar potential reads

$$V = 2h^2X^{-8} - 4\sqrt{2}hgX^{-3} - 2g^2X^2\,. \tag{2.4}$$

The equations of motion for the scalar field and the 3-form are given by[2]

$$\begin{aligned} \partial_\mu\left(\sqrt{-g}\,X^{-1}g^{\mu\nu}\partial_\nu X\right) - \frac{\sqrt{-g}}{5}X^4\,\mathcal{F}_4^2 - \frac{\sqrt{-g}}{10}\,X\partial_X V = 0,\\ \partial_\mu\left(X^4\sqrt{-g}\,\mathcal{F}_4^{\mu\nu\rho\sigma}\right) + h\sqrt{-g}\,\mathcal{B}_3^{\nu\rho\sigma} = 0, \end{aligned} \tag{2.5}$$

while the 7d Einstein's equations have the form

$$R_{\mu\nu} - 5X^{-2}\partial_\mu X\partial_\nu X - \frac{1}{2}X^4\left(\mathcal{F}_{4\mu}\cdot\mathcal{F}_{4\nu} - \frac{3}{5}\mathcal{F}_4^2 g_{\mu\nu}\right) - \frac{1}{5}V\,g_{\mu\nu} = 0\,. \tag{2.6}$$

Importantly, due to the Stückelberg term in the 7d Lagrangian, the 3-form must respect the self-duality condition

$$X^4 \star_7 \mathcal{F}_4 = -2h\mathcal{B}_3\,. \tag{2.7}$$

This relation ensures that the 3-form has the right number of degrees of freedom on-shell.

The theory has a supersymmetric AdS_7 vacuum at $X=1$ and $\mathcal{B}_3=0$. This vacuum preserves 8 (complex) supercharges. Its uplift to eleven dimensions gives rise to the $\mathrm{AdS}_7\times S^4$ vacuum associated to a stack of M5 branes.

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[1] Strictly speaking, in the 7d supergravity multiplet there are also non-abelian vectors A^i. These are associated to possible gaugings of the R-symmetry. We will not consider these fields in our analysis.

[2] We use the notation $\mathcal{F}_k^2 = \frac{1}{k!}\mathcal{F}_{\mu_1\ldots\mu_k}\mathcal{F}^{\mu_1\cdots\mu_k}$ and $\mathcal{F}_{k\,\mu}\cdot\mathcal{F}_{k\,\nu} = \frac{1}{(k-1)!}\mathcal{F}_{k\,\mu\,\mu_2\ldots\mu_k}\mathcal{F}_{k\,\nu}{}^{\mu_2\cdots\mu_k}$.

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2.1 Warm up: charged domain walls

A class of 7d solutions that has been extensively studied in the literature is the class of so-called charged domain walls. These are $\mathcal{N}=(0,4)$ $\mathrm{AdS}_3\times S^3$ solutions fibered over a line, featured by a non-trivial polarisation of the 3-form within the 7d bulk. The holographic interpretation in string theory of these backgrounds has been deeply explored. Specifically, it has been shown that string dualities allow to relate these domain walls to backgrounds of the type $\mathrm{AdS}_3\times S^2$ [15, 21, 23, 24], $\mathrm{AdS}_2\times S^3$ [25, 26] and $\mathrm{AdS}_2\times S^2$ [17, 26, 27]. In this section we present a slightly more general charged domain wall solution of the type $\mathrm{AdS}_3\times S^3$ fibered over a line.

We consider the following 7d backgrounds

$$\begin{aligned} ds_7^2 &= e^{2U(x)}\left(ds^2_{\mathrm{AdS}_3}+ds^2_{S^3}\right)+e^{2V(x)}dx^2\,,\\ \mathcal{B}_3 &= b(x)\,\left(\mathrm{vol}_{\mathrm{AdS}_3}+\mathrm{vol}_{S^3}\right)\,,\\ X &= X(x)\,, \end{aligned}\tag{2.8}$$

where we take the radii of AdS_3 and S^3 to be equal and normalised to one. It can be shown that the Killing spinor associated to the above geometries has the standard form

$$\epsilon = Y(x)\,\epsilon_0\,,\tag{2.9}$$

where ϵ_0 is a constant spinor in 7d. The BPS equations were obtained in [28], and are given by

$$U'=\frac{2}{5}e^V f,\qquad X'=-\frac{2}{5}e^V X^2 D_X f,\qquad Y'=\frac{Y}{5}e^V f,\qquad b'=-\frac{2\,e^{2U+V}}{X^2}\,.\tag{2.10}$$

A crucial element in the integration of the BPS equations is the following gauge choice

$$\partial_x\left(e^{4(U+V)}X^3\right)=0.\tag{2.11}$$

If we now choose for f the BPS superpotential (2.3), the equations become exactly solvable and we find the solution

$$\begin{aligned} e^{2U} &= \frac{8x^{8/5}(C+x^4)^{1/10}}{g^2}\,, & e^{2V} &= \frac{8}{g^2x^{2/5}\left(C+x^4\right)^{2/5}}\\ X &= \frac{(C+x^4)^{1/5}}{x^{4/5}}\,, & b &= -\frac{16\sqrt{2}\sqrt{C+x^4}}{g^3}\,. \end{aligned}\tag{2.12}$$

This background breaks half of the 7d supersymmetries, preserving 4 supercharges that realise *small* $\mathcal{N}=(0,4)$ supersymmetry in three dimensions [19]. The solution depends on the tuning of the parameter C. In this sense it represents an extension of the charged domain wall solution presented in [28], which corresponds to the case $C<0$. This can be explicitely shown by observing that (2.12) matches with (4.23) in [28] if the relation $g^{3/2}=2^3 2^{1/4}\sqrt{-C}$ holds. On the contrary, for $C\geq 0$, one has $x\in[0,\infty)$ and the solutions (2.12) are new. Even if the local profile of the solution is similar, the extension to $C>0$ values is quite important for its stringy interpretation. In fact, as discussed in the massive IIA context [21], the branch with $C<0$ implies the presence of smeared orientifolds (ONS5 planes), while smeared orientifolds are absent in the $C>0$ case.

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3 A new AdS_3 Janus solution in AdS_7

In this section we construct new Janus solutions to the seven-dimensional theory introduced above, realised as $\mathrm{AdS}_3 \times S^3$ foliations over a line. We provide extra details to the integration of the BPS equations, which were firstly derived in [28]. The final result is a one-parameter family of solutions describing a continuous deformation of the AdS_7 geometry. These geometries are asymptotically locally AdS_7 and entirely smooth in the 7d bulk.

3.1 The AdS_3 geometry and the BPS equations

We start taking the following prescription for the seven-dimensional geometry

$$\begin{aligned} ds_7^2 &= e^{2U(r)} L^2 ds^2_{\mathrm{AdS}_3} + e^{2W(r)} \kappa^2 ds^2_{S^3} + e^{2V(r)} dr^2 \,, \\ \mathcal{B}_3 &= k(r) L^3 \mathrm{vol}_{\mathrm{AdS}_3} + l(r) \kappa^3 \mathrm{vol}_{S^3} \,, \\ X &= X(r) \,. \end{aligned} \tag{3.1}$$

Here we explicitly introduced the AdS_3 and S^3 radii, L and κ, respectively, since they will not be equal on-shell, in contrast with the simpler previous case. We consider a Killing spinor of the following form [28]

$$\epsilon = Y(r) \left[\cos\left(\frac{\theta(r)}{2}\right) \mathbb{1}_8 + \sin\left(\frac{\theta(r)}{2}\right) \gamma^{012} \right] \epsilon_0 \,, \tag{3.2}$$

where ϵ_0 is a constant spinor in 7d.[3] We point out that even if we introduced the new dynamical variable θ in the Killing spinor, the corresponding equations preserve the same amount of supersymmetries of the charged domain wall, namely four complex supercharges in seven dimensions. As we will see, these will be organised into *large* $\mathcal{N} = (0, 4)$ supersymmetry in the explicit solutions. As noticed in [28], where the consistent set of BPS equations for the above geometries was first derived, the $\theta = 0$ case corresponds to a solitonic branch of solutions in which $U = W$ and $k = l$. This situation corresponds to the charged domain walls discussed in the previous section.

[3] We refer to [28] for more details on the derivation of the BPS equations from the SUSY variations. In (3.2) ϵ_0 is a constant Dirac spinor in 7d, featured by 8 independent complex components. The flat basis is chosen as (e^m, e^3, e^i), with $m = 0, 1, 2$ and $i = 4, 5, 6$ associated to the AdS_3 and S^3 directions and e^3 to the radial direction. To obtain a consistent set of BPS equations one needs to impose a single projection condition along the radial direction, i.e. $\epsilon_0 = \gamma^3 \epsilon_0$.

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The BPS equations read [28]

$$U' = \frac{e^V}{25}\sec\theta\left((3\cos 2\theta + 7)\, f + 6\sin^2\theta X D_X f - \frac{10}{L}e^{-U}\sin\theta\right), \tag{3.3a}$$

$$W' = -\frac{e^V}{25}\sec\theta\left(2\,(\cos 2\theta - 6)\, f + 4\sin^2\theta X D_X f + \frac{10}{L}e^{-U}\sin\theta\right), \tag{3.3b}$$

$$Y' = \frac{Y e^V}{50}\sec\theta\left((3\cos 2\theta + 7)\, f + 6\sin^2\theta X D_X f - \frac{10}{L}e^{-U}\sin\theta\right), \tag{3.3c}$$

$$\theta' = -\frac{2}{5}e^V\sin\theta\,(f - X D_X f)\,, \tag{3.3d}$$

$$k' = \frac{e^{3U+V}}{5X^2}\left(2\tan\theta\,(2f + 3X D_X f) - \frac{10}{L}e^{-U}\sec\theta\right), \tag{3.3e}$$

$$l' = \frac{e^{3W+V}}{5X^2}\left(4\sin\theta\,(f - X D_X f) - \frac{10}{L}e^{-U}\right), \tag{3.3f}$$

$$X' = -\frac{e^V X}{25}\sec(\theta)\left(2\,(4+\cos 2\theta)\, X D_X f + 4\sin^2\theta f - \frac{10}{L}e^{-U}\sin\theta\right). \tag{3.3g}$$

In order to be consistent, the above equations must be supplemented by the following algebraic constraint,

$$\frac{2}{\kappa} - \frac{2}{L}\frac{e^{W-U}}{\cos\theta} + \frac{2}{5}e^W\tan\theta\,(X D_X f + 4f) = 0\,, \tag{3.4}$$

which is a dynamical relation linking the radii of the AdS_3 and S^3 factors. In what follows we will provide some details on the integration of the BPS equations.

3.2 Constructing the solutions

First, we notice that equations (3.3) imply the following algebraic constraints between the dynamical variables

$$\begin{aligned} Y &= Y_0\, e^{U/2}\,, \\ e^{U-W}\cos\theta &= C_1\,, \\ e^W\tan\theta\,(X\partial_X f + 4f) &= C_2\,, \end{aligned} \tag{3.5}$$

where Y_0, C_1, C_2 are constant parameters. Taking the derivative of the third relation it can be shown that the superpotential f must be the one given in (2.3). The particular solution presented in [28] is featured by the very specific choice $C_1 = 1$ and $C_2 = \frac{5g}{\sqrt{2}}$. The goal of this section is to show that this particular solution can be extended to an infinite class of AdS_3 solutions associated to parametric deformations of the AdS_7 vacuum.

To this aim we can rewrite the algebraic constraint (3.4) using the conditions (3.5). This leads to

$$\frac{1}{\kappa} - \frac{1}{LC_1} + \frac{C_2}{5} = 0\,. \tag{3.6}$$

In this way the dynamical constraint (3.4) takes a purely parametric form involving the radii of AdS_3 and S^3. To obtain the above expression we wrote the metric factors e^U and e^W as

$$e^U = \frac{\sqrt{2}\,C_1C_2}{5g}\,X^{-1}\sin^{-1}\theta \qquad \text{and} \qquad e^W = \frac{\sqrt{2}\,C_2}{5g}X^{-1}\cos\theta\sin^{-1}\theta\,. \tag{3.7}$$

It is quite natural at this point to trade the r coordinate with the function $\theta(r)$ associated to the spinor (3.2). We can use the BPS equation for θ in (3.3) for that purpose, and write the relation

$$e^V dr = -\frac{2\sqrt{2}}{g}\,\frac{X^4 d\theta}{\sin\theta}\,. \tag{3.8}$$

Using (3.8) we can now explicitly integrate the BPS equations in θ. Firstly, we evaluate (3.8) within the equation for X and integrate in θ. We obtain the following expression

$$X = \left(1 + \left(1 - \frac{10}{LC_1C_2}\right)\tan^2\theta + C_X\sin^2\theta\tan^2\theta\right)^{-1/5}\,. \tag{3.9}$$

Finally, using (3.9) and integrating in θ the equations for k and l, we obtain

$$\begin{aligned} k &= k_0 + \left(\frac{\sqrt{2}\,C_1C_2}{5g}\right)^3\left(4\sin^{-2}\theta\left(1 - \frac{10}{LC_1C_2}\right) + C_X\tan^2\theta\right)\,, \\ l &= l_0 + \left(\frac{\sqrt{2}\,C_2}{5g}\right)^3\left(\sin^{-2}\theta\left(1 - \frac{10}{LC_1C_2}\right) - C_X\sin^2\theta\right)\,, \end{aligned} \tag{3.10}$$

where C_X, k_0, l_0 are integration constants and the coordinate θ is defined over the interval $\theta \in [0, \frac{\pi}{2}]$.

3.3 Smooth deformations of the AdS$_7$ vacuum

We now show that for a sub-class of the solutions obtained in the previous section the 7d background is regular at any point. The explicit solution of the BPS equations (3.3) is given by

$$\begin{aligned} e^U &= \frac{\sqrt{2}\,C_1C_2}{5g}\,X^{-1}\sin^{-1}\theta\,, \\ e^W &= \frac{\sqrt{2}\,C_2}{5g}X^{-1}\cos\theta\sin^{-1}\theta\,, \\ e^V &= \frac{2\sqrt{2}}{g}X^4\sin^{-1}\theta\,, \\ X &= \left(1 + \left(1 - \frac{10}{LC_1C_2}\right)\tan^2\theta + C_X\sin^2\theta\tan^2\theta\right)^{-1/5}\,, \\ k &= k_0 + \left(\frac{\sqrt{2}\,C_1C_2}{5g}\right)^3\left(4\sin^{-2}2\theta\left(1 - \frac{10}{LC_1C_2}\right) + C_X\tan^2\theta\right)\,, \\ l &= l_0 + \left(\frac{\sqrt{2}\,C_2}{5g}\right)^3\left(\sin^{-2}\theta\left(1 - \frac{10}{LC_1C_2}\right) - C_X\sin^2\theta\right)\,. \end{aligned} \tag{3.11}$$

We observe that the 3-form must satisfy the self-duality condition (2.7), implying that $k_0 = 0$ and $l_0 = -2\left(\frac{\sqrt{2}C_2}{5g}\right)^3\left(1-\frac{10}{LC_1C_2}\right)$. It follows that the solution (3.11) depends on five parameters C_1, C_2, C_X, L, κ. These constants must satisfy the constraint (3.6). As we said the above solutions are defined over the interval $\theta \in [0, \frac{\pi}{2}]$. We may start considering the limit $\theta \to \frac{\pi}{2}$. In this limit we can write explicitly the behaviour of the scalar field

$$X^{-5} = \frac{C_X + 1 - \frac{10}{LC_1C_2}}{(\theta - \frac{\pi}{2})^2} + \frac{1}{3}\left(1 - 5C_X + \frac{20}{LC_1C_2}\right) + \mathcal{O}\left(\theta - \frac{\pi}{2}\right)^2 . \tag{3.12}$$

It is now immediate to see that if one imposes the relation

$$C_X = -1 + \frac{10}{LC_1C_2}\,, \tag{3.13}$$

the scalar field becomes regular. One can check that the same holds true for the scalar potential. In turn, we can expand the metric around $\theta = \frac{\pi}{2}$, obtaining a regular $\mathrm{AdS}_3 \times \mathbb{R}^4$ geometry

$$ds^2_7 \sim \left(\frac{\sqrt{2}\,LC_1C_2}{5g}\right)^2 X_0^{-2} ds^2_{\mathrm{AdS}_3} + \frac{8X_0^8}{g^2}\left(d\theta^2 + \left(\theta - \frac{\pi}{2}\right)^2 ds^2_{S^3}\right)\,, \tag{3.14}$$

where X_0 is the finite order value of the scalar field given in (3.12). Using the relation (3.13), this is given by $X_0^{-5} = 2 - \frac{10}{LC_1C_2}$. Finally, one can also check that the 4-form $\mathcal{F}_4 = d\mathcal{B}_3$ is regular in the $\theta \to \frac{\pi}{2}$ limit.
Introducing now the parameter

$$\lambda = 1 - \frac{10}{C_1C_2L}\,, \tag{3.15}$$

and using the algebraic contraint (3.6), we can rewrite the AdS_3 and S^3 radii as

$$L = \frac{10}{C_1C_2(1-\lambda)}\,, \qquad \kappa = -\frac{10}{C_2(1+\lambda)}\,. \tag{3.16}$$

The solution (3.11) takes then the following form

$$\begin{aligned} ds^2_7 &= \frac{8}{g^2X^2\sin^2\theta}\left(\frac{1}{(1-\lambda)^2}ds^2_{\mathrm{AdS}_3} + \frac{\cos^2\theta}{(1+\lambda)^2}ds^2_{S^3} + X^{10}d\theta^2\right)\,,\\ X &= \left(1+\lambda\sin^2\theta\right)^{-\frac{1}{5}}\,,\\ \mathcal{B}_3 &= \frac{16\sqrt{2}\,\lambda\,(1+\sin^2\theta)\sin^{-2}\theta}{g^3(1-\lambda)^3}\,\mathrm{vol}_{\mathrm{AdS}_3} - \frac{16\sqrt{2}\,\lambda\,\cos^4\theta\sin^{-2}\theta}{g^3(1+\lambda)^3}\mathrm{vol}_{S^3}\,. \end{aligned} \tag{3.17}$$

This represents a new family of solutions defined for $\lambda \in (-1, 1)$, which reduces to the global AdS_7 vacuum when $\lambda = 0$, parametrised as

$$ds^2_7 = \frac{8}{g^2}\sin^{-2}\theta\left(ds^2_{\mathrm{AdS}_3} + \cos^2\theta ds^2_{S^3} + d\theta^2\right). \tag{3.18}$$

We can then interpret the solutions as a one-parameter family of deformations of the AdS_7 vacuum geometry. As we showed above, this family of solutions is smooth in the $\theta \to \frac{\pi}{2}$

limit. At the other extrema $\theta \to 0$ the geometry is asymptotically locally AdS$_7$. More explicitly, in the $\theta \to 0$ limit we find

$$\begin{aligned} ds_7^2 &\sim \frac{8}{g^2\theta^2}\left(\frac{1}{(1-\lambda)^2}ds^2_{\mathrm{AdS}_3} + \frac{1}{(1+\lambda)^2}ds^2_{S^3} + d\theta^2\right), \\ X &= 1 - \frac{\lambda}{5}\,\theta^2 + \mathcal{O}(\theta^3), \end{aligned} \tag{3.19}$$

plus

$$\mathcal{F}_4 \sim -\frac{32\sqrt{2}\,\lambda}{g^3(1-\lambda)^3\,\theta^3}d\theta\wedge \mathrm{vol}_{\mathrm{AdS}_3} + \frac{32\sqrt{2}\,\lambda}{g^3(1+\lambda)^3\,\theta^3}d\theta\wedge\mathrm{vol}_{S^3}\,. \tag{3.20}$$

This reproduces locally the AdS$_7$ vacuum geometry, which means that the contributions associated to the flux are subleading in the equations of motion. This can be easily verified for the Einstein's equations by looking at the behaviour of the Ricci scalar in the $\theta \to 0$ limit,

$$R_7 = -\frac{21g^2}{4} + \mathcal{O}(\theta^4)\,, \tag{3.21}$$

whose leading term is, manifestly, the AdS$_7$ curvature. It can be further verified that in the $\theta \to 0$ limit all the other contributions to the equations of motion with the exception of the 7d vacuum energy are subleading. In particular, for the $|\mathcal{F}_4|^2$ term, the leading contribution is proportional to λ and goes to zero as $\mathcal{O}(\theta^4)$.

It is well-known that Janus solutions are good supergravity duals for defect CFTs. At this stage of the analysis, we can identify the defect locus holographically. First we go to the AdS$_7$ boundary, located at $\theta = 0$, and we choose the Poincaré coordinates for AdS$_3$

$$ds^2_{\mathrm{AdS}_3} = \frac{ds^2_{\mathbb{R}^{1,1}} + dz^2}{z^2}\,, \tag{3.22}$$

where $ds^2_{\mathbb{R}^{1,1}}$ is the metric over the 2d Minkowski slices. Next we go to the AdS$_3$ boundary, located at $z \to 0$. One can see that the 7d metric (3.19) can be written as follows

$$ds_7^2 \sim \frac{8}{g^2\theta^2 z^2}\left[ds^2_{\mathbb{R}^{1,1}} + (1-\lambda)^{-2}\left(dz^2 + \left(\frac{1-\lambda}{1+\lambda}\right)^2 z^2 ds^2_{S^3}\right) + z^2 d\theta^2\right], \tag{3.23}$$

where we rescaled the Minkowski metric by a factor $(1-\lambda)^2$. First we note that for $\lambda = 0$ we recover the AdS$_7$ global geometry written in the conformally flat coordinates on AdS$_3 \times S^3$. If $\lambda \neq 0$ we observe a conical singularity at the origin of the $\mathbb{R}^4$ space parametrised by (z, S^3). We point out that the presence of this singularity is crucially related to the fact that AdS$_3$ and S^3 have different radii. More explicitly, the deficit angle is given by $\delta = \frac{4\pi\lambda}{1+\lambda}$, which must be defined in $(0, 2\pi)$.

From the previous analysis we conclude that when $\lambda \neq 0$, the AdS$_7$ isometries are broken and the resulting dual CFT becomes two-dimensional. From (3.23) it is manifest that the z-direction describes the physical distance of an observer from the defect (at least in the $\theta \to 0$ limit). As far as the θ-direction is regarded, it describes the embedding of the 2d degrees of freedom of the defect within the "mother" SCFT$_6$. More specifically, from the point

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of view of the 6d theory, the 2d defect can be viewed as a position-dependent deformation parametrised by the θ-direction. In this sense, moving towards the $\theta \to \frac{\pi}{2}$ limit, which is where the space closes off smoothly, might correspond to an RG flow towards a 2d CFT. In order to develop a more quantitative analysis on this point, it might be useful to construct the full Minkowski-sliced solution in (θ, z). This is however far beyond the scope of this work.

4 The Janus solution in M-theory

In this section we study the M-theory uplift of the 7d Janus solution found in the previous section. We present the explicit 11d geometry obtained by using the uplift formulas in [29], to then show that it belongs to the general classification of $\mathrm{AdS}_3 \times S^3 \times S^3 \times \Sigma_2$ solutions to M-theory of [8–10, 22, 30, 31], preserving *large* $\mathcal{N} = (4,4)$ supersymmetry. Moreover, exploiting the recent insights of [32], we relate our Janus solution to the 7d charged domain wall discussed in section 2.1, which preserves *small* $\mathcal{N} = (4,4)$ supersymmetry. As in [32] the class of *small* solutions is obtained as a particular limit of the class of *large* solutions.

Finally, we compute the holographic central charge. The naive derivation of this quantity exhibits a divergent behaviour close to the AdS_7 boundary, which needs to be regularised within a regularisation scheme that explicitly depends on λ [33]. We propose a concrete strategy to impose a cut-off over the non-compact direction, and obtain a central charge which is monotonic and well-behaved in λ.

4.1 The M-theory uplift

Minimal 7d supergravity can be obtained via consistent truncation in various ways. We will consider here the case of M-theory reductions over 4-spheres [29]. The 11d metric can be written as

$$\begin{aligned} ds^2_{11} &= \Delta^{1/3}\, ds^2_7 + 2g^{-2}\, \Delta^{-2/3}\, ds^2_4\,, \\ ds^2_4 &= X^3\, \Delta\, d\xi^2 + X^{-1}\, c^2\, ds^2_{S^3}\,, \end{aligned} \tag{4.1}$$

where $\Delta = X\, c^2 + X^{-4}\, s^2$ with $c = \cos\xi$ and $s = \sin\xi$. The 11d 4-flux takes the form

$$\begin{aligned} G_4 = &\frac{4}{\sqrt{2}}\, g^{-3}\, c^3\, \Delta^{-2}\, W\, d\xi \wedge \mathrm{vol}_{S^3} + \frac{20}{\sqrt{2}}\, g^{-3}\, \Delta^{-2}\, X^{-4}\, s\, c^4\, dX \wedge \mathrm{vol}_{S^3} \\ &- s\, \mathcal{F}_{(4)} - \sqrt{2}\, g^{-1} c\, X^4 \star_7 \mathcal{F}_{(4)} \wedge d\xi, \end{aligned} \tag{4.2}$$

where $W = X^{-8}\, s^2 - 2X^2\, c^2 + 3\, X^{-3}\, c^2 - 4\, X^{-3}$ and we imposed the relation $h = \frac{g}{2\sqrt{2}}$ between the 7d gauge parameters. Using these expressions we can rewrite the 7d AdS_7 vacuum as

$$\begin{aligned} ds^2_{11} &= ds_{\mathrm{AdS}_7} + 2g^{-2}\, ds^2_{S^4}\,, \\ G_4 &= 6\sqrt{2} g^{-3} c^3\, d\xi \wedge \mathrm{vol}_{S^3}\,, \end{aligned} \tag{4.3}$$

where the radius of AdS_7 is expressed in terms of the 7d coupling as $L^2_{\mathrm{AdS}_7} = 8\, g^{-2}$. This geometry is the $\mathrm{AdS}_7 \times S^4$ Freund-Rubin vacuum associated to a stack of M5 branes.

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Using expressions (4.1) and (4.2) we can now uplift the 7d solution (3.17), to obtain

$$\begin{aligned} ds_{11}^2 &= \frac{8\,\Delta^{1/3}}{g^2\,X^2\sin^2\theta}\left(\frac{1}{(1-\lambda)^2}ds^2_{\mathrm{AdS}_3}+\frac{\cos^2\theta}{(1+\lambda)^2}ds^2_{S^3}+X^{10}d\theta^2\right)\\ &\quad+\frac{2\Delta^{-2/3}}{g^2}\left(X^3\Delta d\xi^2+X^{-1}c^2 ds^2_{\tilde{S}^3}\right),\\ G_4 &= \frac{4}{\sqrt{2}}\,g^{-3}\,c^3\,\Delta^{-2}\,W\,d\xi\,\wedge\,\mathrm{vol}_{\tilde{S}^3}+\frac{20}{\sqrt{2}}\,g^{-3}\,\Delta^{-2}\,X^{-4}\,s\,c^4\,dX\,\wedge\,\mathrm{vol}_{\tilde{S}^3}-d(\sin\xi\mathcal{B}_3)\,, \end{aligned} \tag{4.4}$$

where to avoid confusion in the notation we renamed $\tilde{S}^3$ the internal 3-sphere in (4.1), (4.2). We recall that $X=(1+\lambda\sin^2\theta)^{-\frac{1}{5}}$. The explicit form of $\mathcal{B}_3$ is given in (3.17).

The above backgrounds are $\mathrm{AdS}_3\times S^3\times\tilde{S}^3\times\Sigma_2$ foliations over a 2d Riemann surface Σ_2, parametrised by the coordinates (θ,ξ). It is easy to see that for $\theta\to 0,\pi$ they reproduce locally the $\mathrm{AdS}_7\times S^4$ M-theory vacuum. Moreover, the solutions are entirely regular at any value of θ and describe the global $\mathrm{AdS}_7\times S^4$ vacuum when $\lambda=0$. Our analysis in the next section shows that they preserve large $\mathcal{N}=(4,4)$ supersymmetry.

4.2 The 7d Janus solution as a $\mathcal{N}=(4,4)$ solution to M-theory

The space of $\mathrm{AdS}_3\times S^3\times S^3\times\Sigma_2$ solutions to M-theory has been extensively explored in the literature. Of particular relevance for our work are the classifications of solutions with *large* $\mathcal{N}=(4,4)$ supersymmetry [8–10, 22, 30, 31]. Specifically, the existence of Janus solutions within $\mathrm{AdS}_7\times S^4$ vacua in M-theory has been discussed in [8, 9, 30, 31].

In this section we show that our Janus solutions in seven dimensions lie in the aforementioned classifications. We focus on the results in [31], where the existing classifications of $\mathrm{AdS}_3\times S^3\times S^3\times\Sigma_2$ backgrounds were extended to a broader class of solutions specified by a reduced set of data consisting on two functions over Σ_2 called $\hat{h},\mathcal{G}$, and a real parameter[4] γ. The two functions satisfy the following differential equations

$$\partial_{\bar{w}}\partial_w\hat{h}=0,\qquad\qquad \partial_w\mathcal{G}=\frac{1}{2}(\mathcal{G}+\bar{\mathcal{G}})\partial_w\ln\hat{h}\,, \tag{4.5}$$

where $w,\bar{w}$ are complex coordinates over the Riemann surface. The 11d metric can be entirely expressed in terms of the above quantities, as follows. We start with the general prescription

$$ds_{11}^2=f^2_{\mathrm{AdS}_3}ds^2_{\mathrm{AdS}_3}+f^2_{S^3}ds^2_{S^3}+f^2_{\tilde{S}^3}ds^2_{\tilde{S}^3}+f^2_{\Sigma_2}ds^2_{\Sigma_2}, \tag{4.6}$$

where the warping functions depend only on the coordinates on the Riemann surface and $ds^2_{\Sigma_2}=dwd\bar{w}$. We now introduce the two combinations [30, 31]

$$W_\pm=|\mathcal{G}\pm i|^2+\gamma^{\pm1}(\mathcal{G}\bar{\mathcal{G}}-1), \tag{4.7}$$

[4]This parameter is associated to the isometry superalgebra $D(2,1;\gamma)\oplus D(2,1;\gamma)$ [31].

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from which the warping functions can be expressed as [8–10, 22, 30, 31]

$$\begin{aligned} f^6_{\mathrm{AdS}_3} &= \frac{\hat{h}^2 W_+ W_-}{\beta_1^6\,(\mathcal{G}\bar{\mathcal{G}}-1)^2}\,, & f^6_{\Sigma_2} &= \frac{|\partial_w \hat{h}|^6}{\beta_2^3\beta_3^3\,\hat{h}^4}(\mathcal{G}\bar{\mathcal{G}}-1)W_+W_-\,, \\ f^6_{\mathrm{S}^3} &= \frac{\hat{h}^2(\mathcal{G}\bar{\mathcal{G}}-1)W_-}{\beta_2^3\beta_3^3\,W_+^2}\,, & f^6_{\tilde{\mathrm{S}}^3} &= \frac{\hat{h}^2(\mathcal{G}\bar{\mathcal{G}}-1)W_+}{\beta_2^3\beta_3^3\,W_-^2}\,. \end{aligned} \tag{4.8}$$

As far as the fluxes are concerned they can also be obtained in terms of the quantities $(\hat{h},\,\mathcal{G},\gamma)$. We refer to [31] for their explicit expressions. The parameters β_1,β_2,β_3 are real parameters such that $\beta_1+\beta_2+\beta_3=0$ and $\gamma=\beta_2/\beta_3$. Introducing real coordinates over the Riemann surface $w=x_1+ix_2$, one can directly relate the uplift (4.4) of our 7d solution to the classification presented above. In particular, performing the following change of coordinates within the Riemann surface,

$$x_1 = 2\,\mathrm{arctanh}\left(\frac{\cos\theta}{\sqrt{1+\lambda\sin^2\theta}}\right) \qquad \text{and} \qquad x_2=\xi\,, \tag{4.9}$$

one can derive the particular form of the functions $\hat{h}$ and $\mathcal{G}$ associated to (4.4). These are given by

$$\hat{h} = \frac{\sqrt{2}(\lambda+1)}{g^3}\sinh x_1\cos x_2 \qquad \text{and} \qquad \mathcal{G} = i\left(2\cos x_2\,\mathrm{sech}(x_1-ix_2)-1\right), \tag{4.10}$$

where $x_1\in[0,+\infty)$ and $x_2\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$. We point out that the 7d parameters must be expressed in terms of the 11d ones in the specific form

$$\beta_1^2 = \frac{1-\lambda}{2}\,, \qquad \beta_2^2=\frac{1+\lambda}{2}\,, \qquad \beta_3^2=1\,, \tag{4.11}$$

leading to the following value of γ,

$$\gamma = \frac{\beta_2}{\beta_3} = -\frac{1+\lambda}{2}\,. \tag{4.12}$$

From this analysis we conclude that the 11d uplift of our Janus solution (4.4) belongs to the general classification in [31] and therefore preserves *large* $\mathcal{N}=(4,4)$ supersymmetry.

An interesting aspect of the solutions with *large* $\mathcal{N}=(4,4)$ supersymmetry is that they give rise to similar classes of $\mathrm{AdS}_3\times S^3\times S^3\times\Sigma_2$ solutions with *small* $\mathcal{N}=(4,4)$ supersymmetry for certain values of the γ-parameter. An interesting example was presented in [32], using as starting point the explicit backgrounds in [15, 28]. In what follows we apply the same procedure to our Janus solution (4.4), to obtain a class of solutions with *small* supersymmetry. More precisely, following [32] our goal is to compute the $\gamma\to-\infty$ limit of the Janus solution (4.4).

We start this analysis by rescaling the 7d gauge coupling as $g\to g\,(1+\lambda)^{1/3}$. With this rescaling the explicit solution for $\hat{h}$ (4.10) does not depends on γ, namely

$$\hat{h} = \frac{\sqrt{2}}{g^3}\sinh x_1\cos x_2\,. \tag{4.13}$$

It is useful to cast the function $\mathcal{G}$ in the form [32]

$$\mathcal{G} = -i\left(1+\gamma^{-1}\mathcal{F}\right)\,, \tag{4.14}$$

where $\mathcal{F}$ is a complex function. To find its explicit expression we can make use of the invariance of (4.5) under $\mathcal{G} \to \mathcal{G} = ia+b\,\mathcal{G}$. If we choose $a = -\frac{p}{\gamma}-1$ and $b = -\frac{p}{\gamma}$, from (4.10) we can write

$$\mathcal{F} = \frac{2p\cos x_2}{\cosh(x_1 - ix_2)}, \tag{4.15}$$

where p is a constant. We are now ready to compute the limit $\gamma \to -\infty$ of our Janus solution (4.4). A key property we need to verify is that our solution is compatible with backgrounds of the type

$$ds^2_{11} = h_1 H^{-1/3}\left(ds^2_{\mathrm{AdS}_3} + ds^2_{S^3}\right) + H^{2/3}\left(dz^2 + d\rho^2 ds^2_{\tilde{S}^3}\right)\,, \tag{4.16}$$

where a new system of coordinates for the 11d background $\{z, \rho\}$ has been introduced. This parametrisation was introduced in [15] to provide the brane picture underlying the 7d charged domain walls in [28]. One can show that our solution can be expressed in the above form through the map

$$u = i\sinh w, \qquad z = \mathrm{Re}(u) = -\cosh x_1 \sin x_2, \qquad \rho = \mathrm{Im}(u) = \sinh x_1 \cos x_2\,. \tag{4.17}$$

After this coordinate transformation, the master equation for $\mathcal{G}$, (4.5), is still solved. This can be shown explicitly using Cauchy-Riemann equations over Σ_2

$$\partial_{x_1} z = \partial_{x_2}\rho, \qquad \partial_{x_1}\rho = -\partial_{x_2} z.$$

Further, we can split $\mathcal{F}$, given in (4.15), into real and imaginary parts as follows

$$\mathcal{F} = \frac{2}{h_1}\sinh^2 x_1 \cos^2 x_2 H + iF_I, \tag{4.18}$$

and rewrite our solution as

$$\begin{aligned} ds^2 &= \frac{h_1}{H^{1/3}}\left(ds^2_{\mathrm{AdS}_3} + ds^2_{S^3}\right) + H^{2/3}\left[\frac{\cos 2x_2 + \cosh 2x_1}{2}(dx_1^2 + dx_2^2) + \sinh^2 x_1 \cos^2 x_2\, ds^2_{\tilde{S}^3}\right], \\ G_4 &= d(b_{\mathrm{AdS}_3}\,\mathrm{vol}_{\mathrm{AdS}_3}) + d(b_{S^3}\,\mathrm{vol}_{S^3}) + d(b_{\tilde{S}^3}\,\mathrm{vol}_{\tilde{S}^3}), \end{aligned} \tag{4.19}$$

where the function H is given by

$$H = \frac{2h_1 p \coth x_1 \mathrm{csch} x_1}{\cosh 2x_1 + \cos 2x_2} \qquad \text{with} \qquad h_1 = \frac{4\sqrt{2}(\lambda+1)}{g^3(1-\lambda)^3}. \tag{4.20}$$

The functions $b_{\mathrm{AdS}_3}, b_{S^3}, b_{\tilde{S}^3}$ associated to the 4-flux have the form

$$b_{\mathrm{AdS}_3} = b_{\mathrm{S}^3} = -2h_1 \cosh x_1 \sin x_2, \qquad b_{\tilde{\mathrm{S}}^3} = \frac{1}{2}h_1 \sinh x_1 \cos x_2 F_I + \hat{\Phi}, \tag{4.21}$$

with

$$F_I = 2p\frac{\sinh x_1 \sin 2x_2}{\cosh 2x_1 + \cos 2x_2} \qquad \text{and} \qquad \hat{\Phi} = 2h_1 p \sin x_2 .$$

The function H satisfies the following master equation

$$\partial^2_{x_2} H + \partial^2_{x_1} H - 3\tan x_2 \partial_{x_2} H + 3\coth x_1 \partial_{x_1} H = 0\,, \tag{4.22}$$

where the coordinates are defined within the intervals $x_1 \in [0, +\infty)$ and $x_2 \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.

We conclude this section by observing that in the $x_1 \to 0$ limit, the background (4.19) takes the form of a stack of smeared M5-branes with metric

$$ds^2 = h_1^{2/3} f(x_2) \left(p^{-1/3} H_5^{-1/3} ds^2_{\mathbb{R}^{1,5}} + H_5^{2/3} p^{2/3} (dx_1^2 + dx_2^2 + x_1^2 ds^2_{\tilde{S}^3}) \right),$$

where

$$H_5 = \frac{1}{x_1^2} \qquad \text{and} \qquad f(x_2) = \cos^{2/3} x_2 .$$

In the opposite limit $x_1 \to +\infty$, we may rename $x_1 = \log r$ and write,

$$ds^2 = h_1^{2/3} \left[\left(\frac{r}{p^{1/3}} \, ds^2_{\mathbb{R}^{1,5}} + 2\, p^{2/3} \frac{dr^2}{r^2} \right) + 2\, p^{2/3} \left(dx_2^2 + \cos^2 x_2 ds^2_{\tilde{S}^3} \right) \right],$$

which is the metric of the $\mathrm{AdS}_7 \times S^4$ vacuum. Therefore, applying the recipe of [32] we have extracted the 11d geometry described by equation (4.19) from the solution (4.4), preserving *large* $\mathcal{N} = (4,4)$ supersymmetry. The background (4.19) preserves *small* $\mathcal{N} = (4,4)$ supersymmetry and is equivalent to the uplift of the charged domain wall (2.12), obtained using the formulas in (4.1). More precisely, the coordinate transformation that shows the equivalence between the backgrounds (4.19) and the uplift of (2.12) is given by

$$x = C^{1/4} \sqrt{\sinh x_1}, \qquad \xi = x_2, \qquad g = \frac{2\, 2^{1/6} C^{1/6}}{h_1^{1/3}}, \qquad p = \frac{1}{4\sqrt{C}}. \tag{4.23}$$

We point out that the above change of coordinates is well-defined only for $C > 0$ (see the discussion at the end of section (2.1)).

4.3 The holographic central charge

In this section we discuss the derivation of the holographic central charge for the 11d Janus solution given by (4.4). We follow the general prescription of [34–36]. To this aim we need to cast our 11d metric (4.4) in the following form

$$ds^2_{11} = a(\zeta, \theta^i)(dx^2_{\mathbb{R}^{1,1}} + b(\zeta) d\zeta^2) + g_{ij}(\zeta, \theta^i) d\theta^i d\theta^j\,, \tag{4.24}$$

where the coordinates θ^i parametrise the 7d internal space $g_{ij} d\theta^i d\theta^j$. In (4.24) the AdS_3 geometry has been written as a foliation of a 2d Minkowski space $\mathbb{R}^{1,1}$ with radial coordinate ζ. The functions $a(\zeta, \theta^i)$ and $b(\zeta)$ are then given by

$$a(\zeta, \theta^i) = \frac{8\, \Delta^{1/3}}{g^2\, X^2 \sin^2\theta} \frac{\zeta^2}{(1-\lambda)^2}\,, \qquad b(\zeta) = \zeta^{-4}\,. \tag{4.25}$$

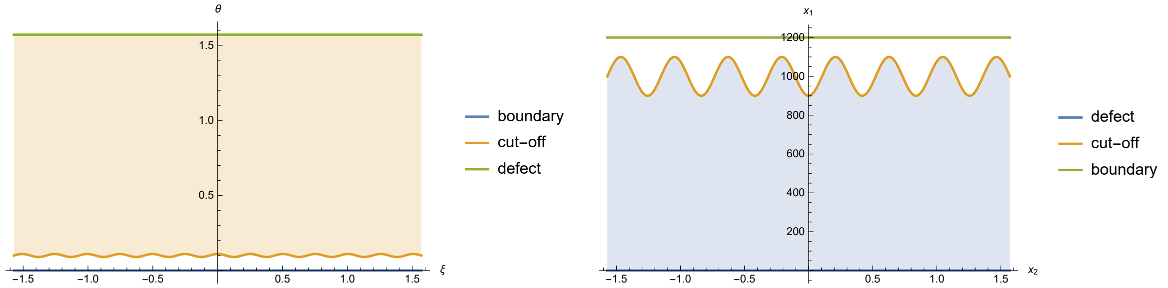

Figure 1. Global structure of the Riemann surface Σ_2. On the left-hand side the Riemann surface is parametrised by (θ, ξ), while in the right-hand side it is parametrised by conformally flat coordinates, (x_1, x_2). The wiggled lines generically denote an x_2 dependent regularisation scheme.

The holographic central charge has the general form [34–36]

$$c_{\text{hol}} = \frac{3}{G_N}\frac{b(\zeta)^{1/2}(\hat{H})^{\frac{3}{2}}}{\hat{H}'} \qquad \text{with} \qquad \hat{H}^{1/2} = \int d\theta^i \sqrt{\det[g_{ij}]}\, a(\zeta, \theta^i)^{1/2}, \tag{4.26}$$

and $\hat{H}'$ the derivative of $\hat{H}$ w.r.t. ζ. G_N is the 11d Newton's constant. Substituting the value of $a(\zeta, \theta^i)$ and the internal 7d metric obtained from (4.4) we arrive at the following expression for the holographic central charge

$$c_{\text{hol}} = \frac{3}{32 G_N\, h^9\, |\lambda - 1|(\lambda+1)^3} \int_0^{\frac{\pi}{2}} d\theta \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\xi \, \cos^3\xi \, \cot^3\theta \sin^{-2}\theta\,, \tag{4.27}$$

where for simplicity of notation we have used the gauge parameter $h = \frac{g}{2\sqrt{2}}$. We immediately observe that the above integral does not converge. Indeed, the function $\cot^3\theta \sin^{-2}\theta$ scales as θ^{-5} in the AdS_7 limit, $\theta \to 0$.

As already mentioned, the solution (4.4) is a $\text{AdS}_3 \times S^3 \times S^3 \times \Sigma_2$ background, with Riemann surface enclosed between the intervals $\theta \in [0, \frac{\pi}{2}]$ and $\xi \in [-\frac{\pi}{2}, \frac{\pi}{2}]$. The AdS_7 boundary is the segment $\theta = 0$, while the defects are located at the opposite segment $\theta = \frac{\pi}{2}$. Going to the conformally flat coordinates (x_1, x_2) introduced in (4.9) the Riemann surface is the strip defined by $x_1 \in [0, +\infty)$ and $x_2 \in [-\frac{\pi}{2}, \frac{\pi}{2}]$. This is depicted in figure 1. Using the inverse transformation of (4.9),

$$\cos\theta = \frac{(1+\lambda)^{1/2}\tanh\left(\frac{x_1}{2}\right)}{\left(1+\lambda\tanh^2\left(\frac{x_1}{2}\right)\right)^{1/2}} \qquad \text{and} \qquad \xi = x_2\,, \tag{4.28}$$

we can then rewrite (4.27) as

$$c_{\text{hol}} = \frac{3}{64 G_N\, h^9}\frac{1+\lambda}{|1-\lambda|}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dx_2 \, \cos^3 x_2 \int_0^{\infty} dx_1 \, \sinh^3\frac{x_1}{2}\cosh\frac{x_1}{2}. \tag{4.29}$$

In these coordinates the divergence at the AdS_7 boundary arises at $x_1 \to +\infty$. Following the prescription in [33] we introduce a cut-off Λ in this direction defined through

$$\cosh\Lambda = \frac{\alpha_0(x_2)}{\varepsilon^2} + \alpha_1(x_2) + \alpha_2(x_2)\varepsilon^2 + \cdots\,, \tag{4.30}$$

where α_i are scheme-dependent functions. Upon choosing e.g. constant α_i's, one obtains

$$c_{\rm hol} = \frac{1}{32 G_N\, h^9} \frac{1+\lambda}{|1-\lambda|} \sinh^4 \frac{\Lambda}{2} \,. \tag{4.31}$$

However, we will simply extract from (4.31) the contribution of the defects to the central charge, and show that this gives a positive monotonic function increasing within the interval $0 < \lambda < 1$. The contribution of the AdS$_7$ boundary is simply obtained by imposing $\lambda = 0$ in the previous result. This gives

$$c_{\rm hol}^{6d} = \frac{1}{32 G_N\, h^9} \sinh^4 \frac{\Lambda}{2} = \frac{1}{\pi^4}\, N_5^3 \sinh^4 \frac{\Lambda}{2} \,, \tag{4.32}$$

where we have rewritten the expression in terms of the M5-brane charge, N_5, using the standard relation $G_N = 16\pi^7 \ell_{11}^9$ together with $h^{-3} = 8\pi \ell_{11}^3 N_5$, with $N_5 = \frac{1}{(2\pi\ell_{11})^3} \int_{S^4} G_4$ and G_4 given by (4.3). This gives the right scaling of the 6d CFT dual to the AdS$_7 \times S^4$ vacuum. Finally, the contribution of the defects to the central charge is given by

$$c_{\rm hol}^{\rm def} = \frac{2}{\pi^4} \frac{\lambda}{|1-\lambda|}\, N_5^3 \sinh^4 \frac{\Lambda}{2} \,. \tag{4.33}$$

We would like to finish this section with a short remark. The central charge for the solutions preserving *small* $\mathcal{N} = (4,4)$ supersymmetry can be obtained from the result for the solutions with *large* $\mathcal{N} = (4,4)$ supersymmetry, given by (4.31), taking the $\lambda \to \infty$ limit, as discussed in the previous section. This gives rise to expression (4.32), namely, to the same central charge as the 6d mother theory. This is an interesting result that we further interpret in our paper [41], where we discuss the physical interpretation of this class of solutions.

Another interesting holographic observable is the regularised on-shell action. Upon imposing the local equations of motion, the whole action may be rewritten as a boundary term, up to a bulk piece coming from the 11d Chern-Simons (CS) action. The boundary piece receives contributions both from the bulk action and from the GHY boundary term. However, in our case, we do have non-trivial contributions from the CS term, due to the three independent pieces of the 3-form appearing in the 11d solution. This, in turn, causes the appearence of *logarithmic divergences* in addition to the usual poles to be subtracted. These log-terms are associated with gravitational anomalies of the 6d dual field theory. In order to overcome this issue and produce a well-defined regularised on-shell action, we would need to perform a holographic renormalisation analysis, in the spirit of [37].

5 The Janus solution in massive IIA

Minimal 7d supergravity is also employed in the literature as a consistent truncation of massive IIA supergravity [38]. This dimensional reduction is performed over an internal manifold M_3 that is topologically a 3-sphere and locally a fibration of a 2-sphere over an interval.

Following the same logic of section 4, in this section we write the uplift of our 7d Janus solution (3.17) to massive Type IIA string theory. This procedure leads to a new class of

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AdS$_3$ solutions in massive IIA. We start by recalling the reduction formulas for the NS-NS sector. The 10d metric is decomposed as [38]

$$\ell^{-1}ds^2_{10} = \frac{1}{8}g^2 X^{-\frac{1}{2}}e^{2A}ds^2_7 + X^{\frac{5}{2}}ds^2_{M_3}\,, \qquad ds^2_{M_3} = \frac{3^8}{2^8}\frac{e^{6A}}{\alpha^2}\pi^2 q^2 dz^2 + \frac{1-x^2}{16w}e^{2A}ds^2_{S^2}\,, \tag{5.1}$$

where $w \equiv X^5(1-x^2)+x^2$, ℓ is the AdS$_7$ radius and g is the 7d coupling. As we said, the internal geometry is topologically equivalent to a 3-sphere, and locally it reproduces the fibration of an S^2 over an interval, parametrised by the z coordinate. X is the 7d scalar field, and is such that $X=1$ at the AdS$_7$ vacuum. As for the M-theory truncation, these reduction formulas are defined for the 7d gauge couplings h and g such that $h=\frac{g}{2\sqrt{2}}$.

A crucial feature of this truncation is the possibility to express all the relevant quantities describing the internal geometry in terms of a single function $\alpha(z)$ [39] such that

$$\dddot{\alpha} = -162\pi^3 F_0\,, \tag{5.2}$$

where the dot denotes the differentiation with respect to z and F_0 is the Romans mass. Specifically, the truncation is described by the following functions

$$\begin{aligned} e^A &= 2^{\frac{7}{4}}\sqrt{\pi}\left(-\frac{\alpha}{\ddot{\alpha}}\right)^{\frac{1}{4}}, \quad & e^{\phi} &= \frac{2^{\frac{5}{4}}3^4\pi^{\frac{5}{2}}}{\sqrt{\dot{\alpha}^2-2\alpha\ddot{\alpha}}}\left(-\frac{\alpha}{\ddot{\alpha}}\right)^{\frac{3}{4}}, \\ x^2 &= \frac{\dot{\alpha}^2}{\dot{\alpha}^2-2\ddot{\alpha}\alpha}, & q &= \frac{1}{4}e^{A-\phi}\sqrt{1-x^2} = -\frac{\ddot{\alpha}}{2\pi^2 3^4}. \end{aligned} \tag{5.3}$$

The 10d dilaton Φ and the B_2-field are given by [38]

$$e^{\Phi} = \frac{2^{\frac{5}{4}}3^4\pi^{5/2}}{\sqrt{\ell}}\frac{X^{5/4}}{(\dot{\alpha}^2-2\alpha\ddot{\alpha}X^5)^{1/2}}\left(-\frac{\alpha}{\ddot{\alpha}}\right)^{3/4} \qquad \text{and} \qquad B_2 = \ell\pi\left(-z+\frac{\alpha\dot{\alpha}}{\dot{\alpha}^2-2\alpha\ddot{\alpha}X^5}\right)\mathrm{vol}_{S^2}\,. \tag{5.4}$$

Using the functions given in (5.3), one can write the R-R sector,

$$\ell^{-1}F_2 = -q\,\mathrm{vol}_{S^2} + \frac{1}{16}w^{-1}F_0\,e^{2A}x\sqrt{1-x^2}\mathrm{vol}_2\,, \tag{5.5a}$$

$$\ell^{-1}F_4 = -\ell g^2\frac{3^4}{2^5}\frac{e^{4A}}{\alpha}\pi q^2 dz\wedge X^4 *_7 \mathcal{F}_4 - \frac{1}{2}e^{3A-\phi}x\mathcal{F}_4\,. \tag{5.5b}$$

From these expressions it is manifest that the 7d flux $\mathcal{F}_4 = d\mathcal{B}_3$ plays a role in defining the 10d 4-flux F_4, while the 2-flux F_2 is entirely determined by the variations of the internal geometry and of the 7d dilaton.

We can now use the formulas just introduced to uplift the Janus solution (3.17) to massive IIA. The result of the uplift is a geometry of the type AdS$_3\times S^3\times S^2$ fibered over two intervals. The 10d solution depends on the deformation parameter λ and it reproduces the AdS$_7$ vacuum to massive IIA for $\lambda=0$. The 10d Janus geometry has the following form

$$\begin{aligned} ds^2_{10} = &\frac{\ell X^{-\frac{5}{2}}e^{2A}}{\sin^2\theta}\left(\frac{1}{(1-\lambda)^2}ds^2_{\mathrm{AdS}_3} + \frac{\cos^2\theta}{(1+\lambda)^2}ds^2_{S^3} + X^{10}d\theta^2\right) \\ &+ \frac{3^8}{2^8}\frac{e^{6A}X^{\frac{5}{2}}}{\alpha^2}\pi^2 q^2 dz^2 + \frac{1-x^2}{16w}X^{\frac{5}{2}}e^{2A}ds^2_{S^2}\,. \end{aligned} \tag{5.6}$$

The 10d dilaton and B_2 field (5.4) can be written as

$$\begin{aligned} e^{2\Phi} &= \frac{2^{\frac{5}{2}}3^8\pi^5}{\ell} \frac{\left(1+\lambda\sin^2\theta\right)^{-1/2}}{\dot{\alpha}^2 - 2\alpha\ddot{\alpha}\left(1+\lambda\sin^2\theta\right)^{-1}} \left(-\frac{\alpha}{\ddot{\alpha}}\right)^{3/2}, \\ B_2 &= -\ell\pi z\,\mathrm{vol}_{S^2} + \frac{\ell\pi\,\alpha\dot{\alpha}}{\dot{\alpha}^2 - 2\alpha\ddot{\alpha}\left(1+\lambda\sin^2\theta\right)^{-1}}\,\mathrm{vol}_{S^2}\,, \end{aligned} \tag{5.7}$$

while the RR fluxes are given by

$$\begin{aligned} F_2 &= \left(\frac{\ell\,\ddot{\alpha}}{3^4 2\pi^2} + \frac{\pi\ell\,F_0\,\alpha\dot{\alpha}}{\dot{\alpha}^2 - 2\alpha\ddot{\alpha}\left(1+\lambda\sin^2\theta\right)^{-1}}\right)\mathrm{vol}_{S^2}\,, \\ F_4 &= -\ell^2\,\frac{2^4\lambda}{3^4\pi}\left(d\left(\dot{\alpha}\,\frac{1+\sin^2\theta}{(1-\lambda)^3\sin^2\theta}\right)\wedge\mathrm{vol}_{\mathrm{AdS}_3} - d\left(\dot{\alpha}\,\frac{\cos^4\theta}{(\lambda+1)^3\sin^2\theta}\right)\wedge\mathrm{vol}_{S^3}\right). \end{aligned} \tag{5.8}$$

From the above expressions we observe that the Janus solution is crucially described in ten dimensions by a dyonic profile for the F_4 flux. From the point of view of the defect interpretation, this is a strong indication that the defect branes, namely the branes responsible for the breaking of the AdS_7 isometries, are a bound state of D2-D4 branes. As we mentioned above, when we impose $\lambda = 0$, $F_4 = 0$ vanishes and the AdS_7 solutions to massive IIA are recovered [40]. We refer to [41] for a more detailed analysis on the brane interpretation of the solution (5.6).

6 Conclusions

The holographic interpretation of surface defects with $\mathcal{N} = (4,4)$ and $\mathcal{N} = (0,4)$ supersymmetry has been intensively explored in recent years. The main focus of this research has been put on the defect interpretation of AdS_3 and AdS_2 string backgrounds related by dualities to the *charged domain wall* solutions discussed in section 2.1 (see *e.g.* [15, 19–21, 23, 24] for AdS_3 domain walls and [17, 25–27] for AdS_2 ones). Based on the observations in [32] it is reasonable to expect that the supergravity solutions describing surface defects ultimately belong to the classifications of $\mathrm{AdS}_3 \times S^3 \times S^3 \times \Sigma_2$ backgrounds in [8–10, 22, 30, 31].

It is noteworthy that most $\mathcal{N} = (4,4)$ AdS_3 solutions with a clear holographic defect interpretation preserve *small* $\mathcal{N} = (4,4)$ supersymmetry. This is related to the fact that the brane solutions that underlie these geometries are only well-understood in terms of *defect branes* ending on *mother branes* (which allows to decouple their equations of motion) when the *defect branes* are localised in the worldvolume of the *mother branes* but are otherwise smeared in the rest of the transverse directions, which gives rise to just SU(2) R-symmetry.

In this paper we have taken a step forward in the defect interpretation of AdS_3 backgrounds with *large* $\mathcal{N} = (4,4)$ supersymmetry. These solutions are Janus-like backgrounds that realise one-parameter deformations of the $\mathrm{AdS}_7 \times S^4$ vacuum geometry of eleven dimensional supergravity. Identifying the fully-backreacted brane solutions underlying these geometries is however a very demanding task. This is due to the fact that the flux structure associated to the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ R-symmetry group is much more intricate than the one associated to the small supersymmetry preserved by charged domain walls. In section 4.2 we showed how to relate the AdS_3 Janus solutions to the charged domain walls. This computation suggests that the physics behind the AdS_3 Janus solutions could be related to M2-M5 defect branes intersecting a stack of M5' mother branes, as it is the case for the charged domain wall solutions (see [15]). The invariance under the $SU(2) \times SU(2)$ isometry group implies that for the Janus solutions the M2-M5 defect branes should be *fully-localised* in their transverse space. This could be the crucial physical difference between $\mathcal{N} = (4, 4)$ AdS_3 solutions with *small* and *large* supersymmetry. We leave the problem of understanding the brane picture and the dual field theory for future work.

Acknowledgments

We would like to thank Niall Macpherson for very useful discussions. AC and YL are partially supported by the grant from the Spanish government MCIU-22-PID2021-123021NB-I00. The work of AC is also supported by a Severo-Ochoa fellowship PA-23-BP22-019. The work of NP is supported by the Israel Science Foundation (grant No. 741/20) and by the German Research Foundation through a German-Israeli Project Cooperation (DIP) grant "Holography and the Swampland". The work of AR has been supported by INFN-UNIMIB, contract number 125370/2022, and by the INFN grant Gauge and String theory (GAST).

Data Availability Statement. This article has no associated data or the data will not be deposited.

Code Availability Statement. This article has no associated code or the code will not be deposited.

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