



Dynamic simulation of a transcritical CO₂ heat pump with vapor injection in reciprocating compressors (Voorhees cycle) under off-design operating conditions

Luigi Fusco ^a, Giulio Lenzi ^b, Michele Manno ^{a,*}, Stefano Mazzoni ^a

^a University of Rome Tor Vergata, Via del Politecnico, 1, Rome, 00133, Italy

^b Officine Mario Dorin spa, Via Aretina, 388, Compiobbi (FI), 50061, Italy

ARTICLE INFO

Keywords:

Heat pump
CO₂ transcritical cycle
Reciprocating compressor
Voorhees cycle
Dynamic simulation
Extremum seeking control

ABSTRACT

Transcritical CO₂ heat pumps equipped with vapor injection in the cylinder of a reciprocating compressor (Voorhees cycle) are a promising low-carbon heating technology, but their off-design behavior and optimal control have not been investigated so far. This paper presents a dynamic model of a CO₂ heat pump based on a Flash-Tank Vapor Injection Voorhees cycle, developed in the Simulink/Simscape environment. The reciprocating compressor, equipped with a solenoid injection valve, is represented by a customized Simscape component in which the volumetric efficiencies, the discharge enthalpy, and the mechanical power are evaluated by an Artificial Neural Network (ANN) trained on a detailed lumped-parameter compressor model; this decouples the fast compressor cycle from the slower thermodynamic cycle and enables stable coupled simulation. An Extremum Seeking Control (ESC) algorithm is implemented to track the gas cooler pressure that maximizes the Coefficient of Performance in real time, without a plant model. Across six off-design boundary conditions spanning both directions of variation of each external variable, the ESC identifies the optimum within 0.5 % of the brute-force maximum, and the resulting trends of intermediate pressure, COP, and flash-tank vapor quality agree with previously published steady-state results, confirming the internal consistency of the model and its suitability for optimal control of transcritical Voorhees heat pumps.

1. Introduction

Over the past decades, CO₂ has emerged as a prominent refrigerant due to its low Global Warming Potential (GWP) and zero Ozone Depletion Potential (ODP) (Wu et al., 2021). In particular, the thermodynamic properties of CO₂ make it highly attractive for transcritical cycles (Kauf, 1999; Liao et al., 2000); this natural refrigerant was reintroduced by Lorentzen and Pettersen (1993) and has since been adopted in many applications, including hot water production and residential space heating (Banks et al., 2022; Stene, 2005). Many modifications of the standard transcritical cycle have been proposed to improve its performance (Adamson et al., 2022), including optimization of the cycle for simultaneous heating and cooling duties (Sarkar et al., 2004). One of these is the Voorhees cycle, which features vapor injection into the cylinder of a reciprocating compressor and was introduced by Voorhees (1927). Historically, this modification has been applied mainly to scroll compressors or to reciprocating compressors equipped with a passive admission port. Lambers et al. (2007) optimized the admission port geometry of a

Voorhees-modified reciprocating CO₂ compressor via indicator-diagram analysis, reporting a theoretical COP improvement potential of about 17 % (10 % demonstrated experimentally) and identifying the admission clearance as the key trade-off between backflow and throttling losses. At the system level, Zha et al. (2008) showed experimentally that a Voorhees economizer can increase R-744 heat pump heating capacity by 1.3 to 2 times at low evaporating temperatures, and is advantageous in both capacity and COP below -20 °C evaporating temperature or at low compressor rotational speed, despite an approximately 5 % reduction in suction volume caused by the modification. Recent developments in solenoid valve technology, however, make it possible to replace the admission port with an electronically actuated valve, thereby providing direct control of the injection phase duration and, consequently, of the mass of vapor introduced into the cylinder. In this context, previous work by Fusco et al. (2025) assessed the performance of the corresponding thermodynamic cycle and compared it with the standard transcritical cycle under design (nominal) operating conditions; that study, however, was limited to steady-state analysis.

* Corresponding author.

E-mail addresses: luigi.fusco@alumni.uniroma2.eu (L. Fusco), g.lenzi@dorin.com (G. Lenzi), michele.manno@uniroma2.it (M. Manno), stefano.mazzoni@uniroma2.it (S. Mazzoni).

<https://doi.org/10.1016/j.ijrefrig.2026.107070>

Received 27 May 2026; Received in revised form 17 July 2026; Accepted 3 August 2026

Available online 5 August 2026

0140-7007/© 2026 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

Latin letters

<i>a</i>	demodulation amplitude
<i>P</i>	mechanical power (W)
<i>b</i>	modulation amplitude
<i>p</i>	pressure (MPa)
<i>D</i>	bore (m)
<i>q</i>	vapor quality
<i>e</i>	control error
<i>R</i>	ratio of connecting rod length to crank radius
<i>h</i>	specific enthalpy (J kg ⁻¹)
<i>S</i>	cross-sectional area (m ²)
<i>k</i>	learning rate
<i>SR</i>	section ratio (valve-to-cylinder)
<i>K</i>	PI controller gains
<i>T</i>	temperature (K)
<i>L</i>	stroke (m)
<i>u</i>	controller output
<i>m</i>	mass flow rate (kg s ⁻¹)
<i>V</i>	compressor displacement (m ³)
<i>n</i>	rotational speed (s ⁻¹)
<i>W</i>	specific work (J kg ⁻¹)
<i>N</i>	number of cylinders

Greek letters

β	pressure ratio
$\Delta\theta$	crank angle interval (°)
ΔT	temperature difference (K)
η	efficiency
ρ	fluid density (kg m ⁻³)
τ	residence time (s)
ω	angular frequency (rad s ⁻¹)
Φ	phase (rad)

Subscripts

<i>a</i>	air
<i>in</i>	inlet
<i>cyl</i>	cylinder
<i>inj</i>	injection
<i>d</i>	discharge
<i>l</i>	low
<i>dis</i>	displacement
<i>nom</i>	nominal
<i>el</i>	electric
<i>P</i>	proportional
<i>ev</i>	evaporator
<i>pp</i>	pinch point
<i>FT</i>	flash tank
<i>s</i>	suction
<i>gc</i>	gas cooler
<i>SH</i>	superheat
<i>h</i>	high
<i>v</i>	volumetric
<i>I</i>	integral
<i>w</i>	water

Acronyms

ANN	Artificial Neural Network
COP	Coefficient Of Performance
ESC	Extremum Seeking Control
FT	Flash Tank
FTVI	Flash-Tank Vapor Injection
GWP	Global Warming Potential
IHX	Internal Heat Exchanger
ODP	Ozone Depletion Potential
PI	Proportional–Integral

The present work proposes a dynamic model of the thermodynamic cycle aimed at assessing its performance under off-design conditions and, specifically, at testing suitable optimal control strategies. Dynamic modeling of CO₂ and transcritical refrigeration systems has been pursued since the early 2000s, using a variety of formulations (Pfaf-ferott and Schmitz, 2004; Rasmussen and Alleyne, 2004). Bendapudi et al. (2008) compared moving-boundary and finite-volume heat exchanger formulations in a centrifugal chiller model, finding that moving-boundary method executed about three times faster while maintaining nearly identical accuracy, though finite-volume was more robust through start-up and load-change transients and gave more accurate refrigerant charge predictions. Building on this, Li and Alleyne (2010) developed a switched moving-boundary modeling approach capable of handling compressor shut-down and start-up transients, using pseudo-state variables to track inactive refrigerant zones and validating the model against experimental stop–start cycles. Haberschill et al. (2007) developed and experimentally validated a dynamic model of a CO₂ air-to-water heat pump spanning both subcritical and transcritical operation, finding good agreement in steady state but a faster simulated transient response than measured, attributed mainly to an oversimplified representation of compressor thermal inertia. This line of work has recently been extended to large-scale industrial heat pumps (Wolscht et al., 2024). This study focuses on the Flash-Tank Vapor Injection (FTVI) layout. To analyze off-design operating conditions, the model relies on a lumped-parameter model of the reciprocating compressor developed by the authors in a companion work (Fusco et al., 2026) and used there to evaluate the main performance indicators of the machine over a wide range of operating conditions (Yang et al., 2013; Tanveer and Bradshaw, 2022). In the dynamic simulation framework presented here, however, the characteristic time scale of the compressor working cycle differs significantly from that of the overall thermodynamic cycle. For this reason, an Artificial Neural Network (ANN) was trained using the results of the detailed compressor model and embedded within the dynamic simulation environment. The ANN enables a fast and reliable estimation of the compressor performance indicators, substantially reducing the computational effort and, most importantly, allowing an effective coupling between the compressor and the remaining components of the thermodynamic cycle.

Within this dynamic modeling framework, particular attention is devoted to the control of the operating variables that most strongly influence the performance of transcritical systems, such as the gas cooler pressure (Liu et al., 2019). The determination of the optimal high-side pressure has been the subject of extensive research. Early studies developed empirical correlations relating the optimal discharge pressure to the gas cooler outlet temperature and evaporating conditions (Kauf, 1999; Liao et al., 2000), although later comparative assessments showed that such correlations can deviate significantly from experimental optima across different systems, with COP penalties of up to 30 % reported for heat pump water heaters under variable water flow rate control, leading the authors to advocate for real-time optimization algorithms as a more robust alternative (Cecchinato et al., 2010). To overcome this limitation, real-time model-based algorithms were subsequently proposed to track the optimal pressure without relying on a fixed correlation (Cecchinato et al., 2012; Liu et al., 2019). The dynamic model proposed in this paper therefore incorporates an adaptive control algorithm aimed at continuously maximizing the cycle performance, expressed in terms of the Coefficient of Performance (COP), through the real-time adjustment of the cycle high-side pressure. The adopted control strategy is Extremum Seeking Control (ESC), a model-free, real-time optimization technique particularly suited to systems characterized by nonlinear behavior and time-varying operating conditions (Cui et al., 2022). ESC has been successfully applied to vapor compression systems for subcooling optimization: Koeln and Alleyne (2014) used a receiver and additional expansion valve to enable independent control of condenser subcooling, with ESC tracking the optimum under changing conditions and achieving a 9 % experimental COP increase over zero-subcooling operation.

Burns et al. (2020) proposed a faster proportional-integral ESC variant, converging at the same timescale as the plant dynamics, and demonstrated a power reduction from 710 W to 440 W on a production room air conditioner despite coarse sensor resolution and discrete actuator steps on the production hardware. ESC has also been applied to transcritical CO₂ heat pump water heaters for COP maximization, converging to within 1 % of the true optimum under varying ambient and water-outlet temperature conditions and avoiding COP losses of up to 33 % compared to fixed-pressure operation (Hu et al., 2015), with reported COP improvements of up to 9 % in experimental tests (Cui et al., 2022). More recently, ESC has been employed for intermediate pressure optimization in two-stage flash-tank heat pumps, achieving convergence to within 1 % of the optimum identified by sequential quadratic programming under both staircase and realistic time-varying ambient temperature profiles (Wang and Li, 2019), and cascade refrigeration systems (Wang et al., 2020), as well as for multi-variable optimization of vapor-injection transcritical CO₂ heat pump water heaters, achieving simultaneous power-consumption reductions and a COP improvement of about 5 % (Cui et al., 2021). Alternative data-driven and model-based strategies, such as Model Predictive Control (MPC), have also been explored for transcritical CO₂ systems (Wang et al., 2021; Zhang et al., 2023; Yin et al., 2024), as well as for related vapor compression applications (Elliott and Rasmussen, 2013). Hybrid approaches combining ESC and MPC have also been proposed (Weiss et al., 2018). ESC is preferred here because it requires no plant model and is therefore directly applicable to the prototype compressor under development, whose dynamic characteristics are not yet fully characterized experimentally.

The original contributions of this paper can be summarized as follows:

1. A dynamic model of a transcritical CO₂ heat pump combining the FTVI cycle topology with a Voorhees reciprocating compressor equipped with an electronically actuated solenoid injection valve is developed in the Simulink/Simscape environment. Dynamic models of FTVI cycles with scroll or screw compressors have been reported (Wang and Li, 2019), but no dynamic model is available for the Voorhees variant, in which the in-cylinder vapor injection couples the compressor working cycle to the system dynamics in a way that scroll-based FTVI models cannot represent.
2. A customized Simscape component of the reciprocating compressor with solenoid injection valve is implemented, in which the compressor performance is evaluated by an ANN surrogate of the detailed lumped-parameter working-cycle model of Fusco et al. (2026); this decouples the fast compressor dynamics from the slower thermodynamic-cycle dynamics while preserving the physical consistency of the working-cycle model.
3. An Extremum Seeking Control algorithm is designed to track the gas cooler pressure that maximizes the COP in real time. ESC has been previously applied to transcritical CO₂ heat pumps with scroll compressors and either separate or sub-cooler vapor injection (Hu et al., 2015; Cui et al., 2021, 2022); its application to a Voorhees cycle, in which the steady-state coupling between intermediate pressure, injection-valve opening angle, and flash-tank vapor quality is imposed by Eq. (13), has not been investigated. The performance of the controller is quantitatively assessed against brute-force optima across a sequence of step changes in the external boundary conditions (water inlet temperature, water temperature rise in the gas cooler, ambient air temperature).
4. Together with the ESC algorithm, the dynamic model enables the optimal control system of the prototype heat pump described in Fusco et al. (2026), currently under construction, to be designed and tested by simulation before the prototype becomes operational for experimental validation. The internal consistency of the proposed framework is verified by comparing the predicted trends of COP, intermediate pressure, and flash-tank vapor quality with the steady-state results of Fusco et al. (2025).

Table 1

Design operating conditions of evaporator and gas cooler.

	Evaporator	Gas cooler
Fluid	Air	Water
$T_{in}/^{\circ}\text{C}$	−6	25
$\Delta T/\text{K}$	−3	40
p/MPa	0.12	0.4
$\Delta T_{pp}/\text{K}$	3	3

Table 2

Pressure levels and injection valve opening angle at design operating conditions.

p_d/MPa	p_{inj}/MPa	p_s/MPa	$\Delta\theta_{inj}/^{\circ}$
11.00	3.400	2.501	40.0

2. Methods

The dynamic model of the FTVI Voorhees cycle implemented in the Simulink/Simscape environment is depicted in Fig. 1. As described in Section 1, the main characteristic of this cycle is the use of a solenoid valve to introduce vapor into the compression chamber. Since the Simulink/Simscape library does not include a component representing a reciprocating compressor equipped with a solenoid injection valve, it was necessary to develop this component to achieve the objective of the paper; the details of its implementation are presented in Section 2.1. The first step in the implementation of the model concerned the main assumptions regarding the external fluids, which are summarized in Table 1.

To avoid unnecessarily lengthening the manuscript, the procedure used to evaluate the thermodynamic states is not reported here, as it has been described in detail in a previous paper (Fusco et al., 2025). After selecting the properties of the external fluids, the thermodynamic states of the cycle were evaluated and used for the design of each component. In an FTVI cycle the gas cooler pressure and the intermediate pressure are design parameters (Fusco et al., 2025), whereas the evaporating pressure depends on the temperature profile of the external fluid and on the selected minimum temperature difference (ΔT_{pp}) in the evaporator. Moreover, an initial value of the opening angle of the injection valve must be selected, bearing in mind that this parameter can be varied at any time during operation. The initial values of the three pressure levels and of the injection valve opening angle are reported in Table 2; the degree of superheating is provided by the Internal Heat Exchanger (IHX) and was set to 10 K.

The authors developed a lumped-parameter model of the reciprocating compressor equipped with a solenoid injection valve. Due to the different time scales of the compressor working cycle and of the overall thermodynamic cycle, an ANN was trained on the results of the lumped-parameter model and incorporated into a customized Simscape component, as described in Section 2.1. Because the compressor completes a working cycle in a few tens of milliseconds, whereas the thermodynamic-cycle dynamics evolve over hundreds of seconds, the compressor operates in a quasi-steady, cycle-averaged regime at every instant of the system-level simulation; the ANN consequently returns the cycle-averaged compressor performance corresponding to the instantaneous boundary conditions, consistently with the quasi-steady representation of positive-displacement components adopted in standard dynamic simulation libraries. The details of the lumped-parameter model and of the ANN training procedure are reported in a companion paper (Fusco et al., 2026). This paper focuses on the final stage of the customization of the Simscape component, on its integration in the dynamic cycle model, and on the adaptive control strategy.

In the heat pump model represented in Fig. 1, four parameters are actively controlled. The first is the water temperature increase in the

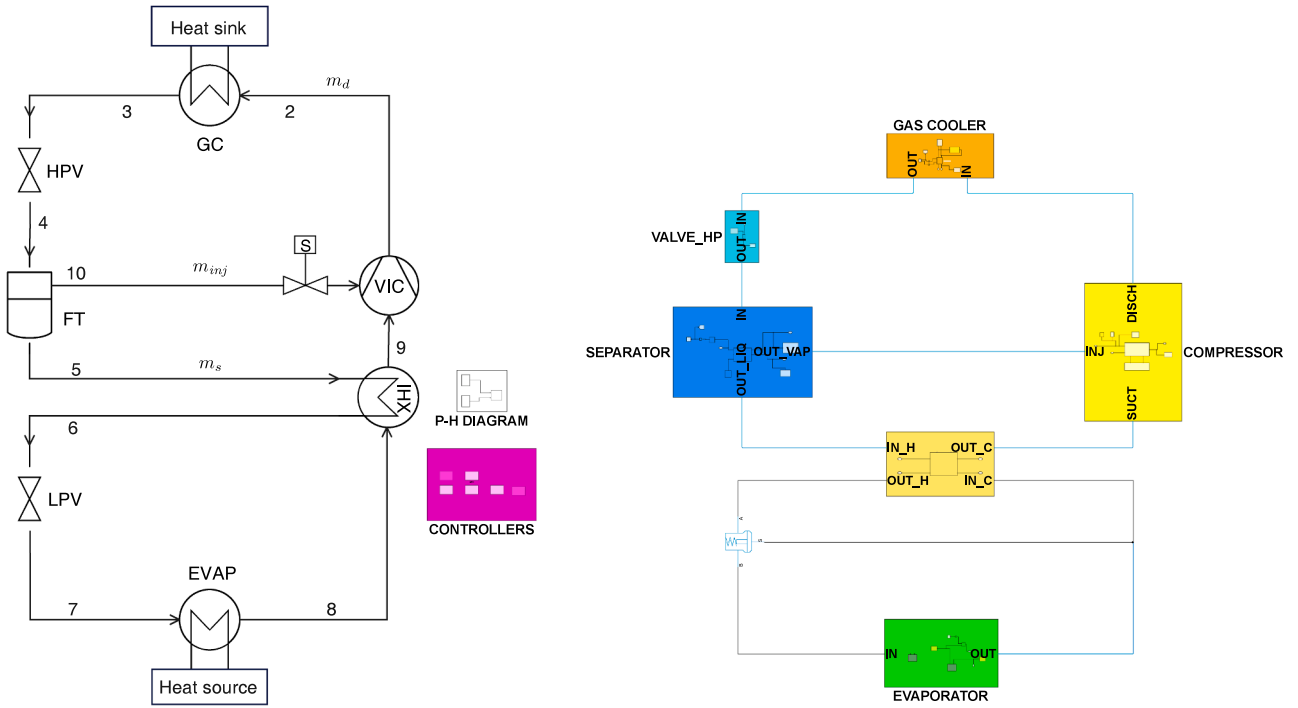


Fig. 1. FTVI Voorhees cycle-based heat pump: schematic layout (left) and dynamic model implementation in Simulink/Simscape (right).

gas cooler, which is regulated by adjusting the water mass flow rate supplied to the gas cooler. The PI controller output is a dimensionless coefficient that is multiplied by the nominal mass flow rate to determine the actual water mass flow rate as follows:

$$\dot{m}_w = c_w \dot{m}_{w,nom} \quad (1)$$

where c_w is the controller output. The second is the air temperature decrease in the evaporator, controlled by adjusting the air mass flow rate in the same way, through a dimensionless coefficient c_a . The third is the discharge (gas cooler) pressure, which is set by the adaptive control system described in Section 2.2 and imposed by means of a back-pressure valve downstream of the gas cooler. Finally, the superheat degree is controlled to maintain a constant value under all operating conditions, using a bypass valve on the cold side of the IHX. Proportional-Integral (PI) controllers were implemented for these four variables, using the parallel form:

$$u(t) = K_p e(t) + K_I \int_0^t e(z) dz, \quad (2)$$

where $e(t)$ is the control error and $u(t)$ is the controller output; PI controller gains are reported in Table 3. For completeness, Fig. 2 illustrates the control strategies described above as implemented in the simulation environment. The thermostatic expansion valve upstream of the evaporator controls the CO₂ outlet temperature from the evaporator; the corresponding Simscape component incorporates an intrinsic control strategy, which ensures that the evaporator outlet state remains either dry saturated vapor or slightly superheated. To keep the outlet condition as close as possible to dry saturated vapor, the maximum allowable degree of superheating is set to 0.2 K. The intermediate pressure level is left free to adjust dynamically in response to the prevailing operating conditions, among which are the CO₂ outlet pressure and temperature from the gas cooler and the injection valve opening angle.

The components of the thermodynamic cycle were modeled using standard blocks from the Simscape Fluids library, whose governing equations and constitutive relations are documented in the library reference. The gas cooler and the evaporator were represented by System-Level Condenser Evaporator blocks, in the two-phase/thermal-liquid (2P-TL) and two-phase/moist-air (2P-MA) configurations respectively;

Table 3

PI controller gains.

Controlled parameter	K_p	K_I
p_{gc}	0.1 MPa ⁻¹	0.1 MPa ⁻¹ s ⁻¹
ΔT_w	1.0 K ⁻¹	0.01 K ⁻¹ s ⁻¹
ΔT_a	0.1 K ⁻¹	0.01 K ⁻¹ s ⁻¹
ΔT_{SH}	0.1 K ⁻¹	0.01 K ⁻¹ s ⁻¹

the internal heat exchanger by a System-Level Heat Exchanger (2P-2P) block; and the flash tank by a Receiver Accumulator (2P) block. In these blocks, the internal volume is the only parameter governing the dynamic response of the model, since it determines the mass and energy accumulation in the component. The internal volume of each component was estimated as:

$$V = (\dot{m}\tau)/\bar{\rho} \quad (3)$$

where $\dot{m}$ is the nominal mass flow rate, $\bar{\rho}$ is the mean fluid density, and τ is an assumed fluid residence time. Residence times of 10 s, 10 s, 2 s, and 60 s were selected for the gas cooler, evaporator, internal heat exchanger, and flash tank, respectively, as a trade-off between simulation speed and numerical stability, and to reflect the buffering role of the flash tank in the cycle. The expansion from the gas cooler pressure to the injection pressure was modeled by an Orifice (2P) block, and the expansion from the injection pressure to the evaporating pressure by a Thermostatic Expansion Valve (2P) block. The reciprocating compressor with injection valve is the customized component described in Section 2.1. The refrigerant thermophysical properties were evaluated from the built-in Simscape CO₂ property tables. Because the component network is acausal, the solution sequence is generated automatically by Simscape from the assembled system of differential-algebraic equations rather than being defined procedurally; this system was integrated with the variable-step ode23t solver, with relative and absolute tolerances of 10⁻³ and 10⁻⁶, respectively.

The level of detail of the component models was deliberately kept to the minimum required by the objective of this work, namely the assess-

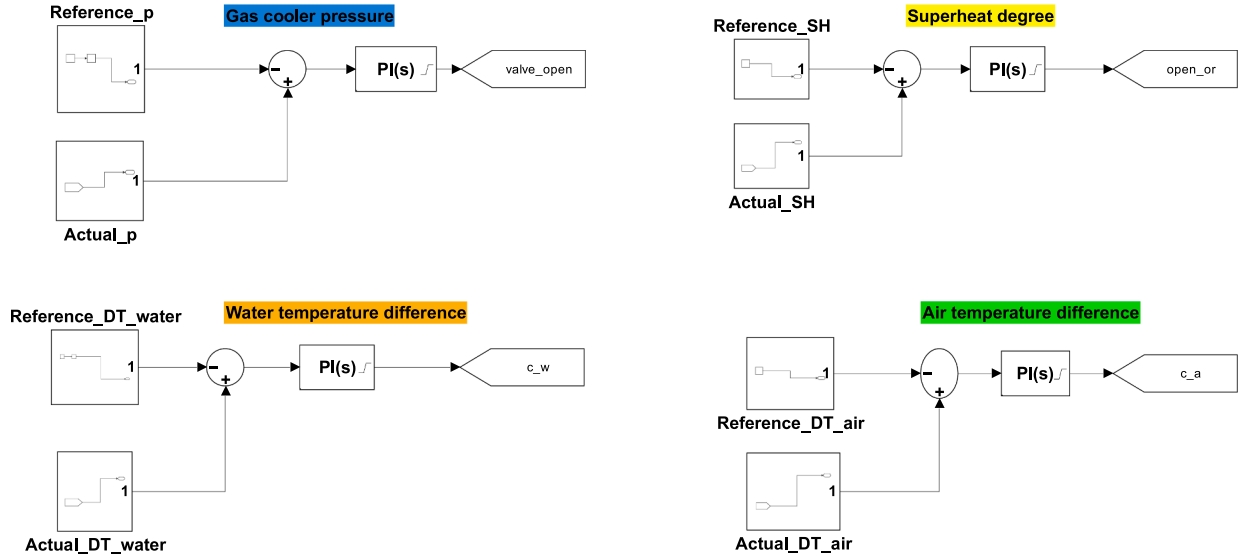


Fig. 2. Control strategy implemented in the simulation environment: the four PI controllers and the ESC loop acting on the plant.

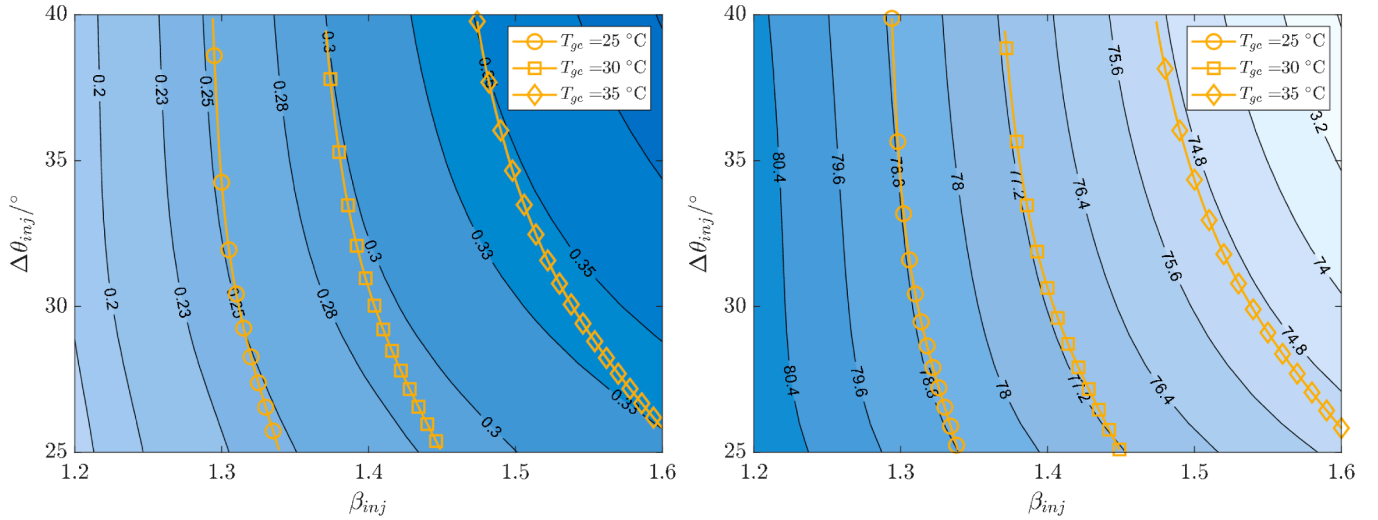


Fig. 3. Operating maps of the Voorhees compressor at the cycle design conditions: ratio of injected to discharged mass flow rate (left) and specific work (right). Yellow curves identify the valve opening angles that satisfy Eq. (13) for different gas cooler outlet temperatures.

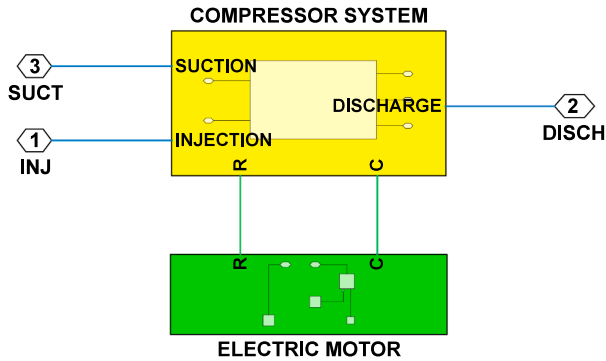


Fig. 4. Customized Simscape component of the reciprocating compressor with injection valve.

ment of the dynamic response and of the model-free optimal control of

the FTVI Voorhees cycle rather than the reproduction of a specific physical installation. Consistently with this general scope, the System-Level heat-exchanger blocks describe the heat exchangers through an overall energy balance rather than a spatially distributed formulation. The quantitative results are therefore a reasonable approximation of those that would be obtained with more detailed component models; in particular, the System-Level representation of the gas cooler does not resolve the strong variation of the CO_2 properties across the pseudocritical region, so that the absolute values of the optimal gas cooler pressure and of the maximum COP are affected by this approximation. This does not affect the conclusions of the present study: the purpose of the model is to reproduce the qualitative dynamic behavior of the cycle and to demonstrate that the ESC reliably tracks the COP optimum, and neither the qualitative response of the system nor the ability of the controller to locate the optimum depends on the fidelity of the heat-exchanger model. Keeping the component models as simple as the objective allows also keeps the focus on the two elements that constitute the contribution of this work: the ANN-based model of the vapor-injection compressor and the ESC algorithm.

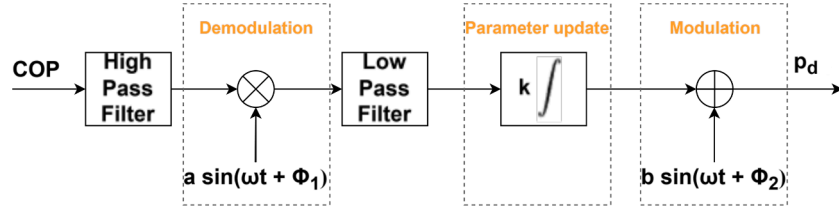


Fig. 5. Block diagram of the Extremum Seeking Control (ESC) algorithm implemented in the Simulink environment.

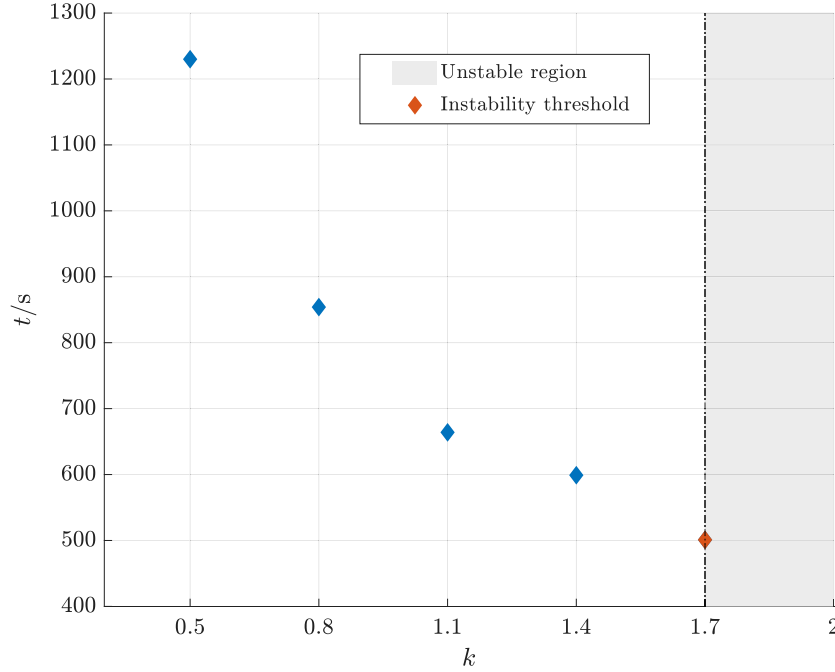


Fig. 6. Convergence time as a function of the learning rate k . Shaded region ($k > 1.7$): onset of undamped oscillations from loss of time-scale separation.

Table 4
Geometric and kinematic parameters of the CD8-3000-150H compressor.

Parameter	Value	Parameter	Value
L/mm	65	SR_s	15.3 %
D/mm	65	SR_d	9.28 %
N_{cyl}	8	R	6.36
n/rpm	1450	P_{el}/kW	186.4

2.1. Implementation of the reciprocating compressor model with injection valve

The reciprocating compressor model refers to a real machine currently under development. The main technical data of the compressor CD8-3000-150H, manufactured by Officine Mario Dorin spa, are summarized in Table 4. In the table, SR is the ratio of the valve flow area to the cross-sectional area of the cylinder S_{cyl} , and R is the ratio between the connecting rod length and the crank radius.

The ANN was developed in the MATLAB environment and implemented in Simulink/Simscape through a dedicated component. Following Fusco et al. (2026), the network takes four input variables:

- Discharge pressure p_d in the range 8 MPa to 15 MPa;
- Injection pressure ratio $\beta_{inj} = p_{inj}/p_s$ in the range 1.05 to 1.7;
- Evaporating temperature T_{ev} in the range -20 – 5 °C (which sets the suction pressure through the saturation relation);
- Injection valve opening angle $\Delta\theta_{inj}$ in the range 25° to 40° .

This parameterization resulted in a total of 1920 operating points, which were randomly partitioned into training (70 %), validation (15 %), and test (15 %) subsets. The network architecture comprises a single hidden layer of 15 neurons with hyperbolic-tangent sigmoid activation, and was trained with the Levenberg–Marquardt back-propagation algorithm using early stopping on the validation set to prevent over-fitting. The training converged in 586 epochs to a best validation mean squared error of 3.95×10^{-6} , and achieved an overall coefficient of determination $R^2 = 0.99994$ and a root-mean-squared error of 1.99×10^{-3} on the independent test set; further details of the training procedure and the full accuracy statistics are reported in Fusco et al. (2026). It is worth noting that the design value $\Delta\theta_{inj} = 40^\circ$ (Table 2) corresponds to the upper boundary of the $\Delta\theta_{inj}$ training range, which is set by the onset of back-flow through the injection valve; this boundary is sampled by the parametric sweep generating the 1920 operating points and is therefore covered by the test statistics reported above.

The outputs of the ANN are the suction volumetric efficiency $\eta_{v,s}$, the injection volumetric efficiency $\eta_{v,inj}$, the discharge enthalpy h_d , and the mechanical power P :

$$\eta_{v,s} = f_1(p_d, \beta_{inj}, T_{ev}, \Delta\theta_{inj}), \quad (4)$$

$$\eta_{v,inj} = f_2(p_d, \beta_{inj}, T_{ev}, \Delta\theta_{inj}), \quad (5)$$

$$h_d = f_3(p_d, \beta_{inj}, T_{ev}, \Delta\theta_{inj}), \quad (6)$$

$$P = f_4(p_d, \beta_{inj}, T_{ev}, \Delta\theta_{inj}). \quad (7)$$

The ANN outputs are a subset of the broader set of compressor key performance indicators defined in Fusco et al. (2026); here we retain only those required to couple the compressor with the thermodynamic cy-

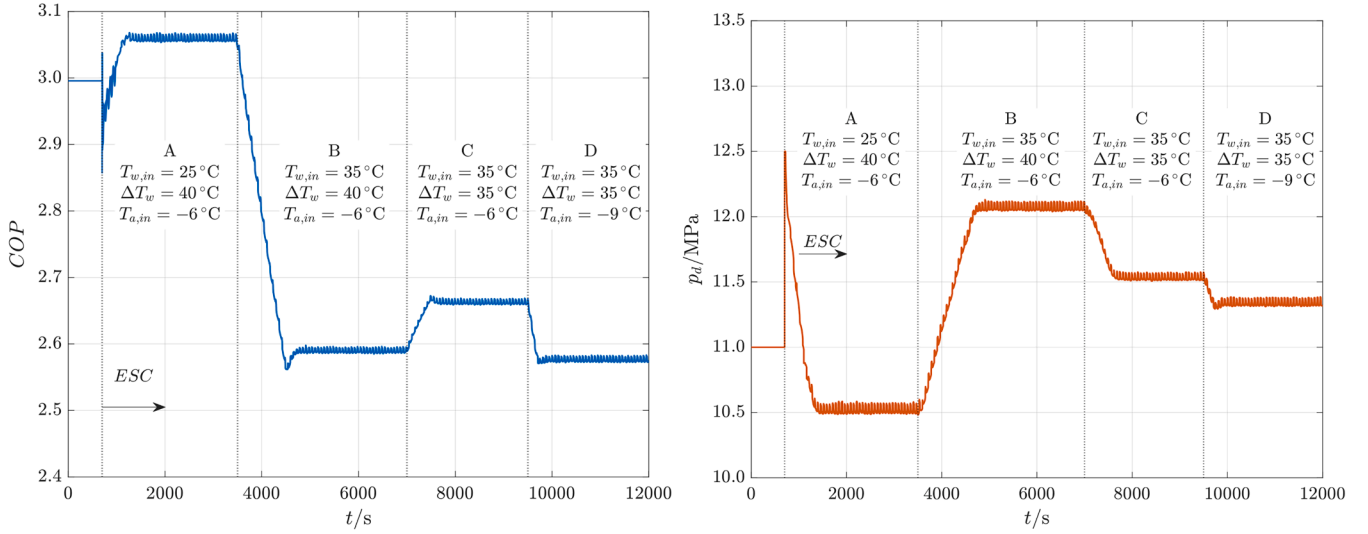


Fig. 7. Dynamic simulation results: evolution of the COP (left) and of the gas cooler pressure p_d (right) during the sequence of step changes in the boundary conditions.

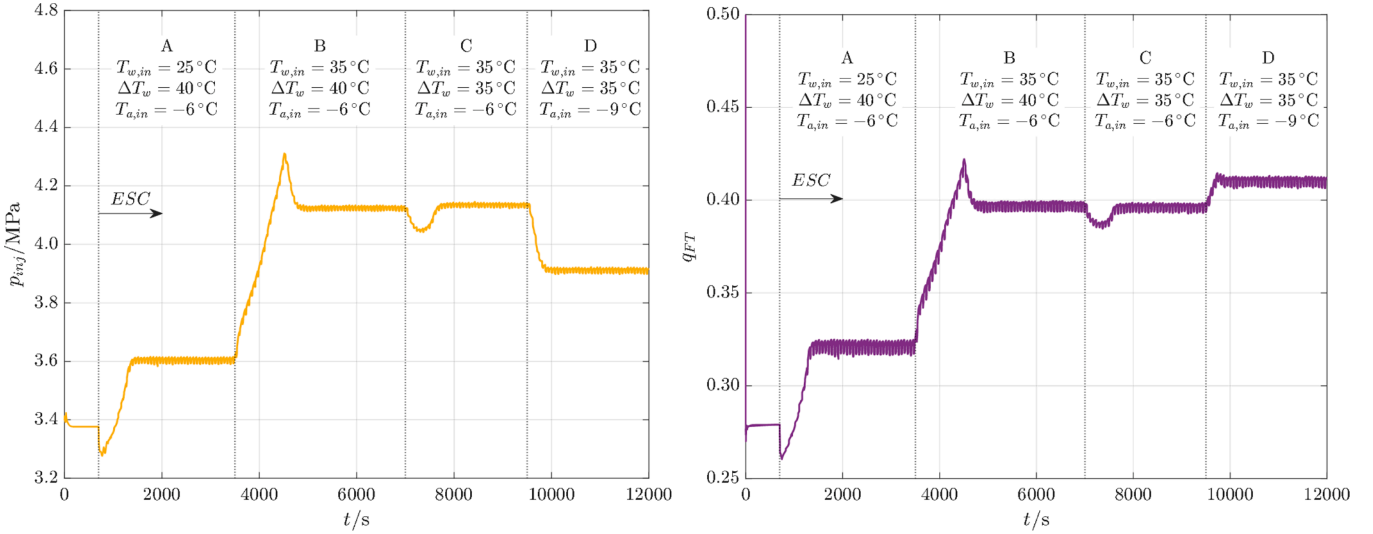


Fig. 8. Dynamic simulation results: evolution of the intermediate (injection) pressure p_{inj} (left) and of the vapor quality in the flash tank q_{FT} (right).

cle dynamics. Exposing the mechanical power P directly as an ANN output, rather than reconstructing it from an isentropic efficiency and the enthalpy difference, avoids the loss of information associated with the constant-volume mixing approximation previously used by Fusco et al. (2025): that approximation is known to overestimate the specific work by approximately 7% to 10.5% across the operating envelope, with respect to the full working-cycle model (Fusco et al., 2026), so predicting P from the trained ANN yields a physically more consistent coupling between the compressor and the thermodynamic cycle.

The suction and injection volumetric efficiencies are defined as:

$$\eta_{v,s} = \frac{\dot{m}_s}{\rho_s V_{dis}}, \quad (8)$$

$$\eta_{v,inj} = \frac{\dot{m}_{inj}}{(\rho_{inj} - \rho_s) V_{dis}}. \quad (9)$$

The definition of $\eta_{v,inj}$ has been introduced in Fusco et al. (2026) as a more practical and more selective descriptor of the injection process, and is used here in addition to the conventional suction volumetric ef-

ficiency. The customized Simscape component, shown in Fig. 4, connects to the system through the suction, injection, and discharge ports. Based on the ANN outputs, the component evaluates the mass flow rates through the following constitutive equations:

$$\dot{m}_s = \eta_{v,s} \rho_s V_{dis} n, \quad (10)$$

$$\dot{m}_{inj} = \eta_{v,inj} (\rho_{inj} - \rho_s) V_{dis} n, \quad (11)$$

$$\dot{m}_d = \dot{m}_s + \dot{m}_{inj}, \quad (12)$$

while the discharge enthalpy h_d and the mechanical power P are read directly from Eqs. (6) and (7). While the compressor can in principle be operated with the intermediate pressure and the injection valve opening angle as independent variables, at the steady-state operation of the entire heat pump these two variables become coupled, because the mass of vapor that can be injected depends on the amount of vapor produced by the expansion in the flash tank. This is expressed by the following steady-state constraint:

$$\frac{\dot{m}_{inj}}{\dot{m}_d} = f(p_d, \beta_{inj}, T_{ev}, \Delta\theta_{inj}) = q_{FT}(p_d, T_{gc}, p_{inj}). \quad (13)$$



Fig. 9. Dynamic simulation results of the PI controllers: water temperature difference and mass flow rate (top); air temperature difference and mass flow rate (middle); superheat degree and bypass valve opening (bottom).

Eq. (13) effectively defines the relationship between intermediate pressure and injection valve opening angle at steady state for given gas cooler and evaporator operating conditions. To illustrate this behavior, Fig. 3 shows the output of the lumped-parameter compressor model of Fusco et al. (2026) at the cycle design conditions of Tables 1 and 2 ($T_{ev} = -12^\circ\text{C}$, $p_d = 11\text{ MPa}$), corresponding to an overall pressure ratio of approximately 4.4. The ratio of injected to discharged mass flow rate and the specific work are represented as functions of the intermediate pressure ratio p_{inj}/p_s and of the injection valve opening angle $\Delta\theta_{inj}$.

When the constraint imposed by Eq. (13) is not considered, increasing either the intermediate pressure or the valve opening angle leads to a higher injected mass flow rate and a corresponding reduction in specific work. However, the yellow curves identify the valve opening angles that satisfy Eq. (13) for different gas cooler outlet temperatures, showing that higher intermediate pressures require a rapid reduction of the opening angle, due to the limited vapor availability from the flash tank, which ultimately leads to an increase in specific work. Therefore, to maximize the COP of the Voorhees cycle the injection valve should remain open

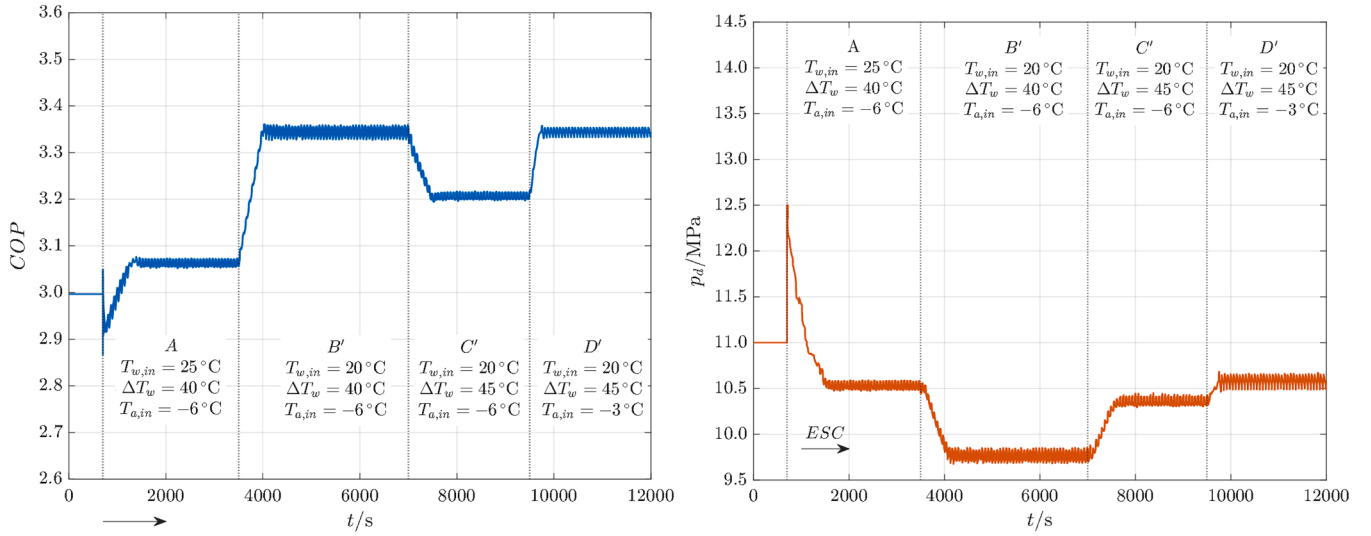


Fig. 10. Dynamic simulation results: evolution of the COP (left) and of the gas cooler pressure p_d (right) during the sequence of step changes in the boundary conditions for the additional off-design scenario.

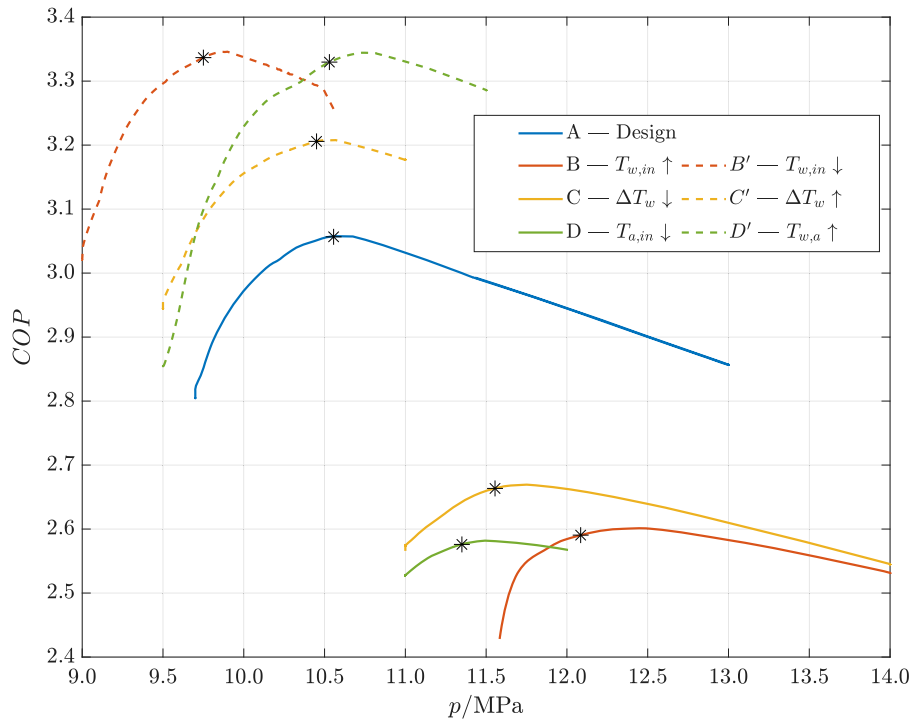


Fig. 11. COP as a function of the gas cooler pressure under the seven sets of boundary conditions considered in the dynamic simulation. The asterisks indicate the gas cooler pressure identified by the ESC algorithm in each case.

for as long as possible, even though this results in a lower intermediate pressure; the upper limit for the injection valve opening angle is essentially set by the onset of back-flow through the valve.

With respect to the rotational subsystem of the compressor, the driving unit was modeled as an idealized electric motor: for the sake of simplicity, its dynamic behavior was represented by an imposed angular velocity source, neglecting electrical transients and control dynamics. This modeling choice allows the analysis to focus on the mechanical and thermodynamic response of the compressor while preserving the essential characteristics of the rotational input.

2.2. Extremum seeking control

As discussed in Section 1, an ESC algorithm has been implemented to maximize the COP. ESC is an adaptive, model-free optimization approach designed to handle scenarios in which the system's optimal operating point is either unknown or changes over time (Mamur et al., 2022). The algorithm operates through four distinct stages to fine-tune a parameter (Ariyur and Krstic, 2004), as represented in Fig. 5. The process begins with a *demodulation* stage, in which the objective function signal is multiplied by a sinusoid of the same frequency as the excitation

Table 5
ESC design parameters.

Parameter	Value
Central (modulation) frequency $\omega/\text{rad s}^{-1}$	0.1
High-pass filter cut-off frequency $\omega_h/\text{rad s}^{-1}$	1.1
Low-pass filter cut-off frequency $\omega_l/\text{rad s}^{-1}$	0.085
Demodulation amplitude a/MPa – phase Φ_1/rad	$0.7 - 0$
Modulation amplitude b/MPa – phase Φ_2/rad	$0.1 - 0$
Learning rate k	1.1
Initial condition $p_{d,0}/\text{MPa}$	11

signal (Hu et al., 2015). This operation extracts the component of the objective function that is correlated with the parameter perturbation (Zhang and Ordóñez, 2012). Prior to or in conjunction with demodulation, an optional high-pass filter can be applied to remove any DC bias in the objective function signal, ensuring that only the relevant oscillatory components contribute to the optimization process (Zhang and Ordóñez, 2012). Subsequently, an optional low-pass filter is employed to attenuate high-frequency components and noise arising from external disturbances in the demodulated signal, thereby improving the stability and accuracy of the estimation process (Hu et al., 2015; Zhang and Ordóñez, 2012); the low-pass filter is also highly effective in preventing spurious parameter updates (Krstić and Wang, 2000). Following the filtering stages, the *parameter update* is carried out by integrating the filtered demodulated signal: the integrator state represents the current estimate of the optimized parameter and evolves in a direction that steers the system toward the extremum of the objective function. This integral adaptation mechanism provides a smooth and continuous adjustment of the parameter, contributing to closed-loop stability and convergence properties. Finally, the *modulation* stage injects a low-amplitude sinusoidal perturbation into the parameter under optimization (Ariyur and Krstić, 2004): this excitation is essential to probe the local sensitivity of the objective function with respect to the parameter, thereby enabling the system to generate informative response variations. The interaction between the injected perturbation and the system dynamics produces measurable oscillations in the objective function, which are then exploited by the demodulation and filtering stages to drive the optimization toward the optimum (Krstić and Wang, 2000).

In principle, the ESC algorithm could be used to control simultaneously the gas cooler pressure and the injection valve opening angle, but in this study it is applied only to the gas cooler pressure, while the injection valve opening angle is kept at the maximum value that avoids the onset of back-flow through the valve. Previous work (Fusco et al., 2025) has indeed shown that, for given operating conditions, the performance is optimal when the amount of vapor injected into the cylinder is maximum: the injection valve opening angle can therefore be set to a constant value, simplifying the operation of the control system. The architecture of the ESC algorithm implemented in Simulink is illustrated in Fig. 5, while the values of the design parameters are reported in Table 5.

The values of the design parameters have been chosen following the tuning guidelines proposed by Ariyur and Krstić (2004) and Krstić and Wang (2000), adapted to the dynamics of the thermodynamic cycle considered in this study, as well as the guidelines provided by MathWorks (The MathWorks, Inc., 2024). In addition, it is important to note that the tuning of both the ESC and the inner PI controllers accounted for the coupling between the two control layers, which are dynamically interconnected through the plant. The modulation (perturbation) frequency ω has been set well below the dominant time constant of the heat pump dynamics to ensure a clear separation of time scales between the perturbation and the slower evolution of the cycle variables: a perturbation period of approximately 63 s (corresponding to $\omega = 0.1 \text{ rad s}^{-1}$) is shorter than the typical settling time observed in the simulation, which is of the order of several hundred seconds, and is at the same time long enough for the thermodynamic cycle to reach a quasi-stationary response to each perturbation cycle (Hu et al., 2015). The high-pass

filter, with cut-off frequency ω_h , removes the DC bias from the perturbed objective function signal, while the low-pass filter, with cut-off frequency ω_l , attenuates the high-frequency components of the demodulated signal (The MathWorks, Inc., 2024). The modulation amplitude b has been kept small (0.1 MPa, that is about 0.9 % of the gas cooler pressure) to avoid a significant perturbation of the plant operation, while the demodulation amplitude a (0.7 MPa) has been chosen large enough to produce a demodulated signal clearly above the numerical noise floor observed in the objective function, consistent with the recommendation $a > b$. The demodulation and modulation phases Φ_1 and Φ_2 have both been set to zero, satisfying the condition $\cos(\Phi_1 - \Phi_2) > 0$. Finally, the learning rate k governs the convergence speed of the integrator in the parameter-update stage: the value adopted here provides a satisfactory compromise between the convergence speed and damping of the oscillations around the optimum, as demonstrated by the simulation results reported in Section 3.

The authors verified in preliminary simulations that varying ω within a factor of two around the adopted value does not change the identified optimum but affects the settling time proportionally, and that increasing k beyond approximately 1.7 resulted in the emergence of a secondary low-frequency oscillation superimposed on the perturbation cycle, consistent with a violation of the time-scale separation assumption underlying the averaging analysis of the ESC loop (Krstić and Wang, 2000), and was therefore avoided. To provide a complete overview of the learning rate value, Fig. 6 shows the convergence time, representing the time elapsed between the activation of the ESC controller and the achievement of convergence, as a function of the learning rate parameter, together with the maximum acceptable value, once the other parameters are fixed. The choice of $k = 1.1$ has been made taking into account a satisfactory convergence speed while at the same time keeping a reasonable margin from the instability threshold.

3. Results and discussion

The heat pump model was simulated starting from the design operating conditions reported in Table 2. During the first 700 s of simulation the ESC controller is kept inactive and the gas cooler pressure is maintained at the design value of 11 MPa, as shown in Fig. 7. The ESC controller is then activated, and the gas cooler pressure is modulated to attain the maximum achievable COP while maintaining all other boundary conditions constant. When the gas cooler pressure is held fixed at 11 MPa the COP is 2.996; under ESC control, the COP rises slightly to 3.06, with the optimal gas cooler pressure identified at 10.55 MPa.

After 3500 s the inlet water temperature to the gas cooler is increased by 10 K, reaching 35 °C. As shown in Fig. 7, this produces an expected decrease in COP, as reported in the literature: an increase in the gas cooler outlet temperature leads to a reduction in cycle performance. The ESC controller reacts by increasing the gas cooler pressure up to 12.085 MPa, for which the COP settles at 2.591. As a consequence of the new operating point, the intermediate pressure and the vapor quality in the flash tank both increase (Fig. 8); the steady-state analysis of Fusco et al. (2025) predicts the same direction of change when the gas cooler outlet temperature rises, which supports the consistency of the dynamic model. It is worth recalling that, when stationary conditions are reached, the ratio of injected to discharged mass flow rates must equal the vapor quality produced by the flash expansion, which means that, for a given injection valve opening angle, the intermediate pressure is automatically determined by the matching between compressor performance and flash-tank expansion represented by Eq. (13).

The system was then subjected to a further change in boundary conditions: with the gas cooler water inlet temperature kept at 35 °C, the water temperature increase imposed by the PI controller is reduced from 40 K to 35 K at $t = 7000$ s. The ESC controller adjusts the gas cooler pressure to optimize the COP, which reaches a value of 2.664 at a gas cooler pressure of 11.56 MPa (Fig. 7), significantly lower than in the previous condition. In contrast, the intermediate pressure and vapor quality ex-

hibit only minor variations in this new configuration (Fig. 8), a behavior that the steady-state results of Fusco et al. (2025) also display when only the water temperature rise is changed.

The final variation in boundary conditions is imposed after 9500 s of simulation: while maintaining the previously imposed water inlet temperature and temperature increase, the air inlet temperature is decreased by 3 K, resulting in an increase of the compressor suction pressure and, consequently, a reduction of the COP. In this case, the ESC controller slightly drops the gas cooler pressure to 11.35 MPa, corresponding to a COP of 2.576 (Fig. 7). The resulting intermediate pressure decreases significantly because it is directly influenced by the evaporation pressure, while the vapor quality increases slightly (Fig. 8); both trends reproduce those of the published steady-state analysis of FTVI Voorhees cycles (Fusco et al., 2025).

Regarding the other implemented PI controllers, which regulate the air and water temperatures as well as the superheat degree, the corresponding results are reported in Fig. 9. The figure presents, on the left, the reference temperature profiles and the actual dynamic response, and, on the right, the corresponding controller outputs (i.e., water and air mass flow rates and valve opening). The reference signals are accurately tracked under all operating conditions, demonstrating the capability of the implemented control strategies to ensure stable and reliable system operation, with only negligible deviations between reference and measured values.

To further assess the robustness of the ESC controller, an additional off-design scenario was simulated, again starting from the design operating condition (condition A) reported in Table 2 and applying a sequence of step changes in the boundary conditions in the opposite direction with respect to those discussed above: the gas cooler water inlet temperature is first decreased, followed by an increase in the water temperature difference, and finally by an increase in the air inlet temperature. For brevity, only the COP and gas cooler pressure trends are reported for this scenario. Condition A is not described again here, as it coincides with the previous scenario. As shown in Fig. 10, after 3500 s the inlet water temperature to the gas cooler is decreased to 20 °C, and the COP increases consistently with the expected trend, reaching a value of 3.337 at a gas cooler pressure of 9.75 MPa. After 7000 s, starting from the previous off-design condition, the gas cooler water temperature difference is increased, reaching 45 K. The gas cooler pressure rises to 10.45 MPa, with the corresponding COP settling at 3.205. After 9500 s, the air inlet temperature is increased by 3 K, reaching -3 °C. The COP increases to 3.329, while the gas cooler pressure also increases, reaching 10.53 MPa. It is worth noting that, also in this scenario, the results obtained are consistent with those presented in Fusco et al. (2025).

To provide a comprehensive overview of the ESC performance, the system was simulated with the ESC deactivated at several fixed values of the gas cooler pressure, under each of the seven sets of boundary conditions considered above. The resulting COP-versus- p_d curves are shown in Fig. 11, where the asterisks mark the gas cooler pressures selected by the ESC algorithm (Fig. 7). The ESC algorithm performs optimally for the first set of boundary conditions reaching the optimum value of COP. Regarding the other sets of boundary conditions, the algorithm still performs very well, although it does not perfectly capture the absolute maximum. However, the corresponding deviations in COP, reported in Table 6, are negligible: they are insignificant compared to the overall COP range and have no practical impact on the operation of the system or on the identification of the optimal operating point.

4. Conclusions

This paper has presented a dynamic model of a transcritical CO₂ Voorhees cycle, based on a Flash-Tank Vapor Injection layout, developed in the Simulink/Simscape environment to investigate the behavior of a transcritical heat pump system under off-design operating condi-

Table 6
COP deviations.

Condition	Deviation	Condition	Deviation
A	0.026 %	B'	0.320 %
B	0.365 %	C'	0.070 %
C	0.221 %	D'	0.419 %
D	0.227 %		

tions and to test a model-free optimal control strategy. The main conclusions of the study are the following:

1. The reciprocating compressor with solenoid injection valve has been represented by a customized Simscape component in which an Artificial Neural Network, trained on the results of a previously developed lumped-parameter working-cycle model (Fusco et al., 2026), provides the volumetric efficiencies, the discharge enthalpy, and the mechanical power. Because the ANN returns the cycle-averaged performance of a compressor whose working cycle is about four orders of magnitude faster than the thermodynamic-cycle dynamics, this approach decouples the two time scales and enables a stable and computationally efficient dynamic simulation of the whole system.
2. An adaptive control system based on an Extremum Seeking Control algorithm has been implemented to track the gas cooler pressure that maximizes the COP without requiring a plant model. Across seven sets of boundary conditions—the design condition and six off-design conditions, obtained by varying the water inlet temperature, the water temperature rise in the gas cooler, and the air inlet temperature to the evaporator, each in both directions—the controller achieves a COP within 0.5 % of the brute-force optimum in every case.
3. The shape of the COP characteristic as a function of the gas cooler pressure ranges from relatively flat to somewhat more pronounced across the operating envelope, yet remains moderately flat in all cases, as can be seen in Fig. 11. The ESC locates the optimum accurately in both regimes, confirming that the controller approaches the true optimum closely under qualitatively different response shapes.
4. The trends of intermediate pressure, COP, and flash-tank vapor quality obtained from the dynamic simulations reproduce those of the steady-state analysis previously published for the same class of cycles, confirming the internal consistency of the dynamic model.

The selected boundary conditions are representative of realistic operation of CO₂ heat pumps: water inlet temperatures of 20–35 °C and water temperature rises of 35 K to 45 K cover both domestic hot water production and space-heating supply, while air inlet temperatures down to -9 °C correspond to cold-climate winter operation, making the cases relevant to both commercial/residential and low-temperature industrial heat pumps.

Some limitations of the present work indicate directions for future research. First, the proposed model has not yet been validated experimentally; experimental validation is planned once the prototype reciprocating compressor described in Fusco et al. (2026) becomes operational. Second, the control strategy regulates only the gas cooler pressure, with the injection valve opening angle kept at its maximum admissible value, as suggested by previous results (Fusco et al., 2026); a natural extension is a multi-variable ESC that simultaneously optimizes the gas cooler pressure and the injection valve opening angle. Third, the ANN surrogate represents the cycle-averaged behavior of the compressor and therefore does not resolve transient intra-cycle phenomena; a time-resolved compressor model would be required to relax the quasi-steady assumption and to extend the framework to substantially faster control loops. Finally, the driving electric motor has been modeled as an ideal angular-velocity source, neglecting electrical transients and motor-control dynamics, whose interaction with the ESC loop at short time scales could be assessed with a more detailed representation.

Overall, the proposed modeling and control framework represents an effective tool for the analysis and the optimization of transcritical heat

pump systems based on a Voorhees cycle, and lays the foundation for the experimental validation of the prototype currently under development.

CRedit authorship contribution statement

Luigi Fusco: Writing – original draft, Visualization, Validation, Software, Methodology, Investigation; **Giulio Lenzi:** Validation, Resources; **Michele Manno:** Writing – review & editing, Visualization, Validation, Supervision, Resources, Methodology, Funding acquisition, Conceptualization; **Stefano Mazzoni:** Writing – review & editing, Validation, Software.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

Michele Manno reports financial support was provided by Ministry of Enterprises and Made in Italy. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

This work was supported by the Ministry of Enterprises and Made in Italy, project no. F/350182/01-02/X60.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at [10.1016/j.ijrefrig.2026.107070](https://doi.org/10.1016/j.ijrefrig.2026.107070)

References

- Adamson, K.-M., Walmsley, T.G., Carson, J.K., Chen, Q., Schlosser, F., Kong, L., Cleland, D.J., 2022. High-temperature and transcritical heat pump cycles and advancements: a review. *Renew. Sustain. Energy Rev.* 167, 112798. <https://doi.org/10.1016/j.rser.2022.112798>
- Ariyur, K.B., Krstic, M., 2004. Real-Time Optimization by Extremum-Seeking Control. Wiley-Interscience, Hoboken, NJ. <https://doi.org/10.1002/0471669784>
- Banks, A., Grist, C., Heller, J., Lim, H., 2022. Field measurement of central CO₂ heat pump water heater for multifamily retrofit. *Sustainability* 14, 8048. <https://doi.org/10.3390/su14138048>
- Bendapudi, S., Braun, J.E., Groll, E.A., 2008. A comparison of moving-boundary and finite-volume formulations for transients in centrifugal chillers. *Int. J. Refrig.* 31 (8), 1437–1452. <https://doi.org/10.1016/j.ijrefrig.2008.03.006>
- Burns, D.J., Laughman, C.R., Guay, M., 2020. Proportional–integral extremum seeking for vapor compression systems. *IEEE Trans. Control Syst. Technol.* 28 (2), 403–412. <https://doi.org/10.1109/TCST.2018.2882772>
- Cecchinato, L., Corradi, M., Cosi, G., Minetto, S., Rampazzo, M., 2012. A real-time algorithm for the determination of R744 systems optimal high pressure. *Int. J. Refrig.* 35 (4), 817–826. <https://doi.org/10.1016/j.ijrefrig.2012.01.005>
- Cecchinato, L., Corradi, M., Minetto, S., 2010. A critical approach to the determination of optimal heat rejection pressure in transcritical systems. *Appl. Therm. Eng.* 30 (13), 1812–1823. <https://doi.org/10.1016/j.applthermaleng.2010.04.015>
- Cui, C., Zong, S., Song, Y., Yin, X., Cao, F., 2021. Multi-variable extreme seeking control for efficient operation of sub-cooler vapor injection trans-critical CO₂ heat pump water heater. *Appl. Therm. Eng.* 184, 116338. <https://doi.org/10.1016/j.applthermaleng.2020.116338>
- Cui, C., Zong, S., Song, Y., Yin, X., Cao, F., 2022. Experimental investigation of the extreme seeking control on a transcritical CO₂ heat pump water heater. *Int. J. Refrig.* 133, 111–122. <https://doi.org/10.1016/j.ijrefrig.2021.09.027>
- Elliott, M.S., Rasmussen, B.P., 2013. Decentralized model predictive control of a multi-evaporator air conditioning system. *Control Eng. Pract.* 21 (12), 1665–1677. <https://doi.org/10.1016/j.conengprac.2013.08.006>
- Fusco, L., Lenzi, G., Manno, M., Mazzoni, S., 2026. Assessment of key performance indicators for a reciprocating compressor with electronically controlled vapor injection. *Appl. Therm. Eng.* 302, 131923. <https://doi.org/10.1016/j.applthermaleng.2026.131923>
- Fusco, L., Manno, M., Mazzoni, S., Vellini, M., 2025. Thermodynamic analysis of CO₂ transcritical cycles with vapor injection in reciprocating compressors (Voorhees cycles) for heat pumps operating at different design conditions. *Int. J. Refrig.* . <https://doi.org/10.1016/j.ijrefrig.2025.09.016>
- Haberschill, P., Guitari, I., Lallemand, A., 2007. Comportement dynamique d'une pompe à chaleur au CO₂ en cycles sous-critique et transcritique. *Int. J. Refrig.* 30 (4), 732–743. <https://doi.org/10.1016/j.ijrefrig.2006.09.002>
- Hu, B., Li, Y., Cao, F., Xing, Z., 2015. Extremum seeking control of COP optimization for air-source transcritical CO₂ heat pump water heater system. *Appl. Energy* 147, 361–372. <https://doi.org/10.1016/j.apenergy.2015.03.010>
- Kauf, F., 1999. Determination of the optimum high pressure for transcritical CO₂-refrigeration cycles. *Int. J. Therm. Sci.* 38 (4), 325–330. [https://doi.org/10.1016/S1290-0729\(99\)80098-2](https://doi.org/10.1016/S1290-0729(99)80098-2)
- Koeln, J.P., Alleyne, A.G., 2014. Optimal subcooling in vapor compression systems via extremum seeking control: theory and experiments. *Int. J. Refrig.* 43, 14–25. <https://doi.org/10.1016/j.ijrefrig.2014.03.012>
- Krstić, M., Wang, H.H., 2000. Stability of extremum seeking feedback for general nonlinear dynamic systems. *Automatica* 36, 595–601. [https://doi.org/10.1016/S0005-1098\(99\)00183-1](https://doi.org/10.1016/S0005-1098(99)00183-1)
- Lambers, K.J., Süß, J., Köhler, J., 2007. Admission (economizer) port optimization of a Voorhees modified reciprocating CO₂ compressor. *HVAC&R Res.* 13 (3), 525–540. <https://doi.org/10.1080/10789669.2007.10390967>
- Li, B., Alleyne, A.G., 2010. A dynamic model of a vapor compression cycle with shut-down and start-up operations. *Int. J. Refrig.* 33 (3), 538–552. <https://doi.org/10.1016/j.ijrefrig.2009.09.011>
- Liao, S.M., Zhao, T.S., Jakobsen, A., 2000. A correlation of optimal heat rejection pressures in transcritical carbon dioxide cycles. *Appl. Therm. Eng.* 20 (9), 831–841. [https://doi.org/10.1016/S1359-4311\(99\)00070-8](https://doi.org/10.1016/S1359-4311(99)00070-8)
- Liu, X., Fu, R., Wang, Z., Lin, L., Sun, Z., Li, X., 2019. Thermodynamic analysis of transcritical CO₂ refrigeration cycle integrated with thermoelectric subcooler and ejector. *Energy Convers. Manag.* 188, 354–365. <https://doi.org/10.1016/j.enconman.2019.02.088>
- Lorentzen, G., Pettersen, J., 1993. A new, efficient and environmentally benign system for car air-conditioning. *Int. J. Refrig.* 16, 4–12. [https://doi.org/10.1016/0140-7007\(93\)90014-Y](https://doi.org/10.1016/0140-7007(93)90014-Y)
- Mamur, H., Üstüner, M.A., Bhuiyan, M.R.A., 2022. Future perspective and current situation of maximum power point tracking methods in thermoelectric generators. *Sustain. Energy Technol. Assess.* 50, 101824. <https://doi.org/10.1016/j.seta.2021.101824>
- Pfafferoth, T., Schmitz, G., 2004. Modelling and transient simulation of CO₂-refrigeration systems with Modelica. *Int. J. Refrig.* 27 (1), 42–52. [https://doi.org/10.1016/S0140-7007\(03\)00098-7](https://doi.org/10.1016/S0140-7007(03)00098-7)
- Rasmussen, B.P., Alleyne, A.G., 2004. Control-oriented modeling of transcritical vapor compression systems. *J. Dyn. Syst. Meas. Control* 126 (1), 54–64. <https://doi.org/10.1115/1.1648312>
- Sarkar, J., Bhattacharyya, S., Ram Gopal, M., 2004. Optimization of a transcritical CO₂ heat pump cycle for simultaneous cooling and heating applications. *Int. J. Refrig.* 27 (8), 830–838. <https://doi.org/10.1016/j.ijrefrig.2004.03.006>
- Stene, J., 2005. Residential CO₂ heat pump system for combined space heating and hot water heating. *Int. J. Refrig.* 28, 1259–1265. <https://doi.org/10.1016/j.ijrefrig.2005.07.006>
- Tanveer, M.M., Bradshaw, C.R., 2022. A review of geometry, mass flow and heat transfer sub-models used in positive displacement compressor simulations. In: 26th International Compressor Engineering Conference at Purdue, p. Paper 2761. <https://docs.lib.purdue.edu/icec/2761/>
- The MathWorks, Inc., 2024. Extremum seeking control. <https://it.mathworks.com/help/sicontrol/ug/extremum-seeking-control.html>
- Voorhees, G.T., 1927. The Compression Refrigerating Machine. Nickerson & Collins co., Chicago.
- Wang, W., Li, Y., 2019. Intermediate pressure optimization for two-stage air-source heat pump with flash tank cycle vapor injection via extremum seeking. *Appl. Energy* 238, 612–626. <https://doi.org/10.1016/j.apenergy.2019.01.083>
- Wang, W., Zhao, Z., Zhou, Q., Qiao, Y., Cao, F., 2021. Model predictive control for the operation of a transcritical CO₂ air source heat pump water heater. *Appl. Energy* 300, 117339. <https://doi.org/10.1016/j.apenergy.2021.117339>
- Wang, W., Zhou, Q., Tian, G., Hu, B., Li, Y., Cao, F., 2020. The intermediate temperature optimization for cascade refrigeration system and air source heat pump via extreme seeking control. *Int. J. Refrig.* 117, 150–162. <https://doi.org/10.1016/j.ijrefrig.2020.05.009>
- Weiss, W.K., Burns, D.J., Guay, M., 2018. Realtime optimization of MPC setpoints using time-varying extremum seeking control for vapor compression machines. In: International Refrigeration and Air Conditioning Conference at Purdue, p. Paper 1951. <https://docs.lib.purdue.edu/iracc/1951>
- Wolscht, L., Knobloch, K., Jacquemoud, E., Jenny, P., 2024. Dynamic simulation and experimental validation of a 35 MW heat pump based on a transcritical CO₂ cycle. *Energy* 294, 130897. <https://doi.org/10.1016/j.energy.2024.130897>
- Wu, D., Hu, B., Wang, R.Z., 2021. Vapor compression heat pumps with pure low-GWP refrigerants. *Renew. Sustain. Energy Rev.* 138, 110571. <https://doi.org/10.1016/j.rser.2020.110571>
- Yang, B., Bradshaw, C.R., Groll, E.A., 2013. Modeling of a semi-hermetic CO₂ reciprocating compressor including lubrication submodels for piston rings and bearings. *Int. J. Refrig.* 36 (7), 1925–1937. <https://doi.org/10.1016/j.ijrefrig.2012.10.017>
- Yin, X., Fang, J., Wang, A., Cao, F., 2024. Real-time multi-variable optimization of the interstage subcooling vapor-injection based transcritical CO₂ HPWH: an integrated model predictive control approach. *Appl. Therm. Eng.* 239, 122099. <https://doi.org/10.1016/j.applthermaleng.2023.122099>
- Zha, S., Hafner, A., Nekså, P., 2008. Investigation of R-744 Voorhees transcritical heat pump system. *Int. J. Refrig.* 31 (1), 16–22. <https://doi.org/10.1016/j.ijrefrig.2007.06.017>
- Zhang, C., Ordóñez, R., 2012. Design of extremum seeking control. In: Zhang, C., Ordóñez, R. (Eds.), *Extremum-Seeking Control and Applications: A Numerical Optimization-Based Approach*. Springer, London, pp. 47–66.
- Zhang, T., Cao, F., Song, Y., Yin, X., He, Y., Fang, J., 2023. The model predictive control strategy of the transcritical CO₂ air conditioning system used in railway vehicles. *Appl. Therm. Eng.* 218, 119376. <https://doi.org/10.1016/j.applthermaleng.2022.119376>