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An Improved Negotiation-Based Approach for Collecting and Sorting Operations in Waste Management and Recycling

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Abstract: This paper addresses the problem of optimal planning for collection, sorting, and recycling operations. The problem arises in industrial waste management, where distinct actors manage the collection and the sorting operations. In a weekly or monthly plan horizon, they usually interact to find a suitable schedule for servicing customers but with a not well-defined scheme. We proposal an improved negotiation-based approach using an auction mechanism for optimizing these operations. Two interdependent models are presented: one for waste collection by a logistics operator and the other for sorting operations at a recycling plant. These models are formulated as mixed-integer linear programs where costs associated with sorting and collection are to be minimized, respectively. We describe the negotiation-based approach involving an auction where the logistics operator bids for collection time slots, and the recycling plant selects the optimal bid based on the integration of sorting and collection costs. This approach aims to achieve an optimization of the entire waste management process. Computational experiments are presented.

Keywords: waste management; optimization; auctions; negotiation approaches

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1. Introduction and Literature Review

The purpose of this study is to provide a model based on negotiation in the field of waste management and recycling. The paper is inspired by previous work [1] in which a similar approach was proposed for a waste collection problem. Several efforts are reported in countries and/or local administrations to incentivize waste collection and material circularity through incentives and negotiation mechanisms. In this area, particular attention has been devoted to the collection and sorting processes. Collection refers to the process of gathering and collecting waste from various sources, such as households, businesses, and public areas. It involves the use of waste collection vehicles and containers to transport waste to designated collection points or facilities. Sorting, on the other hand, involves the separation and categorization of different types of waste materials. This process is necessary to ensure that different recyclable materials (i.e., fractions of recyclable materials) are properly identified and separated from non-recyclable waste. Sorting can be performed manually or with the help of automated systems and technologies.

The main research gap covered in this work is the proposal of an improved combinatorial auction system that takes into account the specific characteristics of industrial waste recycling, leveraging the interactions among logistics and sorting operators. In particular, the main innovation of the proposal lies in the different and improved design of the bidding phase. Thanks to the improved auction system, the bidders can generate more different bids, covering a larger solution space. In particular, bids are generated not only by looking at the objective of the logistics operator but also at the objective of the sorting decision maker to obtain more robust solutions. This led to solving the winner determination problem more effectively compared to standard methods, as we will show in the following.

The interaction between collection and sorting is crucial for waste management. Here is how they work together.

- Waste collection systems need to be well planned and organized to ensure that waste is collected regularly and efficiently. Proper collection routes and schedules are established based on the type of waste, the volume of waste generated, and the specific needs of the area. By implementing effective collection practices, waste can be gathered and transported to sorting facilities in a timely and cost-effective manner.
- Sorting plays a vital role in separating recyclable materials from non-recyclable waste. Once waste reaches the sorting facility, trained personnel or automated systems sort through the waste to identify and separate materials such as paper, plastic, glass, metal, and organic waste. This segregation allows for the recycling of valuable materials and prevents them from being disposed of in landfills or incinerators.
- By effectively sorting waste, recyclable materials can be diverted from landfills and sent to recycling facilities. Sorting also enables the recovery of valuable resources from waste, promoting a circular economy and reducing the need for raw material extraction.
- Not all waste materials can be recycled. Sorting ensures that non-recyclable waste is properly identified and disposed of in appropriate ways, such as landfilling or waste-to-energy facilities. By separating non-recyclable waste, the focus can be placed on maximizing the recycling potential of the remaining materials.

In summary, collection and sorting are interconnected steps in waste management. The efficient collection ensures that waste is gathered and transported to sorting facilities, while sorting allows for the separation of recyclables from non-recyclable waste. This collaboration is essential for effective waste management practices that promote recycling, resource conservation, and proper disposal.

Negotiation plays an important role in waste management, especially when it comes to addressing the interests of various stakeholders and finding mutually beneficial solutions. Here is how negotiation works in waste management.

- The first step in the negotiation process is identifying the relevant stakeholders involved in waste management. This may include government agencies, waste management companies, environmental organizations, local communities, and residents.
- Each stakeholder will have different interests, objectives, and concerns when it comes to waste management. It is crucial to understand these interests and objectives to facilitate effective negotiations. For example, waste management companies may prioritize cost-efficiency and profitability, while environmental organizations may focus on sustainability and minimizing environmental impact.
- Successful negotiations require building relationships and establishing trust among the stakeholders. This involves open communication, active listening, and acknowledging the perspectives of all parties involved. Building trust creates a collaborative environment where stakeholders are more willing to work together to find common ground.
- Transparency and sharing of relevant information are essential in waste management negotiations. This includes data on waste generation, recycling rates, waste treatment technologies, and regulatory requirements. Sharing information helps stakeholders make informed decisions and facilitates the development of effective waste management strategies.
- Negotiations in waste management aim to find win–win solutions that address the interests and concerns of all stakeholders involved. For example, waste management companies may agree to invest in more environmentally friendly technologies in exchange for regulatory incentives or public support.
- In complex negotiations, mediators or facilitators may be involved to assist in the negotiation process. They help manage conflicts, guide discussions, and ensure that all

parties have an equal opportunity to express their views. Mediators can help bridge the gap between different stakeholders and facilitate productive negotiations.
- Once an agreement is reached, it needs to be implemented and monitored to ensure compliance. This may involve developing waste management plans, implementing new policies or regulations, and monitoring progress toward waste reduction and recycling targets. Regular review and evaluation of the agreement can help identify areas for improvement and ensure its effectiveness over time.

The literature addressed solid waste management in different ways, and several authors analyzed the topic from the strategic, tactical, and operational side. Cerqueira-Streit et al. [2] detail in their survey how waste management is a critical factor in turning the circular economy into a profitable business. To address the complexity of waste management processes, Cardoso et al. [3] propose a method based on a literature review and apply evidence-based decision making to resolve conflicts in electronic waste management. However, profitability can only be reached if the underlying waste management processes are optimized. For a comprehensive review of the literature on strategic and tactical issues in the management of solid waste, the readers are referred to the following surveys [4–6].

Combinatorial auctions (CAs) are a well-consolidated method to mathematically formalize and solve shared resource allocation problems between different actors. In particular, they are useful when the bidder is interested in bidding not only on a single item but also on a combination of them [7]. In recent years, we assisted a renewed interest in CAs as methods to solve multi-actor problems in shared economy or for eco-sustainable service systems. Triki [8] uses CAs to formalize and optimize the interactions among actors in crowd shipping, where occasional drivers are considered and the winner determination problem is used to solve the vehicle routing problem. The problem of allocating energy storage to optimize social welfare and prices is addressed by Zhong et al. [9] using CAs. In [10], the design of online CAs is considered when supply chain costs are considered. Another recent application of CAs is in the renewal energy industry and market. As illustrated by the survey of Ehrhart et al. [11], the characteristics of complementary and substitute relationships between renewal energy projects can be exploited by CAs. CAs show good perspectives in circular economy, reverse logistics, and green supply chains. Ma et al. [12] use CAs to improve the performances of an online used-car platform with the aim of social welfare maximization and considering incentive compatibility conditions. The authors use both price and non-price attributes in bid generation, such as environmental compliance, service level, and car rating. CAs in the circular economy are also used in [13] to optimally compute, through the winner determination problem, the reselling prices in secondary markets. The remanufacturing of end-of-life products is the application case for an automated demand–supply matching negotiation algorithm proposed by Fernández et al. [14]. Reverse CAs are studied by Triki et al. [15] for real applications in food industries. Incentives for recycling in the international supply chain is the topic addressed in the work by Bimonte et al. [16], where the strategic decision is addressed with a Stackelberg game. In [17], auctions are used to regulate the relations between a carrier and several shippers in the waste collection process. Auction allocation and routing problems are solved by metaheuristics. The paper proposes the auction framework as a carrier-centric pattern in addition to solving synchronization issues. Differently, our proposal focuses on the relations between the logistics operators and the sorting/recycling plant. The work in [18] studies auctions for the battery recycling industry. The authors propose a multi-unit trade reduction mechanism that is demonstrated to be efficient for resource allocation in the electric vehicle battery recycling market. The work is aimed at supporting the decision-maker in evaluating policies for developing the EVBR market. In [19], a multi-objective approach for the circular economy is proposed. The importance of using a similar approach to the one presented in the previous paper is underlined in [20] where the literature on zero defect manufacturing has been classified. The investigation on how to combine optimization and negotiation to minimize recycling costs, taking into account the collection process and material sorting cost, has also been studied in [21] where the authors proposed a mixed-integer linear formulation for the multiperiod planning problem.

We are interested in solution approaches that use market-based mechanisms, as described in [22], negotiations, cooperation, and auctions. Auctions and CAs make it possible to formally describe the problem of assigning shared and limited resources among customers and suppliers, as shown in [7]. Some applications can be found in flexible manufacturing [23], healthcare [24] and value creation in manufacturing [25]. Simulation is used in [26] to study negotiation protocols in a closed supply chain and to assess reusing policies. Ref. [27] consider the importance of multiple criteria analysis in waste collection. Another study is presented in [17] in which waste collection is solved using an auction-based system. The authors assume that the carrier plays the auctioneer role to decide on who wins the bids and the corresponding payments, while the shippers are the bidders.

Notwithstanding the renewed interest in combinatorial auctions in applications such as the shared economy and eco-sustainability, there are a few applications in waste recycling and in particular bidding auction systems specifically designed to improve waste management operations. In contrast, in our manuscript, we propose a model based on combinatorial auctions able to efficiently allocate resources in the joint process of industrial solid waste collection and sorting.

The remainder of the paper is as follows. Section 2 presents the mathematical models of the two problems. Section 3 formalizes the negotiation-based approach to integrate these models. Section 4 details the experiments and results; Section 5 provides the conclusions.

2. Models for Sorting and Logistics Operators

In this section, we model collection and sorting planning in an industrial solid waste management system. The processes of the case studied are sketched in Figure 1 for transport and in Figure 2 for sorting. Transport is carried out by a specialized logistics operator that serves customers by transporting empty containers to their location and picking full containers to be delivered to the sorting plant. Several industries produce mixed waste that must be sorted. In particular, warehouses and transport operators are a source of recyclable waste where different types of plastic and paper are grouped. In the analyzed sorting plant, waste goes through a sorting cabin where it is visually inspected and separated into its components. After several other sequential operations, the separated waste is transformed into secondary raw material and can be sent to other processing plants.

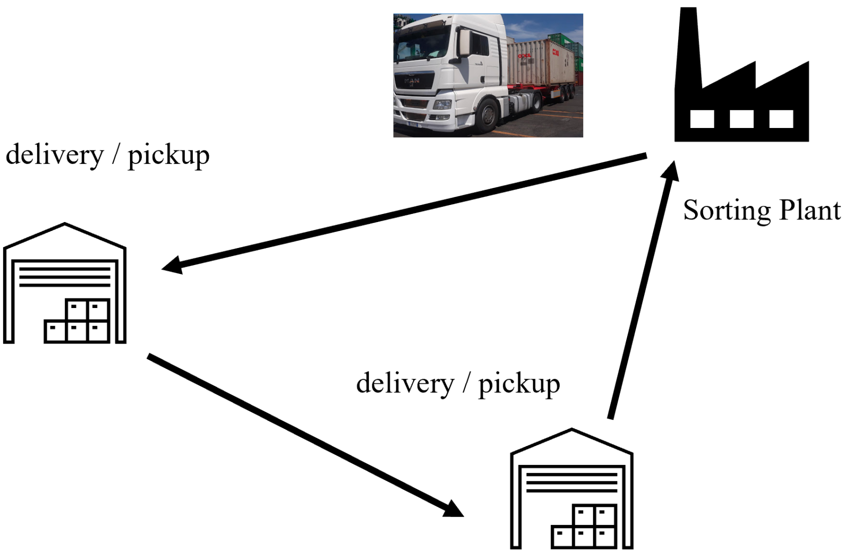

Figure 1. Diagram of the process operated by the carriers.

Different technologies can be used to improve the automation and digitization phases of the collection and sorting processes both at the operational and planning levels. Arebey et al. [28] surveyed how tracking technologies, such as RFID and GPS, could support waste management. In particular, RFID and image recognition applied to bins are used to track waste production and improve planning processes. Recently, Abdallah et al. [29] surveyed the research literature considering artificial intelligence applications to municipal waste management. Artificial intelligence and computer vision can be applied to sorting to automatically separate waste [30]. In this context, our case study is related to an industrial waste recycling network, where RFID is used to track the position of containers located in customer locations or in a sorting plant.

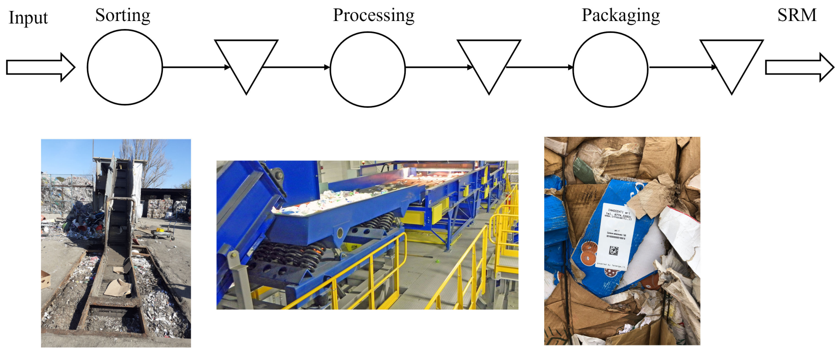

Figure 2. Diagram of sorting plant with the main waste treatment phases from the input of solid waste to secondary raw materials (SRM).

The sorting process is still not automated, and visual inspection coupled with manual sorting at the cabin is performed. Customers, the logistics operator, and the sorting operator are connected through email and the Enterprise Resource Planning (ERP) system. The planning of the two processes is scheduled every week/month independently by the sorting and the logistics operators. This leads to peaks in daily sorting operations because the customers interact directly only with the logistics operator. In the studied case, only a not formally specified interaction between logistics and sorting operator occurs, mostly by phone call, to synchronize transports with sorting operations.

Starting with these premises, we decided to focus on negotiation as a way to manage our waste management problem. In particular, the decision process that needs optimization and is studied in this paper is as follows. Business customers establish contracts with recycling operators. They generate waste during their business processes, which is stored in local buffers (e.g., containers). Once the buffers are full, a recycling request is generated to the logistics operator in charge to define a transportation plan. Once the transportation schedule is established and communicated to the sorting operator, the latter can plan the sorting operations at the plant. The plant operates sequential sorting stations that require setup and staffing. Additionally, waste buffers have limited capacity and incur occupancy costs. After the sorting/recycling operations, the recycled material can be directed to secondary raw material markets. Although the planning of these two subproblems (collection and sorting) is usually completed sequentially and separately, this study proposes an effective method to optimize and integrate both planning phases. We use an improved auction-based scheme to solve the sorting and collection problems in an integrated way and obtain an optimal multiperiod schedule for the two sub-processes. In particular, concerning the approach described in [1], the bid generation scheme is designed to produce a more robust transport plan and thus greater opportunities for the sorting operators to optimize the sorting plan.

The sorting operator aims to minimize the cost function of the sorting operations in the plant formed of variable and fixed components. The variable component considers both sorting and inventory costs based on the amount of commodity processed, while the fixed component considers the setup cost of the working stations involved. In fact, the model is a variant of lot sizing applied to a reverse logistics problem.

The commodity to be sorted in the sorting plant must be moved from waste generation points, where customer demands are assumed to be generated; to this end, a logistics operator aims to minimize such collection cost by optimally routing a fleet of vehicles. To model its decision problem, the logistics operator has to estimate some core parameters such as the planning time horizon and its discretization in time slots, where customers are located, their demands, the unit collection cost (time and customer dependent), the fleet of vehicles, and their capacities.

The connection between the two problems (collection and sorting) is that the optimal quantity a_t to be sorted in each time slot t should be equal to the total amount of commodity collected by the logistics operator from customers in the same time slot. In the following, we first define the formulation of the two problems, and next, we show how to tunnel information between the latter.

2.1. Formulation of the Sorting Problem

The sorting problem aims to define the optimal amount of waste to be sorted within a planning horizon, discretized into $|T_s|$ time slots, for every sorting facility. To this end, we introduce a nonnegative variable x_{jt} defining the quantity of material to be sorted in time slot $t \in T_s$, and a binary decision variable y_{jt} that is equal to 1 if the sorting station (facility) $j \in J$ is opened in $t \in T_s$ and 0 otherwise. Opening a sorting facility in $t \in T_s$ implies a setup time, as detailed in the following; therefore, it is important to correctly evaluate which facilities must operate and which must be closed. I_{jt} is the level of inventory of the upstream buffer of station $j \in J$ in time slot $t \in T_s$ with the condition $I_{j0} = 0$, $\forall j \in J$. Here are the parameters:

- c_t^s, f_j, h_j are the variable sorting costs in time slot $t \in T_s$, the fixed setup time for station $j \in J$, and the inventory holding cost for station $j \in J$, respectively;
- a_t is the quantity of material arriving at the sorting plant at time slot $t \in T_s$;
- E_j, M_j are the minimum and maximum allowed sorting quantity for station $j \in J$, respectively, in case it is activated;
- α is the loss factor considering the mean percentage of material that is discarded after selection at each sorting stage;
- LC_j is the inventory capacity for station $j \in J$.

The P1 sorting model is the following:

$$\min \; Z_s = \sum_{j \in J} \sum_{t \in T_s} c_t^s \cdot x_{jt} + \sum_{j \in J} \sum_{t \in T_s} f_j \cdot \max(0, y_{jt} - y_{j,t-1}) + \sum_{j \in J} \sum_{t \in T_s} h_j \cdot I_{jt} \tag{1}$$

s.t.

$$E_j y_{jt} \leq x_{jt} \leq M_j y_{jt}, \quad \forall j \in J, \forall t \in T_s, \tag{2}$$

$$I_{1,t} = I_{1,t-1} + a_{t-1} - x_{1,t-1}, \quad \forall t \in T_s \setminus \{0\}, \tag{3}$$

$$I_{j,t} = I_{j,t-1} + \alpha x_{j-1,t-1} - x_{j,t-1}, \forall t \in T_s \setminus \{0\}, j \in J \setminus \{1\}, \tag{4}$$

$$I_{jt} \leq LC_j, \quad \forall j \in J, t \in T_s, \tag{5}$$

$$x_{jt} \geq 0, \; y_{jt} \in \{0, 1\}, \forall t \in T_s, \forall j \in J. \tag{6}$$

Equation (1) is the objective function of the problem aiming at minimizing the sorting cost, the setup time cost, and the inventory holding cost. Constraint (2) limits x_{jt} values. Constraints (3) and (4) define the inventories. We note that (3) embeds the quantities to be collected from the customers. Constraints (5) limit the inventory level to a maximum capacity, while (6) defines the variable domains.

Model P1 has a nonlinearity in the objective function that can be easily linearized as follows:

$$\min\ Z_s = \sum_{j\in J}\sum_{t\in T_s} c^s_t \cdot x_{jt} + \sum_{j\in J}\sum_{t\in T_s} f_j \cdot \bar{y}_{jt} + \sum_{j\in J}\sum_{t\in T_s} h_j \cdot I_{jt} \tag{7}$$

s.t.

$$E_j y_{jt} \le x_{jt} \le M_j y_{jt}, \quad \forall j \in J, \forall t \in T_s, \tag{2'}$$

$$I_{1,t} = I_{1,t-1} + a_{t-1} - x_{1,t-1}, \quad \forall t \in T_s \setminus \{0\}, \tag{3'}$$

$$I_{j,t} = I_{j,t-1} + \alpha x_{j-1,t-1} - x_{j,t-1}, \forall t \in T_s \setminus \{0\}, j \in J \setminus \{1\}, \tag{4'}$$

$$I_{jt} \le LC_j, \quad \forall j \in J, t \in T_s, \tag{5'}$$

$$x_{jt} \ge 0,\ y_{jt} \in \{0,1\}, \forall t \in T_s, \forall j \in J. \tag{6'}$$

$$\bar{y}_{jt} \ge y_{jt} - y_{j,t-1}, \quad \forall j \in J, t \in T_s, \tag{8}$$

$$\bar{y}_{jt} \ge 0, \quad \forall j \in J, t \in T_s. \tag{9}$$

2.2. Formulation of the Collection Problem

In the collection problem, we have a set I of customers who require the collection of quantities $q_i, \forall i \in I$, of waste material to be sorted/recycled during the time horizon T_c with $T_c \le T_s$. The logistics operator uses a fleet H of vehicles, which each have a limited capacity C. The cost c^c_{it} measures the expenses for the logistics operator to collect the waste commodity from customer $i \in I$ in time slot $t \in T_c$. The binary decision variable $z_{iht} \in \{0,1\}$ is equal to 1 if customer $i \in I$ is serviced by vehicle $h \in H$ in time slot $t \in T_c$, and it is zero otherwise. The resulting collection model P2 is the following:

$$\min\ Z_c = \sum_{i\in I}\sum_{h\in H}\sum_{t\in T_c} c^c_{it} \cdot z_{iht} \tag{10}$$

s.t.

$$\sum_{h\in H}\sum_{t\in T^c} z_{iht} = 1, \forall i \in I, \tag{11}$$

$$\sum_{i\in I} q_i \cdot z_{iht} \le C, \forall h \in H, \forall t \in T_c, \tag{12}$$

$$z_{iht} \in \{0,1\}, \forall i \in I, h \in H, t \in T_c. \tag{13}$$

Equation (22) is the objective cost function. Constraint (11) imposes that each customer has to be serviced only once, while (12) defines capacity restrictions for each vehicle in each timeslot. Constraint (13) defines the domain of variables z_{iht}. While model P1 is a variant of the lot sizing model and a variant of the model presented in [21], model P2 defines a multiperiod allocation problem which can be considered a variant of the knapsack problem.

3. The Negotiation-Based Approach

As mentioned previously, the two models are interdependent because the quantity to be sorted in each time slot is determined based on the decision on when to collect materials from customers. In particular, the binding between the two models is defined by the following equation:

$$a_t = \sum_{i\in I}\sum_{h\in H} q_i \cdot z_{iht}. \tag{14}$$

Since the two problems are addressed, in general, by two different decision-makers, a negotiation-based approach can support the selection of a solution, taking into account the distinct objectives of the two actors of the system. Negotiation-based approaches have been applied in several forms in the literature (see, e.g. [7]) to address different applications. In this paper, we solve the problem by proposing a negotiation scheme with an auction model depicted in Figure 3 and described in the following. The interactions among the actors start from the customers and the sorting operator sides simultaneously; indeed, the first send requests q_i and the latter send the available capacities over time. Once all the data are received, the logistics operator defines the bids and sends them to the sorting operator. Then, the plant operator, based on the bids received, chooses the best bid solving a winner determination problem. Once the best bid is made, the sorting operator sends this information to the logistics operator who prepares the collection plan and informs the customers of the collection schedule. In the following, we detail how bids are generated and how the winner determination problem is defined.

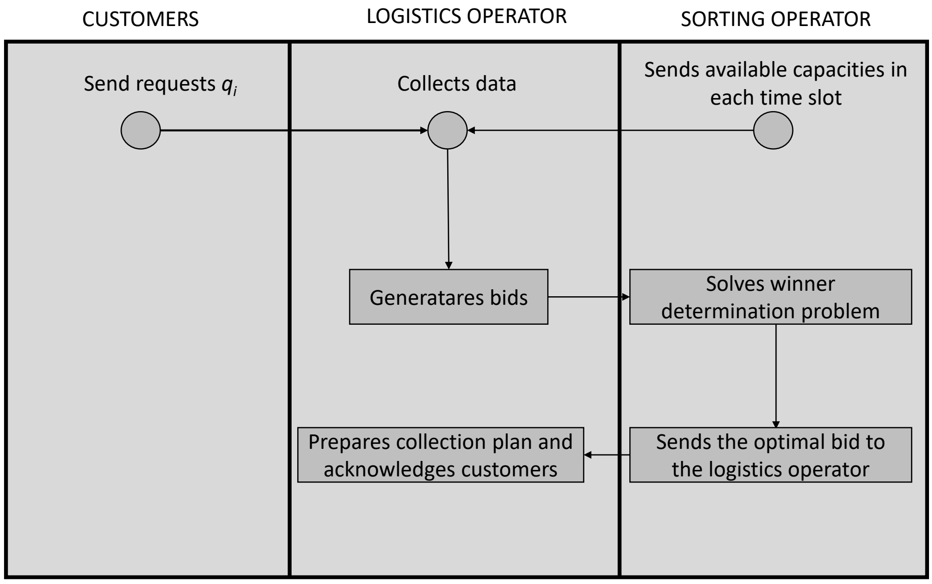

Figure 3. Scheme of the auction-based negotiation.

3.1. The Bid Generation Problem

The bid is the vector (γ, τ) where γ is a row vector of dimensions $|I|$ that corresponds to the quantities to be processed from each customer and τ is a vector of dimensions $|I|$ of the time slots chosen in the bid to serve each customer. A set of K bids is generated by iteratively solving the problem P2. In particular, at each iteration k, the solution obtained in iteration $(k-1)$ is totally or partially prohibited in problem P2 by adding a specific constraint in the k-th bid generation problem. The proposed bid generation scheme guarantees different bids that are feasible for the logistics operator and near-optimal. At each iteration k, let $N^{k-1} = \{(i,h,t) : z^*_{iht} = 1\}$ in bid $k-1$ (clearly $N^0 = \emptyset$). To generate a bid at iteration $k \geq 1$, we introduce a tabu list L^k_{iht} such that

$$L^k_{iht} = \begin{cases} 0, & \text{if } k = 1, \\ L^{k-1}_{iht} - 1, & \text{if } k > 1 \text{ and } (i,h,t) \in N^{k-1} \text{ and } L^{k-1}_{iht} > 1, \\ r, & \text{if } k > 1 \text{ and } L^{k-1}_{iht} = 0. \end{cases}$$

List L acts as a tabu list; in fact, the idea is to prevent client i from being serviced by vehicle h at time t in iteration k if it has been serviced by the same vehicle at the same time in the previous iteration. This allows for diversifying the solutions associated with the bids to be given in input to the collection problem in the next winner-determination problem. However, in order not to possibly prohibit all the assignments of the previous iteration, guaranteeing both diversification and intensification, we define an aspiration criterion such that

$$z_{iht} \leq \rho^k_{iht} + \zeta^k_{iht} \tag{15}$$

where

$$\rho_{iht}^{k} = \begin{cases} 0, & \text{if } L_{iht}^{k} > 0, \\ 1, & \text{if } L_{iht}^{k} = 0. \end{cases}$$

and

$$\zeta_{iht}^{k} = \begin{cases} 1, & \text{if } Uniform(0,1) > 1 - \frac{Z_c^{(k-1)} - Z_c^{(k-2)}}{Z_c^{(k-2)}}, \\ 0, & \text{otherwise.} \end{cases}$$

Parameter ζ_{iht}^{k} has the following effect: assume that the algorithm is processing iteration k in the bid generation problem. Let $Z_c^{(k-1)}$ and $Z_c^{(k-1)}$ be the optimal solution value of the bid generation problem in iterations $(k-1)$ and $(k-2)$, respectively. The ratio $\frac{Z_c^{(k-1)} - Z_c^{(k-2)}}{Z_c^{(k-2)}}$ is the percentage of variation of the optimal objective of the bids associated with the iterations mentioned. If the variation is positive and large, that is, the cost is increased by a sufficiently large amount, the algorithm is not prone to intensify this solution; therefore, if an assignment that may restore a previous more profitable solution is prohibited, the algorithm tries to activate an aspiration criterion to override the tabu list status. As can be seen, if ρ_{iht}^{k} is zero and the ratio $\frac{Z_c^{(k-1)} - Z_c^{(k-2)}}{Z_c^{(k-2)}}$ is large, $1 - \frac{Z_c^{(k-1)} - Z_c^{(k-2)}}{Z_c^{(k-2)}}$ is small, say close to zero, and there is a greater chance that ζ_{iht}^{k} becomes 1 by its definition.

The overall problem to solve to find the k-th bid is the following.

$$\min \sum_{i \in I} \sum_{h \in H} \sum_{t \in T_c} c_{it}^{c} \cdot z_{iht} + \sum_{t \in T_c} \max\{0, [a_t - \sum_{j \in J} (LC_j - I_{jt})]\} \tag{16}$$

s.t.

$$\sum_{h \in H, t \in T_c} z_{iht} = 1, \forall i \in I, \tag{17}$$

$$\sum_{i \in I} q_i \cdot z_{iht} = a_t, \forall h \in H, \forall t \in T_c, \tag{18}$$

$$a_t \leq C, \forall t \in T_c, \tag{19}$$

$$z_{iht} \leq \rho_{iht}^{k} + \zeta_{iht}^{k}, \forall i \in I, \forall h \in H, \forall t \in T_c, \tag{20}$$

$$z_{iht} \in \{0,1\}, \forall i \in I, \forall h \in H, \forall t \in T_c. \tag{21}$$

Note that the objective function takes into account two components. The first component is exactly the collection cost; the second part of the objective tries to infer the state of occupancy of the resources that the sorting operator will use. In fact, the term $[a_t - \sum_{j \in J}(LC_j - I_{jt})]$ penalizes scenarios in which there is no residual capacity in the sorting facilities at time t. Instead, if a_t is smaller than the residual capacity at time t, then the second term of the objective is zero. Note that even though this second term is nonlinear, it can be easily linearized as follows:

$$\min \sum_{i \in I} \sum_{h \in H} \sum_{t \in T_c} c_{it}^{c} \cdot z_{iht} + \sum_{t \in T_c} R_t \tag{22}$$

s.t.

$$\sum_{h \in H, t \in T_c} z_{iht} = 1, \forall i \in I, \tag{23}$$

$$\sum_{i \in I} q_i \cdot z_{iht} = a_t, \forall h \in H, \forall t \in T_c, \tag{24}$$

(25)

$$a_t \leq C, \forall t \in T_c, \tag{26}$$

$$z_{iht} \leq \rho^k_{iht} + \zeta^k_{iht}, \forall i \in I, \forall h \in H, \forall t \in T_c, \tag{27}$$

$$R_t \geq [a_t - \sum_{j \in J}(LC_j - I_{jt})], \forall t \in T_c, \tag{28}$$

$$R_t \geq 0, \forall t \in T_c, \tag{29}$$

$$z_{iht} \in \{0,1\}, \forall i \in I, \forall h \in H, \forall t \in T_c. \tag{30}$$

3.2. The Winner Determination Problem

The winner determination problem starts from problem P1 and considers Equation (14). To embed the bids into the model, we consider the following reformulation. The k-th bid $(\gamma, \tau)^k$ can be rewritten by introducing parameter q^k_{it}, $\forall i \in I, \forall t \in T_c, \forall k \in K$, such that

$$q^k_{it} = \begin{cases} q_i, & \text{if customer } i \text{ is serviced in timeslot } t \in T_c \text{ in bid } k, \\ 0, & \text{otherwise.} \end{cases}$$

Note that $q^k_{it} = 0$, $\forall t \in T_s \setminus T_c$. Moreover, a bid selection binary variable $u^k \in \{0,1\}, \forall k \in K$, is needed such that

$$u^k = \begin{cases} 1 & \text{if bid } k \text{ is selected,} \\ 0 & \text{otherwise.} \end{cases}$$

Thus, Equation (3) can be rewritten as

$$I_{1,t} = I_{1,t-1} + \sum_{i \in I, k \in K} q^k_{i,t-1} \cdot u^k - x_{1,t-1} \quad \forall t \in T_s \setminus \{0\} \tag{31}$$

To ensure only one bid is the winner, the following constraint is added

$$\sum_{k \in K} u^k = 1 \tag{32}$$

The winner determination problem is then:

$$\min \mathrm{Z_c} \tag{33}$$

s.t.
Equations (2), (4), (5), (6), (31) and (32),
$u^k \in \{0,1\}$, $\forall k \in K$.

4. Experimental Results

The algorithm and the models have been implemented in Python 3.12.0 and solved with Gurobi$^{\text{TM}}$ on a Windows machine equipped with an Intel Core$^{\text{TM}}$ i7-12700K @ 3.60 GHz, 12 core and 32 GB RAM. The instance used for experimentation is adapted from a real recycling plant composed of two stations. The number of customers is 30. The collection time horizon is 25 days; the sorting problem time horizon is 30 days. The cost of collection is set to a uniform distribution such that $c^c_i \in [5,15]$, while quantities are chosen uniformly at random in the interval $[3,5]$, and the maximum capacity for trucks is fixed and equal to 10. The variable selection cost c^s_i is set to 3, while the setup cost is 5 and the inventory storage cost per unit of quantity and time is set to 1. The inventory capacity is 50, while the minimum and maximum production capacity for stations for a unit of time are 0 and 10, respectively. No loss is considered, i.e., $\alpha = 1$.

Figure 4 reports the results when the algorithm is run with the number of bids generated ranging from 1 to 20. If we have only one bid, the logistics operator proposes its optimal solution to the recycling plant that is used to solve the plant planning problem. In that case, we have the minimum cost for the collection process and the maximum cost for the sorting process. In the other cases, the winner determination problem defines the bid to be selected. Figure 4a shows the number of the selected bids. The objective values of the collection and sorting problems are reported in Figures 4b and 4c, respectively. The total cost, that is, the sum of the two objectives, is reported in Figure 4d.

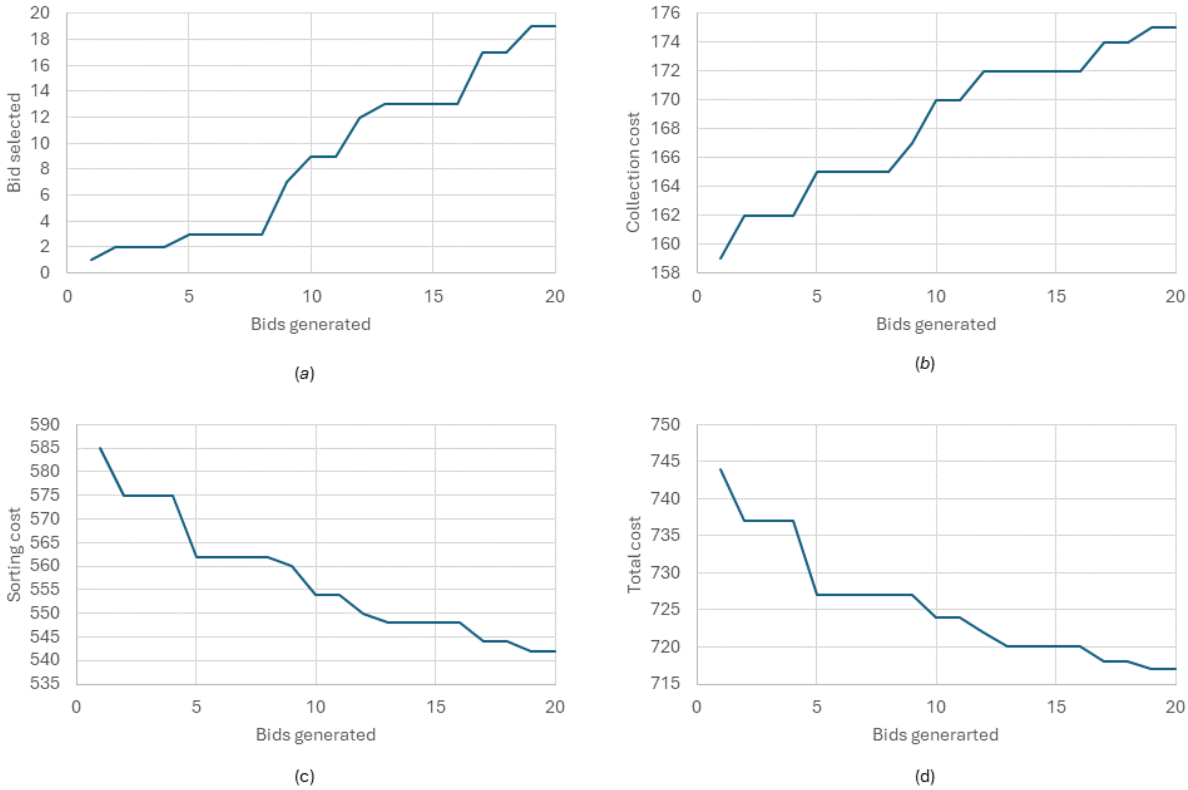

Figure 4. Results obtained by the proposed approach. (**a**) shows the number of bids generated and the number of selected bids; (**b**) shows the number of bids generated and the collection cost; (**c**) shows the number of bids generated and the sorting cost; (**d**) shows the number of bids generated and the total cost.

As can be seen by analyzing the results, as the number of bids generated by the algorithm increases, the collection cost tends to increase as well (see Figure 4b), while the sorting cost shows a nonincreasing behavior (see Figure 4c). However, in general, they produce a decreasing total cost. In fact, even though the percentage of increase and the percentage of reduction of the two cost functions are substantially the same, that is, about 10%, the total cost exhibits a 5% overall reduction (see Figure 4d).

For comparison, in Figure 5, we report the results obtained by implementing a competing approach [1] on the same testbed. As can be seen, while the sorting cost decreased with the increasing number of bids and the collection cost was reduced instead, similarly to what happened with our approach, the overall total cost increased by about 2%. This shows the effectiveness of the ingredients used in the proposed algorithm and the models to improve the overall efficiency of the system.

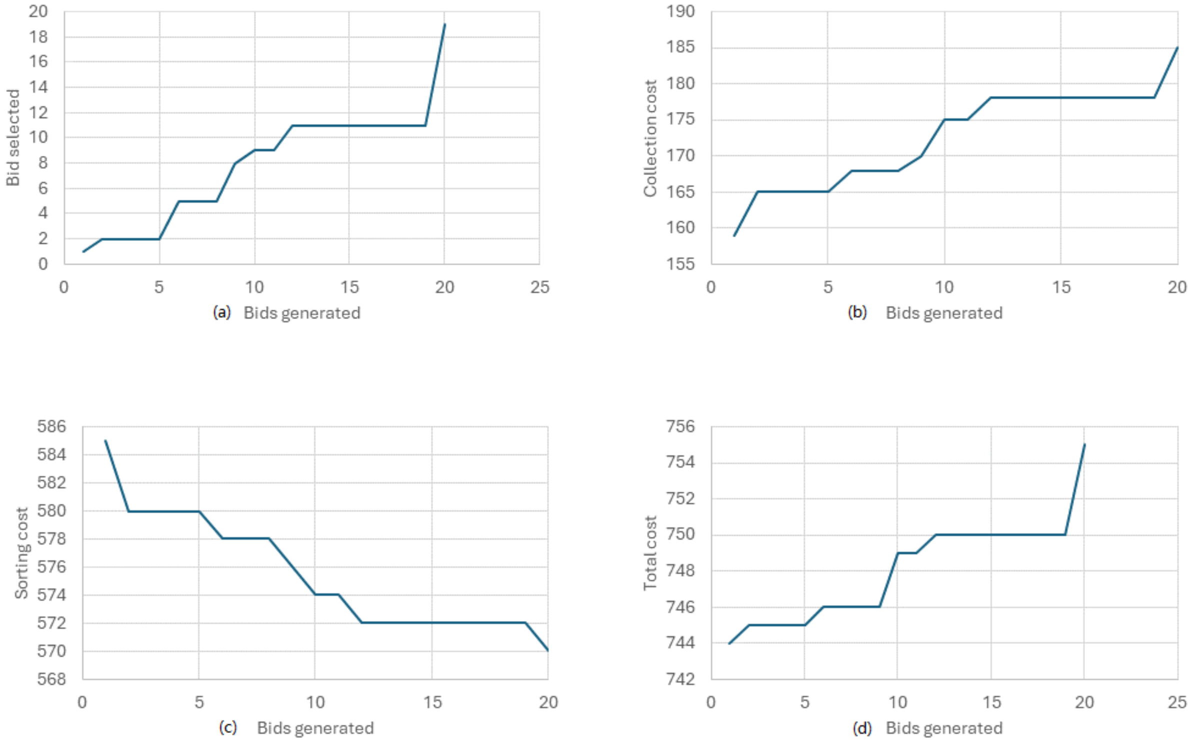

Figure 5. Results obtained by a competing approach. (**a**) shows the number of bids generated and the number of selected bids; (**b**) shows the number of bids generated and the collection cost; (**c**) shows the number of bids generated and the sorting cost; (**d**) shows the number of bids generated and the total cost.

Further Comparison with an Integrated Model and Increasing Instance Size

In this section, we compare the proposed model with a reference integrated model defined in a single decision-maker environment where the decision-maker optimizes simultaneously the two (i.e., collection and sorting) problems, minimizing the objective of the sorting problem. The model (denoted P3) is formed by the constraints of both model P1 and model P2 and is detailed in the following.

$$\min Z_c = \sum_{i \in I} \sum_{h \in H} \sum_{t \in T_c} c^c_{it} \cdot z_{iht}$$

$$\text{s.t.}$$

$$E_j y_{jt} \le x_{jt} \le M_j y_{jt}, \forall j \in J, \forall t \in T_s,$$

$$I_{1,t} = I_{1,t-1} + \sum_{i \in I} \sum_{h \in H} q_i \cdot z_{ih,t-1} - x_{1,t-1}, \forall t \in T_s \setminus \{0\},$$

$$I_{j,t} = I_{j,t-1} + \alpha x_{j-1,t-1} - x_{j,t-1}, \forall t \in T_s \setminus \{0\}, j \in J \setminus \{1\},$$

$$I_{jt} \le LC_j, \forall j \in J, t \in T_s,$$

$$\sum_{h \in H} \sum_{t \in T^c} z_{iht} = 1, \forall i \in I,$$

$$\sum_{i \in I} q_i \cdot z_{iht} \le C, \forall h \in H, \forall t \in T_c,$$

$$x_{jt} \ge 0,\ y_{jt} \in \{0,1\}, \forall t \in T_s, \forall j \in J,$$

$$z_{iht} \in \{0,1\}, \forall i \in I, h \in H, t \in T_c.$$

We tested P3 in the same test scenario used before and found that P3 produces an optimal solution value of 542, which is the same solution value as that found by our approach when 20 bids are generated.

Repeating the same test considering the variant of P3 where the objective function is replaced by the objective of the collection problem P1 leads to the same results as the one obtained when only one bid is allowed.

To complete our experimental campaign, we tested our approach by increasing the instance size considering 30 and 70 customers. The number of bids is 20. The results of these tests are reported in Table 1. As can be seen in the last column, the gain in terms of the total cost (the sum of collection and sorting costs) of our approach compared to the competing approach [1] increases with an increasing number of customers, passing from 4.9% (30 customers) to 9.4% (70 customers), indicating the effectiveness of our approach.

Table 1. Test for an increasing number of customers.

Customers	Collection Time Horizon (Days)	Sorting Time Horizon (Days)	Collection Cost	Sorting Cost	Gain in the Total Cost
30	25	30	175	542	4.9%
40	25	30	222	721	5.4%
50	30	35	278	812	7.5%
60	30	35	329	1025	8.9%
70	35	40	389	1144	9.4%

5. Conclusions

This paper addressed a problem that arises in waste management when recycling operations must be optimized while coordinating a logistics operator that must minimize the collection cost of the commodity to be disposed of and a recycling plant to minimize sorting operations. We formulated two multiperiod planning problems and proposed a new negotiation scheme based on auctions. The results show that the approach is promising, as the total cost can be lowered when the number of bids generated by the logistics operator increases. As a follow-up of this approach, further large-scale tests are planned, and additional details in the formulations of the two problems will be considered.

This research has two main shortcomings. The first is that the proposed methodology needs the actors to be integrated with an information system (a virtual marketplace) where the bids can be exchanged automatically. A second flaw is that we did not consider the possibility of having the participation of spot actors, such as occasional logistics operators. Future development of the paper could consider these kinds of extensions to our proposal as well as the opportunity to measure the robustness of the approach to varying the input to the problem.

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