Journal of Economics (2025) 146:237–262
https://doi.org/10.1007/s00712-025-00917-x

To discriminate or not to discriminate: how to enforce unverifiable quality in repeated procurement

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Received: 29 January 2025 / Accepted: 5 July 2025 / Published online: 25 July 2025

Abstract

We analyse a model of repeated procurement whereby a buyer may elicit unverifiable quality by relying on two types of competitive procedures. The first type is non-discriminatory, namely a low-price auction with a public reserve price, whereas the second type is a scoring auction that includes a non-financial, discriminatory dimension based on past performance. We first provide sufficient conditions for the existence of relational procurement contracts under which the buyer can elicit the desired level of quality. We then assess which mechanism is preferable in terms of (i) the buyer's preferences and (ii) the equilibrium existence conditions. As for (i), we establish the conditions whereby the two procedures yield the buyer the same utility as well as those under which a non-discriminatory procedure ensures a lower cost of the project, although this comes with a lower quality and a positive probability of the project not being delivered altogether. As for (ii), no clear-cut results can be established. Indeed, the range of values of the project net-of-quality utility for which an equilibrium exists under the non-discriminatory procedure is always larger than under the discriminatory one. Conversely, the two procedures have a different ranking in terms of stringency of equilibrium existence requirements for the discount factor and the net social value of quality.

Keywords Public procurement · Relational contracts · Unverifiable quality · Past performance · Reserve price · Handicap

JEL Classification H57 · D44 · D86 · K23

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1 Introduction

Quality enforcement in procurement is essential for transforming contractual promises into tangible value for the buyer. While measurable quality dimensions - say, the number of pages per minute that a photocopying machine can process or the speed of the CPU of a personal computer - can be enforced at some cost, unverifiable dimensions can significantly impact overall performance as contractual clauses cannot be specified in advance. Examples of unverifiable, thus non-contractible, dimensions in public procurement abound, ranging from food palatability in catering services to the user-friendliness of software solutions.

Since procurement is ordinarily a repeated activity, relational contracts have been advocated as a potential instrument to elicit unverifiable quality. Relational contracts are informal agreements and unwritten codes of conduct that are sustained by the value of future relationships and are applicable in cases where the outcome of a repeated relationship is based on some unverifiable variables. Thus, quality provision may emerge as an outcome of cooperation between the buyer and the contractor in a long-term relationship.[1]

A relational procurement contract (RPC, henceforth) may be implemented by designing a competitive mechanism. This implies carrying out the procurement activity by means of an auction in each period. In future tenders, the buyer would punish (in a way that we will describe better below) the firm(s) who failed to deliver the expected quality level and/or the firms would punish the buyer who failed to stick to the cooperative behaviour, typically reducing the rent the firm(s) would obtain under cooperation. An equilibrium RPC entails that all parties stick to their cooperative strategies to avoid future punishments.

We consider two scenarios, each with a different punishment mechanism and, consequently, a different type of procurement auction. The first scenario entails the use of handicaps as a punishment device. In each period, the buyer runs a scoring auction where each firm's score depends upon both by its bid and its past performance. Past performance enters the score under the form of a handicap (a negative component of the score) that the buyer assigns to a contractor who failed to deliver quality in the past. In the second scenario, a reserve price acts as a punishment device. In each period, the buyer runs a standard low-price auction with a publicly announced reserve price, The reserve price is set to reward or punish firms depending on whether quality has been provided in the past.

These two scenarios are theoretically relevant and of tangible practical importance. From a theoretical perspective, they represent real-world translations of broader

[1] The literature on relational contracts originates from the seminal papers of Bull (1987) and MacLeod and Malcomson (1989). More recent contributions to this literature are Levin (2003), who first characterises the optimal relational contract with adverse selection and moral hazard and Levin (2002), who studies the relationship between bilateral and multilateral contracts. Relational contracts have been applied in several fields: labour market (MacLeod 2003; Levin 2003 and Li and Matouschek 2013), the interaction between/within firms (Baker et al. 2002 and Rayo 2007), regulation (Cesi et al. 2012), environmental economics (Cesi and D'Amato 2023) and experimental economics (Fehr and Schmidt 2004 and Bigoni et al. 2014). A relational contract with observable but unverifiable quality in procurement was first studied by Taylor and Wiggins (1997) and Doni (2006) and extended to private and public procurement by Albano et al. (2017), and (2023) Andrews and Barron (2016), Board (2011) and Calzolari and Spagnolo (2020).

classes of mechanisms. Indeed, using the language of Jehiel and Lamy (2015), an auction with a handicap—that is, a past-performance score—is an instance of a *discriminatory* auction, whereas no discrimination occurs in the case of an auction with a reserve price. Additionally, the handicap-based RPC is an instance of what Levin (2003) defines as a bilateral relation contract, in that a deviation by one party causes a breakdown of cooperation between the latter and its counterpart without affecting the relationship between the other parties. In contrast, the reserve price RPC represents a multilateral relational contract, where deviation by any party triggers a breakdown of all relationships.

From a practical standpoint, both types of procurement mechanisms are widely used. The U.S. Federal Acquisition Regulations (FAR) foresees that "[p]ast performance should be an important element of every evaluation and contract award for commercial items", providing (federal) agencies with broad discretion in deciding how they will consider firms' *prior experience*—whether the firms have done similar work before—and *past performance*—how well they have done that work (FAR, part 12.206). In a 2011 report, the U.S. Government Accountability Office (GAO) emphasised that "in almost all procurements [...] the contracts were awarded to the offerors that received the highest rating for nonprice factors, such as prior experience or past performance" (U.S. Government Accountability Office (2011), p. 2). In our model, we will analyse a competitive procedure in which the buyer evaluates both a price and a non-price factor, where the latter is captured by a score assigned to the contractor who has carried out the project in the previous period.

The other type of competitive tendering is a standard low-price auction in which the buyer sets a public reserve price. While not ubiquitous, the reserve price is one major strategic dimension in procurement design (see, for instance, Albano et al. 2006). In some cases, setting and announcing the reserve price is mandatory for procurement procedures. This is the case in Italy, where the national public procurement law foresees the use of the reserve price for acquisitions above a predetermined threshold value.

We analyse an infinitely repeated game in which, in each period, a buyer uses a competitive tender to procure a project of fixed size and variable and unverifiable quality. Two firms participate in the auction in each period. Within this stylised setting, we address the question concerning which instrument (handicap vs. reserve price) a buyer *should* select. The two procedures are compared both in terms of the buyer's welfare along the cooperative path and of the strictness of the conditions that make cooperation feasible in a long-term relationship.[2] An additional contribution of the paper is to extend the existing literature by providing a unified framework to study both mechanisms, thus making the comparison between the two RPCs feasible. Indeed, while the analysis of the handicap-based mechanism in Albano et al. (2017) assumed a buyer fully committed to her strategy, the procurement contract with a reserve price was studied in Albano et al. (2023) for the case of a fully strategic buyer who chooses based on her individual incentives at each moment in time. In this paper, we extend the analysis of the handicap-enriched auction to the case of an uncommit-

[2] We abstract away from regulatory considerations that may limit the class of feasible solutions.

ted buyer, making it fully comparable with the mechanism in which the punishment is carried out through a reserve price.

To illustrate the welfare comparison between the two types of contracts, it is first necessary to describe the nature of equilibria in each case. Under a non-discriminatory procedure (i.e., with a reserve price), there exist two types of equilibria depending on the level of the reserve price set by the buyer along the equilibrium path. With a "high" reserve price, all firms can bid for the contract regardless of their efficiency level; the reserve price is always slack, and bids (and rents) are limited by competition only. First-best quality may be enforced. The features of this equilibrium replicate those obtained under a discriminatory procedure (i.e., with handicap) since the handicap is never applied along the equilibrium path. As a result, along the equilibrium path, the buyer's utility is identical under a discriminatory and a non-discriminatory RPC with a high reserve price.

The comparison between a discriminatory and a non-discriminatory procedure with a "low" reserve price is more involved. With a "low" reserve price, only efficient firms are able to submit a bid that allows them to recover their cost, leaving the buyer face the risk of no delivery of the project when all firms are inefficient. Only a suboptimal level of quality may be enforced since the higher the quality the larger the rent that should be granted to firms to induce cooperation. A non-discriminatory procedure limits the firms' rent, but yields a sub-optimal quality level and induces the risk of not getting the project delivered. On the other hand, a discriminatory procedure is more costly to the buyer, but allows the buyer to implement the socially optimal quality level. In general, the comparison is based on the intrinsic value of the project and the net quality assessment. When these are large, a discriminatory procedure is preferred as the social cost of the larger rent to the firm is outweighed by the security of delivery of the project with a first-best quality level. The opposite holds when the value of the project and its quality are less relevant to the buyer.

When comparing the strictness of the conditions guaranteeing the existence of an equilibrium, the ranking between the two mechanisms becomes less clear-cut. We show that the ranking depends upon i) the firms' critical level of the discount factor to deliver the quality required by the contract, and ii) the minimum net social value of quality required by the buyer to enforce a cooperative contract. As to the former, we show that the critical discount factor is highest in the case of a non-discriminatory procedure with a "low" reserve price, whereas the conditions on the net social value of quality are most stringent in the case of the discriminatory procedure with a handicap. Hence, while making the set of the critical discount factors the most restrictive a non-discriminatory auction with a "low" reserve price makes the condition on the net social value of the quality the weakest. Overall, even within a simple analytical framework, a definitive ranking between the two classes of competitive mechanisms can be established only on specific dimensions.

The paper is organised as follows. In Sect. 2, we lay out the model. In Sect. 3 we derive the sufficient conditions for an equilibrium RPC to exist under a non-discriminatory procedure. The case of a discriminatory procedure is analysed in Sect. 4, where we outline the sufficient conditions under which a pair of self-enforcing RPCs exist. In Sect. 5, we carry out the comparison exercise between the two procedures. Section6 concludes.

2 The model

Players. A buyer wishes to procure a single project of variable quality q, where $q \in [0, +\infty)$. This procurement activity takes place at time t and it is repeated for an infinite number of periods, so that $t = 0, ..., \infty$. The value of the project to the buyer is given by $v + q$, where v denotes the intrinsic value of the project and q is the value of its quality. The utility of the buyer can thus be written as $U(v, q, p) = v + q - p$, where p is the price the buyer pays when procuring the project, and zero otherwise.

Two firms, 1 and 2, can compete for the project. At each t, firm i's cost is $C\left(\theta_i^t, q_i^t\right) = \theta_i^t + \psi\left(q_i^t\right)$, where $i = 1, 2$, and zero in case of no production (we will drop the superscript t whenever this is possible). This implies that the total cost of the contract at time t consists of a fixed firm-specific component, θ_i^t, and of a quality-dependent component, $\psi\left(q_i^t\right)$, which is the cost of quality. The fixed firm-specific cost parameter is a draw from a discrete random variable whose two possible realisations at any time t are θ_L and θ_H, with $\theta_L < \theta_H$. We denote with $\Delta \equiv \theta_H - \theta_L$; hence, Δ is the largest cost advantage that one of the two firms may have on the fixed-cost component. For both firms, the probability of realisation of θ_L is fixed and time-invariant; we denote this probability with β, where $\beta \in (0, 1)$. Hence, for $i = 1, 2$ and $t = 0, \ldots, \infty$, $Pr(\theta_i^t = \theta_L) = \beta$. The other cost component $\psi(.)$ is the time-invariant cost of quality and is identical across firms when the quality they provide is also identical. We assume that $\psi(.)$ is increasing and convex in its argument and satisfies $\psi\left(0\right) = 0$. Firm i's profits are given by $\pi_i = p - \theta_i - \psi(q_i)$ when it provides the project and zero otherwise. Firms do not have access to credit, so that a non-negativity profit constraint holds for them in each period. All players are risk-neutral and have a common discount factor equal to δ, where $\delta \in (0, 1)$, capturing their time preference and the uncertainty about the existence of future opportunities of trade.

We let the net social value of quality be defined as $B(q)$, where $B(q) \equiv q - \psi(q)$. We assume that $B(q) \geq 0$ for any q in the range relevant to our analysis, so that the provision of quality is (socially) beneficial, and $v \geq \theta_H$, so that the buyer is always better off by procuring the project.

In a first-best environment, the buyer's utility is given by $v + B(q) - \min\{\theta_i\}$, with θ_i being firm i's realisation of the fixed cost component, with $i = 1, 2$. The first-best quality level that maximises it, denoted by q^{FB}, is implicitly defined by the standard optimality condition for quality, $\psi'\left(q^{FB}\right) = 1$.

Competitive tendering We analyse two different formats of competitive tendering. First, we assume the buyer awards the project by means of a *non-discriminatory* competitive procedure, where in each period the buyer awards the project by running a low-price auction with a public reserve price. This competitive procedure requires the buyer to announce a public reserve price r and then firms simultaneously submit their bids b_i, for $i = 1, 2$. Any firm bidding above the reserve price is excluded from the auction. The project is awarded to the low-price firm of the remaining firms; in case of a tie, the project is awarded by using a fair random device.

We also analyse the case in which the buyer awards the project by means of a *discriminatory* procedure, where in each period the buyer awards the project by means of a low-score scoring auction with a handicap. This competitive procedure requires the buyer to announce the handicaps to the two firms, h_i, and each firm to submit a bid, denoted as b_i (for $i = 1, 2$). The buyer uses a scoring rule to take into account both the firms' bids and their handicaps. This scoring rule is given by $S(b_i, h_i) = b_i + h_i$. In case of a tie, that is, if $S(b_1, h_1) = S(b_2, h_2)$, the buyer awards the project randomly with equal probabilities.

In both formats of the competitive procedure, bids are mono-dimensional, and firms bid only on price. This is because quality is assumed to be non-contractible (see below).[3] Both the bids and the handicaps are assumed to be discrete numbers, thus they are multiples of the smallest unit, ν, which is assumed to be arbitrarily small.[4] We also assume that firms bear no bidding costs.[5]

Informational structure The informational structure is common to both formats. We consider a game of incomplete information. The buyer has incomplete information on the realisation of the firms' fixed cost parameters; conversely, each firm knows its own and the rival's cost parameters when realised. All players observe the bids made by all firms once the contract is awarded and the quality delivered by the contractor after the latter has carried out the project. Although quality is perfectly observable, the lack of verifiability makes it non-contractible and non-enforceable in a court of law. This simple informational structure makes our problem tractable and isolates the buyer's main problem, that is, selecting the most efficient firm in the bidding stage of the procedure, and inducing the required level of project quality.

3 Non-discriminatory procedure

In this section, we provide the equilibrium analysis of our game in the case of a non-discriminatory procedure. The analysis follows closely Albano et al. (2023) for the case of two firms. We thus keep the discussion to a minimum and concentrate only on aspects relevant to our comparison between the two types of competitive procedures (for further details, see Albano et al. 2023).

The stage game is the following:

Stage 1 The buyer auctions off a project through a low-price sealed-bid auction with a public reserve price *r*;

Stage 2 (The *bidding stage*) firms learn their cost parameters, and submit sealed bids; all bids strictly greater than the reserve price *r* are deemed invalid and thus rejected; the lowest-bid firm is selected as the contractor out of the remaining firms (if any); all bids are made public.

[3] If firms were to bid also on quality, they both would always find it optimal to bid the highest possible level of quality, wiping out all the effects of the quality component in the bids.

[4] Discrete bids are necessary for the existence of a pure-strategy equilibrium in our auctions under complete information. Indeed, when bids are real numbers a pure strategy Nash equilibrium does not exist. Alcalde and Dahm (2011) and (2013) show that one can derive the intuitive outcome whereby the low-cost firm wins the auction at the cost of the high-cost firm by taking the limiting equilibrium of a modified game in which bids are expressed as multiple of the smallest monetary unit when the latter goes to zero and ties are randomly broken. As to the effect of the discrete nature of the handicap, see the discussion in Sect. 4.2.

[5] This assumption would apply, for instance, when participation costs are of an order of magnitude much lower than the cost of the project itself.

Stage 3 (The *execution stage*) the contractor chooses the quality level and delivers the project; the buyer pays the contractor a price equal to its bid; quality is publicly observed, and all payoffs are collected.

3.1 The constituent game

The following proposition provides the equilibrium analysis of the constituent game.

Proposition 1 Let

$$\overline{v} \equiv \frac{2\beta}{1-\beta}\Delta + \theta_H. \tag{1}$$

The static game admits a perfect Bayesian equilibrium whereby the buyer selects the reserve price, r^S, as follows: if $v \leq \overline{v}$, then $r^S = \theta_L$; if instead $v > \overline{v}$, then $r^S \geq \theta_H$. If the project is awarded, the contractor delivers a quality equal to zero.

As in Albano et al. (2023), in a static setting firms do not have any incentive to provide quality. Reasoning backward, the contractor always finds it optimal to deliver no quality in the final stage of the game. Hence, in the second stage, all firms bid anticipating to deliver no quality. In the first stage, the buyer anticipates that no quality will be provided and sets a reserve price that solves the standard trade-off in a low-price auction: a high reserve price is such that both low- and high-cost firms participate in the auction, but the price paid by the buyer is only limited by competition. On the other hand, a low reserve price reduces the cost of the project as only low-cost firms can submit a valid bid.

3.2 The dynamic game

We now turn our attention to the dynamic game given by an infinite repetition of the constituent game analysed in the previous section. We focus on a relational procurement contract (RPC) between the buyer and the two firms that describes, for any history of the game, the reserve price the buyer sets, the bids the firms submit, and the quality the contractor chooses.

Before describing the players' strategies, we introduce the following notation. Denote with q^t the actual quality delivered at time *t* by the contractor in that period. Also, denote with ϕ_i^t the quality level that, at the bidding stage, firm *i* anticipates delivering at the execution stage. Also, let

$$\rho_{low}(q) \equiv [\theta_L + \psi(q), \theta_H + \psi(q))\,; \tag{2}$$

$$\rho_{high}(q) \equiv [\theta_H + \psi(q), +\infty)\,. \tag{3}$$

These two intervals for the reserve price will be relevant for the rest of the analysis. When the reserve price lies in the interval $\rho_{low}(q)$, only an efficient firm can cover its cost when providing quality; on the other hand, when the reserve price lies in

$\rho_{high}(q)$, any firm, irrespective of its efficiency level, is able to cover its cost when providing quality.

We carry out our analysis with players adopting the following grim trigger strategies:

- *Buyer*: At $t = 0$, the buyer sets $r = r^C$. At $t \geq 1$, the buyer sets $r = r^C$ if $r^\tau = r^C$ and $q^\tau = q^C$, for all $\tau = 0, \ldots, t-1$; otherwise, the buyer sets $r = r^P = \theta_L$.
- *Firm i ($i = 1, 2$)*: At any $t \geq 0$, firm i with cost $\tilde{C}_i^t \equiv \tilde{C}(\theta_i^t, \phi_i^t) = \theta_i^t + \psi(\phi_i^t)$ bids an amount equal to $\max\{\tilde{C}_i^t, \tilde{C}_j^t\}$, $j \neq i$, and is awarded the contract if $\tilde{C}_i^t < \tilde{C}_j^t$, otherwise firm *j* is awarded the contract, where
  - $\phi_j^t = q^C$ if $r^\tau = r^C$ for all $\tau = 0, \ldots, t$, and $q^{\tau'} = q^C$ for all $\tau' = 0, \ldots, t-1$;
  - $\phi_j^t = 0$ otherwise.

If awarded the project at time *t*, firm *i* delivers quality $q^t = \phi_i^t$.

One immediate remark is in order. Given the discrete nature of bids, at the bidding stage the low-cost firm optimally bids an amount marginally lower than the high cost. Since the monetary unit is negligibly small, we can safely assume that both bids are equal to the $\max\{\tilde{C}_i^t, \tilde{C}_j^t\}$, and the contract is awarded to the low-cost firm.

Let these strategies be denoted as σ_B^r for the buyer and σ_i^r for firm *i*. They are functions that map—at each stage of each repetition of the game—the publicly observable history into players' actions.

The above strategies describe a relational procurement contract (RPC) whose main elements are a "cooperative" and a "punishment" reserve price, r^C and r^P, respectively, as well as a "cooperative" level of quality q^C. More precisely, the buyer starts by setting r^C and firms submit bids. If the selected firm delivers quality q^C then the game proceeds to the next period in which the buyer sets again r^C. If any deviation from r^C and/or q^C occurs, the punishment path unfolds in which the buyer sets a reserve price r^P and the selected firm does not deliver quality.[6] This RPC is self-enforcing if the players' strategies form a perfect public equilibrium of the repeated game. We emphasise the multi-lateral nature of the contract whereby a deviation by any player, be it the buyer or either firm, leads to a breakdown of cooperation among all players (as in Levin, 2002). We carry out the equilibrium analysis of the game by checking the absence of profitable one-shot deviations (POSDs henceforth) for each player (Mailath and Samuelson 2006), on and off the equilibrium path.

The next proposition characterises the conditions under which a self-enforcing relational contract exists and its features. Before presenting our results, we introduce some thresholds that will be referred to in the proposition:

[6] The punishment reserve price equal to θ_L derives from the reversal to the static equilibrium in case of a deviation. Indeed, as shown in Proposition 1, when $v \leq \overline{v}$, in the static equilibrium, $r = \theta_L$. When instead $v > \overline{v}$, in the static equilibrium $r \geq \theta_H$. However, this reserve price is a punishment not harsh enough to induce cooperation in the repeated game since it would equalize the firms' expected profits during the cooperative and punishment phases. This motivates the upper limit on *v* for equilibrium existence introduced in Proposition 2 (for further details, see Albano et al. 2023).

$$\delta_r \equiv \frac{\psi\left(q^C\right)}{\psi\left(q^C\right)+\beta(1-\beta)\left[\min\{r^C-\psi\left(q^C\right),\theta_H\}-r^P\right]}; \tag{4}$$

$$B_{r_{high}} \equiv (1-\beta)^2(\overline{v}-v); \tag{5}$$

$$B_{r_{low}} \equiv \frac{2\beta(1-\beta)}{1-(1-\beta)^2}\left(r^C-\psi\left(q^C\right)-r^P\right). \tag{6}$$

Then,

Proposition 2 A vector of strategies $(\sigma_B^r, \sigma_1^r, \sigma_2^r)$ defines a self-enforcing RPC if $v \in [\theta_H, \overline{v}]$, $\delta \geq \delta_r$ and either (*i*) $r^C \in \rho_{high}(q^C)$ and $B\left(q^C\right) \geq B_{r_{high}}$; or (*ii*) $r^C \in \rho_{low}\left(q^C\right)$ and $B\left(q^C\right) \geq B_{r_{low}}$.

Among those q^C and r^C satisfying the conditions above, let $\hat{q}^C$ and $\hat{r}^C$ denote those values yielding the highest (expected) single-period utility to the buyer. Then:

(i) when $r^C \in \rho_{high}(q^C)$, $\hat{q}^C = q^{FB}$ and $\hat{r}^C$ is any $r \in \rho_{high}(q^C)$;
(ii) when $r^C \in \rho_{low}(q^C)$, $\hat{q}^C$ and $\hat{r}^C$ satisfy

$$\psi'\left(\hat{q}^C\right) = \frac{1-(1-\beta)^2}{1-(1-\beta)^2+\frac{2(1-\delta)}{\delta}} < 1, \tag{7}$$

and

$$\hat{r}^C = \psi\left(\hat{q}^C\right)\left(1+\frac{1-\delta}{\delta}\frac{1}{\beta(1-\beta)}\right)+\theta_L. \tag{8}$$

The first part of Proposition 2 illustrates the sufficient conditions for the existence of two types of equilibria, depending on the level of r^C. In both cases, along the equilibrium path, the buyer sets a reserve price equal to r^C, those firms able to participate in the procedure bid anticipating to deliver quality q^C and the selected contractor (if any) always delivers it. This outcome is sustained by the threat of reverting to the equilibrium of the static game described in Proposition 1 when $v < \overline{v}$. Thus, off the equilibrium path, the reserve price r^P is set equal to θ_L, and no quality is delivered.

The first type of equilibrium arises when r^C is high (i.e., $r^C \in \rho_{high}(q^C)$). In this case, the reserve price allows even an inefficient firm to submit a bid covering the cost of quality. Hence, the project is always delivered. The second type occurs when r^C is low (i.e., $r^C \in \rho_{low}(q^C)$). A low reserve price makes an inefficient firm unable to recover its cost when delivering quality. In the event of all firms being inefficient, all bidders submit prices above the reserve price. The buyer rejects all bids and the project is not delivered.[7]

Given the multiplicity of equilibria described in the first part of the Proposition, the second part illustrates the optimal reserve price and quality levels that are part of a self-enforcing RPC and that deliver the highest utility to the buyer. Notice that this optimal quality may depart from the first-best level since our buyer has to leave some rent to the contractor to induce the required quality. When the reserve price is low (i.e. $r^C \in \rho_{low}$) we find that the optimal quality is lower than its first-best level. Also, the optimal reserve price is chosen to make the contractor's IC just binding. Then, a trade-off in the choice of the optimal policy arises: a higher quality gives more utility to the buyer, but also generates a higher cost for the contractor; this in turn requires a higher reserve price to induce cooperation, hence the buyer's cost for the project is higher. Notice that this increase in the reserve price is larger the lower the discount factor, since the contractor has to be granted a larger rent when it is less patient. This same trade-off disappears when the reserve price is high (i.e. $r^C \in \rho_{high}$), in which case the buyer's cost for the project is not related to the quality level and is only determined by competition between the two firms. Hence the buyer is able to elicit the first-best quality level.

4 Discriminatory procedure

In this section, we provide the equilibrium analysis of our game in the case of a discriminatory procedure. The analysis extends Albano et al. (2017) in that we consider the case of a non-committed buyer. We assume the buyer to be fully strategic, and able to react optimally to the other players' actions.

The equilibrium analysis focuses on the novel aspects deriving from the full strategic role of the buyer and on the aspects that are relevant to our comparison between the two types of competitive procedures (for further details, see Albano et al. 2017).

The stage game is the following:

Stage 1 The buyer auctions off a project through a low-score sealed-bid auction; the buyer publicly announces the handicap h_1 and h_2;

Stage 2 (The *bidding stage*): firms learn their cost parameters and submit their bids; the lowest-score firm is selected as the contractor; all bids are made public;

Stage 3 (The *execution stage*) the contractor chooses the quality level and delivers the project; the buyer pays the contractor a price equal to its bid; quality is publicly observed and all payoffs are collected.

4.1 The constituent game

The following proposition provides the equilibrium analysis of the static game.

[7] Alternatively, one could assume that high-cost bidders willing to deliver quality will refrain from bidding altogether.

Proposition 3 The static game admits a perfect Bayesian equilibrium whereby the buyer selects any identical handicap for both firms. Once awarded the project, the contractor delivers a quality equal to zero.

As in Albano et al. (2017), in a static setting, firms do not have an incentive to provide quality. Reasoning backward, the contractor always delivers zero quality at the execution stage. At the bidding stage, firms bid based on their costs and handicaps, which are all common knowledge. Since the price of the project is an increasing function of the difference in the firms' costs, inclusive of the handicaps, the buyer optimally sets the same handicap to both firms and picks any quality level, anticipating that it will not be delivered.

4.2 The dynamic game

We turn our attention to the dynamic game. In this game, an RPC between the buyer and a single firm describes, for any history of the game, the handicap that the buyer sets to the firm, the bid that this firm submits and the quality that it chooses, if awarded the contract. A pair of RPCs is self-enforcing if it describes a public perfect equilibrium of the game.

Before describing the players' strategies, we introduce the following notation. Denote with h_i^t the handicap set to firm *i* at time *t*. Also, recall that q^t denotes the actual quality delivered at time *t* by the contractor in that period and ϕ_i^t is the quality level that, at the bidding stage, firm *i* anticipates to offer at the execution stage.

We concentrate our analysis on players adopting the following grim trigger strategies:

- *Buyer* At $t = 0$ the buyer sets $h_i = h^C$, $i = 1, 2$. At $t \geq 1$ the buyer sets $h_i = h^C$ if $h_i^\tau = h^C$ and $q^\tau = q^C$ whenever the project was awarded to firm *i*, for all $\tau = 0, \ldots, t-1$; otherwise, the buyer sets $h_i = h^P$.
- *Firm i (for $i = 1, 2$)*: At any $t \geq 0$, firm *i* with (handicap-augmented) cost $\tilde{C}_i^t \equiv \tilde{C}(\theta_i^t, \phi_i^t, h_i^t) = \theta_i^t + \psi(\phi_i^t) + h_i^t$ bids an amount equal to $\max\{\tilde{C}_i^t, \tilde{C}_j^t\}$, $j \neq i$, and is awarded the contract if $\tilde{C}_i^t < \tilde{C}_j^t$, otherwise firm *j* is awarded the contract, where
  - $\phi_j^t = q^C$ if $h_j^\tau = h^C$ for all $\tau = 0, \ldots, t$, and $q^{\tau'} = q^C$ when the project was awarded to firm *j*, for all $\tau' = 0, \ldots, t-1$;
  - $\phi_j^t = 0$ otherwise.

If awarded the project at time *t*, firm *i* delivers quality $q^t = \phi_i^t$.
Let these strategies be denoted as σ_B^h for the buyer and σ_i^h for firm *i*, with $i = 1, \ldots, N$. They are functions that map—at each stage of each repetition of the game—the publicly observable history into players' actions.

The above strategies describe a pair of RPCs whose main elements are two "cooperative" and two "punishment" handicaps, h_i^C and h_i^P $(i = 1, 2)$, respectively, as well as two "cooperative" quality levels q_i^C. More precisely, the buyer starts by setting $h_i^C = h^C$ and firms submit bids. If the selected firm delivers quality $q_i^C = q^C$ then the game proceeds to the next period in which the buyer sets again $h_i^C = h^C$. If any deviation from h^C and/or q^C occurs, the punishment path unfolds in which the buyer sets a handicap h_i^P to the deviating firm and the latter does not deliver quality if awarded the contract. Notice that the nature of these RPCs fulfils the requirement of a bilateral relational contract under which a deviation leads to a breakdown of cooperation between the involved parties only as in Levin (2002). As before, we carry out the equilibrium analysis of the game by checking the absence of profitable one-shot deviations (POSDs henceforth) for each player (Mailath and Samuelson 2006), on and off the equilibrium path.

The formal characterisation of this equilibrium is contained in the following proposition. As in the previous section, before stating the Proposition, we define two thresholds that will be referred to in the Proposition:

$$\delta_h \equiv \frac{\psi(q^C)}{\psi(q^C) + \beta(1-\beta)\min\{\Delta, h^P - \psi(q^C)\}}; \tag{9}$$

$$B_h \equiv \Delta \frac{2 - 3\beta\,(1-\beta)}{2 - \beta\,(1-\beta)}. \tag{10}$$

Then,

Proposition 4 A vector of strategies $(\sigma_B^h, \sigma_1^h, \sigma_2^h)$ defines a pair of self-enforcing RPCs when $\delta \geq \delta_h$, $h^C = 0$, $h^P = \psi(q^C) + \Delta$ and $B(q^C) \geq B_h$. Along the equilibrium path, the first-best quality level q^{FB} is always enforceable.

The proposition illustrates the sufficient conditions for a pair of RPCs to be able to elicit the provision of unverifiable quality. These conditions are on the players' time preferences, mandating that players are sufficiently patient, and on the net social value of quality, which has to be sufficiently high.

As to the level of the handicap, the buyer sets it equal to zero along the equilibrium path, whereas two different punishment handicaps make q^C enforceable: the first one is equal to $h^P = \psi(q^C) + \Delta$, whereas the second is only marginally higher than the former, $h^P = \psi(q^C) + \Delta + \nu$. However, as the smallest discrete unit, ν, becomes negligible, the two punishment values coincide.[8] The two different handicap levels, though, induce a different market interaction off the equilibrium path when one of the two firms is punished for a past deviation. When $h^P = \psi(q^C) + \Delta$, the punished firm drawing a low cost may still be awarded the contract with positive probability, although it would not provide quality. Thus competition between the two firms bounds the price of the contract. When, instead, h^P is marginally higher than $\psi(q^C) + \Delta$, the punished firm is *de facto* debarred from the market in that it cannot be awarded the contract under any cost configuration. As a result, the price of the contract is then (marginally) higher in the latter than in the former case.

[8] This result mirrors the results with a fully committed buyer in Albano et al. (2017), where also two types of equilibria may exist.

Which of these two punishments is part of an equilibrium would depend on the relative importance of the net social value of quality, which captures the benefit of cooperation, and the difference in the fixed cost parameter, Δ, which is the cost of eliciting quality. When $B(q^C) < \Delta$, the net social value of quality is relatively low, thus the buyer prefers to set a handicap that does not debar a deviating firm. As a result, she reaps the benefit from competition at the cost of awarding the contract to a deviating firm that will not provide quality. The handicap is then set exactly equal to $\psi(q^C) + \Delta$. When, instead, $B(q^C) \geq \Delta$, the buyer prefers a handicap that debars the deviating firm, thus keeping only the rival firm in the market. This eliminates the benefit from competition (in terms of a lower price), but ensures the provision of quality under all circumstances.

5 Comparing the discriminatory and the non-discriminatory procedure

Suppose that a buyer is free to choose either procedure. Would she select the handicap-enriched procedure or add a reserve price to a standard low-price auction? The desirability of the two types of procedures may be compared in several ways. The first type of comparison relates to the buyer's expected utility along the equilibrium path; this analysis aims to determine which procedure ensures the highest level of (expected) social welfare. Another type of comparison relates, instead, to the conditions for the existence of an equilibrium; this analysis aims to establish under what conditions one procedure may elicit the desired quality level whereas the other may not.

Our comparison exercise needs to be conducted for a given level of quality. For each procedure, we choose the quality level that delivers the highest utility for the buyer. In the case of the discriminatory procedure with a handicap and the non-discriminatory procedure with a high reserve price, we select the first-best level of quality, which has been shown to be implementable. For the non-discriminatory procedure with a low reserve price, instead, we select the quality level (and the reserve price) maximising the buyer's expected utility, which has been shown to be different from the first-best level.

Expected utility along the equilibrium path We compare here the equilibrium expected utility under the two types of procedure. We start noting that both in the case of a discriminatory procedure and of a non-discriminatory one with a high reserve price, along the equilibrium path the handicap and the reserve price do not constrain firms' choices. Thus the buyer's expected utility is determined by market conditions only. Formally,

$$EU_h = EU_{r_{high}} = v + q^C - \psi(q^C) + \beta^2\Delta. \tag{11}$$

Hence, the two types of procedure can enforce the socially optimal quality level and are also equivalent in terms of social welfare.

The two types of procedure can be more meaningfully compared when the buyer uses a low reserve price. In this case, we evaluate the buyer's expected utility at the optimal reserve price, $\hat{r}^C$, as given in (8). In this case, the buyer's equilibrium expected utility is given by

$$EU_{r_{low}} = \left(v + q^C - \psi\left(1 + \frac{1-\delta}{\delta}\frac{1}{\beta(1-\beta)}\right) - \theta_L\right) 2\beta(1-\beta) + \\ + \left(v + q^C - \psi(q^C) - \theta_L\right)\left(1 - (1-\beta)^2 - 2\beta(1-\beta)\right). \tag{12}$$

As illustrated in Proposition 2, at the optimal reserve price, quality is lower than the first-best. Thus, when choosing between a non-discriminatory procedure with a low reserve price and a discriminatory one with a handicap (or, equivalently, a non-discriminatory procedure with a high reserve price), the buyer faces a multidimensional trade-off. Choosing a procedure with a low reserve price rather than the one with a handicap ensures: i) lower quality (rather than its first-best level); ii) lower price for the project (constrained by the reserve price rather than by competition); iii) positive probability that the project is not delivered (rather than the project being always delivered).

To further explore this trade-off, we assume a specific functional form for the cost of quality, letting $\psi(q) = \frac{1}{2}q^2$. This allows us to state the following

Proposition 5 Assume $\psi(q) = \frac{1}{2}q^2$. Let

$$v^* \equiv \bar{v} - \frac{1}{2} - \frac{(1-(1-\beta)^2)(1-\delta)}{(1-\beta)^2(2(1-\delta)+\delta(1-(1-\beta)^2))}. \tag{13}$$

When $v < v^*$, the buyer prefers a non-discriminatory procedure with a low reserve price, where $\hat{r}^C$ and $\hat{q}^C$ are as in (ii) in Proposition 2. When instead $v^* < v$, the buyer is indifferent between a discriminatory procedure with a handicap, as described in Proposition 4, and a non-discriminatory procedure with a high reserve price, as described in (i) in Proposition 2; either procedure is preferred to a non-discriminatory one with a low reserve price as described in (ii) in Proposition 2.

This proposition illustrates that, when the intrinsic value of the project *v* is large, the buyer is indifferent between a non-discriminatory procedure with a high reserve price or a discriminatory one with a handicap. Both procedures make sure that the project is always delivered and quality is at its first-best level, even if they are more costly. When, instead, the intrinsic value of the project is small, the buyer prefers a non-discriminatory procedure with a low reserve price as it limits the rent accruing to the contractor, even if it comes at the cost of not having the project delivered under some cost conditions.

Existence conditions In the remainder of this section, we explore which procedure requires less stringent conditions for an equilibrium to arise. The conditions refer to *i*) the critical level(s) of the discount factor, *ii*) the net social value of quality (i.e., the threshold values of *B*), and *iii*) the intrinsic value of the project *v*.

We denote as $\hat{\delta}_{r_{low}}$, $\hat{\delta}_{r_{high}}$ and $\hat{\delta}_h$ the threshold values of the discount factor in the case of a non-discriminatory procedure with a low and a high reserve price, and of a discriminatory procedure, respectively. These values are computed at equilibrium and at the level of quality (and reserve price, where applicable) that gives the highest utility to the buyer. This implies that $\hat{\delta}_{r_{low}}$ is evaluated at $\hat{r}^C$ and $\hat{q}^C$, as in part *ii*) of Proposition 2, whereas the other two threshold levels are evaluated at the first-best quality level. A similar notation is used to denote the threshold values of the net social value of quality, as reported in (5), (6) and (10); these are given by $\hat{B}_{r_{low}}$, $\hat{B}_{r_{high}}$ and $\hat{B}_h$. Our results are summarised in the following

Proposition 6 (i) The existence conditions on the discount factor are the most stringent in the case of a non-discriminatory procedure with a low reserve price. They are identical in the two cases of a non-discriminatory procedure with a high reserve price and of a discriminatory procedure with a handicap. That is,

$$\hat{\delta}_{r_{low}} > \hat{\delta}_{r_{high}} = \hat{\delta}_h; \tag{14}$$

(ii) The existence conditions on the net social value of quality are the most stringent in the case of a discriminatory procedure. That is,

$$\hat{B}_h > \max\left\{\hat{B}_{r_{low}}, \hat{B}_{r_{high}}\right\}; \tag{15}$$

(iii) The existence conditions on the intrinsic value of the project are the most stringent in the cases of a non-discriminatory procedure, both with a high and low reserve price.

We discuss each of these results in turn.

(i) discount factor Part (*i*) of the proposition shows that, all other things being equal, under a non-discriminatory procedure with a low reserve price the existence of a self-enforcing RPC is ensured under a smaller range of the discount factor values than under a discriminatory procedure or a non-discriminatory procedure with a high reserve price.

To interpret this result, focus on the comparison between the non-discriminatory procedure with a low reserve price and the discriminatory one with a handicap. The same argument applies when the former is compared to the non-discriminatory procedure with a high reserve price. First, notice that under both types of procedure, the deviating firm's continuation payoff is always zero. Indeed, under the non-discriminatory procedure with both a low and a high reserve price, firms are constrained by a reserve price equal to θ_L, thus limiting to zero the contractor's profits. The same effect is obtained under the discriminatory procedure with a handicap, regardless of whether the latter takes the form of a *de facto* debarment. Conversely, the two procedures lead to different levels of the expected rent accruing to the firms along the equilibrium path. The rent is higher under the discriminatory procedure since the handicap is never binding, whereas the low reserve price limits the firms' profits.

Consequently, for a firm to be willing to cooperate, the critical discount factor needs to be higher in the case of a non-discriminatory procedure.

(ii) net social value of quality. Part (*ii*) of the proposition also shows that the discriminatory procedure requires a more stringent condition on the value of quality for the existence of an equilibrium. The intuition becomes straightforward by comparing the discriminatory procedure and the non-discriminatory procedure with a high reserve price. Along the equilibrium path, the buyer's expected payoff coincides under the two formats. Should a deviation occur, the buyer's expected utility in the continuation game would depend on the nature of the procedure. A non-discriminatory procedure always entails no quality but at the lowest possible rent to the firm. A deviation in the discriminatory format, instead, would lead to a breakdown of cooperation only with the punished firm. The non-punished firm would continue delivering quality, although at a higher price induced by the competitive advantage resulting from the handicap levied on the punished firm. This extra cost for obtaining quality induces a higher threshold level of the net social value of quality for an equilibrium to exist. A similar, albeit more convoluted line of reasoning would apply when comparing the discriminatory procedure with the non-discriminatory one with a low reserve price.

(iii) intrinsic value of the project. Finally, Part (*iii*) of the Proposition maintains that the conditions for the existence of a self-enforcing RPC are less stringent under a discriminatory procedure than under a non-discriminatory one. The motivation for this result lies in the upper value of v that ensures that a self-enforcing contract based on the reserve price exists, whereas such a restriction does not appear in the case of a handicap-based contract. The reason is simple: under a non-discriminatory procedure with a reserve price (both high or low), should the contractor deviate, the punishment entails a reserve price equal to the lowest possible cost, θ_L, which does not include the cost of quality. In this event, the buyer does not obtain the delivery of the project when all firms are inefficient. Thus, only in the case of a sufficiently low valuation of the project, the buyer is willing to leave to the firms the rent necessary to obtain the cooperative outcome.[9] On the other hand, when a deviation occurs in a discriminatory contract, the project will always be delivered, and the above-mentioned trade-off does not arise.

Overall, our analysis shows that it is not possible to single out a preferred instrument, neither in terms of equilibrium expected utility nor in terms of availability of the instruments. The choice of the optimal instrument will then depend on the specific circumstances of the competitive procedure. Nevertheless, we try to summarise our results in a way that provides some guidelines that may be useful to buyers:

- *Feasibility* There are parameter configurations for which an RPC equilibrium cannot be sustained by the proposed mechanisms. This occurs, for instance, in a non-discriminatory procedure with a reserve price when the intrinsic value of the project is too large (i.e, $v > \overline{v}$); and in a discriminatory procedure with a handicap when the net benefit of quality is not too large (i.e., $B\left(q^{FB}\right) < \hat{B}_h$);
- *The role of the intrinsic value of the project* When the proposed mechanisms

[9] See the discussion in footnote 6.

sustain an RPC equilibrium, the intrinsic value of the project v, albeit in conjunction with other considerations, plays a crucial role in determining the preferred mechanism:

- when the intrinsic value of the project is sufficiently small (i.e., $v < v^*$ in our model), the buyer should adopt an (optimal) non-discriminatory procedure with a low reserve price. However, a cooperative outcome can be sustained as an equilibrium only provided that all players are sufficiently patient (i.e., $\delta > \hat{\delta}_{r_{low}}$) and the net benefits of quality are sufficiently large (i.e, $B(\hat{q}^C) > \hat{B}_{r_{low}}$),
- when the intrinsic value of the project is sufficiently large (i.e., $v > v^*$ in our model), the non-discriminatory procedure with a high reserve price and the discriminatory procedure can both sustain an outcome equivalent RPC and are preferred to the non-discriminatory procedure with a low reserve price. Furthermore, when players are only moderately patient (i.e., $\delta < \hat{\delta}_{r_{low}}$), these are the only possible mechanisms that can sustain an RPC. However, the two mechanisms differ in terms of the conditions under which an RPC equilibrium can arise. Indeed, only a discriminatory procedure can sustain an RPC equilibrium when the intrinsic value of the project is large (i.e., $v > \overline{v}$); whereas, when this is relatively small (i.e., $v \leq \overline{v}$) only the non-discriminatory procedure with a high reserve price can be applied when the net benefit of quality is relatively small (i.e, $B(q^{FB}) < \hat{B}_h$).

6 Conclusions

A growing body of literature has proved that relational contracts in repeated procurement may solve the problem of enforcing unverifiable quality. Relational contracts may have a different nature depending on the buyer's ability to use some form of discrimination among firms. We have studied two simple mechanisms replicating the features of real-world procurement auctions: a low-score auction with past performance evaluation (discriminatory), and low-price auction with a public reserve price (non-discriminatory). By assuming a fully strategic buyer, we have tackled the question concerning which mechanism a buyer should prefer. The question is more than a theoretical exercise since, for instance, the European public procurement regulation rules out the possibility of using past performance as an award criterion—that is, it cannot be evaluated alongside other financial and non-financial dimensions of a tender—whereas this is a perfectly legitimate option according to the U.S. (federal) procurement regulation.

We have compared the two different types of RPCs arising from using the two different mechanisms under several dimensions: the buyer utility, the strictness of the existence conditions (critical discount factor and minimum net social value of quality), the induced level of quality, and the risk of failing to deliver the project.

Since in a discriminatory procedure the handicap is applied only off equilibrium, both an auction with a "high" reserve price and a discriminatory auction yield the buyer the same expected utility, induce the first-best level of quality, and guarantee that the project is always delivered. Under the equilibrium with a "low" reserve price, though, the buyer runs the risk of not having the project delivered although she can achieve higher savings when procuring the project. When the intrinsic value of the project and the net social value of quality are high, a non-discriminatory RPC with a "high" reserve price is preferred, as a higher rent to the firms is outweighed by the project being always delivered at the first-best quality level. Moreover, compared with the discriminatory procedure, a non-discriminatory auction with a "low" reserve price induces a less stringent existence condition on the net social value of the quality, although this comes with the most stringent condition of the critical discount factor.

Overall, we find that a clear-cut ranking cannot be established without first defining the relative importance of the dimensions affecting the buyer's choices. Consider, for instance, the case of a public buyer. If the buyer's choices are mainly driven by financial considerations, say, the urgency of reigning public spending, then a non-discriminatory procedure with a "low" reserve price should be preferred. When, instead, getting as a high level of quality as possible is paramount—which might happen in *private* procurement—either a discriminatory procedure or a non-discriminatory one with a "high" reserve price should prevail.

Appendix: derivations and proofs

Proof of Proposition 1 See the proof of Proposition 1 in Albano et al. (2023). □

Proof of Proposition 2 For the first part of the Proposition, see the proof of Proposition 2 in Albano et al. (2023). For the second part of the Proposition, see the proof of Proposition 3 in Albano et al. (2023). □

Proof of Proposition 3 See the proof of Proposition 1 in Albano et al. (2017). □

Proof of Proposition 4 We now derive the conditions under which no one-shot profitable deviation (POSD) exists for any player in any subgame, on and off the equilibrium path.

A. No POSD for firms on and off the equilibrium path

In a set-up similar to the one in this paper and with similar trigger strategies, Albano et al. (2017) show that, with a buyer committed to her strategy s_B, provided that $B(q^C) \geq 0$ and $\delta \geq \delta_h$, as in (9), no POSD exists for the firms when $N \geq 2$, $h^C = 0$, $h^P \geq \psi(q^C)$. Clearly, this applies also to the case of $N = 2$, as in this paper.

B. No POSD for the buyer on the equilibrium path

This part of the proof shows whether and under what conditions there exists a POSD for the buyer on the equilibrium path. Two types of deviations are possible for the buyer. First, the buyer may deviate by setting a strictly positive handicap to one of the two firms (part B.1); w.l.o.g, in the rest of the proof, we take this firm to be firm 1. Second, the buyer may deviate by setting a strictly positive handicap to both firms

(part B.2). Denote by $EU(h_1, h_2)$ the buyer's expected utility when setting handicap h_i to firm i, $i = 1, 2$.
B.1: $h_1 = h > 0$ *and* $h_2 = 0$. No POSD exists for the buyer when

$$EU(0,0) + \frac{\delta}{1-\delta}EU(0,0) \geq EU(h,0) + \frac{\delta}{1-\delta}EU(h^P,0), \tag{A-1}$$

where the second term in the RHS reflects the one-shot nature of the buyer's deviation.

Focus now on *EU*(*h*, 0). This should be evaluated at the level of *h* which provides the highest expected utility for the buyer. The characterisation of this *h* is provided in the following Lemma. □

Lemma 1 *Assume* $h_1 = h > 0$ *and* $h_2 = 0$. *Assume that firm i* $(i = 1, 2)$ *bids* $max\{\tilde{C}_i, \tilde{C}_j\}(j \neq i)$*where* $\tilde{C}_1(\theta_1, 0) = \theta_1 + \psi(0) + h$ *and* $\tilde{C}_2(\theta_2, q) = \theta_2 + \psi(q)$, *for any admissible* $q > 0$, *and the contract is awarded to whichever firm has a (handicap-augmented) cost equal* $min\{\tilde{C}_i, \tilde{C}_j\}$. *Let* $\hat{h}$ *be the value of h that maximises the buyer's expected utility and let* $\hat{EU} \equiv EU(\hat{h}, 0)$. *Then,*

- When $B(q) < \Delta$ (and ν is infinitesimally small), $\hat{h} = \psi(q) + \Delta$, and $\hat{EU}$ is as given in (A-16);
- When $B(q) \geq \Delta$, $\hat{h} = \psi(q) + \Delta + \nu$ with $\nu \to 0$, and $\hat{EU}$ is as given in (A-17).

Proof The value of *EU*(*h*, 0) depends on the level of *h*, which determines the firms' equilibrium bids. Three ranges of *h* are of interest; in each of these cases, we will first determine *EU*(*h*, 0) and then establish the level of *h*, within the range, which maximises *EU*(*h*, 0). We will denote as *LH* the case in which the realisation of the fixed-cost component for firm 1 (respectively, firm 2) is equal to *L* (respectively, *H*); similarly for the other possible cases *LL*, *HL* and *HH*.

- $h = h_H$ where $h_H > \psi + \Delta$: depending on the realisations of the firms' cost parameters and based on their equilibrium bids, the buyer's profits are given by

$$U(h_H, 0) = \begin{cases} v + q - h - \theta_L & \text{if } LL, LH; \\ v + q - h - \theta_H & \text{if } HL, HH, \end{cases} \tag{A-2}$$

which, depending on the probabilities of the different combinations of the firms' cost realisation, gives rise to

$$EU\,(h_H, 0) = v + q - h_H - \theta_L\beta - \theta_H(1-\beta) \tag{A-3}$$

Let $\hat{h}_H$ be the maximand of $EU(h_H, 0)$, and define $\hat{EU}_H \equiv EU(\hat{h}_H, 0)$. Since *EU*(*h*, 0) is decreasing in *h*, when ν is infinitesimally small, then $\hat{h}_H = \psi(q) + \Delta$ and

$$\hat{EU}_H = v + q - \psi(q) - \Delta - \theta_L\beta - \theta_H(1-\beta). \tag{A-4}$$

- $h = h_I = \psi(q) + \Delta$: depending on the realisations of the firms' cost parameter and based on their equilibrium bids, the buyer's profits are given by

$$U(h_I, 0) = \begin{cases} v - \theta_L + \frac{1}{2}(q-h) & \text{if } LH; \\ v + q - h - \theta_L & \text{if } LL; \\ v + q - h - \theta_H & \text{if } HL, HH, \end{cases} \tag{A-5}$$

which, depending on the probabilities of the different combinations of the firms' cost realisations, gives rise to

$$EU\left(h_I, 0\right) = v - \theta_H + \beta\Delta + \frac{q-h}{2}(2 - \beta(1-\beta)) \tag{A-6}$$

Let $\hat{h}_I$ the maximand of $EU(h_I, 0)$, and define $\hat{EU}_I \equiv EU(\hat{h}_I, 0)$. Then, trivially, $\hat{h}_I = \psi(q) + \Delta$ and $\hat{EU}_I = EU\left(\hat{h}_I, 0\right) = EU\left(h_I, 0\right)$.

- $h = h_L$ where $\psi \leq h_L < \psi + \Delta$: depending on the realisations of the firms' cost parameter and based on their equilibrium bids, the buyer's profits are given by

$$U(h_L, 0) = \begin{cases} v - \psi(q) - \theta_H + h & \text{if } LH; \\ v + q - h - \theta_L & \text{if } LL; \\ v + q - h - \theta_H & \text{if } HL, HH, \end{cases} \tag{A-7}$$

which, depending on the probabilities of the different combinations of the firms' cost realisation, gives rise to

$$\begin{aligned} EU\left(h_L, 0\right) =& (v - \psi(q) - \theta_H + h_L)\beta(1-\beta) + \\ &+ (v + q - \theta_L - h_I)\beta^2 + (v + q - \theta_H - h_I)(1-\beta). \end{aligned} \tag{A-8}$$

Let $\hat{h}_L$ be the maximand of $EU(h_L, 0)$, and define $\hat{EU}_L \equiv EU(\hat{h}_H, 0)$. Since $EU(h_L, 0)$ is decreasing in *h*, then $\hat{h}_L = \psi(q)$ and

$$\hat{EU}_L = (1 - \beta + \beta^2)(q - \psi(q)) + v - \theta_H + \beta^2\Delta \tag{A-9}$$

Simple but tedious computations allow to show that: i) $\hat{EU}_L < \hat{EU}_I$ for any value of q, and ii) $\hat{EU}_H \geq \hat{EU}_I$ if and only if $B(q) \geq \Delta$. □

The buyer's utility along the equilibrium path writes

$$EU(0, 0) = \begin{cases} v + q - \psi(q) - \theta_H & \text{if } LH, HL, HH; \\ v + q - \psi(q) - \theta_L & \text{if } LL, \end{cases} \tag{A-10}$$

which, depending on the probabilities of the different combinations of the firms' cost realisation, gives rise to

$$EU(0,0) = (v + q - \psi(q) - \theta_L)\beta^2 + (v + q - \psi(q) - \theta_H)(1 - \beta^2). \quad \text{(A-11)}$$

Simple algebra allows to show that, when $B(q) < \Delta$, then $EU(0,0) > \hat{EU}_I$; instead, when $B(q) \geq \Delta$, then $EU(0,0) > \hat{EU}_H$. Hence, no POSD of this kind exists for the buyer.

B.2: $h_1 > 0$ and $h_2 > 0$. No POSD exists for the buyer when

$$EU(0,0) + \frac{\delta}{1-\delta}EU(0,0) \geq EU(h_1, h_2) + \frac{\delta}{1-\delta}EU(h^P, h^P), \quad \text{(A-12)}$$

where the second term in the RHS reflects the one-shot nature of the buyer's deviation.

We start by showing that any buyer's deviation with different but positive handicaps to the two firms is strictly dominated, in expected terms, by a deviation with identical handicaps to both firms. Indeed, when setting a positive handicap to both firms, the buyer induces a deviation from their cooperative strategy by both firms, whose (handicap-augmented) costs are then equal to $\tilde{C}(\theta_j, 0) = \theta_j + \psi(0) + h_j$ for $j = 1, 2$. This implies that firms, ex-ante symmetric in terms of their actual costs, are made ex-post asymmetric because of the different handicaps, thus raising the cost of the project for the buyer. Consequently, we can restrict our attention to the case of identical handicaps to the two firms, that is, $h_1 = h_2 = h > 0$.

The buyer does not deviate by setting an identical handicap to both firms when

$$EU(0,0) + \frac{\delta}{1-\delta}EU(0,0) \geq EU(h, h) + \frac{\delta}{1-\delta}EU(h^P, h^P). \quad \text{(A-13)}$$

Compare the first term in the LHS and in the RHS. $EU(0, 0)$ is given in (A-11), while $EU(h, h)$ is also given in (A-11), but for $q = 0$. Since quality is socially desirable, then $EU(0,0) > EU(h, h)$. Hence, no POSD of this kind exists.

C. No POSD for the buyer off the equilibrium path.

This part of the proof shows whether and under what conditions there exists a POSD for the buyer from the prescribed punishment when a previous deviation has occurred. Two types of past deviation are possible. First, only one firm deviated in the past: w.l.o.g., in the rest of the proof, we take this firm to be firm 1 (part C.1). Second, both firms deviated in the past (part C.2).

C.1: only firm 1 deviated in the past. Assume only firm 1 deviated in the past. The buyer's strategy would entail setting a handicap h^P to firm 1 only. The buyer may then deviate in three ways. First, she may charge to firm 1 a handicap different from what prescribed by her strategy, say $h_1 \neq h^P$ and $h_1 > 0$, and keep a handicap equal to 0 to the rival firm. Second, she may charge a strictly positive handicap to both firms, say $h_1 > 0$ and $h_2 > 0$, with $h_1, h_2 \neq h^P$. Following the same argument we used in part *B.2* of this proof, we restrict our attention to deviations with identical handicaps to both firms, so that $h_1 = h_2 = h > 0$. Third, she may charge to firm 1 a handicap different from what prescribed by her strategy and equal to zero, and set a positive handicap $h_2 > 0$ to the other firm.

C.1.1: $h_1 = h \neq h^P$ and $h_1 > 0$, and $h_2 = 0$. The buyer's IC reads as

$$EU(h^P,0) + \frac{\delta}{1-\delta}EU(h^P,0) \geq EU(h,0) + \frac{\delta}{1-\delta}EU(h^P,0). \quad \text{(A-14)}$$

The RHS of the IC takes note of the one-shot nature of the buyer's deviation. Notice that the second terms in the LHS and RHS are identical and the IC reduces

$$EU(h^P,0) \geq EU(h,0). \quad \text{(A-15)}$$

Lemma 1 illustrates that $h^P = \psi + \Delta$ is a necessary and sufficient condition for (A-15) to hold. Notice that the value for $EU(h^P,0)$ are given according to the value of h^P as it follows:

- when $B(q^C) \geq \Delta$,

$$\hat{EU} = v + q^C - \psi(q^C) - \Delta - \theta_L\beta - \theta_H(1-\beta); \quad \text{(A-16)}$$

- when $B(q^C) < \Delta$,

$$\hat{EU} = v - \theta_H + \beta\Delta + \frac{q^C - \psi(q^C) - \Delta}{2}(2 - \beta(1-\beta)). \quad \text{(A-17)}$$

C.1.2: $h_1 = h_2 = h > 0$. The buyer IC reads as

$$EU(h^P,0) + \frac{\delta}{1-\delta}EU(h^P,0) \geq EU(h,h) + \frac{\delta}{1-\delta}EU(h^P,h^P). \quad \text{(A-18)}$$

The RHS instead takes note of one-shot nature of the buyer's deviation.

The two expressions for *EU*(*h*, *h*) and $EU(h^P,h^P)$ in the RHS of the above IC do not depend on the level of *h* and are given in (A-11) when setting $q = 0$. Hence, the IC in (A-18) reduces to

$$EU(h^P,0) \geq EU(h,h). \quad \text{(A-19)}$$

From Lemma 1, when $B(q) \geq \Delta$, then (A-19) is satisfied when $B(q) \geq \Delta\,(1 - \beta\,(1-\beta))$, which is implied by the initial condition on *B*(*q*) and Δ. Also from Lemma 1, when $B(q) < \Delta$, then (A-19) is satisfied when $B(q) \geq \Delta\frac{2-3\beta(1-\beta)}{2-\beta(1-\beta)}$, which gives the condition (10) in Proposition 4.

C.1.3: $h_1 = 0$ *and* $h_2 = h > 0$. The buyer IC reads as

$$EU(h^P,0) + \frac{\delta}{1-\delta}EU(h^P,0) \geq EU(0,h) + \frac{\delta}{1-\delta}EU(h^P,h^P). \quad \text{(A-20)}$$

The RHS instead takes note of one-shot nature of the buyer's deviation. Consider the first term of the RHS. Any $h > 0$ levied on firm 2 would cause the latter to bid anticipating not to deliver quality. Thus the buyer would get zero quality regardless of the identity of the winner. As the expected cost of the project is increasing in the value of h, the buyer would optimally set an infinitesimally small h, hence A-20) coincides with (A-19).

C.2: both firms deviated in the past. Assume both firms deviated in the past. The buyer's strategy would entail setting a handicap h^P to both firms. The buyer may then deviate charging a handicap different from h^P to either firm. Following the same argument we used in part *B.2)*, we restrict our attention to deviations with identical handicaps to both firms.

The buyer IC reads as

$$EU(h^P, h^P) + \frac{\delta}{1-\delta} EU(h^P, h^P) \geq EU(h, h) + \frac{\delta}{1-\delta} EU(h^P, h^P). \quad \text{(A-21)}$$

The LHS of the IC takes note of the fact that both firms deviated in the past. The RHS instead takes note of the one-shot nature of the buyer's deviation.

As discussed in point *C.1.2*, *EU*(*h*, *h*) is independent of *h* and thus the buyer's IC always holds as an equality. □

Proof of Proposition 5 Once the equivalence in (11) is established, the proof is identical to the one of Proposition 4 in Albano et al. (2023). □

Proof of Proposition 6 We need to compare the critical threshold levels of the discount factors, computed at the optimal quality level, that is, q^{FB} for the discriminatory and non-discriminatory procedure with a high reserve price, and $\hat{q}^C$, for the non-discriminatory procedure with a low reserve price.

First, notice that both $\hat{\delta}_{r_{high}}$ and $\hat{\delta}_h$ only depend on q via $\psi(q)$, and enforce the same first-best quality level. Then $\hat{\delta}_{r_{high}} = \hat{\delta}_h$.

To show that $\hat{\delta}_{r_{low}} \geq \hat{\delta}_h$ first rewrite the two critical thresholds as follows:

$$\hat{\delta}_{r_{low}} = \frac{1}{1 + \frac{\beta(1-\beta)[\hat{r}^C - \psi(\hat{q}^C) - \theta_L]}{\psi(\hat{q}^C)}},$$

and

$$\hat{\delta}_h = \frac{1}{1 + \frac{\beta(1-\beta)\Delta}{\psi(q^{FB})}}.$$

Then

$$\hat{\delta}_{r_{low}} \geq \hat{\delta}_h \Leftrightarrow \frac{\Delta}{\psi(q^{FB})} \geq \frac{\left[\hat{r}^C - \psi(\hat{q}^C) - \theta_L\right]}{\psi(\hat{q}^C)}.$$

After substituting the expression (8) for $\hat{r}^C$ in the RHS of the inequality, the condition becomes

$$\delta \geq \frac{\psi(q^{FB})}{\psi(q^{FB}) + \beta(1-\beta)\Delta},$$

which is exactly the condition for the existence of an equilibrium under a discriminatory format as shown in Proposition 4.

Next we show that

$$\hat{B}_h > \hat{B}_{r_{high}}.$$

Observe first that both $\hat{B}_h$ and $\hat{B}_{r_{high}}$ do not depend on q and that

$$\max_v \hat{B}_{r_{high}} = (1-\beta)^2 \frac{2\beta}{1-\beta}\Delta = 2\beta(1-\beta)\Delta.$$

Since

$$\hat{B}_h = \Delta \frac{2 - 3\beta(1-\beta)}{2 - \beta(1-\beta)} > 2\beta(1-\beta)\Delta, \, \forall \beta \in (0, 1)$$

the statement holds.

Finally, from the expressions (6) and (10) we are able to show that there exists a level of r defined by

$$r_{max} = \Delta \frac{2 - 3\beta(1-\beta)}{2 - \beta(1-\beta)} \frac{1 - (1-\beta)^2}{2\beta(1-\beta)} - \psi(q) - \theta_L$$

such that

$$\hat{B}_h > \hat{B}_{r_{low}} \text{ if } r < r_{max}.$$

Indeed by substituting r with $\hat{r}^C$ and solving the inequality with respect to δ we get

$$\delta > \delta' = \frac{\psi(q)\beta\frac{2\beta(1-\beta)}{1-(1-\beta)^2}}{\Delta(1-\beta)\frac{2-3\beta(1-\beta)}{2-\beta(1-\beta)} + \psi(q)\beta\frac{2\beta(1-\beta)}{1-(1-\beta)^2}}.$$

Notice that this last condition holds because $\hat{\delta}_{r_{low}} \geq \hat{\delta}_h$ (from the first part of the proof) and $\delta' < \hat{\delta}_h$.

Rewrite $\hat{\delta}_h$ and δ' as follows:

$$\hat{\delta}_h = \frac{1}{1 + \frac{\beta(1-\beta)\Delta}{\psi(q^{FB})}},$$

and

$$\delta' = \frac{1}{1 + \Delta(1-\beta)\frac{2-3\beta(1-\beta)}{2-\beta(1-\beta)}\frac{1-(1-\beta)^2}{2\beta^2(1-\beta)}\frac{1}{\psi(\hat{q}^C)}}.$$

This allows us to compare only the second terms of the denominators in both expressions. Notice that q^{FB} and $\hat{q}^C$ are such that $\psi'(q^{FB}) = 1$ and $\psi'(\hat{q}^C) < 1$. Since $\psi(\cdot)$ is a strictly increasing and convex function then $\hat{q}^C < q^{FB}$, and $\psi(q^{FB}) > \psi(q^C)$. Consequently, the sufficient condition for $\hat{\delta}_h > \delta'$ becomes

$$\frac{2-3\beta(1-\beta)}{2-\beta(1-\beta)}\frac{1-(1-\beta)^2}{2\beta^2(1-\beta)} > \beta,$$

which always holds given the assumption on β. □

Funding Open access funding provided by Università degli Studi di Roma Tor Vergata within the CRUI-CARE Agreement. We acknowledge financial support under theNational Recovery and Resilience Plan (NRRP), Mission 4, Component 2, Investment 1.1, Call for tender No. 104 published on 2.2.2022 by the ItalianMinistry of University and Research (MUR), funded by the European Union –NextGenerationEU – Project Title: Auctions: Theory and Applications – CUP E53D23006040006-Grant Assignment Decree No. 104 adopted on 02-02-2022 by the Italian Ministryof Ministry of University and Research (MUR).

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