CEIS Tor Vergata Research Papers

Università di Roma Tor Vergata

ISSN 2610-931X

CEIS Tor Vergata

RESEARCH PAPER SERIES

Vol. 20, Issue 7, No. 550 – December 2022

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On the role of bargaining power in Nash-in-Nash bargaining: when more is less*

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December 21, 2022

Abstract

In bargainings, the parties' bargaining powers (BPs) may determine not only how the surplus is shared (share effect), but also the size of the aggregate surplus (size effect). Since the size effect may be positive or negative, the sign of the effect on a party's payoff of a change in her BP is in principle undetermined. We first look at a general model with a party (the principal) negotiating with two counterparts. At the Nash-in-Nash solution, we show that the equilibrium payoff of the principal may be decreasing in her BP. Necessary conditions for this to occur are an asymmetric bargaining model and a sufficiently large difference in the way the bargained upon variables affect the principal's payoff. We then revisit a standard linear vertical industry with one upstream firm, downstream Cournot competition, and public contracts. A negative effect on the upstream firm's profits deriving from an increase in her BP is always found when the firm has different BPs across the negotiations and final goods are complements. We map these conditions to those characterised in the general model.

Keywords: bargaining power, Nash-in-Nash, vertical relations.
Jel codes: D21; D43; D86.

*We are grateful to Federico Boffa, Daniel Cardona Coll, József Sákovics and Juha Tolvanen for very helpful insights. We also wish to thank the participants at the 2021 (virtual) OLIGOWORKSHOP in Maastricht and at JEI 2022 in Las Palmas de Gran Canaria. Marc Escrihuela-Villar and Walter Ferrarese acknowledge the financial support from the Spanish Ministerio de Ciencia y Educación and Ministerio de Universidades (Agencia Estatal de Investigación) through the project PID2019-107833GB-I00/MCIN/AEI/10.13039/501100011033.

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1 Introduction

In bilateral economic interactions, it is often the case that both parties have the ability to affect the variable(s) upon which the outcome of the interaction depends. Negotiation between the parties then determines the terms of trade. One of the most common ways of modelling this interaction is the Nash bargaining, which axiomatically determines the bargained upon variable. The outcome of a Nash bargaining depends on a number of variables/modelling choices, including the bargaining power of the parties, which captures some undefined ability of the parties to affect the outcome of the negotiation.

When one party has to negotiate with several counterparts, a commonly used framework adopted in the economic literature is the Nash-in-Nash, where this party conducts several simultaneous and independent bilateral Nash bargainings and the outcome of the negotiations is given by the Nash equilibrium in the Nash bargainings. In this context, we derive the main result of the paper: in a Nash-in-Nash equilibrium, the equilibrium payoff of a party may be decreasing in her bargaining power.

We start from the simple observation that, in many economically relevant situations, the outcome of the negotiation determines not only how the two parties share the surplus generated by their interaction, but also the aggregate surplus. Thus, the effect of a change in the party's bargaining power on her equilibrium payoff may be decomposed in an effect on the aggregate surplus (which we call *size effect*) and an effect on the share this agent appropriates of the total surplus (which we call *share effect*). We first apply this idea in a very simple yet very general bilateral negotiation setting. We show that the share effect always dominates and a party's payoff is always increasing in her bargaining power. We then show that a less clear-cut result may be obtained in the case of many simultaneous bilateral negotiations. In this context, we show that a larger bargaining power may result in a lower equilibrium payoff. We observe that this possibility depends on the effect that one bargained upon variable has on the other bargained upon variable.

We then apply these concepts to the analysis of a standard problem in industrial organization, that is the one of a negotiation in a vertical industry over a linear input price. We look at this problem from the viewpoint of the upstream firm. In a bilateral monopoly, in line with our previous results, we show that the upstream firm is always better off when it has larger bargaining power. Instead, in the context of multiple bilateral negotiations, Cournot competition and public contracts, we show that the nature of downstream competition affects the relationship between the upstream firm's bargaining power and its equilibrium payoff. A negative relationship may emerge when downstream firms produce complementary goods.

This is in line with intuition. A lower bargaining power for the upstream firm in a specific bilateral negotiation is associated with a lower negotiated input price. This in turn improves the competitive position of the downstream firm, which increases its quantity. The resulting increase in quantity by the rival firm makes sure that the size of the aggregate profits in the downstream market increases, pushing up the aggregate industry profits. Hence, the negative *share* effect suffered by the upstream firm is more than compensated by a positive *size* effect,

The Nash Bargaining (NB, henceforth) is very often used in the economic literature (Nash, 1950). In a NB, the so-called Nash solution is a function which selects the unique outcome which maximises the geometric average of the gains that the players realize by reaching an agreement instead of settling for the disagreement payoffs. By satisfying a given set of axioms, this solution is shown to characterise the result of an efficient bargain between two parties. Horn and Wolinsky (1988) extend the Nash bargaining to a situation of one-to-many agents, introducing the so-called Nash-in-Nash, where the outcome of the negotiation is given by the Nash equilibrium between many independent and simultaneous bilateral negotiations.[1] Since Roth (1979) and Binmore (1980), bilateral negotiations are allowed to be asymmetric. This implies that the Nash solution is given by the outcome which maximises the *weighted* geometric average of the players' (net) payoff. The weights used in the geometric average capture some imprecisely defined differences in "bargaining power", where a large exponent is interpreted as representing a relatively high bargaining power of a party.

[1] Collard-Wexler et al. (2019) extend further this set-up to the case of a many-to-many situation.

In spite of the relevance in the economic literature both of the Nash bargaining and the Nash-in-Nash approaches, the nature of the bargaining power and its relationship to the outcome of the negotiation(s) has been a relatively unexplored issue.

Binmore et al. (1986) provide microfoundations to the otherwise axiomatic approach of the NB. They show the equivalence between the Nash Solution and the equilibrium of a strategic bargaining game with alternate moves. This equivalence is shown both to a strategic model in which players have time preference and the length of bargaining period goes to zero, and to a strategic model in which parties have von Neumann-Morgestern utility functions and there is an uncertain termination of bargaining. In this context, different bargaining powers are motivated uniquely by the existence of asymmetries in the bargaining procedure or in the parties' beliefs. In the first case, there is an asymmetry in the time lengths between the alternate offers of the two parties; in the second case, parties have different beliefs on the probability of termination of the bargaining game.

We found no trace in the literature of further attempts to explain the nature of the bargaining power, if not a generic reference to the vague concept of negotiation skills. On the other hand, the recent empirical literature has given it a central role in its econometric analysis. A few papers have attempted to provide an estimation of the bargaining power, disentangling it from the other effects deriving from the market structure, which in turn affects both the nature of the firms' profits and their outside options (Draganska et al., 2010; Grennan, 2014; Richards et al., 2018). Other papers have included the bargaining power as an explanatory variable of their structural econometric models of specific industries, concurring with the explanation of different bargaining results in simultaneous negotiations (Crawford et al., 2012; Crawford et al., 2018).

Most of the existing theoretical literature has similarly downplayed the nature and the role of bargaining power. Many papers have abstracted away from it, studying a symmetric Nash bargaining, that is a situation in which parties are assumed to have equal bargaining parties (see, for instance, Horn and Wolisnky, 1988; and Dobson and Waterson, 1997). In other cases, parties have been allowed to have different bargaining powers, without however neither motivating this assumption nor having carried out an in-depth analysis of its effects on the bargaining outcome (see, for instance, Iozzi and Valletti, 2014). The only notable exception belongs to the household economics literature. In a general equilibrium model in which the intra-household allocation is determined through a Nash bargaining, Gersbach and Haller (2009) investigate the intra- and inter-household effects of a change in the bargaining power of an individual. They find that the key determinant of this change is the magnitude of the price responses. When price effects are small, an individual always benefits from an increase in her bargaining power, while all other individuals in the same household are damaged. When instead price effects are large, a change in the bargaining power of an individual may harm her and all other individuals in the same household.

In this paper, we first analyse a general framework in which very limited restrictions are imposed on the nature of the relationship between the bargained upon variable(s) and the payoff agents derive from it (them). We first focus on the case of a single bilateral negotiation, that we use a useful reference point. We find that, in line with the common interpretation of the nature of a party's bargaining power, her equilibrium payoff always increases with her bargaining power. We then turn to analyse the case of multiple bilateral negotiations in which one agent (referred to as the principal) negotiates with many counterparts. We solve for the equilibrium outcome by the Nash-in-Nash approach. we characterise the necessary conditions for a negative effect of on the principal's payoff due to an increase in her bargaining power. We find that two conditions are required for this result. First, the bargaining model must be asymmetric. This implies that the principal is allowed to have a different bargaining power from her counterpart in each negotiation and a different bargaining power in each negotiation. Asymmetry also implies that the bargained upon variables affect differently the principal's payoff and, also, the payoffs of the counterparts. The second condition is that, in equilibrium, the bargained upon variables affect in a significantly different manner the principal's payoff. We are able to formalise this latter condition in terms of the (relative) degree of convexity of the payoff functions of each negotiating pair.

We then revisit the case of a vertical industry with two downstream Cournot competitors, linear demand and input prices, and public contracts. In line with the result obtained in the general model, when the model is symmetric, a change in the upstream bargaining power never reduces her equilibrium payoffs. On the contrary, we confirm that, in an asymmetric model, the upstream firm's equilibrium profits may be decreasing in her bargaining power. The nature of the downstream market turns out to be crucial. The negative relationship between the upstream firm's bargaining power and her equilibrium profits may occur only when goods offered by the two downstream competitors are complements. We are able to map this condition on the nature of the final good with the conditions established in the general model.

Our paper thus contributes both to the theoretical and empirical literature using the Nash-in-Nash approach. On the theoretical side, our main contribution is highlighting a possible negative effect of a change in an agent's bargaining power. The first natural consequence of this is the call for a re-evaluation of the interpretation of the bargaining power parameter. Indeed, this has so far been envisaged as illustrating unexplained asymmetries in the negotiation, with a larger value always associated with a more favourable outcome of the negotiation. The second consequence is the highlight of a further, yet unexplored, unwanted effect of the symmetry hypothesis used in many applied theory papers. Our paper suggests that the use of a symmetric model may hide some potentially interesting ways in which the bargaining power may shape the outcome of multiple negotiations. Our results should also be useful to empirical papers using the Nash-in-Nash approach to estimate the outcome of simultaneous negotiations. Indeed, we point out the necessity to adopt a flexible estimation method that takes into account the possibility of a non-monotone relationship between the outcome of the negotiation and the bargaining powers of the parties, possibly also dealing with the existence of multiple solutions in the estimation of the bargaining power parameters.

The structure of the paper is as follows. Section 2 analyses the general case of single and multiple bilateral Nash bargaining(s). Section 3 analyses a linear vertical industry under Cournot competition and public contracts. All proofs are relegated to an Appendix.

2 The general case

In this section, we first look at the case of a single bilateral negotiation in which two agents, S and A, bargain upon a variable a. We derive the outcome of this negotiation by using the asymmetric Nash bargaining. We then turn our attention to the case in which agent S is involved in n simultaneous and independent negotiations with n counterparts. Each negotiation is on a single variable that, however, affects the payoffs of all $n+1$ agents. In this case, we derive the equilibrium outcome of these negotiations by using the so-called Nash-in-Nash solution (Horn and Wolinsky, 1988): the equilibrium bargained upon variables are the Nash equilibrium of n independent and simultaneous negotiations, whereby the outcome of each negotiation is the solution of a bilateral Nash bargaining that takes as given the outcome of the remaining $n-1$ negotiations. Both in the case of a single negotiation and of multiple negotiations, we impose minimal requirements on the nature of the agents' payoffs.

A single bilateral negotiation. We assume two agents S and A. An agent's payoff $\pi^i(a)$ is such that $\pi^i : \mathbb{R}_+ \to \mathbb{R}$ with $i = \{S, A\}$. Payoffs are twice continuously differentiable. The Nash product writes:

$$\Omega\left(a\right) \equiv \left[\pi^S(a)\right]^{\alpha} \times \left[\pi^A(a)\right]^{1-\alpha}, \tag{1}$$

where $\alpha \in [0,1]$ and $1-\alpha$ is the bargaining power in the negotiation for agent S and A, respectively. Both agents are assumed to have no outside options.[2]

We assume that the solution to our bargaining problem exists and it is unique and we denote it by $\hat{a}$. Formally,

$$\hat{a} = \underset{a \in \mathbb{R}_+}{\operatorname{argmax}}\ \Omega(a). \tag{2}$$

Let $\hat{\pi}^i \equiv \pi^i(a)|_{a=\hat{a}}$, with $i = \{S, A\}$. At an interior solution, $\hat{a}$ is implicitly defined

[2]Given the very general nature of the agents' payoffs, the outside option for each player can be subsumed into π^i.

by the first order condition:

$$\Omega_a \equiv \alpha\, \hat{\pi}^S_a\, \hat{\pi}^A + (1-\alpha)\, \hat{\pi}^S\, \hat{\pi}^A_a \equiv 0, \tag{3}$$

where we express the derivative using subscripts of the independent variable. Notice that, for (3) to hold, it is necessary that

$$\text{sign}\left[\hat{\pi}^S_a\right] = -\,\text{sign}\left[\hat{\pi}^A_a\right], \tag{4}$$

since $\hat{\pi}^S > 0$ and $\hat{\pi}^A > 0$. This illustrates the conflict of interests between the two parties intrinsic to the negotiation. Second-order conditions at the Nash solution are assumed to hold throughout the entire analysis, so that $\Omega_{aa} < 0$.

In this framework, we are interested in the effect of a change in agent S's bargaining power on her own equilibrium payoff. Since agent's payoff ultimately depends on the bargained variable a, we can write:

$$\frac{d\hat{\pi}^S}{d\alpha} = \hat{\pi}^S_a \hat{a}_\alpha. \tag{5}$$

From the equilibrium conditions of the Nash problem, it is possible to show that $\text{sign}[\hat{\pi}^S_a] = \text{sign}[\hat{a}]$. Thus, the derivative in (5) cannot have a negative sign, implying that, when involved in a single negotiation, an increase in agent S's bargaining power never reduces her equilibrium payoff.

In spite of this clear cut result, some useful insights may be obtained by manipulating (3) to obtain:

$$\pi^S \equiv -\frac{\alpha}{1-\alpha}\Phi(\hat{a}), \tag{6}$$

where $\Phi \equiv \hat{\pi}^A\, \frac{\hat{\pi}^S_a}{\hat{\pi}^A_a}$.

By differentiating w.r.to α the RHS of (6), we may obtain an alternative expression for the marginal effect of a change in α on the equilibrium profits of agent S. This is given by

$$\frac{d\hat{\pi}^S}{d\alpha} = -\frac{1}{(1-\alpha)^2}\,\Phi - \frac{\alpha}{1-\alpha}\Phi_a \hat{a}_\alpha. \tag{7}$$

Equation (7) illustrates that the total variation in agent S's payoff w.r.to α can be disentangled in two effects. The first term in the RHS of (7) accounts for the direct effect of a larger bargaining power α on agent S's payoff, independent of any readjustment in the terms of bargain $\hat{a}$. It captures the variation in the share of the surplus obtained by agent S, hence we refer to this effect as the *share effect*. From (3) and (4), it can be shown to be non-negative.

The second term instead considers the effect of a larger α on agent S's payoff deriving from a change in the terms of bargain $\hat{a}$. In other words, it describes how a change in agent S's bargaining power impacts on the overall surplus of the game, and how much of this new surplus can be appropriated by agent S keeping constant her initial bargaining power. We refer to this effect as the *size effect*.

Define now $\sigma^i \equiv -a\frac{\hat{\pi}^i_{aa}}{\hat{\pi}^i_a}$ as the curvature of agent i's equilibrium payoff with respect to a; this is a standard measure of the curvature (or, equivalently, of the degree of convexity) of the function $\hat{\pi}^i$. Also, let $\epsilon^i \equiv -a\frac{\hat{\pi}^i_a}{\hat{\pi}^i}$ be the elasticity of agent i's equilibrium payoff with respect to a. It is possible to show that equation (7) may be rewritten as

$$\frac{d\hat{\pi}^S}{d\alpha} = -\frac{1}{(1-\alpha)^2}\,\Phi - \frac{\alpha}{1-\alpha}\hat{\pi}_a \hat{a}_\alpha \left(1 + \frac{\sigma^S_a - \sigma^A_a}{\epsilon^A_a}\right). \tag{8}$$

Equation (8) provides a further way of illustrating the marginal effect on agent S's equilibrium profits of a change in her bargaining power. It shares with equation (7) the first addend, which we showed to illustrate the *share effect*, always non-negative. Focus now on the second addend which we interpreted to illustrate the *size effect*. It is, in turn, the product of two terms. The first term is given by $-\frac{\alpha}{1-\alpha}\hat{\pi}_a\hat{a}_\alpha$, and, when commenting (5), we have already shown it to be always non positive. More interesting is the second term, $1+\frac{\sigma^S-\sigma^A}{\epsilon^A}$. The ratio illustrates the relative change in curvature of the two agents' equilibrium payoff expressions, normalised by the payoff elasticity w.r.t. a of the player's A payoff. Intuitively, it provides a measure of the rate of change of agent S's payoff, relative to the same measure for the other player, due to a change in the terms of bargain. The second-order condition of our Nash problem implies that $\frac{\sigma^S-\sigma^A}{\epsilon^A} \geq -\frac{1}{\alpha}$. Thus, the sign of $1+\frac{\sigma^S-\sigma^A}{\epsilon^A}$ is ambiguous, which in turn implies that the sign of the whole addend illustrating the *size effect* is ambiguous.

Combining the discussion on equations (5) and (8), we conclude that, in the single bilateral negotiation case, a change in an agent's bargaining power may give rise to two possibilities: either an increase in α provides a larger share of a larger pie, or it provides a larger share of a decreasing pie, with the latter negative effect being small enough not to offset the positive share effect.

Multiple bilateral negotiations. We now move to the case of multiple bilateral negotiations between agent S and n counterparts. For simplicity, we assume $n = 2$. The two counterparts of agent S are denoted by A and B, who negotiate with S simultaneously and independently over the two variables a and b, respectively. Both variables determine agent i's payoff.

We assume that the payoff functions $\pi^i : \mathbb{R}^2_+ \to \mathbb{R}$, with $i = \{S, A, B\}$, are twice continuously differentiable. We denote by $\alpha \in [0, 1]$ and $1-\alpha$ the bargaining powers of agent S and A, respectively, in their bilateral negotiation. Similarly, we denote by $\beta \in [0, 1]$ and $1-\beta$ the bargaining powers of agent S and B, respectively, in their bilateral negotiation.

The Nash products for the two negotiations are:

$$\Omega^A(a,b) \equiv \left[\pi^S(a,b) - \bar{\pi}^{SA}\right]^{\alpha} \times \left[\pi^A(a,b)\right]^{1-\alpha}, \tag{9}$$

$$\Omega^B(a,b) \equiv \left[\pi^S(a,b) - \bar{\pi}^{SB}\right]^{\beta} \times \left[\pi^B(a,b)\right]^{1-\beta}, \tag{10}$$

where $\bar{\pi}^{SA} \geq 0$ and $\bar{\pi}^{SB} \geq 0$ denote the agent's S outside options in case of breakdown of the negotiation with agent A and B, respectively. Since agents A and B do not have an alternative partner, their disagreement payoffs are simply zero.

Let $\hat{\pi}^i \equiv \pi^i(a,b)|_{(a,b)=(\hat{a},\hat{b})}$, with $i = \{S, A, B\}$. The following first-order conditions are necessary for an interior solution of each of the two negotiations:

$$\Omega_a^A \equiv \alpha\, \hat{\pi}^A\, \hat{\pi}_a^S + (1-\alpha)\,(\hat{\pi}^S - \bar{\pi}^{SA})\, \hat{\pi}_a^A \equiv 0; \tag{11}$$

$$\Omega_b^B \equiv \beta\, \hat{\pi}^B\, \hat{\pi}_b^S + (1-\beta)\,(\hat{\pi}^S - \bar{\pi}^{SB})\, \hat{\pi}_b^B \equiv 0. \tag{12}$$

We assume the second-order conditions to hold in each of the two problems. We also assume the following:

Assumption 1. $|\Omega_{aa}^A| > |\Omega_{ab}^A|$ *and* $|\Omega_{bb}^B| > |\Omega_{ba}^B|$.

This assumption implies that, in the first-order conditions defining the negotiated a, the effect of a change in a on the marginal effect of a on the outcome of the negotiation is always larger than the effect of a change in b. In words, it says that the second-order effect of a on the outcome of the negotiation is always larger than the second-order effect of the other variable.

Since $\hat{\pi}^S, (\hat{\pi}^S - \bar{\pi}^{SA}), (\hat{\pi}^S - \bar{\pi}^{SB}), \hat{\pi}^A, \hat{\pi}^B$ are all positive terms, for (11) and (12) to hold, it is needed that, respectively,

$$\text{sign}\left[\hat{\pi}_a^S\right] = -\,\text{sign}\left[\hat{\pi}_a^A\right], \tag{13}$$

$$\text{sign}\left[\hat{\pi}_b^S\right] = -\,\text{sign}\left[\hat{\pi}_b^B\right], \tag{14}$$

that illustrate the conflict of interest intrinsic to each negotiation.

A Nash-in-Nash solution is a pair $\left(\hat{a}, \hat{b}\right) \in \mathbb{R}_+^2$ such that:

$$\hat{a} \in \underset{a\in\mathbb{R}_+}{\text{argmax}}\; \Omega^A\left(a, \hat{b}\right); \tag{15}$$

$$\hat{b} \in \underset{b\in\mathbb{R}_+}{\text{argmax}}\; \Omega^B\left(\hat{a}, b\right). \tag{16}$$

We assume that the solution to this bargaining game exists and it is unique.

As in the case of a bilateral monopoly, we are interested in the effect of a change in agent S's bargaining power on her own equilibrium payoff. W.l.o.g., focus on the negotiation with player A. Since agent S's payoff ultimately depends on the bargained variables a and b, we can write

$$\begin{aligned}\hat{\pi}^S_\alpha &= \hat{\pi}^S_a \hat{a}_\alpha + \hat{\pi}^S_b \hat{b}_\alpha \\ &= \left(\hat{\pi}^S_a + \hat{\pi}^S_b \hat{b}_a\right) \hat{a}_\alpha\end{aligned} \tag{17}$$

where the second line of (17) is obtained making use of the identity $\hat{b}_\alpha \equiv \hat{b}_a \hat{a}_\alpha$, where $\hat{b}(a)$ is the outcome b of the other negotiation as a function of a, as implied by (12). Notice that a standard stability condition requires $\hat{b}_a \in (-\infty, 1)$, see Horn and Wolinsky (1988).

Equation (17), albeit simple, already illustrates the main determinants of the effect on $\hat{\pi}^S$ of a change in α and that the sign of this effect may be negative.

To illustrate, first notice that it is possible to show that, in equilibrium,

$$\text{sign}\left[\hat{\pi}^S_a\right] = \text{sign}\left[\hat{a}_\alpha\right]. \tag{18}$$

Take now the case of a positive $\hat{\pi}^S_a$. Condition (18) and Assumption 1 imply that $\hat{a}_\alpha > 0$; thus, a sufficient condition for $\hat{\pi}^S_\alpha$ to be positive is that $\hat{\pi}^S_a > -\hat{\pi}^S_b \hat{b}_a$. This condition is clearly unwarranted. Indeed, whether or not this condition holds depends on the relative magnitude of the marginal effects $\hat{\pi}^S_a$ and $\hat{\pi}^S_b$ and on the sign and magnitude of $\hat{b}_a$ which clearly also depend on the degree of asymmetry in the bargaining powers across the two negotiations.

A further, yet inconclusive, illustration of the forces at play may be obtained by rearranging equations (11) and (12) and solving them for $\hat{\pi}^S$ to obtain:

$$\hat{\pi}^S = -\frac{\alpha}{2(1-\alpha)}\Phi^A - \frac{\beta}{2(1-\beta)}\Phi^B + \frac{\bar{\pi}^{SA} + \hat{\pi}^{SB}}{2}. \tag{19}$$

where $\Phi^A \equiv \hat{\pi}^A \frac{\hat{\pi}^S_a}{\hat{\pi}^A_a}$ and $\Phi^B \equiv \hat{\pi}^B \frac{\hat{\pi}^S_b}{\hat{\pi}^B_b}$.

Differentiating (19) with respect to α gives

$$\hat{\pi}^S_\alpha = -\frac{1}{2(1-\alpha)^2}\Phi^A - \frac{\alpha}{2(1-\alpha)}\Phi^A_a - \frac{\beta}{2(1-\beta)}\Phi^B_a + \frac{\partial}{\partial\alpha}\frac{\bar{\pi}^{SA} + \bar{\pi}^{SB}}{2}. \tag{20}$$

Define now $\sigma_x^i \equiv -x\frac{\hat{\pi}_{xx}^i}{\hat{\pi}_x^i}$ as the directional curvature coefficient of player i's equilibrium payoff with respect to x, with $x \in \{a, b\}$; this is a standard measure of the curvature of $\hat{\pi}^i$ at x and along the x-axis. Also, let $\epsilon_x^i \equiv -x\frac{\hat{\pi}_x^i}{\hat{\pi}^i}$ be the elasticity of player i's equilibrium payoff with respect to x. Finally, for simplicity, assume $\bar{\pi}_\alpha^{SA} = \bar{\pi}_\alpha^{SB} = 0$.[3] Thus, equation (20) may be rewritten as

$$\begin{aligned}\hat{\pi}_\alpha^S = & -\frac{1}{2(1-\alpha)^2}\Phi^A + \\ & -\frac{\alpha}{2(1-\alpha)}\hat{\pi}_a^S\hat{a}_\alpha\left(1+\frac{\sigma_a^S-\sigma_a^A}{\epsilon_a^A}\right)-\frac{\beta}{2(1-\beta)}\hat{\pi}_b^S\hat{b}_a\hat{a}_\alpha\left(1+\frac{\sigma_b^S-\sigma_a^B}{\epsilon_b^B}\right).\end{aligned} \tag{21}$$

This equation is the direct counterpart of (8) in the case of a single bilateral negotiation. It disentangles the effect on agent S's payoff due to a change in α in the *share effect* and the *size effect*.

The first addend in the RHS of (21) illustrates the *share effect*: it accounts for the direct effect of a larger bargaining power α on agent S's payoff, independent of any readjustment in the terms of bargain a and b. It is identical to the first term of (8) and, using the same argument as in the case of the single bilateral negotiation, it can be shown to be non-negative.

The second and the third addend in (21) illustrate the *size effect.* They account for the effect of a change in the agent S's bargaining power α on the aggregate payoff while keeping fixed the bargaining powers of the two parties. More specifically, the second addend relates to the change in the aggregate payoff of the negotiation between S and A, while the third term relates to the change in the aggregate payoff of the negotiation between S and B. The interpretation of these two addends is very much in line with the interpretation of the second term in (8). The third addend however features a new element in the first term, $\hat{b}_a$, which reflects the way in which a change in a reflects in a change in the outcome of the other negotiation, b.

It is useful to focus on the case of a symmetric model. Intuitively, we define a

[3] In the absence of this assumption, the value and the sign of this derivative would depend on the way the outside option is modeled.

symmetric model as a situation in which agent S's bargaining power is identical in both negotiations and her payoff depends on a and b in an equal way; also, the payoff of agent A depends on a in the same way as the payoff of agent B depends on b. More formally, conditions for our model to be symmetric are i) $\alpha = \beta$, ii) $\pi^S(a,b) = \pi^S(b,a)$ for all a and b; and iii) for $i = A, B$ and $i \neq j$, then $\pi^i(a,b) = \pi^j(b,a)$ for all a and b.

In the case of a symmetric model, (20) reduces to

$$\hat{\pi}^S_\alpha = -\frac{1}{2(1-\alpha)^2}\Phi^A - \frac{\alpha}{2(1-\alpha)}\hat{\pi}^S_a \hat{a}_\alpha (1 + \hat{b}_a)\left(1 + \frac{\sigma^S_a - \sigma^A_a}{\epsilon^A_a}\right). \tag{22}$$

Equation (22) is almost identical to the one derived in the case of a single bilateral negotiation. The only difference is the term $1+\hat{b}_a$, that is always positive. Indeed, under Assumption 1 and the stability condition for the existence of Nas-in-Nash solution, it can be shown that $\hat{b}_a \in (-1, 1)$. In light of this, (22) illustrates that, in a symmetric Nash-in-Nash model, an increase in α never reduces the equilibrium payoff of agent S.

Next Proposition sums up the discussion provided so far and illustrates if and under what conditions a size effect may be negative and possibly outplay the share effect, thus rendering ambiguous the sign of the total effect of a change in α on the agent S's payoff.

Proposition 1. *i) In the case of a single bilateral negotiation, $\hat{\pi}^S$ is a non-decreasing function in α.*
ii) In a symmetric Nash-in-Nash model, $\hat{\pi}^S$ is a non-decreasing function in α.
iii) In an asymmetric Nash-in-Nash model, necessary conditions for $\hat{\pi}^S_\alpha < 0$ are

- $\mathrm{sign}[\hat{\pi}^S_a] = \mathrm{sign}[\hat{\pi}^S_b] > 0$ *and* $\Omega^B_{ba} < 0$;
- $\mathrm{sign}[\hat{\pi}^S_a] = \mathrm{sign}[\hat{\pi}^S_b] < 0$ *and* $\Omega^B_{ba} > 0$;
- $\mathrm{sign}[\hat{\pi}^S_a] = -\,\mathrm{sign}[\hat{\pi}^S_b]$ *and* $\Omega^B_{ba} > 0$.

This Proposition illustrates if and under what conditions an increase in her own bargaining power may reduce the equilibrium profits of a party. It illustrates this by disentangling the effect that the change in the bargaining power has on the *share* the same party can appropriate in the bargain and on the *size* of the total aggregate surplus of the bargaining parties. It shows that this never happens in a single bilateral negotiation. Even if we show that an increase in a party's bargaining party may affect positively her share and, at the same time, affect negatively the size of the aggregate surplus, we show that the former effect always outplays the latter. We also show that this very same result applies in the case of a symmetric model with multiple bilateral negotiations when bargains occur simultaneously and independently and the outcome of the negotiation is a Nash equilibrium in the Nash bargaining.

The picture changes dramatically in the case of an asymmetric model of multilateral bargaining. We derive the necessary conditions for the size effect to outplay the share effect. Intuitively, this depends on the way the change in bargained upon variables affects the relative payoffs of the parties, and, in particular, on the difference in the degree of curvature of the equilibrium payoffs.

3 A linear vertical industry

In this section, we provide an application of our general analysis. We consider an industry in which a single upstream supplier sells an intermediate good to 2 downstream firms. The upstream supplier is denoted by U and the downstream firms are denoted by i, with $i \in \{1, 2\}$. Downstream firms use this input to produce differentiated goods and sell them to final consumers. The ratio of input to output is identical for the two downstream firms and is normalized to one. Each downstream firm i pays a linear input price t_i to the upstream supplier and does not incur any other cost. The costs of the upstream supplier are normalized to zero.

Demand. We assume a linear demand for the final good. Denoting by q_i and q_j the output offered by firm i and j respectively, the inverse demand for firm i is given by

$$p_i(q_i, q_j) = 1 - q_i - bq_j, \text{ for } i, j = 1, 2 \text{ and } i \neq j. \tag{23}$$

whenever this is positive. As it is well-known, the parameter b describes the relationship between the two goods produced by downstream firms: when $b \in (0, 1]$ goods are substitute (perfect substitutes in case of $b = 1$), when $b = 0$ goods are independent, when $b \in [-1, 0)$ goods are complements.

The game. Competition in the industry is described as a two-stage game. In the first stage, the upstream firm negotiates a linear input price with each of its counterparts. Negotiations are simultaneous and independent, implying that while bargaining, both parties in the negotiation treat the other input price as given. Each bargain is obtained using the two-person Nash solution. Thus, the outcome is a set of input prices that are a Nash equilibrium in the Nash bargains. In the second stage of the game, we assume observable contracts. That is, the outcome of the negotiations that occurred in the first stage of the game is perfectly observable by all parties. Given the values of the negotiated linear prices from stage 1, Cournot competition takes place. We derive the pure strategy subgame perfect Nash equilibrium of this game and proceed by backward induction.

Bargaining. For $i \in \{1, 2\}$ and $i \neq j$, denote by $\tilde{\pi}_i(t_i, t_j)$ the profit in the last stage of downstream firm i and by $\tilde{\pi}_U(t_i, t_j)$ the profit in the last stage of the upstream monopoly firm, where t_i and t_j are the negotiated input prices to firm i and j, respectively. Also, let $\overline{\pi}_U$ be the disagreement payoff for the upstream firm. Since each downstream firm i has no alternative supplier, its disagreement payoff is zero. In the first stage of the game, the upstream supplier and each downstream firm i form a separate bargaining unit over the linear input price t_i. The Nash product of this negotiation is given by

$$\Omega^i = [\hat{\pi}_U(t_i, t_j) - \overline{\pi}_U]^{\alpha_i} \times [\hat{\pi}_i(t_i, t_j)]^{1-\alpha_i} \tag{24}$$

where $\alpha_i \in (0, 1]$ denotes the bargaining power of the upstream firm relative to that of the downstream firm i. The linear input price t_i resulting from the negotiation is obtained by maximising $\Omega^i(t_i, t_j)$ w.r.t. t_i. The equilibrium of the bargaining stage is found as the Nash solution to the two separate bargaining problems. In other words, it is given by the pair of tariffs (t_i^*, t_j^*) such that

$$t_i^* = \arg\max_{t_i} \Omega^i(t_i, t_j^*) \tag{25}$$
$$t_j^* = \arg\max_{t_j} \Omega^j(t_j, t_i^*)$$

In each negotiation, we assume that breakdowns are unobservable by the rival downstream firm (Iozzi and Valletti, 2014). The outside option of the upstream firm is obtained taking into account that, in case of breakdown of the negotiation with firm i, it can still sell to firm j. Its outside option is then equal to $\overline{\pi}_U = t_j \overline{q}_j$, where t_j is the anticipated equilibrium level of the input price resulting from the negotiation, and $\overline{q}_j$ is the quantity that firm j sells in the event of a disagreement. Unobservability of breakdowns affects this quantity $\overline{q}_j$. When the breakdown of the negotiation between firm U and i is unobservable, rival firm j is not able to adjust its behavior to the absence of firm i in the downstream market. Firm j thus sticks to its optimal quantity as if both firms were present in the downstream market. Therefore, $\overline{q}_j$ is the last-stage anticipated quantity in a 2-firm equilibrium, calculated at the anticipated equilibrium input prices, and is, therefore, independent of the currently negotiated t_i.

3.1 Equilibrium Analysis

In the second stage of the game, each retailer i sets its final quantity to maximize $\pi_i = (p_i - t_i)q_i$, where p_i is given by (23). The besbehaviourfunction for firm i, whenever positive, is given by

$$q_i^{BR} = \frac{1}{2}\left(1 - t_i - bq_j\right). \tag{26}$$

By solving the system of best-reply functions implied by (26), we obtain the second-stage equilibrium quantities in the subgame where the pair of tariffs (t_i, t_j) has emerged from the bargaining stage:

$$\tilde{q}_i(t_i, t_j) = \frac{2 - 2t_i - b\left(1 - t_j\right)}{4 - b^2}. \tag{27}$$

These quantities determine the second stage equilibrium payoffs of the downstream retailers and of the upstream supplier in the first stage of the game, which are given, respectively, by:

$$\tilde{\pi}_i\left(t_i, t_j\right) = \left(\frac{2 - 2t_i - b\left(1 - t_j\right)}{4 - b^2}\right)^2; \tag{28}$$

$$\tilde{\pi}_U\left(t_i, t_j\right) = \sum_i t_i\left(\frac{2 - 2t_i - b\left(1 - t_j\right)}{4 - b^2}\right). \tag{29}$$

In the negotiation stage, anticipating the outcome of the following stage, the Nash product between U and downstream firm i is:

$$\Omega^i = \left[\tilde{\pi}_U\left(t_i, t_j\right) - \overline{\pi}_U\right]^{\alpha_i} \times \left[\tilde{\pi}_i\left(t_i, t_j\right)\right]^{(1-\alpha_i)}. \tag{30}$$

Because of the unobservability of breakdowns, $\overline{\pi}_U = t_j\tilde{q}_j(t_i, t_j)$; that is, the upstream firm profits from selling to the remaining retailer j are calculated by the anticipated second-stage equilibrium quantities and are therefore independent of the negotiated input price.

From the first order conditions of this Nash problem, we obtain the input price that solves this bilateral negotiation between U and firm i, as a function of the outcome of the negotiation between U and firm j. This is given by

$$t_i(t_j) = \frac{\alpha_i\left(2(1 + bt_j) - b\right)}{4}, \tag{31}$$

The Nash-in-Nash solution is obtained by solving the system of two first-order conditions of the two Nash problem. We get the equilibrium linear input prices

$$t_i^* = \frac{\alpha_i(2 - b)\left(2 + \alpha_j b\right)}{8 - 2\alpha_i\alpha_j b^2}. \tag{32}$$

We are interested in the effect that a change in the upstream supplier's bargaining power in one of the negotiations has on its equilibrium payoff. Replacing (32) in (28) and (29) we get the equilibrium profits for the upstream supplier:

$$\pi_U^* = \frac{8(2-b)}{2(2+b)\left(4-\alpha_i\alpha_j b^2\right)^2} \times \\ \left((\alpha_i+\alpha_j) - \alpha_i^2\alpha_j^2 b^2(2+b) + 4\alpha_i\alpha_j b\left(3-\alpha_i-\alpha_j\right) - 4\left(\alpha_i^2+\alpha_j^2\right)\right) \tag{33}$$

The effect of this profits due to a change in the upstream firm bargaining power is captured by the following expression:

$$\frac{d\pi_U^*}{d\alpha_i} = \frac{2(2+\alpha_j b)(2-b)(\alpha_i\alpha_j(1-\alpha_j)b^2 + 2\alpha_j(2-\alpha_i-\alpha_j)b + 4(1-\alpha_i))}{(2+b)(4-\alpha_i\alpha_j b^2)^3}. \tag{34}$$

We can now state the main result of this section:

Proposition 2. *In our linear vertical industry,*
i) in a symmetric model, $\left.\frac{d\pi_U^*}{d\alpha_i}\right|_{\alpha_i=\alpha_j} > 0$;
ii) in an asymmetric model, for any α_j, π_U^* *is concave in* α_i *and*

- *when* $b \in (0,1]$, $\frac{d\pi_U^*}{d\alpha_i} > 0$ *for any* $\alpha_i \in [0,1]$;
- *when* $b = 0$, $\frac{d\pi_U^*}{d\alpha_i} > 0$ *for any* $\alpha_i \in [0,1)$, *and* $\frac{d\pi_U^*}{d\alpha_i} = 0$ *for* $\alpha_i = 1$;
- *when* $b \in [-1,0)$, $\frac{d\pi_U^*}{d\alpha_i} = 0$ *at* α_i^*, *where*

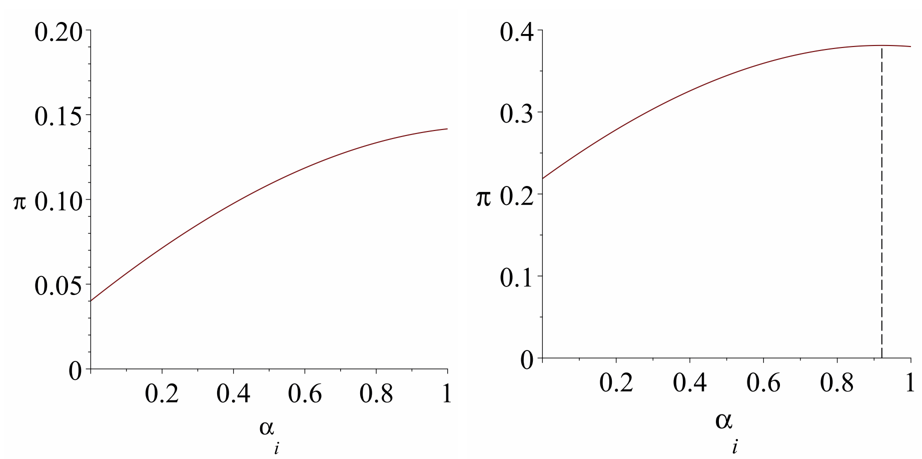
$$\alpha_i^* = \frac{2(2+\alpha_j(2-\alpha_j)b)}{4+\alpha_j b(2+\alpha_j b - b)}, \tag{35}$$

and $\alpha_j < \alpha_i^* < 1$.

The Proposition illustrates if and under what conditions a change in the upstream firm's bargaining power in one of the two negotiations leads to a variation of its equilibrium profits with an opposite sign. Results are shown to be different depending on the symmetric or asymmetric nature of the model.

In line with the general analysis of Section 2, in a symmetric model with multiple negotiations, an increase in the upstream supplier's bargaining power cannot lower his payoff. It is also worth noting that such result is independent of the nature of the downtsream goods. Things are different when the supplier has asymmetric bargaining powers towards the two downstream firms. Proposition 1 confirms that such asymmetry is a necessary yet not sufficient condition for our main result to hold. In particular, only under complementary goods (and if α_i is sufficiently large relative to α_j) the supplier could be hurt with a larger bargaining power. The role of the downtsream goods for the outcome of a Nash-in-Nash model has been pointed out by Horn and Wolinsky (1988) who show that a monopolistic supplier as a result of a merger in the upstream market obtains a larger (resp. lower) payoff when goods are substitutes (resp. complements) with respect to the case of two suppliers, each negotiating with one downstream firm. However a comparative static analysis like ours cannot be performed due to the restriction of a symmetric Nash bargaining approach. In this regard, we believe that our result is relevant even in light of the fact that, beyond Horn and Wolinsky (1998), bargaining powers' symmetry across multiple negotiations is a typical assumption in many relevant papers adopting the Nash-in-Nash approach (see, among others, Aghadadashli *et al.* (2016), Iozzi and Valletti (2014), Gaudin (2017), Symeonidis (2008; 2010)). Figure 1 provides a graphical illustration.

Figure 1: The upstream equilibrium payoff in the asymmetric model with substitute goods ($b = 0.8$, left panel) and complementary goods ($b = -0.8$, right panel), when $\alpha_j = \frac{1}{2}$.



Finally, in order to create a connection with the taxonomy presented in Section 2, we have that, at the Nash-in-Nash solution, whenever $\frac{\partial \pi_U^*}{\partial \alpha_i} < 0$, then:

$$\frac{\partial \pi_U^*}{\partial t_i} = \frac{2(1-\alpha_i)(2+\alpha_j b)}{(2+\alpha_j)\left(4-\alpha_i\alpha_j b^2\right)} > 0; \tag{36}$$

$$\frac{\partial \pi_U^*}{\partial t_j} = \frac{2(1-\alpha_j)(2+\alpha_i b)}{(2+\alpha_j)\left(4-\alpha_i\alpha_j b^2\right)} > 0; \tag{37}$$

$$\frac{dt_i^*}{dt_j} = \left.\frac{\alpha_i b}{2}\right|_{b\in(-1,0)} < 0; \tag{38}$$

$$\frac{dt_i^*}{d\alpha_i} = -\frac{2(-2+b)(2+\alpha_j b)}{\left(-4+\alpha_i\alpha_j b^2\right)^2} > 0, \tag{39}$$

which corresponds to the first point presented in Proposition 1, in the case of an asymmetric Nash-in-Nash model.

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Appendix

Proof of Proposition 1. We first look at the case of a single negotiation. Our main object of analysis is equation (5). By implicitly differentiating (3), we get:

$$\hat{a}_\alpha = -\frac{\Omega_{a\alpha}}{\Omega_{aa}}. \tag{A-1}$$

The numerator of (A-1) can be obtained differentiating (3) with respect to α, which gives:

$$\Omega_{a\alpha} = \hat{\pi}_a^S \hat{\pi}^A - \hat{\pi}^S \hat{\pi}_a^A. \tag{A-2}$$

Because of (4) and $\Omega_{aa} < 0$ we get that:

$$\text{sign}\,[\Omega_{a\alpha}] = \text{sign}\,[\hat{a}_\alpha] = \text{sign}\left[\hat{\pi}_a^S\right]. \tag{A-3}$$

Hence, when $\hat{\pi}_a^S \geq 0\,$(resp. ≤ 0), then $\hat{a}_\alpha \geq 0\,$(resp. ≤ 0) and the first part of the statement is proved. Next, we move to the case of two simultaneous negotiations and focus first on the asymmetric case. Our main object of analysis is equation (17).

Second-order conditions imply:

$$\Omega_{aa}^A = \alpha\,\pi^A\,\pi_{aa}^S + \pi_a^S\,\pi_a^A + (1-\alpha)(\pi^S - \pi^{SA})\,\pi_{aa}^A < 0 \tag{A-4}$$

$$\Omega_{bb}^B = \beta\,\pi^B\,\pi_{bb}^S + \pi_b^S\,\pi_b^B + (1-\beta)(\pi^S - \pi^{SB})\,\pi_{bb}^B < 0. \tag{A-5}$$

Implicitly differentiating the first order conditions in (11) and (12), we obtain:

$$\hat{a}_\alpha = -\frac{\Omega_{a\alpha}^A\,\Omega_{bb}^B - \Omega_{ab}^A\,\Omega_{b\alpha}^B}{\Omega_{aa}^A\,\Omega_{bb}^B - \Omega_{ab}^A\,\Omega_{ba}^B}; \tag{A-6}$$

$$\hat{b}_a = \frac{\Omega_{aa}^A\,\Omega_{b\alpha}^B - \Omega_{a\alpha}^A\,\Omega_{ba}^B}{\Omega_{a\alpha}^A\,\Omega_{bb}^B - \Omega_{ab}^A\,\Omega_{b\alpha}^B}. \tag{A-7}$$

Also, from the differentiation of (12), we obtain:

$$\Omega^B_{b\alpha} = 0, \tag{A-8}$$

so that we can re-write (A-6) and (A-7) as follows:

$$\hat{a}_\alpha = -\frac{\Omega^A_{a\alpha}\,\Omega^B_{bb}}{\Omega^A_{aa}\Omega^B_{bb} - \Omega^A_{ab}\Omega^B_{ba}}; \tag{A-9}$$

$$\hat{b}_{\hat{a}} = -\frac{\Omega^B_{ba}}{\Omega^B_{bb}}. \tag{A-10}$$

Notice that under Assumption 1, (A-3) holds in the multiple negotiation case as well. We now analyze all cases in which $\frac{d\hat{\pi}^S}{d\alpha}$ could be negative.

Case 1: $\pi^S_a > 0$ and $\pi^S_b > 0$. In this case π^S_α could be negative only if $\hat{b}_a < 0$, which, from (A-10), can only occur if $\Omega^B_{ba} < 0$.

Case 2: $\pi^S_a < 0$ and $\pi^S_b < 0$. In this case π^S_α could be negative only if $\hat{b}_a > 0$, which, from (A-10), can only occur if $\Omega^B_{ba} > 0$.

Case 3: $\pi^S_a > 0$ and $\pi^S_b < 0$ or $\pi^S_a <$ and $\pi^S_b > 0$. In this case π^S_α could be negative only if $\hat{b}_a > 0$which, from (A-10), can only occur if $\Omega^B_{ba} > 0$.

Let's focus now on the symmetric case $\pi^S_a = \pi^S_b$.

Case 1: $\pi^S_a > 0$. In this case $\pi^S_\alpha < 0$ only if $\dot{b}_a < -1$. This implies that $\Omega^B_{ba} < \Omega^B_{bb} < 0$, which contradicts Assumption 1.

Case 2: $\pi^S_a < 0$. In this case $\pi^S_\alpha < 0$ only if $\dot{b}_a > 1$, which cannot hold in any Nash-in-Nash solution. ■

Proof of Proposition 2. Trivial and therefore omitted. ■

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