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A fractal analysis method to characterise rock joint morphology

G. Guida^{1*}, F. Casini², G.M.B. Viggiani²

¹⁾ Politecnico di Milano

²⁾ Università degli Studi di Roma Tor Vergata

*) giulia.guida@polimi.it

Abstract. Rock joints morphological characteristics strongly influence the shear strength dilatancy response of the discontinuity. Morphological features of rock joints are commonly identified with a roughness descriptor along standard 100 mm-length profile detected by instrumentation such as profilometers. This work extends a method proposed for fractal analysis of particle contours to describe rock joint profiles in terms of quantitative descriptors of their roughness. It is well-established that natural surfaces have a fractal nature, self similar over a wide range of scales. This implies that the measured length of their outline is a function of the measurement scale: the smaller the measurement scale, the longer the profile length. Based on the interpretation of the fractal analysis of rock joint profiles, relating the length of the profile to the measurement scale, descriptors identifying the roughness and its characteristic scale are proposed. The method is first applied to some artificial profiles, and later to real rock joint profiles.

1. Introduction

The morphological features of natural rock discontinuities strongly influence the hydro-mechanical behaviour of a jointed rock mass. The former is strongly affected by the discontinuity condition, in terms of opening, roughness, filling and alteration [1]. This work focuses the attention on the characterisation of joint roughness.

Roughness generally describes the morphology of a joint surface by means of two distinct components [2, 3]: a small scale component named "unevenness" related to the textural surface irregularities and a large scale component named "waviness" related to the surface curvature. The first is thought to affect mostly the shear strength of the discontinuity in accordance to the following criterion [4, 5, 6]:

$$\tau = \sigma \tan(\phi_r + i_{eff}), \quad (1)$$

where τ and σ are the peak shear strength and the normal stress respectively, ϕ_r is the residual friction angle, while i_{eff} is a dilatancy angle related to the discontinuity features, equal to $i_{eff} = JRC \log(JCS/\sigma)$ for $\sigma \leq JCS$ and $i_{eff} = 0$ for $\sigma > JCS$, at which point asperities are "flatten out" and $\phi = \phi_r$. The failure surface in equation (1) is non linear, as its slope depends on σ . The larger the value of JRC (Joint Roughness Coefficient), that ranges between 0 and 20, the more pronounced is the curvature of the failure envelope for $\sigma < JCS$ (Joint Compression Strength). The JRC gives a qualitative indication of the roughness of a rock joint profile. It is generally estimated by a visual comparison between the rock joint profile, obtained experimentally from *e.g.* profilometers, and the 10 Barton's paradigmatic profiles related to JRC



values ranging from 0 to 20 [5]. However, this procedure is affected by an element of subjectivity and provides at best an indication of the profile roughness. In recent years, several quantitative methods are proposed for the determination of roughness [7, 8]. The fractal dimension, D_f , ranging between 1 for a perfectly smooth profile and a maximum value of less than 2 for an extremely rough profile, is considered one of the most promising [9, 10] and several empirical correlations exist to estimate JRC from D_f [8].

This paper proposes a new method to characterise the morphology of the joint profiles, still based in their fractal analysis but also considering the characteristic scale of the asperities.

2. Fractal analysis

Fractal analysis stems from the observation that the measured length of a profile, p , is a function of the measurement scale, b , and that the smaller the measurement scale, the longer the measured length [11]. The algorithm adopted, coded in Matlab, was initially developed by Guida *et al.* [12] to describe the morphology of closed particle contours, and is adapted here to open profiles. Given the coordinates of a profile, the algorithm computes its length by adopting segments of fixed size, b , as units. As an example, figure 1(a) considers Barton [5] profile #10, showing how decreasing the length of the measurement scale, the profile is fitted with greater detail. Even if providing the least accurate estimate of the actual profile length, large segment units, $b \sim 15$ mm, still carry relevant information about its curvature.

The logarithm of the normalised profile length, p/D , is plotted as a function of the logarithm of the corresponding normalised stick length, b/D . The normalising quantity $D = 100$ mm is a characteristic length, assumed equal to the profile width. Each point of the plot in figure 1(b) is related to a different stick length b adopted for the fractal analysis, in the range: $b/D = 0.001 - 1$. Starting from $b/D = 1$ and moving to the left, as b/D decreases, p/D increases until the computed length saturates becoming horizontal, when the segment unit reaches the resolution of the profile, in this case $b_{min}/D = 0.003$. It is evident that the greater the resolution of the profile, the more information can be extracted from the analysis. If the plot of $\log(p/D)$ vs. $\log(b/D)$ is linear with slope s , the profile is self-similar [13]:

$$p = b^{-s}, \quad (2)$$

where $D_f = 1 - s$ is the fractal dimension. equation (2) implies that the length of any truly self-similar profile diverges to infinity as the measurements scale tends to zero. Natural surfaces

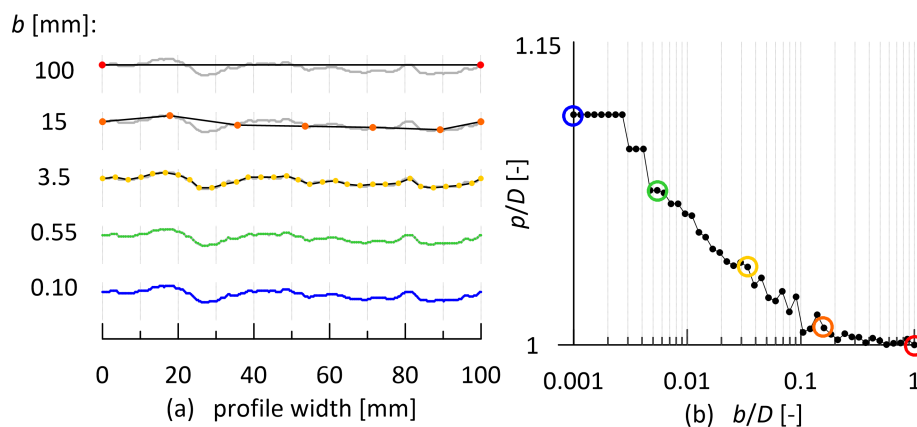


Figure 1. (a) Computation of the length of Barton profile #10 by different segment lengths. (b) Normalised profile length p/D as a function of normalised stick length b/D in a log-log.

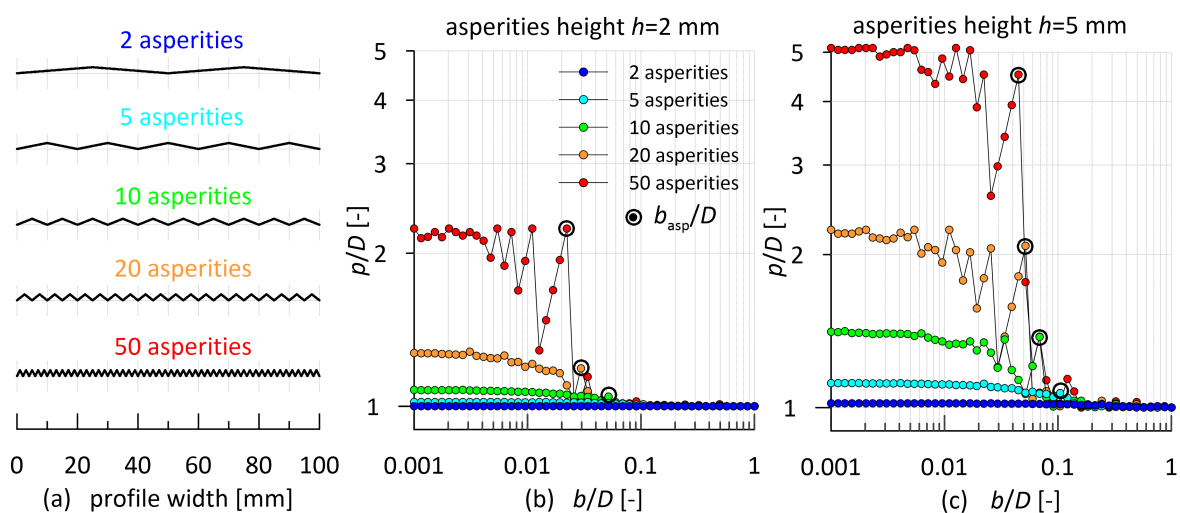
Table 1. Normalized quantities of profiles length and size of the asperities for the artificial saw-tooth profiles.

# asperities	$h = 2 \text{ mm}$		$h = 5 \text{ mm}$	
	b_{asp}/D [-]	p/D [-]	b_{asp}/D [-]	p/D [-]
2	0.25	1.00	0.25	1.02
5	0.10	1.02	0.11	1.12
10	0.05	1.08	0.07	1.41
20	0.03	1.28	0.06	2.24
50	0.02	2.24	0.05	5.10

may have a multi-scale nature [14] self-similar over a broad range of scales. On the other hand, figure 1(b) shows that a plot of $\log(p/D)$ vs. $\log(b/D)$ for Barton template profile #10 can be approximated by two straight lines. The first straight line, characterised by a relatively small slope and small fractal dimension fits the data at larger scales, $b/D \approx 0.1 - 1.0$, whereas the second straight line, at $b/D \approx b_{min} - 0.1$, is characterised by a larger slope and fractal dimension.

3. Artificial Joint Profiles

Figure 2 shows the results of the fractal analysis for two set of artificial joint profiles obtained superimposing to a smooth line 100mm-wide line an artificial saw-tooth profile consisting of (a) 2, 5, 10, 20, 50 asperities, with height of (b) 2 and (c) 5 mm. The computed normalised length of the profiles, p/D , is equal to 1 ($p = D$) until the measurement length reaches the size of the asperities b_{asp}/D , reported in table 1 and in figure 2(b-c) with a symbol; here the plot increases abruptly attaining the theoretical value of the normalised length reported also in table 1. Despite some oscillations, the fractal analysis is able to identify the size of the asperities, although no fractal subset of linear portion emerges on the plots because the profile is not self similar.

**Figure 2.** (a) Artificial saw-tooth profiles. Results of the fractal analysis of the saw-tooth profiles considering an asperities height of (b) $h = 2$ mm and (c) $h = 5$ mm.

To investigate the fractal analysis results for fractal profiles, modules of an Euclidean approximation of the fractal of Koch [15] artificially built in series are analysed. The construction of a fractal of Koch consist on dividing a linear segment of length L into three equally spaced parts, and by substituting the middle part by other two sub-segments of the same length creating an equilateral peak as shown in figure 3(a), with characteristic size $L_1 = L/3$. Repeating the same operations on the sub-segments of length L_1 obtained by the first-order of approximation, a second, third, fourth and fifth orders of the fractal of Koch approximation is gradually created, obtaining for each order a smaller size of the asperities, $L_n = L/3^n$. The fractal of Koch is characterised by repeating this procedure infinite times.

Figure 3(c-d) shows the results of the fractal analysis conducted on profiles 100 mm wide profiles in which an approximation of the fractal of Koch is artificially constructed. In particular, figure 3(c) shows the trend of the profile length p/D over the segment unit adopted b/D , on a profile composed by 5 modules of the fractal of Koch, each one constructed over a segment equal to $L_0 = D/5$, where $D = 100$ mm, at increasing order of Euclidean approximation n from 1 to 5 (see figure 3a). The logarithm of the profile length starts to increase linearly when $b/D < L_0$, and then it becomes constant at a value of $b/D = L_n/D$ where L_n represents the minimum size of the asperities of the fractal of Koch of order n . The slope of all plots is related to the fractal dimension of Koch, $D_f = 1.262$. Thus, the fractal analysis of profiles quantitatively recognises both the morphology irregularity by the fractal dimension, the characteristic scale of the asperities b_{asp} , and the minimum scale available b_{min} . Figure 3(c) reports the fractal analysis results conducted on the profiles showed in figure 3(b), characterised by a construction of a fractal of Koch at the 4th order over segments $L_0 = D/5, D/10, D/20, D/50$, where 5, 10, 20, 50 are the modules in series considered. The trends are characterised by the same slope, equal to $D_f - 1 = 0.262$, that start increasing for segment units smaller than the characteristic scale of the asperity L_0 , different for each of the cases reported (see table 2) and saturate at a stick unit $b_{min}/D = L_4/D$. The oscillations of the trend presented are related to the discrete nature of the profiles that are defined by points.

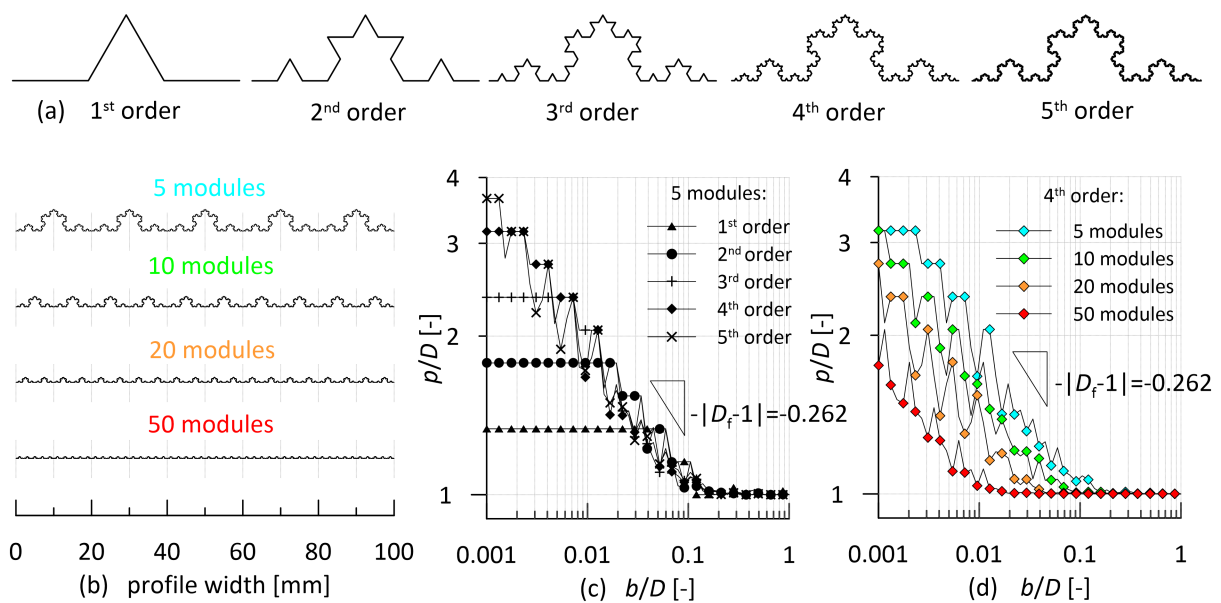


Figure 3. (a) The construction of a fractal of Koch module by Euclidean different order of approximation: from the 1st order to the 5th. (b-d) Fractal analysis of fractal profiles obtained placing in series several modules of Euclidean approximation of the fractal of Koch.

Table 2. Characteristic size for a fractal of Koch profile.

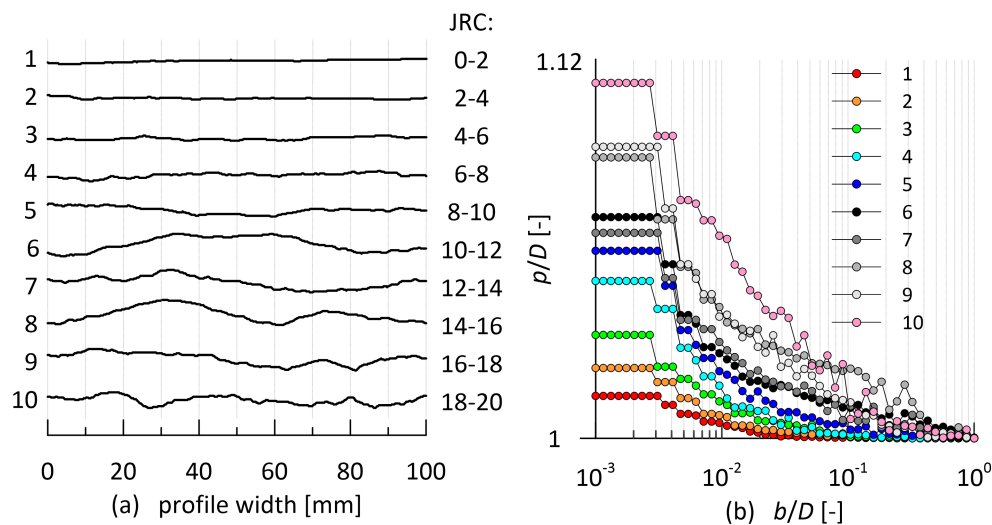
modules	L_0/D	1 st order L_1/D	2 nd order L_2/D	3 rd order L_3/D	4 th order L_4/D	5 th order L_5/D
5	0.2	6.7×10^{-2}	2.2×10^{-2}	7.4×10^{-3}	2.5×10^{-3}	8.2×10^{-4}
10	0.1	3.3×10^{-2}	1.1×10^{-2}	3.7×10^{-3}	1.2×10^{-3}	4.1×10^{-4}
20	0.05	1.7×10^{-2}	5.6×10^{-3}	1.9×10^{-3}	6.2×10^{-4}	2.1×10^{-4}
50	0.02	6.7×10^{-3}	2.2×10^{-3}	7.4×10^{-4}	2.5×10^{-4}	8.2×10^{-5}

4. Barton's Templates

Figure 4 shows the results of the fractal analysis conducted on the template profiles of Barton [5] reported in figure 4a, commonly adopted for the characterisation of JRC . The results of the fractal analysis indicate an increasing complexity of the profile features (figure 4b), with the final profile length and the slope of both linear portions of the plot increasing with profile number and JRC . Moreover, rougher profiles start increasing at greater length scales. This means that rougher profile are also characterised by a greater size of the asperities.

Figure 5(a) shows the definition of several morphological descriptors based on the interpretation of the fractal analysis of Barton profiles, and figure 5(b-c) report their trend as a function of JRC . All the quantitative morphology features that can be deduced interpreting the fractal analysis of the profile #10 are graphically illustrated in figure 5(a), and described below:

- m is the absolute value of the slope of the first linear subset and it measures the macro asperities, it is equal to 0 for a flat profiles, and assumes greater values as much the profiles deviate from its flat trend. For Barton template profiles it is generally close to zero, $m < 0.1$ (see figure 5b), but for closed curves, such as *e.g.* the outline of a particle, m can assume larger values [16, 12].
- μ is the absolute value of the slope of the second linear subset and it measures the

**Figure 4.** (a) Barton profiles and their fractal analysis.

roughness of the profiles, it is equal to 0 for smooth profiles, increasingly large than zero for increasingly complex textural features. Barton profiles show slightly increasing values of μ until profile #6, then they stabilised around $\mu \sim 0.025$.

- α is the absolute value of the slope of the linear regression among $b/D = 0.03 - 0.5$ and it is related to the overall fractal dimension of the curve as $D_f = \alpha + 1$.
- b_m/D indicates the characteristic scale dividing the two linear subsets and identifying the characteristic scale of the asperities. Most of the Barton profiles are characterised by a b_m/D ranging between 0.01 and 0.03 except for the last one, reported also in figure 5(a) characterised by $b_m/D \sim 0.1$.
- $b_{min}/D = 0.001$ indicates the resolution scale of profile at which the trend saturates, that is linked to the resolution of the image [5] from which the profiles are extracted.
- M is defined as the normalised increase of the profile length between the scale $b/D = b_{min}/D - 1$ and gives an overall indication of the profile irregularities.

The roughness of a profile can be characterised by an index of complexity related to the slopes of the different linear subsets of the plot of $\log(p/D)$ vs. $\log(b/D)$, and its overall slope, *e.g.* m , μ , and α and by a range of scales characterising those asperities. The values of the slopes are generally very close to 0 [17]. The fractal of Koch, which appears to have a much more irregular outline than Barton's template profiles, is characterised by a slope of 0.262.

The only two quantities showing a clear monotonic trend with *JRC* are α and M (see figure 5b-c). Descriptors α and M do not distinguish between the micro and macro scale of asperities. Further research must be done to define properly the relation between the hydro-mechanical behaviour of the rock joints with the introduced descriptors of the profile μ , m and the characteristic scale of asperities b_m/D .

5. Conclusions

Fractal analysis is a simple quantitative way to describe a profile morphology over a range of experimentally accessible scales. The fractal analysis was applied first on artificial contours to show its ability to detect the scale of the asperities and the fractal dimension. Subsequently, the analysis was applied to the standard Barton profiles commonly used to characterised the Joint Roughness Coefficient (*JRC*). The results indicate that the normalised computed length

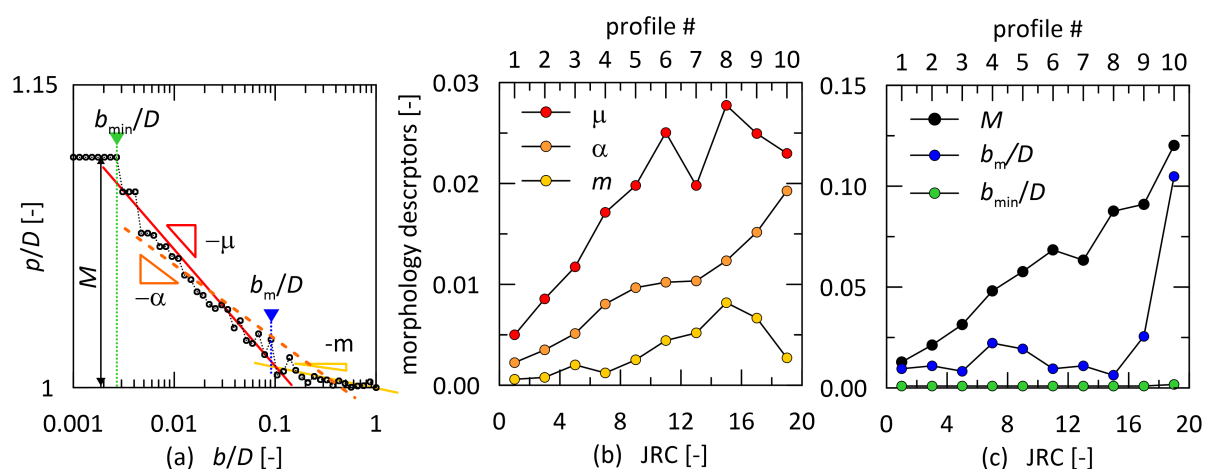


Figure 5. (a) Fractal analysis interpretation and definition of morphology descriptors. (b-c) Trends of the morphology descriptors as a function of the parameter *JRC*.

of Barton profiles increases as the normalised measuring length decreases. Two linear subsets can be recognized characterising the macro and the micro scale features of the profile. The measuring length dividing the two linear subsets gives information about the characteristic scale of the asperities. Several quantities describing the fractal analysis plot are considered for the Barton profiles under examination, in order to establish which one may be the most effective for the characterisation of the Joint Roughness Coefficient and the consequent characterisation of the strength-dilatancy response of the rock discontinuity. A further investigation on the relative roles of the micro and macro scale is required in order to link the mechanical behaviour of the jointed rock mass fractal descriptors.

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