

A dynamic Bayesian network approach for multi-component fragility of aging bridges

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ABSTRACT: Assessing life-cycle seismic safety of aging reinforced concrete bridges is a challenging engineering task. Deterioration phenomena reduce structural capacity, exacerbating poor design choices that are typical of old bridges, while also being characterized by major uncertainties. Management of engineering systems in highly uncertain environments can be efficiently addressed through Markov decision processes, which rely on dynamic Bayesian networks to model the deteriorating system's life-cycle. However, there is still a gap in developing virtual environments that can seamlessly fit in such advanced algorithmic decision-making frameworks, especially under life-cycle seismic behavior considerations. In this study, we develop a dynamic Bayesian network capable of incorporating disparate uncertainties related to chloride-induced corrosion and seismic action, aiming at providing fragility curves over the bridge service life. The framework is applied to a prototype bridge encapsulating key risk-prone features. Using a multi-component approach, the developed network provides valuable insights into the fragility evaluation of both the system and individual components. Markovian transitions among component deterioration states are computed by combining corrosion initiation and propagation models with non-stationary Gamma processes. Subsequently, state-dependent fragilities are obtained through probabilistic seismic assessment based on non-linear dynamic analyses and multinomial logistic regression. Results show that the approach sheds light on the risk interplay mechanisms between components and the system, and on how different corrosion scenarios affect the system fragility. Discussion is finally provided on how these risk considerations can be interpreted for decision-making, allowing for better repair and retrofit strategies.

1 INTRODUCTION

Bridges, vital constituents of transport systems, must resist continuous and sudden stresses over time. Their long-term safety and resilience assessment is a challenging task, especially in the case of existing reinforced concrete (RC) bridges, owing to varied uncertainties in materials, deterioration, and design. The copresence of steel rebar corrosion and seismic actions, in particular, can pose significant risks, triggering catastrophic collapses and major network disruptions with high socio-economic impacts. Therefore, in recent years, focus on life extension and management of aging structures has intensified.

Steel corrosion in reinforced concrete occurs in two, largely discrete, phases: (i) initiation, where aggressive agents breach the concrete cover, and (ii) propagation, where steel rebar cross-section decreases due to material mass loss (Bertolini et al. 2004; Tuutti, 1982). Identifying chlorides as one of the most severe aggressive agents for RC structures, studies by Stewart & Rosowsky (1998) and Vu & Stewart (2000) proposed analytical models to be adopted in probabilistic assessment, taking into account both key environmental and structural factors, such as chlorides exposure, concrete cover depth and concrete water cement ratio. Eq. (1) describes initiation based on Fick's law, whereas Eq. (2) models propagation based on Faraday's law.

$$C(x, t) = \chi C_s \left[1 - \operatorname{erf} \left(\frac{x}{2\sqrt{D_c t}} \right) \right] \quad (1)$$

$$\lambda(t) = 0.37 * \frac{(1 - \frac{w}{c})^{-1.64}}{x} (t - T_{\text{corr}})^{0.29} \quad (2)$$

where x is the concrete cover depth; C_s is the equilibrium chloride concentration at the concrete surface; erf is the Gaussian error function; χ is a factor to account for model uncertainties; t the time in years; w/c is the water cement ratio; and T_{corr} is the time for corrosion initiation in the structure's life-cycle. Corrosion initiation is postulated when the chloride content at the rebar depth $C(x, t)$ reaches a critical value C_{cr} , while $\lambda(t)$ is the uniform corrosion penetration.

Apart from chronic deterioration, extreme events like earthquakes critically impact bridge structural reliability. Knowledge from past earthquakes indicates several types of structural damage, such as deck unseating from bearing failures, column damages, and pounding between deck components (Priestley et al. 1996; Ramirez et al. 2000). Accordingly, several studies have proposed seismic risk assessment frameworks for aging RC bridges (Biondini & Frangopol 2016; Ghosh & Padgett 2010; Nielson & DesRoches 2006; Shekhar & Ghosh 2021). These probabilistic approaches predict time-based risks by merging: (i) probabilistic deterioration assessment, using Eqs. (1)-(2); and (ii) seismic fragility evaluation using nonlinear finite element analysis (NLFEA). This is accomplished initially by predicting the deterioration condition of the structure for designated time-points within the life-cycle. Subsequently, the mechanical properties of the materials are adjusted to conduct fragility analysis under the deteriorated state, aimed at quantifying the seismic risk and its rise at these specific time-points.

However, infrastructure decision-making can be more precise if joint time- and state-based models are considered. This allows for better selection of repair and retrofit actions, but also for updates of uncertainties and fragility functions based on data from monitoring and inspections. Recent advances in algorithmic decision-making reinforces the need for such models, through assimilation of probabilistic models, Bayesian updating, and stochastic optimal control algorithms (Andriotis & Papakonstantinou 2019; 2021; Andriotis et al. 2021). This approach uses partially observable Markov decision processes for inspection and maintenance planning, leveraging upon state-based formulations and dynamic Bayesian networks (DBN). Towards this, in Molaioni et al. (2023), we recently introduced a DBN approach for evaluating the seismic safety of aging bridges throughout their lifespan. This model combines non-stationary transition matrices for corrosion conditions of structural components and consistent, state-dependent seismic fragility functions for both individual components and the entire system. This provides a detailed view of how corrosion and seismic actions affect bridge safety over time. Component and system fragilities, $P_{\text{SDS},t}$, over the life cycle, are computed as:

$$P_{\text{SDS},t} = P(\text{SDS}_t \geq s | \text{IM}_{0:t}, \text{CDS}_{0:t}, \text{SDS}_{0:t-1}) \quad (3)$$

where s represents a damage threshold identifying the seismic damage state (SDS); IM is the intensity measure of the seismic action; CDS represents the corrosion damage state; and t is time, considering the structural life-cycle discretized in yearly time-slices.

In this paper, the effectiveness of the proposed approach is further evaluated by applying it to a case study with focus on demonstrating: (i) the method’s potential to consider multiple deterioration scenarios within the life-cycle; (ii) the ability to capture how corrosion impacts the seismic safety of the case-study bridge and its components overtime; (iii) its utility in pinpointing vulnerable components and guiding retrofit strategies; (iv) its adaptability potential in incorporating real-time data and its compatibility with Markov decision processes-based decision-making.

2 METHODOLOGY

The DBN introduced by Molaioni et al. (2023) and adopted in this study to quantify aging RC bridges’ seismic safety over their life cycle is shown in Figure 1. Variables of the model (blue circles in Figure 1) include: (i) the IM vector of the seismic action, assumed as the peak ground acceleration (PGA); (ii) the CDS vector of structural components, representing the corrosion intensity in a discrete space; (iii) the SDS vector of structural components, encapsulating structural damage of components based on thresholds of Engineering Demand Parameters (EDPs). Transitions among variables (red arrows in Figure 1) represent conditional probabilities among states. These include: (i) the Markovian transition for CDSs over time; and (ii) the state-dependent fragility functions for seismic damage, with the possibility of considering the carry-over effect of damage without repairs (omitted here for simplicity). This way, the adopted approach allows us to break down the problem into smaller, easier to compute segments, by independently analyzing each probabilistic dependency. The aging RC bridge is modeled as a multi-component series system, focusing on the key components, such as columns (COL), High Type Fixed Bearings (HTFB), High Type Expansion Bearings (HTEB), and Low Type Fixed Bearings (LTFB) (Mander et al. 1996; Shekhar & Ghosh 2021). SDSs for these components are classified from “Slight” to “Complete”, based on FEMA (2003) definitions and quantified by thresholds from Nielson & DesRoches (2006), as shown in Table 1. CDSs range from “Sound” to “Critical”, describing the corrosion intensity and its impact on structural components, as shown in Table 2. Considered corrosion intensity parameters are the average mass loss for steel rebars/bolts, M_{loss} [%], steel plate thickness reduction, PTR [mm], and bearing coefficient of friction, k_{corr} [-].

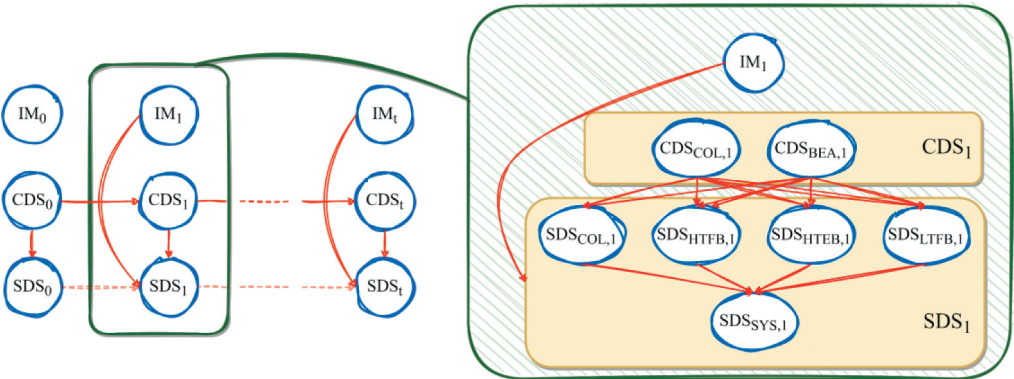


Figure 1. DBN for life-cycle seismic fragility of deteriorating bridges. Left: Model including nodes and transitions. Right: Interactions of components and system states within a single time step.

3 APPLICATION TO A CASE STUDY BRIDGE

The methodology is showcased for a 4-span bridge, typical of many pre-70s seismic-prone and deterioration-susceptible RC bridges in existing transportation networks. The bridge is characterized by poorly detailed RC columns and steel bearings, making it a relevant case study for seismic risk. For detailed information about the case study, its visual representation, and the modeling assumptions for conducting NLFEA, the interested reader is referred to Molaioni et al. (2023).

Table 1. Definition of probabilistic seismic damage states.

Component	EDP	Slight		Moderate		Extensive		Complete	
		M	D	M	D	M	D	M	D
COL	Curvature ductility [-]	1.29	0.59	2.1	0.51	3.52	0.64	5.24	0.65
BEA - Expansion	Displacement [mm]	37.4	0.6	104.2	0.55	136.1	0.59	187	0.65
BEA - Fixed	Displacement [mm]	6	0.25	20	0.25	40	0.47	187	0.65

* M = median of the lognormal distribution, D = Dispersion of the lognormal distribution.

Table 2. Definition of corrosion deterioration states.

CDS		0: Sound	1: Initial	2: Progressive	3: Critical
Component	Parameter	LB-UB	LB-UB	LB-UB	LB-UB
COL	M _{loss} [%]	0-0	0-15	15-30	30-45
BEA	M _{loss} [%]	0-0	0-15	15-30	30-45
BEA	PTR [mm]	0-0	0-3.5	3.5-5.1	5.1-6.5
BEA	k _{corr} [-]	0-0	0-0.35	0.35-0.64	0.64-0.92

* M_{loss} = rebar/bolt mass loss [%], PTR = plate thickness reduction [mm], k_{corr} = additive coefficient of friction for expansion bearings [-].

3.1 Non-stationary transition matrix for corrosion

Non-stationary transitions among CDSs are derived using a probabilistic analysis of the corrosion intensity, with respect to rebars/bolts mass loss, M_{loss} [%], over the structure's lifetime, applying Eqs. (1)-(2). These transitions, for initiation and propagation phases, respectively, calculate the conditional probability of moving from CDS=*i* at a given time-slice, *t*, to CDS=*j* at the next one, *t+1*. Monte Carlo simulation is conducted to propagate uncertainties in environmental conditions, material properties and geometry, thus, eventually to evaluate yearly transitions, as in Molaioni et al. (2023):

$$P(\text{CDS}_{t+1} = j | \text{CDS}_t = i) = \frac{\sum_{\text{samples}} (\text{CDS}_{t+1} = j \cap \text{CDS}_t = i)}{\sum_{\text{samples}} (\text{CDS}_t = i)} \quad (4)$$

Figure 2 illustrates the resulting non-stationary transition matrices for chloride corrosion under "splash" environmental conditions for the case study components. These matrices, showcasing Markovian probabilities of transitioning from one CDS to another over time, are upper-triangular due to the irreversible nature of damage, as long as no repair interventions are applied. In determining these, crucial is the time of corrosion initiation T_{corr}, calculated using Eq. (1). Among samples, 97.1% experienced corrosion initiation within 100 years, with a mean T_{corr} of 13.7 years and a standard deviation of 20.7 years. Assuming T_{corr} as a lognormally distributed random variable in the propagation model (Eq. (2)) significantly impacts transition probabilities, particularly from state 0 to 0 and from state 0 to 1. Sharp inflection points for these transitions are noted, as T_{corr} statistics suggest corrosion onset is more likely in the early years of the life-cycle.

3.2 State-dependent fragility curves

State-dependent and time-invariant fragility functions for both structural components and system have been fitted, corresponding to capacity limit state EDP thresholds, as shown in Table 1. Demand parameters have been obtained from dynamic NLFEA, developing three-dimensional finite element models in OpenSees (McKenna, 2011), incorporating non-linear

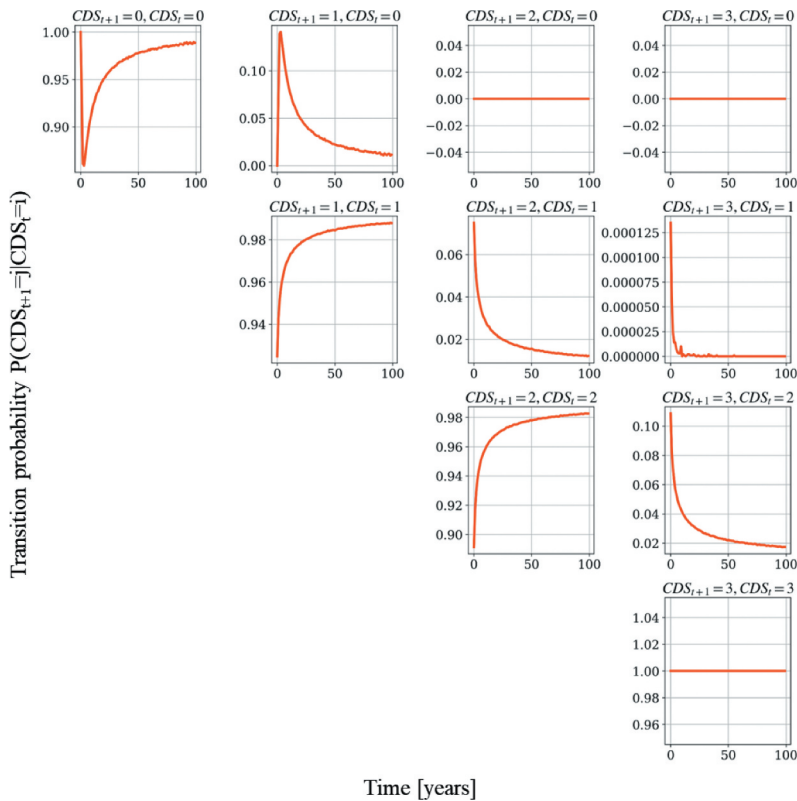


Figure 2. Non-stationary transition matrix for CDSs of components under “splash” conditions.

behavior of key components (COL, HTFB, HTEB, LTFB). A synthetic ground motion suite of 400 records has been applied on 50 statistically varied bridge models, thus propagating uncertainties related to the seismic action, the materials and geometrical properties of the bridge, as well as the corrosion deterioration of components. This way 20'000 EDPs were obtained and then compared with 50 capacity data points sampled from the distributions reported in Table 1, thus eventually creating a set of 1 million SDS labels for each component. A multinomial logistic regression model is trained on this dataset to predict the probability of exceeding a specific SDS as a function of the IM, via the softmax function as in Andriotis & Papakonstantinou, (2018).

For the sake of brevity, in this paper, results related to the “Complete” SDS and corrosion scenarios 00 (As-built), 03 (“Sound” COL, “Critical” corrosion in BEA), 30 (“Critical” corrosion in COL, “Sound” BEA), 33 (“Critical” corrosion in both COL and BEA) are presented. State-dependent fragility curves, shown in Figure 3, reveal that among bridge components, HTEB and COL exhibit the highest fragility across several SDSs and corrosion scenarios, namely, the combination of CDSs for COL and BEA components. Furthermore, a deterioration-driven increase in column fragility is observed due to their own deterioration, while this fragility slightly drops with the onset of corrosion in BEA. Conversely, HTEBs are not significantly influenced by the corrosion of COL alone and do only mildly so due to their own corrosion. This diverse impact of corrosion on the fragility of components leads to an inversion in their hierarchy of strengths. This is evident when comparing the state-dependent curves of scenarios ‘00’ and ‘03’, where the HTEB (dash-dot line) emerges as the most fragile component, with those of scenarios ‘30’ and ‘33’, where COL (thin continuous line) exhibits greater fragility. This highlights the importance of accounting for the multitude of deterioration scenarios that could arise throughout the lifespan of the structure. This understanding is

essential to grasp the potential evolution of the structural behavior, which, in turn, is fundamental in implementing effective enhancements in the bridge’s seismic safety through precise retrofit interventions based on the deterioration of components.

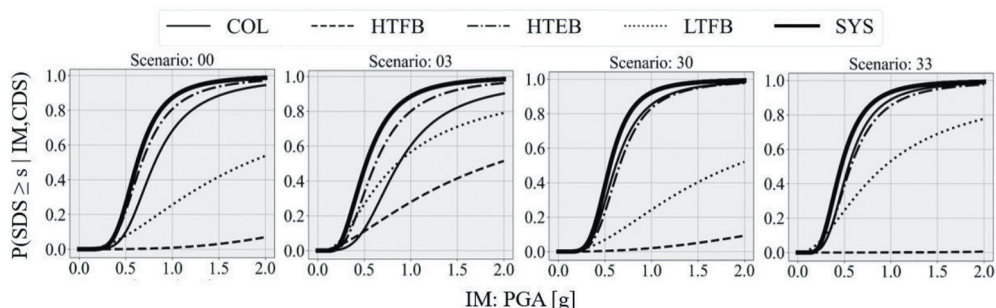


Figure 3. State-dependent fragility curves for both components and system. Corrosion scenarios ‘00’, ‘03’, ‘30’, ‘33’.

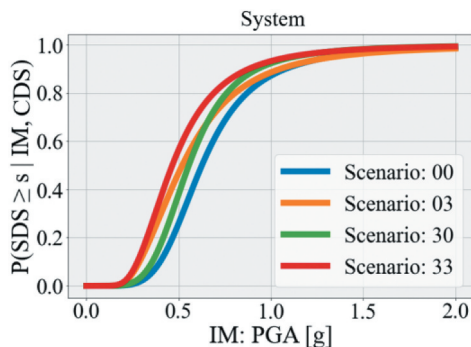


Figure 4. System’s state-dependent fragility curves. Corrosion scenarios ‘00’, ‘03’, ‘30’, ‘33’.

Focusing on the impact of corrosion at the system level, Figure 4 illustrates the system fragilities for these four scenarios. The fragility curves reveal that maximum fragility is consistently observed when corrosion is widespread across the structure (Scenario 33). Additionally, component deteriorations translate into an increase in fragility, which varies according to the deterioration scenario: the system is affected more by corrosion in bearings for $PGA < 0.6$, and more by the corrosion of columns for $PGA > 0.6$, as shown in Figure 4 (green and orange curve, respectively). These findings underscore the distinct role each component plays in influencing the system’s fragility, reinforcing the utility of the adopted method in retrofit planning.

3.3 Life-cycle fragility

The longitudinal fragility of the bridge is determined by marginalizing out the corrosion deterioration state variables, CDSs, for all different components of the system (Figure 4 only displays scenarios 00, 30, 03, 33):

$$P_r(SDS \geq s | IM) = \sum_{CDS_{COL}} \sum_{CDS_{BEA}} P(SDS \geq s | IM, CDS_{COL}, CDS_{BEA}) P_r(CDS_{COL}, CDS_{BEA}) \quad (5)$$

where $P_t(\text{CDS}_{\text{COL}}, \text{CDS}_{\text{BEA}})$ represents the probability of having a given corrosion scenario (i.e., a combination of CDS for column and bearing components). This can be evaluated by considering the Markovian transitions among CDS and the independence of CDS_{COL} and CDS_{BEA} as:

$$\begin{aligned}
 & P_t(\text{CDS}_{\text{COL}}, \text{CDS}_{\text{BEA}}) \\
 &= [P_0(\text{CDS}_{\text{COL}}) \prod_{\tau=0}^{t-1} P(\text{CDS}_{\text{COL},\tau+1} | \text{CDS}_{\text{COL},\tau})] [P_0(\text{CDS}_{\text{BEA}}) \prod_{\tau=0}^{t-1} P(\text{CDS}_{\text{BEA},\tau+1} | \text{CDS}_{\text{BEA},\tau})]
 \end{aligned}
 \tag{6}$$

To illustrate the evolution of longitudinal fragilities, computations are performed at three time-points throughout the structural lifespan: at the as-built condition (0 years), and subsequently at the 25-, and 100-year time-points. The curves reported in Figure 5, delineating the probability of exceeding specific seismic damage states at specific years and IMs, highlight the significant impact of “splash” environmental conditions on the structural integrity over time. A salient observation is the escalation of fragility curves, notable already at year 25 since construction, a trend that reveals the inadequate component detailing and susceptibility to corrosion. Also, the influence of corrosion manifests differently across the four damage thresholds. In the case of the “Slight” SDS, the increase in fragility over time is relatively modest, in contrast to other damage states, where this increase is more significant. This is linked to the inherent seismic vulnerability of the bridge by design, predisposing the structure to readily exceed the ‘Slight’ threshold. On the other hand, the progression of corrosion evokes higher non-linear demands, thereby significantly escalating the probability of exceeding more severe damage states, such as ‘Moderate’ and ‘Extensive’. Furthermore, for all SDSs, an upward trajectory in the bridge fragility over time is evident, reflecting a growing risk of encountering more advanced damage with time. These findings demonstrate the capacity of proposed DBN to propagate corrosion deterioration effects at the component level to the global seismic system risk level over the structural lifespan.

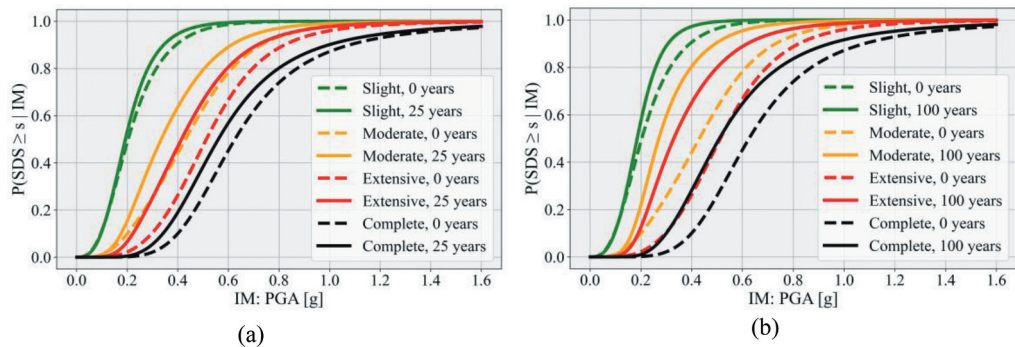


Figure 5. System System fragilities over the life-cycle under “Splash” conditions. (a) 25 years; (b) 100 years.

4 DISCUSSION

The results highlight several advantages of the proposed method. First, its potential to accommodate different deterioration scenarios over the life-cycle of the structure, allows us to define state-dependent fragility functions for possible combinations of component CDSs. This is particularly effective when applying the methodology to real-world bridge structures, which may present non-uniform deterioration distributions across components. For instance, deterioration

may concentrate more on some components than on others, either due to highly differential deterioration phenomena or partial repairs and retrofits applied at different times.

Moreover, the proposed methodology identifies the components' susceptibility to corrosion phenomena, but also how their effects act mutually on the fragilities of the components of the system. As shown by the results in Figure 3, non-uniform deterioration scenarios affect the safety of the structure in several ways. Also, the importance of modeling the bridge as a multi-component system is shown to be key for several insights. Individual component vulnerability can be evaluated through the DBN allowing us to identify the ones most at risk, such as the HTEBs and COLs for this case study. The change in hierarchy of strengths among components can thereby be tracked. This level of detail in the representation of the structural behavior can guide targeted maintenance and reinforcement interventions for the structure.

Finally, the developed DBN can be seamlessly integrated with decision-making frameworks that also involve field observations and actions. The structure of the DBN does not only allow the full description of seismic safety over time through detailed probabilistic transitions among states (i.e., CDS and SDS), but also the observation-based updating of these, which can be in turn propagated to update the fragility functions, together with changes due to repair interventions. This way, the problem can be readily defined as a partially observable Markov decision process and solved through deep reinforcement learning or other appropriate techniques to obtain a near-optimal management policy for the bridge. This has been recently considered in Metwally et al. (2024) which makes use of the realistic bridge system environment proposed here and extends it including repair actions and related costs. The study tests the applicability of multi-agent architectures in identifying dynamic component-level maintenance policies for optimizing system-level cost and reliability objectives and constraints.

5 CONCLUSION

In this paper, a dynamic Bayesian network (DBN) is developed to model the life-cycle seismic fragility of aging RC bridges, with a specific focus on encapsulating the uncertainties associated with corrosion deterioration and seismic actions as stressors to these structures. The methodology conceptualizes the bridge as a multi-component system, considering components that are crucial in determining the global seismic risk, i.e., RC columns and various types of steel bearings. The development of the DBN is facilitated by representing both deterioration and seismic damage in discrete spaces, defining corrosion deterioration states (CDSs) and seismic damage states (SDSs). This way, a structured approach to learning the evolving condition of the bridge components over time is laid out, allowing for efficient description of the complex interplay between deterioration and seismic action for the structural integrity of the bridge. Owing to the DBN properties, this complex learning task is distilled into a neat decomposition consisting of two primary sub-tasks: (i) calculating non-stationary Markovian transitions that portray the progression of CDSs; and (ii) formulating time-invariant and state-dependent fragility curves that capture the probability of exceeding SDSs given the IM and the combination of CDSs of structural components. The approach is applied to a 4-span bridge, which features characteristics typical for pre-70sRC bridges. These include inadequate RC column detailing and steel bearings. The analysis for the case study quantifies the detrimental effect of the environmental "splash" condition, which specifically instigates corrosion with an average onset time of 13.7 years, a critical factor in the structural performance overtime.

The computed time-invariant and state-dependent fragility curves for the case study underscore the high sensitivity of seismic responses to corrosion, providing valuable insights into how corrosion influences the seismic vulnerability of the bridge at various potential scenarios reachable through its life cycle. Furthermore, the approach of mechanically idealizing the bridge as a multi-component system, when combined with the DBN framework, allows us to identify the most at-risk components within the system across a spectrum of corrosion scenarios, which can lend itself to targeted inspection and retrofit intervention planning. Towards this, the capability of the approach to decipher changes in the hierarchy of strength and disentangle complex interactions among components, depending on environmental conditions, ground motion intensities and differential deterioration, provides engineers and decision-makers with insights into the structural

behavior over time. Finally, the presented DBN can be practical in enhancing fragility functions with field observations, thereby allowing for refined seismic risk inference. Due to its structure, it can be also compatible with advanced decision-support systems employing Markov decision processes that enable dynamic structural integrity management. Along these lines, current research trajectories are focused on expanding the capabilities of the DBN model with deep reinforcement learning approaches aimed at optimizing maintenance policies while ensuring structural safety and considering various performance metrics.

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