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Quantum Effects in Gravity Beyond the Newton Potential from a Delocalized Quantum Source

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Recent progress in tabletop experiments offers the opportunity to show, for the first time, that gravity is not compatible with a classical description. In all current experimental proposals, such as the generation of gravitationally induced entanglement between two quantum sources of gravity, gravitational effects can be explained solely with the Newton potential, namely, in a regime that is consistent with the weak-field limit of general relativity and does not probe the field nature of gravity. Hence, the Newtonian origin of the effects is a limitation to the conclusions on the nature of gravity that can be drawn from these experiments. Here, we identify two effects that overcome this limitation: They cannot be reproduced using the Newton potential, and they are independent of graviton emission. First, we show that the interaction between two generic quantum sources of gravity, e.g., in wide Gaussian states, cannot be reproduced with the Newton potential nor with a known classical theory of gravity. Second, we show that the quantum commutator between the gravitational field and its canonically conjugate momentum appears as an additional term in the relative phase of a generic quantum source interacting with a test particle. Observing these two effects would give further quantitative information and stronger evidence, compared to experiments only involving the Newton potential, to show that gravity is nonclassical. More broadly, identifying stronger quantum aspects of gravity than those reproducible with the Newton potential is crucial to prove the nonclassicality of gravity and to plan a new generation of experiments testing quantum aspects of gravity in a broader sense than what has been proposed so far.

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I. INTRODUCTION

Understanding the fundamental nature of gravity—and, in particular, whether we have to abandon its classical description—is one of the deepest open questions in fundamental physics. Until recently, this question has mostly been addressed theoretically because it was not conceivable to devise an experiment that would give a different outcome depending on whether gravity is classical or quantum. However, with the tremendous progress in quantum technologies, it is now possible to plan a new generation of tabletop experiments that could measure, for the first time, a physical effect that cannot be explained by any classical theory of gravity.

Starting from a thought experiment proposed by Feynman at the Chapel-Hill conference in 1957 [1,2], there has been a long debate [3–46] on the possibility to entangle two massive quantum systems via their gravitational interaction. The core of the argument, already discussed by Feynman, is the following: If gravity is a quantum interaction, it can entangle the quantum systems; if it is a classical interaction, it cannot.

The opposite implication does not logically follow. More explicitly, if we observe the generation of gravitationally induced entanglement (GIE) in an experiment, we cannot immediately conclude that gravity is quantum, for several reasons. For instance, if entanglement is generated via the Newton potential, we need to further assume that gravity acts as a mediator [10,11,25]. However, it is, in general, subtle to prove in an experiment which gravitational degrees of freedom are responsible for the entanglement. Second, from a logical point of view, we cannot exclude that the experimental results can be justified by some other description that does not require the quantization of the gravitational field. This problem is analogous to the debate (that started with the discovery of the photoelectric effect)

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concerning which observation would be a convincing proof that electromagnetism is quantum. Notably, this debate was closed after over half a century of discussion, thanks to an experiment by Clauser [47] in 1974. To date, there is no agreement on what would constitute an analogously convincing (and realistic) observation of gravity.

Recently, many different arguments have been put forward to specify in which sense observing GIE would imply that gravity is not classical. For instance, if the LOCC theorem (namely, the impossibility to generate entanglement via local operations and classical communication) is applied to GIE, then, assuming that gravity mediates the interaction, one can prove that gravity is not classical [10,11,42]. Other authors have adopted theory-independent methods [25,41,48,49], e.g., to formulate no-go theorems that, based solely on the experimental outcomes and on some general principles, can exclude a classical description of gravity. In a theory-specific approach, it is possible to use arguments resting on the notion of locality [36], on the reference-frame-dependent nature of the gauge [31], or on the explicit characterization of the field degrees of freedom [35]. Such theory-specific arguments do not prove the quantum nature of gravity, but they provide a theoretical framework in which the results can be interpreted.

Despite all the progress achieved in the past few years, there is still a lack of consensus on whether observing GIE is a convincing enough observation that gravity is not classical. Such a lack of consensus derives from the fact that, in all proposed experiments [10,11,13,15,16,19,21–23,26–28,32–36,38,40,42,45,46], all gravitational effects can be explained using solely the Newton potential. The Newton potential is the solution of the classical Einstein's equations in the nonrelativistic and weak-field regime. Hence, it does not provide any quantitative indication on how the description of gravity should be modified if it is not a classical theory. Importantly, as a nonlocal potential, the Newton interaction can generate entanglement *without* requiring a mediator. Therefore, the Newtonian limit evades the assumption of the LOCC theorem and perfectly explains GIE without invoking any quantum feature of gravity. The application of the LOCC theorem to interpret GIE requires the additional assumption that classical gravity behaves as a local field. While this property holds in the theory of general relativity, it has *not* yet been experimentally verified in this regime of study. So far, there has been no experimental proposal that proves that gravity is quantum without involving additional assumptions on the field nature of gravity, i.e., the fact that gravity cannot be reduced to the Newton potential alone.

Regardless of the personal take on this discussion, measuring GIE via the Newton potential would be a seminal result because it would be the first measurement that cannot be explained using the classical theory of general relativity (as is the case for all experiments performed so far). However, it is now crucial to search for stronger evidence of quantum aspects of gravity that could be tested in future experiments. Identifying richer effects in this regime would have a crucial impact on the scope and relevance of future tests of the quantum nature of gravity: If they exist, this will open a new phenomenological window on quantum effects in gravity.

Here, we provide, for the first time, two examples of such general effects, which are of the same order as the Newton potential in the gravitational coupling. Importantly, these effects are also independent of graviton emission. We explicitly derive the two effects using a field-basis formulation of linearized quantum gravity in the Schrödinger representation [35]. Note that this is a physical regime [50,51], where it is expected that all nonperturbative quantum gravity theories agree in the low-energy limit. In the first case, we consider two quantum sources of gravity prepared in a wide delocalized state, and we show that the gravitational interaction of such sources cannot be reproduced with the Newton potential nor with a known classical theory of gravity. Hence, observing this phase would require either an *ad hoc* modification to classical gravity or its quantum description. In the second case, we consider a simple physical scenario involving a source and a moving test particle, and by carefully analyzing all terms coming from the full Hamiltonian description of the interaction between matter and gravity, we show that the quantum commutator between the gravitational field and its canonically conjugate momentum appears in the relative phase accumulated during time evolution. This case is in contrast with the traditional expectation that the physical effect coming from the gravitational commutators is only relevant at very-high-energy scales. We notice that the phase due to the commutator would not appear if gravity were a classical field. Hence, probing this additional term in the phase would be a test of the gravitational field as a quantum mediator.

The paper is organized as follows. In Sec. II, we spell out the assumptions that are usually taken in the analysis of the GIE proposal, which lead to identifying the Newton potential as the origin of entanglement. In Sec. III, we show how these assumptions can be abandoned, and we describe the gravitational field associated with a general quantum source. In Sec. IV, we calculate the phase arising from the interaction between two quantum sources and show that it cannot be reproduced via the Newton potential nor via other known semiclassical descriptions of gravity. In Sec. V, we show how the commutator between the gravitational field and its canonically conjugated momentum contributes to the relative dynamical phase of the interaction between a quantum source and a test particle.

II. SEMICLASSICAL LOCALIZED SOURCE AND ITS GRAVITATIONAL FIELD

In this section, we identify the assumptions one needs to make on the matter source to obtain the Newton potential,

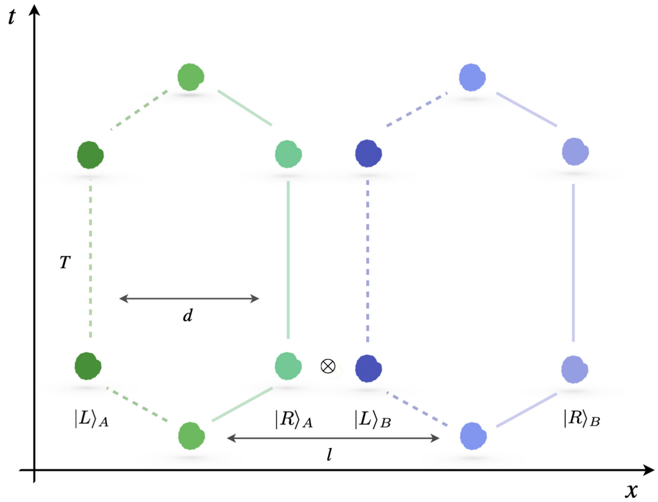

FIG. 1. GIE protocol. Two sources of gravity, A and B, are initially separable, with each being prepared in a quantum superposition of two localized states. After interacting gravitationally for a time T, the full quantum state is measured to certify entanglement. The relative phase at the end of the interferometer contains information about the Newton potential.

and we review the calculation to obtain the quantum state of gravity for semiclassical localized sources in Ref. [35].

The GIE proposal [10,11] considers an interferometric setting in which two sources of gravity, A and B, of mass m are each initially prepared in a quantum superposition of two localized states in the position basis (see Fig. 1),

$$|\psi_0\rangle = \frac{1}{2}(|L\rangle_A + |R\rangle_A) \otimes (|L\rangle_B + |R\rangle_B). \tag{1}$$

Here, $|L\rangle_A$ is the quantum state of particle A in the left path, sharply peaked in position x_0; $|R\rangle_A$ is the quantum state of particle A in the right path, sharply peaked in $x_0 + d$; $|L\rangle_B$ is the quantum state of particle B in the left path, sharply peaked in $x_0 + l$; and $|R\rangle_B$ is the quantum state of particle B in the right path, sharply peaked in $x_0 + d + l$. In addition, d is the delocalization of the single wave packet of each particle, and l is the distance between the center of mass of the two particles. The state of the system is initially separable. The sources then interact gravitationally for some time T. If the system is evolved according to the Schrödinger equation, the application of the Newton potential $\hat{V}_N(\hat{x}_A - \hat{x}_B) = -[(Gm^2)/(|\hat{x}_A - \hat{x}_B|)]$ on the initial state leads to the accumulation of relative phases due to the different distances between the four configurations of the masses, namely,

$$|\psi_T\rangle = \frac{1}{2}[|L\rangle_A(|L\rangle_B + e^{i\Delta\phi_+}|R\rangle_B) + |R\rangle_A(|R\rangle_B + e^{i\Delta\phi_-}|L\rangle_B)], \tag{2}$$

where $\Delta\phi_\pm = [(Gm^2T)/(\hbar l)]\{[l/(l \pm d)] - 1\}$. It is easy to see that, if $\Delta\phi_+ \neq -\Delta\phi_-$, the Newton potential generates entanglement between A and B. In quantum information theory, one can prove that the generation of entanglement is incompatible with local operations (LO) and classical communication (CC). This is known as LOCC theorem. Hence, if entanglement is measured (for instance, via an entanglement witness [10]) and gravity is assumed to be the mediator of the interaction between A and B, gravity cannot be classical. However, as discussed in the Introduction, the conclusions that one can derive from this experiment are limited by (1) the fact that the experiment can be explained with the Newton potential, which is a limit of general relativity, and (2) the fact that the field character of gravity is not probed by this experiment. Therefore, applying the LOCC theorem requires an additional assumption—that gravity acts as a local mediator. Notably, the GIE proposal can be described without the need to explicitly calculate the quantum state of gravity associated with each position of the quantum source.

It is possible to calculate such a quantum state of gravity by using the framework of linearized quantum gravity [50–52] to the first order in the perturbation. The result for sources having the initial state in Eq. (1) was derived in Ref. [35]. The quantized gravitational field can be obtained, for instance, by canonically quantizing the linearized gravity Hamiltonian. Such a Hamiltonian can be obtained by linearizing the Arnowitt-Deser-Misner (ADM) Hamiltonian about the Minkowski background and keeping terms up to κ^2, with $\kappa = 16\pi G/c^4$, where G is the gravitational constant and c the speed of light in vacuum. Following this derivation, one finds a Hamiltonian with four constraints, where the three vector constraints are the linearized momentum constraints, and the scalar constraint is what remains of the Hamiltonian constraint. We detail this description in Appendix A. This theory has a close parallel to quantum electrodynamics, as discussed in detail in Ref. [35], where both the electromagnetic and gravitational descriptions are discussed in depth.

To understand the assumptions required to obtain the Newton potential, let us now focus on a single quantum source. The quantum states $|L\rangle$ and $|R\rangle$ are assumed to be extremely well localized in both position and momentum, so the quantum particles in the interferometer approximately follow a classical path. These localized states are more realistically described by coherent states [53]. Let us denote such a coherent state as $|\alpha_i\rangle_S$, with $i = 1, 2$, and consider it to be localized around position x_i and momentum $p_i = 0$. For any parameter $\alpha_i = \alpha_i^R + i\alpha_i^I$, the mean position and momentum of the coherent state are related to α via $x_i = \langle\hat{x}_S\rangle_\alpha \propto \alpha_i^R$ and momentum $p_i = \langle\hat{p}_S\rangle_\alpha \propto \alpha_i^I$. We say that a source is in a semiclassical localized state if

$$\hat{x}_S|\alpha_i\rangle_S \approx x_i|\alpha_i\rangle_S, \qquad \hat{p}_S|\alpha_i\rangle_S \approx p_i|\alpha_i\rangle_S. \tag{3}$$

Physically, Eq. (3) holds when, for any relevant operational procedure, the commutator between the position and momentum of the source is negligible compared to the precision of the measurement device and so is the width of the Gaussian in the position basis.

For a static source, the only nonzero component of the energy-momentum tensor $T_{\mu\nu}$ is the T_{00} component. In quantum theory, $\hat{T}_{00}$ is, in general, a function of both the position and momentum operator, namely, $\hat{T}_{00}(\hat{x}_S, \hat{p}_S)$ [54]. For a semiclassical localized source $|\alpha_i\rangle_S$, we have

$$\hat{T}_{00}|\alpha_i\rangle_S \approx \rho_i(\vec{x}-\vec{x}_i, t)|\alpha_i\rangle_S, \tag{4}$$

where $\rho_i(\vec{x}-\vec{x}_i, t)$ is the classical energy density in general relativity. For a static pointlike source, $\rho_i(\vec{x}-\vec{x}_i, t) \propto mc^2\delta(\vec{x}-\vec{x}_i)$, with m being the rest mass of the source. Notice that, since we are assuming the source to be static, we do not need any condition on the time evolution of the source.

Under these assumptions, we can calculate the quantum state of gravity $|g_\alpha\rangle$ associated with a coherent state $|\alpha\rangle_S$ in the temporal gauge, i.e., $h_{0\mu} = 0$ (see Appendix A for details). In the Newtonian regime, in which there is no emission of radiation, the gravitational field is the ground state of the linearized quantum gravity Hamiltonian. Following the calculations of Ref. [35], which we also summarize in Appendix A, we find that

$$|\Psi_\alpha\rangle_{S+G} = |\alpha\rangle_S |g_\alpha\rangle_G, \tag{5}$$

where

$$\begin{aligned} |\Psi_\alpha\rangle_{S+G} &= \eta \int \mathcal{D}[h_{ij}]\delta[h^T - \mathsf{h}^T_{\rho_\alpha}]\Psi_{\mathrm{vac}}[h_{ij}]|\alpha\rangle_S|h_{ij}\rangle_G \\ &= \eta' \int \mathcal{D}[\pi_{ij}] \exp\left\{-\frac{i}{2\hbar}\int\frac{d^3k}{(2\pi)^3}\pi_T(\vec{k})\mathsf{h}^T_\rho(\vec{k})\right\} \\ &\quad \times \Psi_{\mathrm{vac}}[\pi_{ij}]|\alpha\rangle_S|\pi_{ij}\rangle_G. \end{aligned} \tag{6}$$

In the expressions above, h_{ij} and π_{ij} are the metric perturbation and its canonically conjugated momentum, and h^{ij}_T and π^{ij}_T are their projections on the transverse direction, i.e., $h^{ij}_T(\vec{k}) \coloneqq P^i_k P^j_l h^{kl}(\vec{k})$ and $\pi^{ij}_T(\vec{k}) \coloneqq P^i_k P^j_l \pi^{kl}(\vec{k})$, with $P^i_j = \delta^i_j - [(k^i k_j)/(|\vec{k}|^2)]$. Finally, $h^T = \delta_{ij}h^{ij}_T$ and $\pi^T = \delta_{ij}\pi^{ij}_T$. In the representation of the canonical momenta π_{ij}, the effect of the matter source on the ground state of gravity is a shift by a phase dependent on the solution of the classical Poisson equation $\partial_j\partial^j h^T(x) = -\kappa\rho_i(x)$ [35] (here, we dropped the t argument in the source because it is static). In the h_{ij} representation, the same condition is imposed via a Dirac delta in the expression of the quantum state.

The functional of the gravitational field Ψ_{vac} is the ground-state solution of the linearized gravitational Hamiltonian with no sources, given in Appendix A, and it reads

$$\begin{aligned} \Psi_{\mathrm{vac}}[h_{ij}] &\propto \exp\left\{-\frac{1}{4\kappa\hbar}\int\frac{d^3k}{(2\pi)^3}|\vec{k}|h^T_{ij}(\vec{k})h^{ij}_T(-\vec{k})\right\}, \\ \Psi_{\mathrm{vac}}[\pi_{ij}] &\propto \exp\left\{-\frac{\kappa}{\hbar}\int\frac{d^3k}{(2\pi)^3}\frac{1}{|k|}\left(\pi^T_{ij}(\vec{k})\pi^{ij}_T(-\vec{k})\right.\right. \\ &\qquad \left.\left. -\frac{1}{2}\pi_T(\vec{k})\pi_T(-\vec{k})\right)\right\}, \end{aligned} \tag{7}$$

in the h_{ij} and π_{ij} bases, respectively.

Equation (4) is crucial to obtain the Newton interaction for the static sources in this regime. However, that assumption is not justified for general quantum sources. Specifically, if we use Eq. (4) for different quantum states of gravity associated with different localized states of the source, we obtain some quantum features that are not realistic. For instance, let us consider two coherent states of the source, $|\alpha_x\rangle_S$ and $|\alpha_{x+\epsilon}\rangle_S$, whose central positions are displaced by an arbitrarily small amount ϵ, with ϵ smaller than the width of the coherent state of the source. Their scalar product is different from zero, i.e., $\langle\alpha_x|\alpha_{x+\epsilon}\rangle \neq 0$. However, the scalar product between the respective full quantum states of matter and gravity, $|\Psi_x\rangle_{S+G}$ and $|\Phi_{x+\epsilon}\rangle_{S+G}$, is zero, namely, $\langle\Psi_x|\Phi_{x+\epsilon}\rangle_{S+G} = 0$ (see Appendix A for details), because, in the semiclassical treatment, the source is modeled in such a way that the resulting quantum gravitational state depends exclusively on the local mass density. Consequently, different mass densities give rise to macroscopically distinct gravitational fields, which leads to an unphysical result when considering more general states of the source: Two sources with overlapping wave functions (hence, with nonzero scalar product) end up having orthogonal quantum states when their gravitational field is also considered. Thus, the description of Eq. (4) cannot be adopted for general quantum states and can only describe superpositions of quantum states of the source that are perfectly distinguishable (e.g., the configurations used in the typical GIE experiment).

Any quantum superposition of such a semiclassical localized source leads to an associated gravity configuration that is not classical, but only in a very limited sense. In particular, for a quantum state $|\psi\rangle_S = \sum_i c_i|\alpha_i\rangle_S$, we have

$$\hat{T}_{00}|\psi\rangle_S = \sum_i c_i\rho_i(\vec{x}-\vec{x}_i, t)|\alpha_i\rangle_S. \tag{8}$$

The resulting quantum state of the gravitational field is then

$$|\Psi_\psi\rangle_{S+G} = \sum_i c_i|\alpha_i\rangle_S|h_i\rangle_G, \tag{9}$$

but the difference between any two gravity states $|h_i\rangle_G$ and $|h_j\rangle_G$, for $i \neq j$, is only in $\mathsf{h}^T_{\rho_{\alpha_i}}$ in Eq. (6), i.e., the solution of the classical Poisson equation for a static source. In this case, all gravitational effects can be simply expressed as a quantum superposition of classical gravitational effects. This is the usual approximation implicitly adopted, for instance, for a quantum source of gravity prepared in a superposition of localized states, as in GIE proposals [10,11]. This approximation should be abandoned to explore more general quantum states of the source and look for stronger signatures of the quantum nature of gravity.

III. GENERAL STATIC QUANTUM SOURCES AND THEIR GRAVITATIONAL FIELD

We now consider the same situation as the previous section but with the initial quantum state of the two sources taken to be a general product state instead of Eq. (1). With this consideration, we show that richer properties of the gravitational field can be obtained because Eq. (4) does not hold anymore. Let us first consider a single, general, static quantum source of gravity (see Fig. 2 for an illustration of the difference from the previous section). The $\hat{T}_{00}$ operator of such a quantum source is, strictly speaking, an operator written as a function of the quantum field of the source. In particular, it is equal to the Hamiltonian density $\hat{\mathcal{H}}(x)$ of a quantum field, such as a Klein-Gordon field [55], i.e., $\hat{T}_{00}(x) = \hat{\mathcal{H}}(x)$. It is convenient to perform the calculations in a basis that diagonalizes $\hat{T}_{00}$, namely, $\hat{T}_{00}|E\rangle_S = E(\vec{x})|E\rangle_S$ [56]. To obtain our result, it is only relevant to describe the quantum state of the source in the first quantization and to know that, since the physical interpretation of $\hat{T}_{00}$ is an energy density, the eigenvalue $E(\vec{x})$ is a local function on spacetime. More specifically, we are interested in the low-energy limit in which the source of gravity is in a static quantum state $|\psi\rangle_S$. This state is more general compared to the coherent states discussed in the previous section; hence, $E(\vec{x})$ has a different functional form from the classical energy density $\rho_i(\vec{x} - \vec{x}_i, t)$. A general quantum state of the source can be expanded as

$$|\psi\rangle_S = \int d\mu(E)\psi(E)|E\rangle_S. \tag{10}$$

The quantum state of the gravitational field associated with such a source can be obtained by a procedure analogous to the one used in Ref. [35]. We only report the main steps and refer the reader to Appendix B for details. The free Hamiltonian of the gravitational field is

$$\hat{H}_G = \kappa \int \frac{d^3k}{(2\pi)^3} (\hat{\pi}_{kl}\hat{\pi}^{kl} - \hat{\pi}^2/2) + \frac{1}{4\kappa} \int \frac{d^3k}{(2\pi)^3} \left(k^2 \hat{h}^T_{ij}(\vec{k}) \hat{h}^{ij}_T(-\vec{k}) - k^2 \hat{h}_T(\vec{k}) \hat{h}_T(-\vec{k}) \right) \tag{11}$$

in which h^{ij}_T is the transverse part of the metric perturbation $h^{ij}_T(\vec{k}) \coloneqq P^i_k P^j_l h^{kl}(\vec{k})$ and $P^i_j = \delta^i_j - [(k^i k_j)/(|\vec{k}|^2)]$. In addition, the physical state has to satisfy the scalar and vector constraints

$$\hat{\mathcal{C}} \coloneqq -\partial_i \partial^i \hat{h}^T - \kappa \hat{T}_{00} = 0, \qquad \hat{\mathcal{C}}^i \coloneqq \partial_j \hat{\pi}^{ij} = 0. \tag{12}$$

We notice that the static source only influences the physical state through the scalar constraint $\hat{\mathcal{C}}$. Solving the constraints in the basis $|E\rangle_S$ for the source and in the basis $|\pi_{ij}\rangle$ of the gravitational field, we obtain

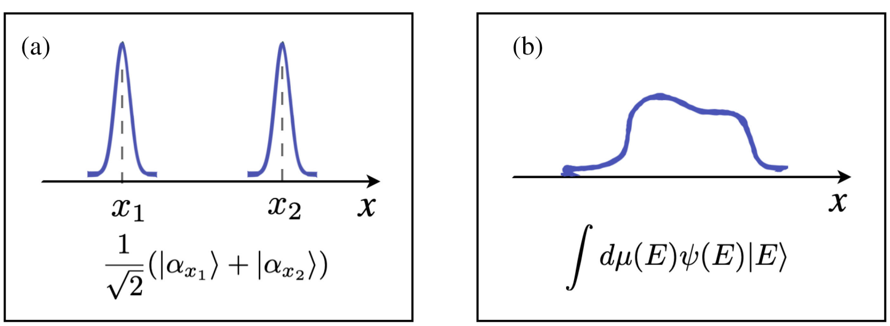

FIG. 2. (a) Superposition of semiclassical localized source. The source is prepared in a quantum superposition of two (or more) Gaussian states. Each state is sharply localized around some central value of position x_i and momentum $p_i = 0$. The resolution of the measurement devices is such that, for all practical purposes, the delocalization of the quantum state in both position and momentum is negligible. Under this assumption, each Gaussian quantum state is an eigenvector of the $\hat{T}_{00}$ operator having the classical energy density $\rho(\vec{x} - \vec{x}_i, t)$ as its eigenvalue. The quantum state of the gravitational field gives rise to the Newton potential. (b) General delocalized quantum source. The source is prepared in an arbitrary delocalized quantum state. The quantum state cannot be approximated as a linear combination of semiclassical localized states but should be solved directly in a convenient basis, i.e., the energy basis $|E\rangle$, which is the basis of eigenvectors of the $\hat{T}_{00}$ operator. The eigenvalues of the $\hat{T}_{00}$ operator do not coincide, in the general case, with the classical energy density; hence, the resulting gravitational interaction is functionally different from the Newton potential.

$$\partial_i\partial^i\hat{h}^T|\Psi_\psi\rangle_{S+G} = -i\hbar\partial_i\partial^i P_{kl}\frac{\delta}{\delta\pi_{kl}}|\Psi_\psi\rangle_{S+G} = -\kappa\int d\mu(E)\psi(E)E(t,\vec{x})|E\rangle_S|g_E\rangle_G. \quad (13)$$

The general solution of Eq. (13) is a joint state of the gravitational field and the source, namely,

$$\begin{aligned}|\Psi_\psi\rangle_{S+G} &= \eta\int d\mu(E)\mathcal{D}[h_{ij}]\psi(E)\delta[h^T - \mathrm{h}_E^T] \\ &\quad\times\Psi_{\text{vac}}[h_{ij}]|E\rangle_S|h_{ij}\rangle_G \\ &= \eta'\int d\mu(E)\mathcal{D}[\pi_{ij}]\psi(E) \\ &\quad\times\exp\left(-\frac{i}{2\hbar}\int d^3x\pi_T(\vec{x})\mathrm{h}_E^T(\vec{x})\right) \\ &\quad\times\Psi_{\text{vac}}[\pi_{ij}]|E\rangle_S|\pi_{ij}\rangle_G, \end{aligned} \quad (14)$$

where η, η' are normalization constants, and $\mathrm{h}_E^T(\vec{x})$ is the solution of the classical Poisson equation with the source being in the eigenstate $|E\rangle_S$ with eigenvalue $E(\vec{x})$. In particular, we have

$$\mathrm{h}_E^T(\vec{x}) = \frac{\kappa}{4\pi}\int d^3y\frac{E(\vec{y})}{|\vec{x}-\vec{y}|}. \quad (15)$$

The integration measure of the metric field can be decomposed into a longitudinal part h_{ij}^L, a transverse-traceless part $\tilde{h}_{ij}^T$, and a transverse-trace part h_T, namely, $\mathcal{D}[h_{ij}] = \mathcal{D}[h_{ij}^L]\mathcal{D}[\tilde{h}_{ij}^T]\mathcal{D}[h_T]$.

Differently from the case of the localized semiclassical source of Sec. II, two overlapping quantum states of the source, $|\psi\rangle$ and $|\phi\rangle$, such that $\langle\psi|\phi\rangle \neq 0$, give rise to quantum states of gravity that are not perfectly distinguishable. This case can be easily seen by directly computing the scalar product of the quantum states of gravity and matter (see Appendix B for details),

$$\langle\Psi_\psi|\Psi_\phi\rangle_{S+G} = \int d\mu(E)\psi^*(E)\phi(E) = \langle\psi|\phi\rangle_S \neq 0, \quad (16)$$

as we would intuitively expect because the quantum state of gravity, in this case, is fully determined by the quantum matter field. The scalar product only depends on the states of matter and does not depend on gravity.

IV. PHYSICAL EFFECTS BEYOND THE NEWTONIAN PHASE FOR A WIDE QUANTUM SOURCE

We now consider a similar scenario to the usual GIE setup but with the sources initially prepared in a general delocalized quantum state. We then let the sources interact gravitationally (see Fig. 3) and calculate the form of the relative phase at the end of the interferometer. Interestingly, we find that the relative phase in this scenario cannot be reproduced with the Newton potential. To see this explicitly, let us define $\hat{h}_{\mu\nu}(x)$ as the linearized quantum gravitational field sourced by the masses and $\hat{T}^A_{\mu\nu}$ and $\hat{T}^B_{\mu\nu}$ as the stress-energy tensors of sources A and B, respectively. The Hamiltonian is the sum of a free and an interaction term, namely,

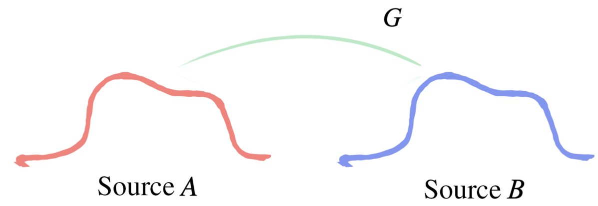

FIG. 3. Gravitational interaction (G) between two delocalized mass sources A and B.

$$\hat{H} = \hat{H}_A + \hat{H}_B + \hat{H}_G^0 + \hat{H}_I^{(\text{tot})}, \quad (17)$$

where $\hat{H}_A$ and $\hat{H}_B$ are the source Hamiltonians and $\hat{H}_G^0$ is the source-free gravitational-field Hamiltonian. For the purpose of this section, the explicit expressions of the source Hamiltonians are not needed. The interaction Hamiltonian between A, B, and gravity is, in its most general form,

$$\hat{H}_I^{(\text{tot})} = -\frac{1}{2}\int d^3x\hat{h}_{\mu\nu}(\vec{x})(\hat{T}_A^{\mu\nu}(\vec{x}) + \hat{T}_B^{\mu\nu}(\vec{x})). \quad (18)$$

For static quantum sources, T_{00} is the only nonzero component of the stress-energy tensor. [59] Since $h_{0\mu} = 0$ in the temporal gauge, this means that the interaction Hamiltonian $H_I^{(\text{tot})}$ is zero at the leading order in G_N expansion. Crucially, the interaction between the source and the gravitational field is fully encoded in the Gauss constraint $\hat{\mathcal{C}} \coloneqq -\partial_i\partial^i\hat{h}^T - \kappa\hat{T}_{00}^A - \kappa\hat{T}_{00}^B = 0$. Expanding both quantum sources in the eigenbasis of $\hat{T}_{00}$, the full quantum state of the gravitational field and the sources is (up to a normalization constant η)

$$|\Psi\rangle_{ABG} = \eta\int d\mu(E_A)d\mu(E_B)\psi(E_A)\phi(E_B)|E\rangle_A|E\rangle_B|g_E\rangle_G, \quad (19)$$

where

$$|g_E\rangle_G = \int\mathcal{D}[\pi_{ij}]\exp\left(-\frac{i}{2\hbar}\int d^3x\pi_T(\vec{x})(\mathrm{h}_{E_A}^T(\vec{x}) + \mathrm{h}_{E_B}^T(\vec{x}))\right) \times\Psi_{\text{vac}}[\pi_{ij}]|\pi_{ij}\rangle_G, \quad (20)$$

with $\mathrm{h}_{E_A}^T(\vec{x})$ and $\mathrm{h}_{E_B}^T(\vec{x})$ given by Eq. (15). The vector $|E\rangle_A|E\rangle_B|g_E\rangle_G$ provides an eigenbasis for the Hamiltonian;

hence, by applying the time evolution, every basis element in Eq. (19) acquires a different phase with a different energy. The energy eigenvalue of the gravitational Hamiltonian corresponding to the vector $|E\rangle_A|E\rangle_B|g_E\rangle_G$ is

$$\mathcal{E}_{ABG} = \mathcal{E}_{\mathrm{vac}} - \frac{\kappa}{8\pi}\int d^3x d^3y \frac{(E_A(\vec{x}) + E_B(\vec{y}))^2}{|\vec{x}-\vec{y}|}, \quad (21)$$

where $\mathcal{E}_{\mathrm{vac}}$ is the vacuum energy of the field. To identify the relevant energy contribution, we need to subtract from the previous expression the vacuum energy $\mathcal{E}_{\mathrm{vac}}$ and the gravitational self-energy of each source, i.e., $\mathcal{E}_S = -[\kappa/(8\pi)]\int d^3xd^3yE_S(\vec{x})E_S(\vec{x})/|\vec{x}-\vec{y}|$, with $S = A,\ B$. Overall, the source-dependent contribution to the entangling phase is

$$\Theta_{AB} = -\frac{\kappa t}{4\pi\hbar}\int d^3xd^3y\frac{E_A(\vec{x})E_B(\vec{y})}{|\vec{x}-\vec{y}|}, \quad (22)$$

which is the generalization for delocalized quantum sources of the phase in Eq. (2) corresponding to the Newton potential [10,11].

Importantly, we would not have obtained this result had we expanded the general source in a basis of coherent states (i.e., those corresponding to semiclassical localized states) and calculated the gravitational field associated with each coherent state. This is the approach taken in Ref. [45], and it only gives rise to Newtonian interactions.

Our result cannot be explained, to the best of our knowledge, by any (potentially modified) classical theory of gravity. In particular, if gravity is assumed to be a classical local field theory and we require that the theory admits a probabilistic description of measurement outcomes (which is a condition required for the experiments), then no classical theory could generate entanglement [10,11,25]. Notice that by "classical" here we do not limit ourselves to general relativity, but this may also refer to modified descriptions of classical gravity as a local field, such as modified gravity theories with higher order derivatives or with extra fields and so on. If we do not assume that gravity is a local field, then the only possibility is to engineer an *ad hoc* and fine-tuned nonlocal potential that reproduces this effect, as we explain in Sec. IV A. However, we are not aware of any modified theory of gravity that displays this form of potential. In the following, we compare our result to the Newton potential and explicitly show the differences between our result and some of the most common models of semiclassical gravity.

We stress that the strength of our result is that it gives additional information on what the description of gravity should be. Suppose that we measure entanglement due to the gravitational interaction between two sources of gravity and we verify that the relative phase in the experiment is Newtonian. In this case, we do not have any quantitative indication of what the correct theory of gravity is because the Newton potential is compatible with a limit of general relativity. For a delocalized quantum source of gravity, on the contrary, no general-relativistic solution for a source particle matches the phase derived in Eq. (22). Hence, from this experiment, we not only deduce that gravity cannot be classical if it is described as a field, but we also have a quantitative prediction for how it differs from general relativity.

A. Interaction via the Newton potential

We now compare our result with other models to describe the gravitational interaction. First of all, we consider the Newton potential, which is the weak-field, nonrelativistic limit of GR and can be expressed via direct coupling through a potential (i.e., it does not exploit the field character of gravity). The standard prescription in the literature so far has been to adopt it for masses in a quantum superposition of localized states [10,11] as well as for delocalized masses in Gaussian states or harmonic oscillators (see, e.g., Refs. [19,24,39,42,45,46]). Here, we show that using the Newton potential to describe these scenarios corresponds to approximating the quantum source as a superposition of localized sources as in Eq. (8). Hence, the Newton potential can be used when the source is in a quantum superposition of semiclassical localized states but not when the source is in a general quantum state with a large delocalization. In the latter case, the functional form of the coupling between the two masses obtained using a complete model of gravity in the quantum regime does not coincide with the one obtained using the Newton potential. To see this explicitly, let us consider the Hamiltonian describing the interaction between two mass sources:

$$\hat{H}_N = \frac{\hat{p}_A^2}{2m_A} + \frac{\hat{p}_B^2}{2m_B} + \hat{V}_N(\hat{x}_A - \hat{x}_B), \quad (23)$$

where $\hat{V}_N(\hat{x}_A - \hat{x}_B) = -G[(m_Am_B)/(|\hat{x}_A - \hat{x}_B|)]$. The action of the Newton potential on an arbitrary quantum state of the sources A and B is

$$\hat{V}_N(\hat{x}_A - \hat{x}_B)|\psi\rangle_A|\phi\rangle_B = -G\int dx_Adx_B\psi(x_A)\phi(x_B)\frac{m_Am_B}{|x_A - x_B|}|x_A\rangle_A|x_B\rangle_B. \quad (24)$$

Let us now compare the above expression with the entangling phase that we obtained from the linearized quantum gravity interacting with two quantum sources. In particular, we remark that Eq. (22) coincides with the Newton potential in Eq. (24) only in a specific limit. First, the quantum state $|E\rangle_S (S = A, B)$ needs to be approximately a position state $|x\rangle_S$ and, more precisely, a coherent state $|\alpha_x\rangle_S$ of the type discussed in Sec. II. Thus, if the source is a delocalized quantum state, using the Newton potential amounts to approximating the full quantum state of S and G as a linear combination of quantum states as in Eq. (9). As we have shown, this approach only gives rise to a quantum superposition of classical gravitational effects. Instead, the phase arising from the full model that we have derived here cannot be reproduced by superposing classical Newtonian effects (i.e., a classical theory of gravity in the weak-field limit). It is easy to check that the Newtonian phase can only be recovered if $E_S(\vec{x}) = m_S \delta(\vec{x} - \vec{x}_S)$, for $S = A, B$, i.e., in the limit in which Eq. (4) holds. In future experiments, one could make the effect more explicit by tuning different functional forms of the eigenvalue of $\hat{T}_{00}$, i.e., the energy density $E(x)$, and observing how the phase changes with the energy density. This experimental setup may be achieved, for instance, by using different potentials for the source.

The only possibility for reproducing the phase via a nonlocal potential, which is, however, different from the Newton potential, is to introduce an *ad hoc* coupling between the two source masses A and B, namely,

$$\hat{V}_{\mathrm{Nloc}} = -G \int d^3x d^3y \frac{\mathrm{Tr}[\hat{T}_A(\vec{x})]\mathrm{Tr}[\hat{T}_B(\vec{y})]}{|\vec{x} - \vec{y}|}. \tag{25}$$

This coupling is an operator on the matter degrees of freedom of the sources and does not require invoking any gravitational-field degree of freedom. However, it would constitute a modification of general relativity, which does not include quantum delocalized sources. In addition, to the best of our knowledge, this coupling is not predicted by any well-known (standard or modified) theory of gravity.

We now ask if the phase of Eq. (22) can be reproduced using a model in which gravity is a classical field and couples to quantum matter. We consider two cases: a genuine classical-quantum coupling such as the one introduced in Ref. [60], and the Schrödinger-Newton equation, which can be seen as the semiclassical limit of the Einstein equations [61].

B. Hybrid model for classical gravity coupled to quantum matter

Let us consider the former case. If gravity is classical and coupled to quantum matter, a general hybrid classical-quantum (CQ) state is [60]

$$\hat{\rho}_{\mathrm{CQ}}(t) = \int dz \rho(z,t) |h_{ij}^z\rangle_G \langle h_{ij}^z| \otimes \hat{\sigma}_S(z,t), \tag{26}$$

where z is a variable that correlates the quantum state of the source and the state of the gravitational field. It was shown in Ref. [60] that, if one requires the dynamical law to preserve positivity, the normalization of probabilities, and the set of CQ states, the most general form of the time evolution is a stochastic open-system dynamics. This type of dynamics is characterized by diffusion and decoherence [62] and thus would not lead to the generation of an entangled state between the source S and the test particle P. Hence, the physical effects due to such a classical-quantum coupling are extremely different from those we obtained by coupling quantum matter to a field theory of gravity in the quantum regime, and the two can be distinguished experimentally, for instance, by measuring the coherence of the quantum state after the interaction. Notice that such an irreversible coupling is the most general CQ coupling that can be obtained under the reasonable assumptions of Ref. [60]. As shown in Ref. [41] using a theory-independent no-go theorem, if the matter source can be prepared in a quantum superposition state, gravity is classical, and the state of gravity is influenced by the quantum state of matter (backreaction), then the coupling has to be irreversible. This immediately implies that any genuine coupling between classical gravity and quantum matter has very different observable effects from those discussed in this work.

C. Schrödinger-Newton equation

Let us now consider the Schrödinger-Newton equation. In the Newtonian limit, the interaction Hamiltonian for two particles of mass m_1 and m_2 interacting gravitationally is [63]

$$H_I^{\mathrm{SN}} \Psi(\vec{x}_1, \vec{x}_2, t) = -G \sum_{a=1,2} \sum_{b=1,2} m_a m_b \int d^3x_1' d^3x_2' \times \frac{|\Psi(\vec{x}_1', \vec{x}_2', t)|^2}{|\vec{x}_a - \vec{x}_b'|} \Psi(\vec{x}_1, \vec{x}_2, t), \tag{27}$$

where $\Psi(\vec{x}_1, \vec{x}_2, t)$ is the joint state of the two particles. In the perturbative regime of gravity, we can simplify the previous expression by taking $\Psi(\vec{x}_1,\vec{x}_2,t) = \Psi^{(0)}(\vec{x}_1,\vec{x}_2,t) + G\Psi^{(1)}(\vec{x}_1,\vec{x}_2,t)$, and the number in brackets refers to the order in perturbation theory in the gravitational constant G. In this case, we have $\Psi^{(0)}(\vec{x}_1, \vec{x}_2, t) = \psi_1(\vec{x}_1)\phi_2(\vec{x}_2)$. The leading order of the wave function $\Psi^{(0)}$ is a product state, and the first order of perturbation $\Psi^{(1)}$ characterizes the entanglement due to gravity. Hence, we find

$$H_I^{\mathrm{SN}} \Psi^{(1)}(\vec{x}_1, \vec{x}_2, t) = -G \sum_{a=1,2} \sum_{b=1,2} m_a m_b \int d^3x_1' d^3x_2' \times \frac{|\psi_1(\vec{x}_1')|^2 |\phi_2(\vec{x}_2')|^2}{|\vec{x}_a - \vec{x}_b'|} \Psi^{(0)}(\vec{x}_1, \vec{x}_2, t). \tag{28}$$

To obtain an expression as close as possible to Eq. (22), we neglect the self-interaction terms in which $a = b$, and we identify $\tilde{E}_1(\vec{x}_1) = m_1 |\psi_1(\vec{x}_1')|^2$ and $\tilde{E}_2(\vec{x}_2) = m_2 |\phi_2(\vec{x}_2')|^2$. Notice that this identification is different from the expression of the eigenvalues $E_{1,2}(\vec{x})$ of the $\hat{T}_{00}$ for a quantum source. In this case, Eq. (28) is still not equivalent to

Eq. (22), which is not surprising: Although the Schrödinger-Newton equation can also be obtained as a mean-field approximation of a theory in which gravity is fundamentally quantum [61], this approximation leads to nonlinearities. Such nonlinearities and other observable effects have a scaling that depends on the specific parameters of the particle (such as its mass and delocalization; see, e.g., Ref. [64]), so the differences between the Schrödinger-Newton equation and our result can be maximized by optimizing over these parameters and by initially preparing the source in a quantum delocalized state in the position basis. However, a distinctive difference with the Schrödinger-Newton equation is that our derivation comes from a quantum description of gravity in the weak-field regime, which preserves the linearity of quantum theory and, hence, unitary evolution. As a consequence, any deviation from unitarity marks a distinction between these two models.

In summary, we have shown that both a genuine CQ coupling and the Schrödinger-Newton equation lead to quantitatively different predictions from our model and can hence be distinguished from our result in an experiment.

V. QUANTUM COMMUTATOR OF THE GRAVITATIONAL FIELD

In this section, we show that the quantum commutator $[\hat{h}_{ij}(\vec{x}), \hat{\pi}^{kl}(\vec{x}')] = i\hbar\delta^k_{(i}\delta^l_{j)}\delta^3(\vec{x}-\vec{x}')$ gives rise to additional terms in the relative phase of an interferometric experiment in which two massive objects become entangled via gravitational interaction. We consider the situation depicted in Fig. 4, where we have a massive static source of gravity S prepared in a general quantum state and a moving test particle P, initially in a product state, interacting gravitationally. For more generality, the position of the source S and the momenta of the probe P can be controlled by external potentials. The Hamiltonian is the sum of a free term and an interaction term, namely,

$$\hat{H} = \hat{H}_S + \hat{H}^0_G + \hat{H}_P + \hat{H}^{(\mathrm{tot})}_I, \tag{29}$$

where $\hat{H}_S$ is the source Hamiltonian, $\hat{H}^0_G$ is the source-free gravitational-field Hamiltonian, $\hat{H}_P$ is the test particle Hamiltonian, and $\hat{H}^{(\mathrm{tot})}_I$ is the interaction Hamiltonian between gravity, the source S, and the probe P. In particular, the source Hamiltonian and the test particle Hamiltonian are

$$\hat{H}_S = \frac{\hat{p}_S^2}{2m_S} + \hat{V}_S(\hat{x}_S), \qquad \hat{H}_P = \frac{\hat{p}_P^2}{2m_P} + \hat{V}_P(\hat{x}_P), \tag{30}$$

and $\hat{V}_S(\hat{x}_S)$ and $\hat{V}_P(\hat{x}_P)$ are externally controlled potentials used to control the quantum states of the source and the test particle. The interaction Hamiltonian between gravity and S and P is

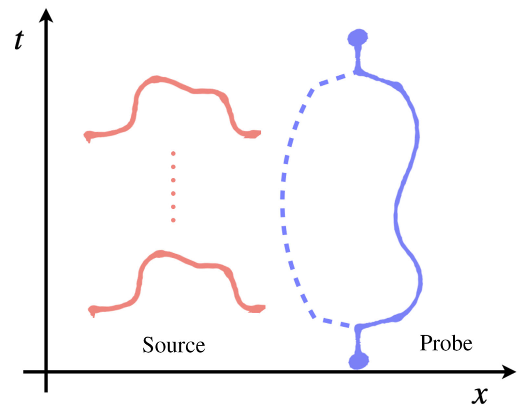

FIG. 4. Source mass S prepared in an arbitrary delocalized state before the start of the experiment at t_0, and a test mass P (the probe) initially in a product state with the rest. The source and the moving probe interact gravitationally, and the source is assumed to be static. Using a weak-field quantum description of gravity, after some time, the full state of the source, the gravitational field, and the probe become entangled. At the end, the source and the probe are measured. The relative phase of the full quantum state encodes the functional form of the gravitational interaction. We find that, for a general quantum state of the source, the gravitational interaction cannot be represented as the Newton interaction and cannot be simulated by a known classical or semiclassical model of gravity. The gravitational commutator appears as an additional term in the relative phase of the quantum source interacting with a moving test particle.

$$\hat{H}^{(\mathrm{tot})}_I = -\frac{1}{2}\int d^3x \hat{h}_{\mu\nu}(\vec{x})(\hat{T}^{\mu\nu}_S(\vec{x}) + \hat{T}^{\mu\nu}_P(\vec{x})). \tag{31}$$

Working in the temporal gauge $h_{0\mu} = 0$, the total Hamiltonian of the source, the test particle, and the gravitational field can be equivalently written as $\hat{H} = \hat{H}_S + \hat{H}_G + \hat{H}_P + \hat{H}_I$, where $\hat{H}_G$ is the Hamiltonian of Eq. (11) with the same constraints as in Eq. (12) [65] and $\hat{H}_I$ is the interaction Hamiltonian,

$$\hat{H}_I = -\frac{1}{2}\int d^3x \hat{h}^{ij}(\vec{x})\hat{T}^P_{ij}(\vec{x}). \tag{32}$$

In the specific case we consider, we make the approximation that the test particle does not backreact on the gravitational field. In other words, we ignore the gravitational field of the probe and only consider its interaction with the gravitational field that the source generates. More concretely, this means that the interaction with the probe does not change the quantum state of the gravitational field sourced by S alone nor the expectation value of the gravitational-field operators. This approximation ensures that adding the interaction Hamiltonian between the gravitational field and the test particle does not map the quantum state of the gravitational field out of the constraint surface; hence, the eigenstate of the initial Hamiltonian remains the same. We take as the initial state

$$|\Xi_0\rangle_{\mathrm{SGP}} = |\Psi_0\rangle_{S+G} \otimes |\phi_0\rangle_P, \tag{33}$$

where $|\Psi_0\rangle_{S+G}$ and $|\phi_0\rangle_P$ are arbitrary quantum states of the source and of the probe, respectively.

The final state at time t is

$$|\Xi_t\rangle_{\mathrm{SGP}} = \hat{U}_{\mathrm{SGP}}(t)|\Xi_0\rangle_{\mathrm{SGP}} = e^{-\frac{it}{\hbar}\hat{H}}|\Xi_0\rangle_{\mathrm{SGP}}. \tag{34}$$

Since the test particle does not backreact on the gravitational field, the dynamical evolution preserves the constraints; namely, the initial physical state is still a solution of the constraint equation after the time evolution. Technically, this means that the interaction Hamiltonian weakly commutes with the constraints, i.e.,

$$[\hat{H}_I,\hat{\mathcal{C}}^i]|\Xi\rangle_{S+G} = -\frac{1}{2}\int d^3k k'_j \hat{T}^P_{lm}(-\vec{k})[\hat{h}^{lm}(\vec{k}),\hat{\pi}^{ij}_L(\vec{k}')]|\Xi\rangle_{S+G} = 0. \tag{35}$$

This process is equivalent to neglecting the commutator between the gravitational-field operator and the longitudinal momentum operator when it acts on the physical state, i.e., $[\hat{h}^{lm}(\vec{k}),\hat{\pi}^{ij}_L(\vec{k}')]|\Xi\rangle_{S+G} = 0$. Conversely, the commutator of the transverse mode $[\hat{h}^{lm}(\vec{k}),\hat{\pi}^{ij}_T(\vec{k}')]|\Xi\rangle_{S+G}$ cannot be neglected because it is responsible for the physical effects. The crucial observation is that the free Hamiltonian of the gravitational field $\hat{H}_G$ and the interaction Hamiltonian $\hat{H}_I$ do not commute when applied to the physical state. This can be seen explicitly (see details in Appendix C), namely,

$$[\hat{H}_G,\hat{H}_I]|\Xi\rangle_{S+G} = i\hbar\kappa\int d^3x\left(\hat{\pi}^T_{ij}(\vec{x}) - \frac{1}{2}P_{ij}\hat{\pi}^T(\vec{x})\right) \times \hat{T}^{ij}_P(\vec{x})|\Xi\rangle_{S+G}. \tag{36}$$

In the following, we focus only on the terms arising from the commutator between $\hat{H}_G$ and $\hat{H}_I$, and we neglect the rest of the commutators (for instance, the commutators between the position and momentum of S and P), which are not relevant for the effect we wish to describe. These terms give rise to an additional phase that can be distinguished experimentally, thanks to its different functional form, from the gravitational commutator terms. In addition, the commutators between $\hat{H}_G$ and $\hat{H}_I$ give rise to a polynomial expression in the time of the experiment t [66]. Here, we evaluate terms up to order t^3. Considering higher order terms is more challenging from a computational perspective, but it has no impact on the final result, namely, the dependence of the relative phase on the commutator between the gravitational-field operator and its canonically conjugated momentum. The full calculation is detailed in Appendix C.

We expand the initial quantum state in its energy eigenbasis as

$$|\Xi_0\rangle_{\mathrm{SGP}} = \int d\mu(E_S)\psi_S(E)|E\rangle_S|g_E\rangle_G|\psi\rangle_P, \tag{37}$$

where $|g_E\rangle_G$ was defined in Eqs. (13) and (14). The state at time t is

$$|\Xi_t\rangle_{\mathrm{SGP}} = \int d\mu(E)\psi_S(E)e^{-\frac{it}{\hbar}\theta_{SP}}e^{\frac{i}{\hbar}(\hat{\Theta}^{(0)}+\hat{\Theta}^{(1)}+\hat{\Theta}^{(2)}+\cdots)} |E\rangle_S|g_E\rangle_G|\psi\rangle_P, \tag{38}$$

in which θ_{SP} is a phase obtained from the action of the source and test particle Hamiltonian on the quantum state, irrelevant for our purposes, and the index $n = 0, 1, 2$ in each $\hat{\Theta}^{(n)}$ is the number of commutators between the gravitational-field operators that give rise to the phase. More explicitly,

$$\hat{\Theta}^{(0)} = -\frac{t}{4}\int\frac{dk^3}{(2\pi)^3}\mathrm{h}^T_E(\vec{k})P_{ij}\hat{T}^{ij}_P(-\vec{k}) - \frac{i\kappa t^2}{8}\int\frac{d^3k}{(2\pi)^3}\frac{1}{|k|}\Big(\hat{T}^P_{ij}(\vec{k})\hat{T}^{ij}_P(-\vec{k}) - 2\hat{\tilde{T}}^{ij}_{PT}(\vec{k})\hat{\tilde{T}}^{PT}_{ij}(-\vec{k})\Big). \tag{39}$$

The first term in the previous expression is an additional part of the entangling phase coming from the coupling between the gravitational field and the momentum of the probe. The second term, which is not a phase but a real exponential term, comes from the integration of the transverse-traceless component of the gravitational field, after evaluating the operators on its quantum state.

The term $\hat{\Theta}^{(1)}$ corresponds to the phase arising from calculating a single commutator of the gravitational-field operators, i.e., $[\hat{H}_G,\hat{H}_I]$, namely,

$$\hat{\Theta}^{(1)} = \frac{\kappa t^3}{8}\int\frac{d^3k}{(2\pi)^3}\hat{T}^P_{ij}(\vec{k})\hat{T}^{ij}_P(-\vec{k}). \tag{40}$$

The term $\hat{\Theta}^{(2)}$ is instead the phase arising due to the double commutators $[\hat{H}_G,[\hat{H}_G,\hat{H}_I]]$ and $[\hat{H}_I,[\hat{H}_G,\hat{H}_I]]$, specifically

$$\hat{\Theta}^{(2)} = \frac{\kappa t^3}{6}\int\frac{dk^3}{(2\pi)^3}\left[\hat{T}^{ij}_P(\vec{k})P^k_iP^l_j\hat{T}^P_{kl}(-\vec{k}) - \frac{1}{2}(P_{ij}\hat{T}^{ij}_P(\vec{k}))(P_{kl}\hat{T}^{kl}_P(-\vec{k}))\right] = \frac{\kappa t^3}{6}\int\frac{d^3k}{(2\pi)^3}\hat{\tilde{T}}^{PT}_{ij}(\vec{k})\hat{\tilde{T}}^{ij}_{PT}(-\vec{k}). \tag{41}$$

Note that the $\hat{\Theta}^{(n)}$ operators, with $n = 0, 1, 2$, do not depend on the gravity degrees of freedom but only on the test particle P. In the second line, $\tilde{T}^{ij}_{PT}$ denotes the transverse-traceless component of T^{ij}_P. In addition, all terms in the phase are of the order of $\kappa/\hbar$, which is natural because we are expanding the gravitational field to the first order in perturbation theory. We are not considering higher order terms giving rise to graviton loops corresponding to the self-interaction of the gravitational field. Hence, the order in the gravitational coupling and $\hbar$ is the same as in the GIE proposals. In addition, $\hat{\Theta}^{(1)}$ and $\hat{\Theta}^{(2)}$ scale differently with time to the leading order phase that does not depend on the commutator; hence, this term can, in principle, be distinguishable experimentally. Therefore, the $\hat{\Theta}^{(n)}$ operators, with $n = 0, 1, 2$, give rise to a relative phase that appears in the interference pattern between different eigenstates of the probe. Probing these additional terms in the phase would be a more direct test of the gravitational field as a quantum mediator.

To observe this relative phase, the probe needs to be prepared in a quantum superposition of states on which the action of the stress-energy tensor gives different values. This setup can be realised, for instance, when the probe is prepared in a quantum superposition of different momentum or energy eigenstates (possibly with a large energy gap), and the superposition is recombined at the end via an interferometric measurement of the source and the probe. It is important to note that, after the interaction with the gravitational field, the full quantum state of the probe, the source, and the gravitational field is entangled. Hence, a joint measurement on the source and the probe, in which the gravitational field follows the quantum state of the source adiabatically (this condition is equivalent to requiring no emission of gravitational radiation) [67], should be performed in order to preserve the coherence of the full state.

VI. DISCUSSION

In this work, we identify two physical effects to test the quantum nature of the gravitational field in tabletop experiments, which cannot be predicted using the Newton potential. The analysis is carried out using a quantum-field formulation of gravity in the weak-field regime. Instead of the standard Fock space representation, we use the Schrödinger representation in the field basis. This formulation is not only convenient to identify quantum states of macroscopically distinct gravitational configurations; most importantly, it is suitable to describe the quantum state of the gravitational field given by any generic massive quantum source. Hence, it provides a framework to investigate rich physical properties and predictions that will be explored in future quantum tests of gravity in tabletop experiments.

Both physical effects that we find influence the relative phase of an interferometer in which two sources of gravity (Sec. IV) or a source particle and a test particle (Sec. V) interact gravitationally.

The first physical effect, in Sec. IV, shows that, when the massive sources are in a generic quantum state, and specifically are strongly delocalized, the dynamical phase arising from their interaction cannot be reproduced by the Newton potential, the Schrödinger-Newton equation, the hybrid model for classical gravity coupled to quantum matter, classical general relativity, or any local field-theoretic formulation of modified classical gravity. In the limit in which the source is in a superposition of semiclassical localized states, as is the case in proposals to observe gravitationally induced entanglement (see, e.g., Refs. [10,11]), the prediction reduces to the one given by the Newton potential. Hence, all our results are consistent with those studied in the literature, when reduced to the same regime. Importantly, this effect is of the same order in the gravitational-field strength as the Newton potential.

The second physical effect, in Sec. V, shows that the quantum commutator of the gravitational field enters the expression of the dynamical phase of an interferometric configuration involving a source particle and a test particle. Specifically, we show that, in the entangling phase between a moving probe and a mass source, the commutators between the linearized gravitational-field operators and its conjugate momenta enter as additional terms. Probing these terms would be a substantial advantage compared to Newton entanglement because it is a direct consequence of gravity being a quantum field.

Overall, these two effects provide a quantitative prediction that gives additional information as to how the gravitational field associated with a general quantum source should be described. Both effects we identified scale as the Newton potential in terms of the gravitational constant. Experimentally, observing gravitational coupling between strongly delocalized masses is extremely challenging. In particular, it requires a significant reduction in decoherence rates and experiments that are capable of distinguishing gravitational effects from other coupling forces [68]. However, efforts are underway to reach a regime where such coupling becomes significant for strongly delocalized massive particles (see, for instance, Refs. [69,70]). Once this goal is achieved, the phase contribution described in Sec. IV must be considered in the interpretation of experiments as an effect that is of the same order as the interaction by Newtonian gravity. The effect we presented in Sec. V will become relevant for successive gravity experiments as relativistic effects should be considered. These effects motivate one to pursue this research direction in the even longer term, as measuring it would provide a stronger argument in favor of the quantization of gravity.

If the proposed predictions are not observed in future experiments, the implications would be significant. All

UV-complete quantum gravity theories are expected to converge to the predictions of linearized quantum gravity in the regime of low-energy tabletop experiments. Therefore, if future experiments witness the entanglement production but the result is incompatible with the two effects we predicted, then it would suggest the presence of something new or highly unexpected for quantum gravity.

It is worth noting that a similar analysis can be conducted for the electromagnetic field, mirroring the methodology in this paper but technically much simpler. The electromagnetic version of these effects may be within the reach of experiments and could serve as a compelling proof-of-concept demonstration for the physical effects identified in this paper. Another possibility for a proof-of-principle experiment is to use an optomechanical system with phonon propagation, e.g., along the lines of Ref. [71]. The study of a concrete experimental implementation of these ideas is left for future work.

In conclusion, we have shown for the first time that, even in the case of static quantum sources of gravity, there are gravitational effects, observable in future tabletop experiments, that give stronger evidence of the quantum nature of gravity than those only requiring the Newton potential. Specifically, such effects cannot be explained using the Newton potential nor, to the best of our knowledge, any known model in which gravity is classical. Identifying these effects is crucial from a conceptual perspective because they increase the impact of current efforts to test the quantum nature of gravity. Identifying and eventually realizing alternative experiments testing stronger and diverse quantum effects in the gravitational field would open an observational window on quantum effects in gravity.

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APPENDIX A: QUANTUM STATE OF THE GRAVITATIONAL FIELD FOR A LOCALIZED SEMICLASSICAL SOURCE

Here, we review the quantization procedure of the weak gravitational field of Ref. [35]. Tabletop experiments for quantum tests of gravity can be described in the regime of a weak gravitational field with nonrelativistic matter sources. In order to provide a suitable theoretical framework for those experiments, we quantize linearized gravity in the field basis [72]. To the first order of the perturbations around the flat metric, i.e., $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, the classical action of linearized gravity [73] coupled to a matter field reads

$$S = \frac{1}{4\kappa}\int d^4x(-\partial_\mu h_{\alpha\beta}\partial^\mu h^{\alpha\beta} + \partial_\mu h\partial^\mu h - 2\partial_\mu h^{\mu\nu}\partial_\nu h + 2\partial_\alpha h_{\mu\nu}\partial^\mu h^{\alpha\nu}) + \frac{1}{2}\int d^4x h_{\mu\nu}T^{\mu\nu}, \tag{A1}$$

in which $\kappa = 16\pi G/c^4$. For canonical quantization, the metric is cast in the $3+1$ decomposition: $ds^2 = -N^2dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$, in which γ_{ij} is the metric of the spacelike foliation. Here, N and N^i are the lapse and the shift vector, respectively; the lapse is a gauge parameter corresponding to the rate of time flow between different foliations, and the components of the shift vector are the gauge parameters relating the spatial coordinates on different foliations at different times. The perturbation in the $3+1$ decomposition is expressed as $\gamma_{ij} = \delta_{ij} + h_{ij}$, $N = 1 + n$, $N^i = 0 + n^i$; therefore, $h_{00} = -2n$, $h_{0j} = \delta_{ij}n^i = n_j$, $h^{0i} = -n^i$. From the action, one can derive the canonical momenta: $\pi^{kl} = [1/(2\kappa)](\dot{h}^{kl} - \dot{h}\delta^{kl})$. The canonical momenta and the linearized gravitational field satisfy the Poisson brackets: $\{h_{ij}(\vec{x}), \pi^{kl}(\vec{x}')\} = \delta^k_{(i}\delta^l_{j)}\delta^3(\vec{x} - \vec{x}')$.

An important feature of our quantization procedure is that we fix the gauge minimally. We only perform the partial gauge fixing $\partial_i n_j = 0$ and $\partial_i n = 0$ and keep spatial diffeomorphism invariance. We obtain the Hamiltonian

$$H_{G+S} = \kappa\int d^3x(\pi_{kl}\pi^{kl} - \pi^2/2) + \frac{1}{4\kappa}\int d^3x\Big[\partial_k h_{ij}\partial^k h^{ij} - \partial_i h\partial^i h - 2\partial^k h_{ik}(\partial_j h^{ij} - \partial^i h) + \frac{1}{2}h_{ij}T^{ij} - 4n_i\mathcal{G}^i - 4n\mathcal{C}\Big], \tag{A2}$$

where the perturbations of the lapse n and the shift vector n_i become Lagrangian multipliers imposing four commuting constraints. In particular, the vector constraints $\mathcal{G}^i$ and scalar constraint $\mathcal{C}_\rho$ are

$$\mathcal{G}^i = 2\partial_j \pi^{ij} + T^{0i} = 0, \quad \mathcal{C}_\rho = -\partial_i \partial^i h^T - \kappa T^{00} = 0. \tag{A3}$$

Since the matter source is quasistatic in the laboratory frame, T^{0i} and T^{ij} are negligible compared to T^{00}. Therefore, the information on the matter source is encoded in the gravitational field only through the scalar constraint $\mathcal{C}_\rho$. If the source S is in a quantum state $|\alpha_i\rangle_S$, i.e., a semiclassical (i.e., no position-momentum commutator effect is measurable on the quantum state up to experimental resolution) and localized quantum state, it is approximately an eigenstate of $\hat{T}_{00}$. In this case, we have [see Eq. (4)]

$$\hat{T}_{00}|\alpha_i\rangle_S \approx \rho_i(x,t)|\alpha_i\rangle_S \approx mc^2\delta^3(\vec{x}-\vec{x}_i)|\alpha_i\rangle_S, \tag{A4}$$

which describes a pointlike particle with rest mass m. The trace of the transverse components of the metric perturbation, h_T, is obtained by solving the Poisson equation through the scalar constraint $\mathcal{C}_\rho$:

$$\begin{aligned} \mathrm{h}^T_{\rho_i}(\vec{x}) &= \frac{\kappa}{4\pi}\int d^3y \frac{\rho_i(\vec{y})}{|\vec{x}-\vec{y}|} + f(\vec{x}) \\ &= \frac{\kappa}{4\pi}\frac{mc^2}{|\vec{x}-\vec{x_i}|} + f(\vec{x}). \end{aligned} \tag{A5}$$

Here, $f(\vec{x})$ can be an arbitrary harmonic function, and it is fixed by the spacetime boundary conditions.

For a quasistatic source, the full Hamiltonian of the gravitational field interacting with the source in Eq. (A2) becomes source-free as in Eq. (11), namely,

$$\begin{aligned} \hat{H}_G = \kappa &\int \frac{d^3k}{(2\pi)^3}(\hat{\pi}_{kl}\hat{\pi}^{kl} - \hat{\pi}^2/2) \\ &+ \frac{1}{4\kappa}\int \frac{d^3k}{(2\pi)^3}\left(k^2\hat{h}^T_{ij}(\vec{k})\hat{h}^{ij}_T(-\vec{k}) - k^2\hat{h}_T(\vec{k})\hat{h}_T(-\vec{k})\right). \end{aligned} \tag{A6}$$

This Hamiltonian can be quantized using the Dirac prescription for the linearized gravitational field (for further details, see Ref. [35]). For a static (semiclassical and localized) source, the quantum state of the gravitational field is the ground state of the gravity Hamiltonian $\hat{H}_G$, and additionally, it has to satisfy the scalar and vector constraints. The quantum state of the source and its gravitational field, in both h_{ij} and π_{ij} representations, reads

$$\begin{aligned} |\Psi\rangle_{S+G} &= \eta\int \mathcal{D}[h_{ij}]\delta[h^T - \mathrm{h}^T_{\rho_\alpha}]\exp\left\{-\frac{1}{4\kappa\hbar}\int\frac{d^3k}{(2\pi)^3}|\vec{k}|h^T_{ij}(\vec{k})h^{ij}_T(-\vec{k})\right\}|\alpha\rangle_S|h_{ij}\rangle_G \\ &= \eta'\int \mathcal{D}[\pi_{ij}]\exp\left\{-\frac{i}{2\hbar}\int\frac{d^3k}{(2\pi)^3}\pi_T(\vec{k})\mathrm{h}^T_\rho(\vec{k}) - \frac{\kappa}{\hbar}\int\frac{d^3k}{(2\pi)^3}\frac{1}{|k|}\left(\pi^T_{ij}(\vec{k})\pi^{ij}_T(-\vec{k}) - \frac{1}{2}\pi_T(\vec{k})\pi_T(-\vec{k})\right)\right\}|\alpha\rangle_S|\pi_{ij}\rangle_G. \end{aligned} \tag{A7}$$

In a more compact notation, since the source is in an eigenstate of $\hat{T}_{00}$, we can denote the full state as $|\Psi\rangle_{S+G} \coloneqq |\alpha\rangle_S|g_\alpha\rangle_G$. If one superposes different quantum states of such matter sources, each of which is centered around a different position x_i, i.e., $|\psi\rangle_S = \sum_i c_i|\alpha_i\rangle_S$, we can solve the Gauss constraint locally for each quantum state $|\alpha_i\rangle$. The resulting solution is a superposition of classical spacetimes, formally expressed as

$$|\Psi\rangle_{S+G} \coloneqq \sum_i c_i|\alpha_i\rangle_S|g_{\alpha_i}\rangle_G. \tag{A8}$$

This result is exactly the description for the matter source and the gravitational field considered in the GIE proposals. The states of the source correspond to semiclassical configurations of matter where the position is localized around a value x_i and the momentum is localized around $p = 0$.

Let us consider two such localized semiclassical quantum states of the source S, $|\alpha_x\rangle_S$ and $|\alpha_{x+\epsilon}\rangle_S$, which are related by a translation and whose central positions differ by an arbitrarily small amount ϵ. The solutions of the Poisson equation, respectively, $\mathrm{h}^T_{\alpha_x}(\vec{x})$ and $\mathrm{h}^T_{\alpha_{x+\epsilon}}(\vec{x})$, are also related by a translation. In particular, the representation of the quantum state in momentum space only differs by a phase $\mathrm{h}^T_{\alpha_x}(\vec{k}) = e^{-ik\epsilon}\mathrm{h}^T_{\alpha_{x+\epsilon}}(\vec{k})$. Such a small difference leads to a vanishing scalar product between the respective full quantum states of matter and gravity, i.e.,

$$\begin{aligned} \langle\Psi_x|\Phi_{x+\epsilon}\rangle_{S+G} &= \eta^2\int \mathcal{D}[\pi^T]\mathcal{D}[\tilde{\pi}^T_{ij}]\mathcal{D}[\pi^L_{ij}] \\ &\quad\times e^{-\frac{i}{2\hbar}\int\frac{d^3k}{(2\pi)^3}\pi_T(\vec{k})(\mathrm{h}^T_{\alpha_x}(\vec{k})-\mathrm{h}^T_{\alpha_{x+\epsilon}}(\vec{k}))} \\ &\quad\times e^{-\frac{2\kappa}{\hbar}\int\frac{d^3k}{(2\pi)^3}\frac{1}{|k|}\tilde{\pi}^T_{ij}(\vec{k})\tilde{\pi}^{ij}_T(-\vec{k})}\langle\alpha_x|\alpha_{x+\epsilon}\rangle \\ &= \delta(\mathrm{h}^T_{\alpha_x}(\vec{k}) - \mathrm{h}^T_{\alpha_{x+\epsilon}}(\vec{k}))\langle\alpha_x|\alpha_{x+\epsilon}\rangle = 0. \end{aligned} \tag{A9}$$

The quantum states of the sources, however, have a non-negligible overlap, i.e., $\langle\alpha_x|\alpha_{x+\epsilon}\rangle \neq 0$; hence, we would expect the scalar product of the source and its gravitational field to be nonvanishing. The vanishing of Eq. (A9) means that the approximation $\hat{T}_{00}|\alpha_i\rangle_S \approx \rho_{cl}(\vec{x})|\alpha_i\rangle_S$, with $\rho_{cl}(\vec{x})$ being the classical energy density, only holds for matter sources prepared in perfectly distinguishable states. Crucially, the vanishing of the scalar product only depends on approximating the eigenvalue of $\hat{T}_{00}$ with the classical function and not on the specific form of the function. For technical simplicity, we have used $\rho_{cl}(\vec{x}) = mc^2\delta^3(\vec{x}-\vec{x}_i)$, but the scalar product would have also vanished had we used a more realistic energy density, for instance, $\rho_{cl}(\vec{x}) = [(mc^2)/\sigma]e^{-\frac{(\vec{x}-\vec{x}_i)^2}{2\sigma^2}}$.

APPENDIX B: DETAILS OF THE CALCULATIONS OF THE SCALAR PRODUCT

Given two generic quantum sources $|\psi\rangle_S$ and $|\varphi\rangle_S$, the inner product of the full quantum state of the gravitational field, together with the source, is

$$\begin{aligned}\langle\Psi_\psi|\Psi_\phi\rangle_{S+G} &= \eta^2 \int dx d\mu(\pi_{ij}) d\mu(E) d\mu(E') \psi_E \phi^*_{E'} \langle E'|x\rangle\langle x|E\rangle e^{-\frac{i}{2\hbar}\int\frac{d^3k}{(2\pi)^3}\pi_T(\vec{k})(\mathrm{h}^T_E(\vec{k})-\mathrm{h}^T_{E'}(\vec{k}))} |\Psi_{\mathrm{vac}}[\pi_{ij}]|^2 \\ &= \eta^2 \int d\mu(\pi_T) d\mu(E) d\mu(E') \psi_E \phi^*_{E'} \langle E'|E\rangle e^{-\frac{i}{2\hbar}\int\frac{d^3k}{(2\pi)^3}\pi_T(\vec{k})(\mathrm{h}^T_E(\vec{k})-\mathrm{h}^T_{E'}(\vec{k}))} \int d\mu(\tilde{\pi}^T_{ij}) d\mu(\pi^L_{ij}) |\Psi_{\mathrm{vac}}[\pi_{ij}]|^2 \\ &= \int d\mu(E)\psi_E\phi^*_E = \langle\psi|\phi\rangle. \end{aligned} \tag{B1}$$

APPENDIX C: DETAILS OF THE CALCULATIONS OF THE QUANTUM COMMUTATOR

In this appendix, we detail the steps to obtain the relative phase in Sec. V due to the commutator of the gravitational-field operators

$$[\hat{h}_{ij}(\vec{k}), \hat{\pi}^{kl}(\vec{p}')] = i\hbar\delta^k_{(i}\delta^l_{j)}\delta^3(\vec{k}+\vec{p}'). \tag{C1}$$

The Hamiltonian of the gravitational field is given in Eq. (11),

$$\hat{H}_G = \kappa\int\frac{d^3k}{(2\pi)^3}\left(\hat{\pi}_{kl}(\vec{k})\hat{\pi}^{kl}(-\vec{k}) - \hat{\pi}^2(\vec{k})/2\right) + \frac{1}{4\kappa}\int\frac{d^3k}{(2\pi)^3}\left(k^2\hat{h}^T_{ij}(\vec{k})\hat{h}^{ij}_T(-\vec{k}) - k^2\hat{h}_T(\vec{k})\hat{h}_T(-\vec{k})\right), \tag{C2}$$

and the interaction is described in Eq. (32),

$$\hat{H}_I = -\frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\hat{h}^{ij}(\vec{k})\hat{T}^P_{ij}(-\vec{k}). \tag{C3}$$

The time evolution operator in Eq. (34), $\hat{U}_{\mathrm{SGP}}(t) \coloneqq e^{-\frac{it}{\hbar}\hat{H}} = e^{-\frac{it}{\hbar}(\hat{H}_S+\hat{H}_P+\hat{H}_G+\hat{H}_I)}$, can be expanded using the Zassenhaus formula. Specifically, for two general quantum operators $\hat{A}$ and $\hat{B}$, we have

$$e^{\hat{A}+\hat{B}} = e^{\hat{A}}e^{\hat{B}}e^{-\frac{1}{2}[\hat{A},\hat{B}]}e^{\frac{1}{6}([\hat{A},[\hat{A},\hat{B}]]+2[\hat{B},[\hat{A},\hat{B}]])}e^{-\frac{1}{24}([[[\hat{A},\hat{B}],\hat{A}],\hat{A}]+3[[[\hat{A},\hat{B}],\hat{A}],\hat{B}]+3[[[\hat{A},\hat{B}],\hat{B}],\hat{B}])}\dots. \tag{C4}$$

The relevant terms for the physical effect we are studying are those coming from the gravitational commutator. Hence, we do not explicitly calculate the terms coming from the commutators $[\hat{x}, \hat{p}]$ of the source and the probe, contained in $\hat{H}_S$ and $\hat{H}_P$. Expanding the time evolution operator through the Zassenhaus formula, we obtain

$$\hat{U}_{\mathrm{SGP}}(t) = e^{-\frac{it}{\hbar}(\hat{\mathcal{H}}_S+\hat{\mathcal{H}}_P)}e^{-\frac{it}{\hbar}\hat{\mathcal{H}}_G}e^{-\frac{it}{\hbar}\hat{\mathcal{H}}_I}e^{\frac{t^2}{2\hbar^2}[\hat{\mathcal{H}}_G,\hat{\mathcal{H}}_I]}e^{\frac{it^3}{6\hbar^3}([\hat{\mathcal{H}}_G,[\hat{\mathcal{H}}_G,\hat{\mathcal{H}}_I]]+2[\hat{\mathcal{H}}_I,[\hat{\mathcal{H}}_G,\hat{\mathcal{H}}_I]])}e^{-\frac{t^4}{24\hbar^4}[[[\hat{\mathcal{H}}_G,\hat{\mathcal{H}}_I],\hat{\mathcal{H}}_G],\hat{\mathcal{H}}_G]}\dots. \tag{C5}$$

We assume that the probe as a test particle does not backreact to the gravitational field. Therefore, the interaction with the probe does not take the quantum state of the gravitational field out of the constraint surface. Formally, this means that the commutator between the interaction Hamiltonian and the vector constraint is effectively negligible,

$$[\hat{h}^{lm}(\vec{k}), k'_i\hat{\pi}^{ij}(\vec{k}')]|\Psi\rangle_{S+G} = [\hat{h}^{lm}(\vec{k}), \hat{\pi}^{ij}_L(\vec{k}')]|\Psi\rangle_{S+G} = 0. \tag{C6}$$

Together with the condition that the quantum gravitational state satisfies the constraint $\hat{\mathcal{G}}_i|\Psi\rangle_{S+G} = 0$, we have

$$\begin{aligned}[\hat{H}_G, \hat{H}_I]|\Psi\rangle_{S+G} &= \left[\kappa \int \frac{d^3k}{(2\pi)^3}\left(\hat{\pi}_{kl}(\vec{k})\hat{\pi}^{kl}(-\vec{k}) - \frac{1}{2}\hat{\pi}(\vec{k})\hat{\pi}(-\vec{k})\right), -\frac{1}{2}\int \frac{d^3k'}{(2\pi)^3}\hat{h}^{ij}(\vec{k}')\hat{T}^P_{ij}(-\vec{k}')\right]|\Psi\rangle_{S+G} \\ &= i\hbar\kappa \int \frac{d^3k}{(2\pi)^3}\left(\hat{\pi}^T_{ij}(\vec{k}) - \frac{1}{2}P_{ij}\hat{\pi}^T(\vec{k})\right)\hat{T}^{ij}_P(-\vec{k})|\Psi\rangle_{S+G} \\ &= i\hbar\kappa \int \frac{d^3k}{(2\pi)^3}\hat{\tilde{\pi}}^T_{ij}(\vec{k})\hat{T}^{ij}_P(-\vec{k})|\Psi\rangle_{S+G},\end{aligned} \tag{C7}$$

where $\tilde{\pi}^T_{ij}(\vec{k})$ is the transverse-traceless part of the momentum of the gravitational field. There are two double commutators: $[\hat{H}_G, [\hat{H}_G, \hat{H}_I]]$ and $[\hat{H}_I, [\hat{H}_G, \hat{H}_I]]$. When they act on the physical state, we obtain

$$\begin{aligned}[\hat{H}_G, [\hat{H}_G, \hat{H}_I]]|\Psi\rangle_{S+G} &= \frac{i\hbar}{4}\left[\int \frac{d^3k}{(2\pi)^3}k^2(\hat{h}^T_{ij}(\vec{k})\hat{h}^{ij}_T(-\vec{k}) - \hat{h}_T(\vec{k})\hat{h}_T(-\vec{k})), \int \frac{d^3k'}{(2\pi)^3}\left(\hat{\pi}^T_{kl}(\vec{k}') - \frac{P_{kl}}{2}\hat{\pi}^T(\vec{k}')\right)\hat{T}^{kl}_P(-\vec{k}')\right]|\Psi\rangle_{S+G} \\ &= -\frac{\hbar^2}{4}\int \frac{d^3k}{(2\pi)^3}k^2(2\hat{h}^T_{ij}(\vec{k}) - P_{ij}\hat{h}^T(\vec{k}))\hat{T}^{ij}_P(-\vec{k})|\Psi\rangle_{S+G} \\ &= -\frac{\hbar^2}{2}\int \frac{d^3k}{(2\pi)^3}k^2\hat{\tilde{h}}^T_{ij}(\vec{k})\hat{T}^{ij}_P(-\vec{k})|\Psi\rangle_{S+G},\end{aligned} \tag{C8}$$

in which $\tilde{h}^T_{ij}$ is the transverse-traceless part of the metric perturbation. The other double commutator is

$$\begin{aligned}[\hat{\mathcal{H}}_I, [\hat{\mathcal{H}}_G, \hat{\mathcal{H}}_I]]|\Psi\rangle_{S+G} &= \left[-\frac{1}{2}\int \frac{d^3k'}{(2\pi)^3}\hat{h}^{ij}(\vec{k}')\hat{T}^P_{ij}(-\vec{k}'), i\hbar\kappa \int \frac{d^3k}{(2\pi)^3}\left(\hat{\pi}^T_{ij}(\vec{k}) - \frac{1}{2}P_{ij}\hat{\pi}^T(\vec{k})\right)\hat{T}^{ij}_P(-\vec{k})\right]|\Psi\rangle_{S+G} \\ &= \frac{\kappa\hbar^2}{2}\int \frac{d^3k}{(2\pi)^3}\left[\hat{T}^{ij}_P(\vec{k})P^k_iP^l_j\hat{T}_{kl}(-\vec{k}) - \frac{1}{2}(P_{ij}\hat{T}^{ij}_P(\vec{k}))^2\right]|\Psi\rangle_{S+G} \\ &= \frac{\kappa\hbar^2}{2}\int \frac{d^3k}{(2\pi)^3}\hat{\tilde{T}}^{PT}_{ij}\hat{\tilde{T}}^{ij}_{PT}|\Psi\rangle_{S+G}.\end{aligned} \tag{C9}$$

In the last line, we have used a compact notation $\tilde{T}^{ij}_{PT}$ to denote the transverse-traceless part of the stress-energy tensor of the probe P (we drop the hat for convenience of notation). Note that this double commutator does not depend on specific details of the source and its corresponding quantum gravitational field.

To obtain the final expression, it is convenient to expand the final quantum state of the full system in terms of the source energy eigenstates since they diagonalize the Gauss constraint,

$$\begin{aligned}|\Psi_t\rangle_{\mathrm{SGP}} &= \int d\mu(E_S)\psi_S(E)\hat{U}_{\mathrm{SGP}}|E\rangle_S|g_E\rangle_G|\psi\rangle_P \\ &= \int d\mu(E_S)d\tilde{\mu}(E'_{S+G})\psi_S(E)|E'\rangle_S|g_{E'}\rangle_G\langle g_{E'}|\langle E'|\hat{U}_{\mathrm{SGP}}|E\rangle_S|g_E\rangle_G|\psi\rangle_P \\ &= \int d\mu(E_S)d\tilde{\mu}(E'_{S+G})\psi_S(E)\langle\hat{U}_{\mathrm{SGP}}\rangle_{EE'}|E'\rangle_S|g_{E'}\rangle_G|\psi\rangle_P,\end{aligned} \tag{C10}$$

in which we have inserted the resolution of identity in the second line. Therefore, to obtain the entangling phase from the evolution, we need to evaluate the matrix element of the time evolution operator $\langle\hat{U}_{\mathrm{SGP}}\rangle_{EE'}$. A requirement to observe the phase is that the experimental timescale is much smaller compared to the coherence time. Here, we truncate the expansion to the t^3 order and neglect higher orders in time. Note that considering higher order terms would add additional terms to the phase, but it would not change the final dependence of the relative phase on the commutator of the gravitational-field operators. We use a bookkeeping parameter α to keep track of the order of the commutator; i.e., we take $[\hat{h}_{ij}(\vec{k}), \hat{\pi}^{kl}(\vec{p}')] = i\alpha\hbar\delta^k_{(i}\delta^l_{j)}\delta^3(\vec{k}+\vec{p}')$. For the final result, we then set $\alpha = 1$. Putting everything together, we have

$$
\begin{aligned}
\langle \hat{U}_{\mathrm{SGP}} \rangle_{EE'} &:= \langle g_E | \langle E | \hat{U}_{\mathrm{SGP}}(t) | E' \rangle_S | g_{E'} \rangle_G \\
&= \langle e^{-\frac{it}{\hbar}(\hat{\mathcal{H}}_S + \hat{\mathcal{H}}_P)} e^{-\frac{it}{\hbar}\hat{\mathcal{H}}_G} e^{-\frac{it}{\hbar}\hat{\mathcal{H}}_I} e^{\frac{t^2\alpha}{2\hbar^2}[\hat{\mathcal{H}}_G, \hat{\mathcal{H}}_I]} e^{\frac{it^3\alpha^2}{6\hbar^3}([\hat{\mathcal{H}}_G,[\hat{\mathcal{H}}_G,\hat{\mathcal{H}}_I]] + 2[\hat{\mathcal{H}}_I,[\hat{\mathcal{H}}_G,\hat{\mathcal{H}}_I]])} \rangle_{EE'} \\
&= e^{-\frac{it}{\hbar}(\hat{\mathcal{H}}_P + E_S + E_G)} e^{\frac{it^3\alpha^2\kappa}{6\hbar}\int \frac{dk^3}{(2\pi)^3} \hat{\tilde{T}}^{ij}_{PT} \hat{\tilde{T}}^{PT}_{ij}} \langle e^{\frac{it}{2\hbar}\int \frac{d^3k}{(2\pi)^3}\hat{h}_{ij}\hat{T}^{ij}_P} e^{\frac{i\kappa t^2\alpha}{2\hbar}\int \frac{d^3k}{(2\pi)^3}(\hat{\pi}^T_{ij} - \frac{1}{2}P_{ij}\hat{\pi}^T)\hat{T}^{ij}_P} e^{-\frac{it^3\alpha^2}{12\hbar}\int \frac{d^3k}{(2\pi)^3} k^2 \tilde{h}^T_{ij}\hat{T}^{ij}_P} \rangle_{EE'}.
\end{aligned}
\tag{C11}
$$

Up to this point, we have isolated the operators of the gravitational field in $\langle \ldots . \rangle_{EE'}$. For convenience in the calculation, we evaluate them in the π_{ij} basis and express the metric operator as a functional derivative $\hat{h}_{ij}(\vec{x}) = i\hbar \frac{\delta}{\delta\pi_{ij}(\vec{x})}$. Therefore, the part of the matrix element that involves gravitational operators is [74]

$$
\begin{aligned}
&\langle e^{\frac{it}{2\hbar}\int \frac{d^3k}{(2\pi)^3}\hat{h}_{ij}\hat{T}^{ij}_P} e^{\frac{i\kappa t^2\alpha}{2\hbar}\int \frac{d^3k}{(2\pi)^3}(\hat{\pi}^T_{ij} - \frac{1}{2}P_{ij}\hat{\pi}^T)\hat{T}^{ij}_P} \rangle_{EE'} \\
&= \eta^2 e^{-\frac{it}{4\hbar}\int \frac{dk^3}{(2\pi)^3} \mathrm{h}^T_{E'} P_{ij} \hat{T}^{ij}_P} \int \mathcal{D}[\pi_{ij}] e^{\frac{t\kappa}{\hbar}\int \frac{dk^3}{(2\pi)^3}\frac{1}{|k|}\tilde{\pi}^T_{ij}\hat{T}^{ij}_P - \frac{t^2\kappa}{4\hbar}\int \frac{d^3k}{(2\pi)^3}\frac{1}{|k|}\hat{\tilde{T}}^{ij}_{PT}\hat{\tilde{T}}^{PT}_{ij} + \frac{i\kappa t^2\alpha}{2\hbar}\int \frac{d^3k}{(2\pi)^3}\tilde{\pi}^T_{ij}\hat{T}^{ij}_P} \Psi_G[\pi_{ij}, E]\Psi^*_G[\pi_{ij}, E']\delta(E - E') \\
&= e^{-\frac{it}{4\hbar}\int \frac{dk^3}{(2\pi)^3}\mathrm{h}^T_{E'}(\vec{k}) P_{ij}\hat{T}^{ij}_P(-\vec{k})} e^{-\frac{t^2\kappa}{4\hbar}\int \frac{d^3k}{(2\pi)^3}\frac{1}{|k|}\hat{\tilde{T}}^{ij}_{PT}(\vec{k})\hat{\tilde{T}}^{PT}_{ij}(-\vec{k})} e^{\frac{t^2\kappa}{8\hbar}\int \frac{d^3k}{(2\pi)^3}\frac{1}{|k|}\hat{T}^P_{ij}(\vec{k})\hat{T}^{ij}_P(-\vec{k})(1 + \frac{it\alpha}{2}|k|)^2},
\end{aligned}
\tag{C12}
$$

where we have used the same normalization condition as in Ref. [35], which determines η,

$$
\eta^2 \prod_k \int d\pi^L_{ij}(\vec{k}) d\tilde{\pi}^T_{ij}(\vec{k}) d\pi^T(\vec{k}) e^{-\frac{2\kappa}{(2\pi)^3\hbar|\vec{k}|}(\tilde{\pi}^T_{ij}(\vec{k}) + bT^P_{ij})(\tilde{\pi}^{ij}_T(-\vec{k}) + bT^{ij}_P)} = \eta^2 \prod_k 2\pi^2 \sqrt{\frac{\hbar|k|}{\kappa}} \int d\pi^L_{ij}(\vec{k}) d\pi^T(\vec{k}) \equiv 1.
\tag{C13}
$$

In the Gaussian integral above, b is an arbitrary function that does not depend on π_{ij}. In our case, $b = -(t/4)\{1 + [(it\alpha)/2]\}|k|)$. Therefore, we can write down the final state, Eq. (C10), explicitly as

$$
\begin{aligned}
|\Psi_t\rangle_{\mathrm{SGP}} = \int d\mu(E_S)\psi_S(E) e^{-\frac{it}{\hbar}(\hat{\mathcal{H}}_P + \hat{\mathcal{H}}_S + \hat{\mathcal{H}}_G)} e^{-\frac{it}{4\hbar}\int \frac{dk^3}{(2\pi)^3}\mathrm{h}^T_E P_{ij}\hat{T}^{ij}_P} e^{\frac{it^3\kappa\alpha^2}{6\hbar}\int \frac{dk^3}{(2\pi)^3}(\hat{T}^{ij}_P P^k_i P^l_j \hat{T}^P_{kl} - \frac{1}{2}(P_{ij}\hat{T}^{ij}_P)^2)} \cdot \\
\cdot\, e^{-\frac{t^2\kappa}{4\hbar}\int \frac{d^3k}{(2\pi)^3}\frac{1}{|k|}\hat{\tilde{T}}^{ij}_{PT}(\vec{k})\hat{\tilde{T}}^{PT}_{ij}(-\vec{k})} e^{\frac{t^2\kappa}{8\hbar}\int \frac{d^3k}{(2\pi)^3}\frac{1}{|k|}T^P_{ij}(\vec{k})T^{ij}_P(-\vec{k})(1 + \frac{it\alpha}{2}|k|)^2} |E\rangle_S |g_E\rangle_G |\psi\rangle_P.
\end{aligned}
\tag{C14}
$$

We isolate the phase operator that depends on the interaction with the probe, which contains the relevant signature that we wish to describe. We denote as θ_{free} the phase obtained from the free Hamiltonian of the probe, source, and gravity. We additionally identify the interaction part of the phase with the operators $\hat{\Theta}^{(n)}$, with $n = 0, 1, 2$, where the label n denotes the nth commutator of the gravitational-field operators:

$$
|\Psi_t\rangle_{\mathrm{SGP}} = \int d\mu(E)\psi_S(E) e^{-\frac{it}{\hbar}\theta_{\mathrm{free}}} e^{\frac{i}{\hbar}(\hat{\Theta}^{(0)} + \hat{\Theta}^{(1)} + \hat{\Theta}^{(2)} \ldots)} |E\rangle_S |g_E\rangle_G |\psi\rangle_P.
\tag{C15}
$$

The zeroth order $\Theta^{(0)}$ reads

$$
\Theta^{(0)} = -\frac{t}{4}\int \frac{dk^3}{(2\pi)^3}\mathrm{h}^T_E(\vec{k}) P_{ij}\hat{T}^{ij}_P(-\vec{k}) - \frac{i\kappa t^2}{8}\int \frac{d^3k}{(2\pi)^3}\frac{1}{|k|}(\hat{T}^P_{ij}(\vec{k})\hat{T}^{ij}_P(-\vec{k}) - 2\hat{\tilde{T}}^{ij}_{PT}(\vec{k})\hat{\tilde{T}}^{PT}_{ij}(-\vec{k})).
\tag{C16}
$$

The first term is the additional entangling phase to Eq. (22) coming from the coupling between the gravitational field and the momentum of the probe. The second term in Eq. (C16) is not a phase but an overall real damping factor.

The first-order correction $\Theta^{(1)}$ to the phase coming from a single gravitational commutator is

$$
\Theta^{(1)} = \frac{\kappa t^3}{8}\int \frac{d^3k}{(2\pi)^3}\hat{T}^P_{ij}(\vec{k})\hat{T}^{ij}_P(-\vec{k}).
\tag{C17}
$$

This quantum correction is proportional to t^3. This term depends on the stress-energy tensor of the probe expressed in the frame corresponding to the temporal gauge.

The correction $\Theta^{(2)}$ coming from the second order of the gravitational commutator is

$$\Theta^{(2)} = \frac{\kappa t^3}{6}\int \frac{dk^3}{(2\pi)^3}\left(\hat{T}^{ij}_P P^k_i P^l_j \hat{T}^P_{kl} - \frac{1}{2}(P_{ij}\hat{T}^{ij}_P)^2\right)$$
$$= \frac{\kappa t^3}{6}\int \frac{d^3k}{(2\pi)^3}\left(\hat{\tilde{T}}^{PT}_{ij}(\vec{k})\hat{\tilde{T}}^{ij}_{PT}(-\vec{k})\right), \qquad \text{(C18)}$$

in which the tilde $\tilde{T}^{ij}_{PT}$ denotes the transverse-traceless component of T^{ij}_P. This term is also proportional to t^3. Since we have kept only terms up to order t^3, we have neglected the terms in $\Theta^{(1)}$ and $\Theta^{(2)}$ of the order t^4 and above.

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