We correct some errors in [CTW22].

The first one is concerned with Proposition 3.3. The constant C' should actually be $C'' = (1+|k|)^{\frac{\beta+1}{2}}\sqrt{\frac{C}{|k|(\beta+1)}}$ instead of $C' = \sqrt{\frac{C}{k(\beta+1)}}$. This does not affect the main results of the paper, because we only need that there is such a constant, and we do not need the specific dependence on k. On the other hand, this specific dependence could be useful for other purposes.

The previous value of C' works for ϕ_k with $k > 0$, but for ϕ_{-k} there is a sign mistake at the commutation relation between ϕ_{-k} and L_0 in the last displayed formula of the proof.

To see that the corrected C'' work for $-k$ (in the assumption k and $-k$ needs to be exchanged but it results in the same inequality) for Ψ_n a generic vector in V_n, we calculate

$$\begin{aligned}
\|\phi_{-k}(L_0+\mathbb{1})^{-\frac{\beta+1}{2}}\Psi_n\|^2 &= \|\phi_{-k}(n+\mathbb{1})^{-\frac{\beta+1}{2}}\Psi_n\|^2 \\
&\leq (1+k)^{\beta+1}\|\phi_{-k}(n+\mathbb{1}+k)^{-\frac{\beta+1}{2}}\Psi_n\|^2 \\
&= (1+k)^{\beta+1}\|(L_0+\mathbb{1})^{-\frac{\beta+1}{2}}\phi_{-k}\Psi_n\|^2 \\
&\leq (1+k)^{\beta+1}\|(L_0+\mathbb{1})^{-\frac{\beta+1}{2}}\phi_{-k}\|^2\cdot\|\Psi_n\|^2 \\
&\leq (1+k)^{\beta+1}\|\phi_k(L_0+\mathbb{1})^{-\frac{\beta+1}{2}}\|^2\cdot\|\Psi_n\|^2 \\
&\leq (1+k)^{\beta+1}(C')^2\|\Psi_n\|^2.
\end{aligned}$$

As $\phi_{-k}(L_0+\mathbb{1})^{-\frac{\beta+1}{2}}\Psi_n$ are mutually orthogonal for different n, for a general Ψ we obtain $\|\phi_{-k}(L_0+\mathbb{1})^{-\frac{\beta+1}{2}}\Psi\|^2 \leq (1+k)^{\beta+1}(C')^2\|\Psi\|^2$, which is what we had to prove with $C'' = (1+k)^{\frac{\beta+1}{2}}\sqrt{\frac{C}{k(\beta+1)}}$.

In addition, in Proposition 3.2, Theorem 3.5, Lemma 3.6, Theorem 3.7, the fields ϕ etc. should be assumed to be hermitian. Note however that in a unitary vertex operator algebra every primary field ϕ with conformal dimension d is equal to $\phi_1 + i\phi_2$ with ϕ_1 and ϕ_2 hermitian primary fields of the same dimension d.

References

[CTW22] Sebastiano Carpi, Yoh Tanimoto, and Mihály Weiner. *Comm. Math. Phys.*, Vol. 390, 169–192 (2022). `https://arxiv.org/abs/2103.16475`.