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**Anomalies,  $U(1)'$  and the MSSM**

Tesi di Dottorato di Ricerca in Fisica

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## Abstract

This Thesis reviews an extension of the MSSM by the addition of an anomalous abelian vector multiplet and contains some original results concerning the phenomenology of an anomalous  $Z'$ .

The review part covers an introduction of the MSSM focusing on its main features, a discussion on the chiral anomalies and how to cancel them in the Standard Model and by the Green-Schwarz mechanism.

Then, the original results are presented. We build the Lagrangian for the Minimal Anomalous  $U(1)'$  Extension of the MSSM where the anomalies are cancelled by the Green-Schwarz mechanism and the addition of Chern-Simons terms, stressing the main differences between our model and the MSSM. The advantage of this choice over the standard one is that it allows for arbitrary values of the quantum numbers of the extra  $U(1)$ . As a first step towards the study of hadron annihilations producing four leptons in the final state (a clean signal which might be studied at LHC) we then compute the decays  $Z' \rightarrow Z_0\gamma$  and  $Z' \rightarrow Z_0Z_0$ . We find that the largest values of the decay rate are  $\sim 10^{-4}$  GeV, while the expected number of events per year at LHC is at most of the order of 10. Then we compute the relic density predicted by our model with a new dark matter candidate, the axino, which is the LSP of the theory. We see that agreement with experimental data implies the assumption that the NLSP is a bino-like particle almost degenerate in mass to the axino.



“There are many path to the top of the mountain.  
But the view is always the same.”

*Asian proverb*

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# Chapter 1

## Introduction

The Standard Model (SM) of particle physics has been confirmed to a great accuracy in many experiments. Despite the fact that the Higgs particle remains experimentally elusive, few scientists doubt that there will be major surprises in this direction. The whole scientific community, however, knows that the SM needs to be improved. Apart from the experimental discover of a tiny neutrino mass, there are also several theoretical issues that make physicists believe that the SM is only an effective manifestation of a more Fundamental Theory.

We know that a new framework must appear at the Planck scale ( $\sim 10^{19}$  GeV) where a theory of quantum gravity reveals. Since the ratio between this scale and the electroweak (EW) scale ( $\sim 100$  GeV) is huge, all quantum corrections will turn out to be many orders of magnitude greater than tree values. This is the so called hierarchy problem, related to fine-tuning and naturalness. Quantum corrections are power-law (usually quadratic) divergent which means that the highest energy physics is most important. More technically, the question is why the Higgs boson is so much lighter than the Planck scale. Indeed one would expect that the large quadratically divergent quantum contributions to the square of the Higgs boson mass would be inevitably make the mass huge, unless there is a fine-tuned cancellation between the quadratic radiative corrections and the bare mass.

Besides the hierarchy problem, there some unexplained features in cosmological observations. Measurements constraint the quantity of baryon matter, dark matter and dark energy in the universe revealing that the known particles make up only a small fraction of the total energy density of the universe. So parti-

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cle physics must provide candidates for dark matter (DM) and dark energy and experiments to check the inherited models.

During the last years many ideas were used to overcome these problems. The most appealing theory is supersymmetry (SUSY), even if its historical motivation was to provide fermions in string theory. In 1981 it was proposed the Minimal Supersymmetric Standard Model (MSSM) i.e. the minimal extension of the SM which provides supersymmetry. SUSY pairs bosons with fermions, so each SM particle has a corresponding partner (unfortunately not discovered yet). SUSY removes the quadratic divergences of the radiative corrections to the Higgs mass. However, there is no understanding of why the Higgs mass is so small from the beginning ( $\mu$  problem). One of the most intriguing feature of the MSSM (and its extensions) is that it provides DM candidates with relic density compatible with experimental data. In fact, if  $R$ -parity is conserved, the lightest supersymmetric particle of the MSSM is stable and makes it a good cold dark matter (CDM) particle.

As cited before, string theory was the framework where SUSY was born. Moreover it is also the most appealing candidate for a quantum theory of gravity. One of the prediction of string theory is the existence of anomalous  $U(1)$ 's. In standard theories the chiral anomalies are cancelled satisfying that the trace of the product of the gauge generators is zero. Instead in string theory this is not always possible, so the  $U(1)$ 's are anomalous, but the theory is still consistent and anomaly free, thanks to the Green-Schwarz (GS) mechanism. These anomalous gauge fields must be massive, otherwise we already discovered them, and the mass is acquired with the Stückelberg mechanism which was the first attempt to give a vector mass preserving gauge invariance, but phenomenologically ruled out in the SM. This procedure involves an axion-like scalar field, which plays also an important role in the GS mechanism for the anomaly cancellation. D-brane models contain several anomalous abelian factors living on each stack of branes. In the presence of these anomalous  $U(1)$ 's, the GS couplings with the axions cancels mixed anomalies<sup>1</sup>, and the Stückelberg mixing renders the “anomalous” gauge fields massive. The masses depend non-trivially on the internal volumes and on other moduli, allowing the physical masses of the anomalous  $U(1)$  gauge bosons to be much smaller than the string scale (even at a few TeV range). An

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<sup>1</sup>Irreducible anomalies are cancelled by the tadpole cancellation.

important role is played by the so-called Generalized Chern-Simons terms (GCS) which are local gauge non-invariant terms. Indeed, these trilinear gauge bosons anomalous couplings are responsible for the cancellation of mixed anomalies between anomalous  $U(1)$ 's and non anomalous factors ensuring the consistency of the theory.

In this Thesis, we are interested in *anomaly related*  $Z'$  bosons and we present the results obtained in [1] and [2]. The theory is non-renormalizable since it has an energy cut-off related to the  $Z'$  mass. More precisely, we study an extension of the MSSM by the addition of an abelian vector multiplet  $V^{(0)}$  and we assume that generically all MSSM particles are charged with respect to the new  $U(1)$ . The anomalies are cancelled using the GS mechanism and the GCS terms, so the anomalous abelian boson becomes massive and behaves like a  $Z'$ .

In approximately one year, the Large Hadron Collider (LHC) at CERN will start to operate at energies of order of 14 TeV in the center of mass (CM). Apart from the search for the Higgs boson, it will probably give us some answers about the parameter space of the physics beyond the SM, the eventual existence of new particles, such as the  $Z'$ , and their nature.

This Thesis is organized as follows. In Chapter 2 we introduce the MSSM. We explain its main characteristics, focusing on the Renormalization Group Equations (RGEs) and the mass spectrum of the theory. In Chapter 3 we discuss the problem of the 1 loop chiral anomalies in field theory. We show how to cancel them with the standard procedure and with the GS mechanism, both in a standard and in a supersymmetric framework. In Chapter 4 we present our model. We discuss the Lagrangian and the differences between our theory and the MSSM. In Chapter 5 we analyze some phenomenological consequences of our model such as the  $Z'$  anomalous decays and the prediction of the relic density with a new dark matter candidate. In Chapter 6 we summarize the conclusions and the results of our work. Finally in the Appendix we give further details of the computations omitted in the main body of the Thesis.



## Chapter 2

# Overview of the MSSM

Supersymmetry is the only possible non trivial unification of internal and space-time symmetries compatible with quantum field theory. It emerges naturally in string theory but it can be introduced apart of it. A SUSY transformation turns a bosonic state into a fermionic state increasing its spin by  $1/2$ , and viceversa. The single particle states of a supersymmetric theory fall into irreducible representations of the SUSY algebra, called *supermultiplets* which, in turn can be organized into *superfield*. Each supermultiplet contains both fermion and boson states that are *superpartners* of each other. One may show that no particle of the SM can be the superpartner of another, so SUSY predicts a list of new particles, none of which has been discovered yet. In reason of this, superpartners cannot be mass degenerate and so SUSY must be a broken symmetry. There are several ways to introduce a breaking of SUSY, however the simplest way is to explicit break it adding *ad hoc* mass terms. This is done in the MSSM. The following treatment of the MSSM is based on the works of Martin [3] and Derendinger [4].

## 2.1 The superpotential

In a renormalizable supersymmetric field theory, the interactions and masses of all particles are determined just by their gauge transformation properties and by the superpotential  $W$ .  $W$  is an analytic function of the complex chiral superfield  $\Phi$ . The gauge quantum numbers and mass dimensions of a chiral superfield are given by these of its scalar component. In the superfield formulation we write the superpotential as

$$W = \frac{1}{2}M^{ij}\Phi_i\Phi_j + \frac{1}{6}y^{ijk}\Phi_i\Phi_j\Phi_k \quad (2.1)$$

Given the supermultiplet content of the theory, the form of the superpotential is guided by the requirement of gauge invariance, so only a subset of the parameters  $M^{ij}$  and  $y^{ijk}$  are allowed to be different from zero. The entries of the mass matrix  $M^{ij}$  can be non zero only for  $i$  and  $j$  such that the superfield  $\Phi_i$  and  $\Phi_j$  have gauge transformation conjugates of each other. Similarly for the Yukawa couplings  $y^{ijk}$ . The interactions implied by the superpotential were shown in Fig. 2.1-2.2.

The particle content of the MSSM is given in Table 2.1-2.2. The superpotential for the MSSM is

$$W_{\text{MSSM}} = y_u^{ij}Q_iU_j^cH_u - y_d^{ij}Q_iD_j^cH_d - y_e^{ij}L_iE_j^cH_d + \mu H_uH_d \quad (2.2)$$

where the indices  $i, j$  run over the three families. For the complete Lagrangian of the MSSM see Appendix B.1.

All of the gauge indices in eq. (2.2) are suppressed. The “ $\mu$  term”, as it is usually called, can be written out as  $\mu(H_u)_\alpha(H_d)_\beta\epsilon^{\alpha\beta}$ , where  $\epsilon^{\alpha\beta}$  is used to tie together  $SU(2)_L$  weak isospin indices  $\alpha, \beta = 1, 2$  in a gauge-invariant way. Similarly for the terms  $Q_iH_u$ ,  $Q_iH_d$  and  $L_iH_d$ .

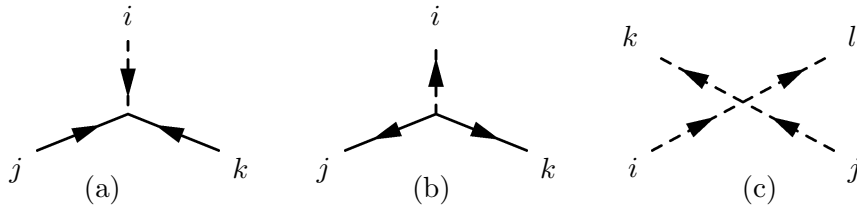


Figure 2.1: The dimensionless non-gauge interaction vertices in a supersymmetric theory: (a) scalar-fermion-fermion Yukawa interaction  $y^{ijk}$ , (b) the complex conjugate interaction  $y_{ijk}$ , and (c) quartic scalar interaction  $y^{ijn}y_{klm}^*$ .

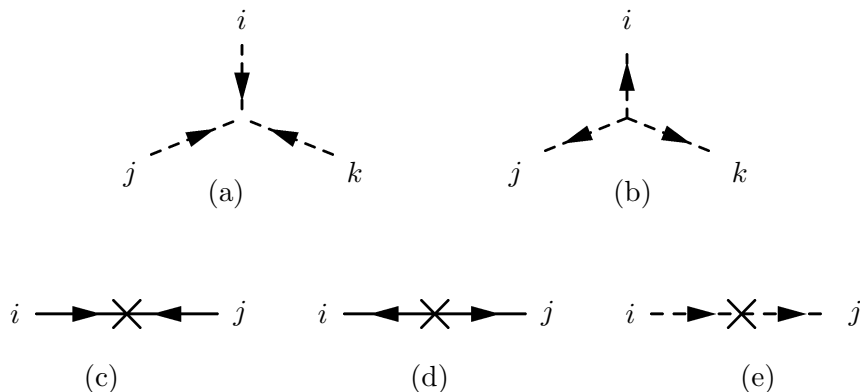


Figure 2.2: Supersymmetric dimensionful couplings: (a) (scalar)<sup>3</sup> interaction vertex  $M_{in}^* y^{jkn}$  and (b) the conjugate interaction  $M^{in} y_{jkn}^*$ , (c) fermion mass term  $M^{ij}$  and (d) conjugate fermion mass term  $M_{ij}^*$ , and (e) scalar squared-mass term  $M_{ik}^* M^{kj}$ .

Supermultiplet			Gauge Representation		
Superfield	Spin 0	Spin 1/2	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$Q_i$	$(\tilde{u}_{iL} \tilde{d}_{iL})$	$(u_{iL} d_{iL})$	<b>3</b>	<b>2</b>	1/6
$U_i^c$	$\tilde{u}_i^c = \tilde{u}_{iR}^\dagger$	$u_i^c = u_{iR}^\dagger$	$\bar{\mathbf{3}}$	<b>1</b>	-2/3
$D_i^c$	$\tilde{d}_i^c = \tilde{d}_{iR}^\dagger$	$d_i^c = d_{iR}^\dagger$	$\bar{\mathbf{3}}$	<b>1</b>	1/3
$L_i$	$(\tilde{\nu}_{iL} \tilde{e}_{iL})$	$(\nu_{iL} e_{iL})$	<b>1</b>	<b>2</b>	-1/2
$E_i^c$	$\tilde{e}_i^c = \tilde{e}_{iR}^\dagger$	$e_i^c = e_{iR}^\dagger$	<b>1</b>	<b>1</b>	1
$H_u$	$(h_u^+ h_u^0)$	$(\tilde{h}_u^+ \tilde{h}_u^0)$	<b>1</b>	<b>2</b>	1/2
$H_d$	$(h_d^0 h_d^-)$	$(\tilde{h}_d^0 \tilde{h}_d^-)$	<b>1</b>	<b>2</b>	-1/2

Table 2.1: Chiral supermultiplets in the Minimal Supersymmetric Standard Model. The spin-0 fields are complex scalars, and the spin-1/2 fields are left-handed two-component Weyl fermions. The index  $i$  runs over the three families.

Supermultiplet			Gauge Representation		
Superfield	Spin 1/2	Spin 1	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$V_3$	$\tilde{G}$ or $\lambda^{(3)}$	$G$	<b>8</b>	<b>1</b>	0
$V_2$	$\tilde{W}^\pm \tilde{W}^0$ or $\lambda_i^{(2)}$	$W^\pm W^0$	<b>1</b>	<b>3</b>	0
$V_1$	$\tilde{B}$ or $\lambda^{(1)}$	$B$	<b>1</b>	<b>1</b>	0

Table 2.2: Gauge supermultiplets in the Minimal Supersymmetric Standard Model. The **1** means that the field is a singlet i.e. uncharged respect to  $SU(n)$  gauge groups.

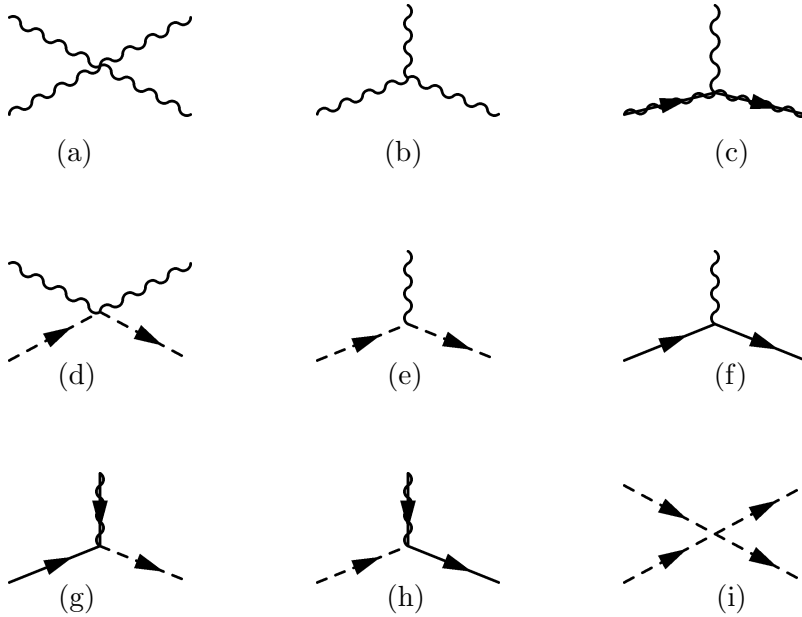


Figure 2.3: Supersymmetric gauge interaction vertices.

The  $\mu$  term in eq. (2.2) is the supersymmetric version of the Higgs boson mass in the SM. It is unique, since terms  $H_u^\dagger H_u$  or  $H_d^\dagger H_d$  are forbidden in the superpotential, because it must be analytic in the chiral superfields. We can also see from the form of eq. (2.2) why both  $H_u$  and  $H_d$  are needed in order to give Yukawa couplings, and thus masses, to all of the quarks and leptons. Since the superpotential must be analytic, for instance the  $QU^c H_u$  Yukawa terms cannot be replaced by something like  $QU^c H_d^\dagger$ . So we need both  $H_u$  and  $H_d$ , even without invoking the argument based on anomaly cancellation (as we will see in Section 3.2).

Since the top quark, bottom quark and tau lepton are the heaviest fermions in SM, it is common to use the approximation that only the third (3,3) component of  $y_u$ ,  $y_d$  and  $y_e$  are important

$$y_u^{ij} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix} \quad y_d^{ij} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix} \quad y_\tau^{ij} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \quad (2.3)$$

In this limit the MSSM superpotential reads in  $SU(2)_L$  components

$$\begin{aligned}
 W_{\text{MSSM}} \approx & y_t(t^c t h_u^0 - t^c b h_u^+) - y_b(b^c t h_d^- - b^c b h_d^0) - y_\tau(\tau^c \nu_\tau h_d^- - \tau^c \tau h_d^0) \\
 & + \mu(h_u^+ h_d^- - h_u^0 h_d^0)
 \end{aligned}
 \tag{2.4}$$

where  $Q_3 = (t \ b)$ ,  $L_3 = (\nu_\tau \ \tau)$ ,  $h_u = (h_u^+ \ h_u^0)$ ,  $h_d = (h_d^0 \ h_d^-)$ ,  $u_3^c = t^c$ ,  $d_3^c = b^c$  and  $e_3^c = \tau^c$ . The minus sign inside the parentheses appear because of the use of  $\epsilon^{\alpha\beta}$  to tie up internal  $SU(2)_L$  indices. These minus signs were chosen so that terms proportional to  $y_t$ ,  $y_b$  and  $y_\tau$  have positive signs when they become the top, bottom and tau masses.

Since the Yukawa interactions  $y^{ijk}$  in a general supersymmetric theory must be completely symmetric under interchange of  $i, j, k$ , we know that  $y_u$ ,  $y_d$  and  $y_e$  imply not only Higgs-quark-quark and Higgs-lepton-lepton couplings as in the SM, but also squark-higgsino-quark and slepton-higgsino-lepton interactions.

There are also scalar quartic interactions with strength proportional to the square of Yukawa couplings. The existence of all the quark and lepton Yukawa couplings in the superpotential (2.2) leads also to the following scalar quartic couplings: (squark)<sup>4</sup>, (slepton)<sup>4</sup>, (squark)<sup>2</sup>(slepton)<sup>2</sup>, (squark)<sup>2</sup>(Higgs)<sup>2</sup> and (slepton)<sup>2</sup>(Higgs)<sup>2</sup>. However, the dimensionless interactions determined by the superpotential are usually not the most important ones of direct interest for phenomenology. This is because the Yukawa couplings are already known to be very small, except for those of the third family (top, bottom, tau). Instead, production and decay processes for superpartners in the MSSM are typically dominated by the supersymmetric interactions of gauge-coupling strength. The couplings of the Standard Model gauge bosons ( $\gamma$ ,  $W^\pm$ ,  $Z^0$  and gluons) to the MSSM particles are determined completely by the gauge invariance of the kinetic terms in the Lagrangian. The gauginos also couple to (squark, quark) and (slepton, lepton) and (Higgs, higgsino) pairs as illustrated in the general case in Fig. 2.3. For instance, each of the squark-quark-gluino couplings is given by  $\sqrt{2}g_3(\tilde{q} T^a q \tilde{g} + \text{h.c.})$  where  $T^a = \lambda^a/2$  ( $a = 1 \dots 8$ ) are the matrix generators for  $SU(3)_C$ .

The dimensionful couplings in the supersymmetric part of the MSSM Lagrangian are all dependent on  $\mu$ . From eq. (2.2), we get the higgsino fermion mass terms

$$- \mathcal{L}_{\text{higgsino mass}} = \mu(\tilde{h}_u^+ \tilde{h}_d^- - \tilde{h}_u^0 \tilde{h}_d^0) + \text{h.c.}
 \tag{2.5}$$

as well as Higgs squared-mass terms in the scalar potential

$$-\mathcal{L}_{\text{susy Higgs mass}} = |\mu|^2 (|h_u^0|^2 + |h_u^+|^2 + |h_d^0|^2 + |h_d^-|^2) \quad (2.6)$$

Since eq. (2.6) is positive definite, we cannot understand electroweak symmetry breaking (EWSB) without including a negative supersymmetry-breaking squared-mass soft term for the Higgs scalars (see Section 2.4.1). Moreover we expect that  $\mu$  should be roughly of order  $10^2$  or  $10^3$  GeV, not so far from the electroweak scale. But why  $|\mu|^2$  is so small compared to  $M_{\text{P}}^2$ ? Why should it be roughly of the same order as  $m_{\text{soft}}^2$ ? The scalar potential of the MSSM seems to depend on two distinct dimensionful parameters, namely the supersymmetry-preserving mass  $\mu$  and the supersymmetry-breaking soft mass terms. The observed value for the electroweak breaking scale suggests, both of these should be within an order of magnitude or so of 100 GeV. This puzzle is called “the  $\mu$  problem”. Several different solutions to the  $\mu$  problem have been proposed, but they all work in roughly the same way<sup>1</sup>; the  $\mu$  term is absent at tree-level before symmetry breaking, and then it arises from the VEV(s) of some new field(s). These VEVs are determined by minimizing a potential that depends on soft terms. Thus, the value of the effective parameter  $\mu$  is related to supersymmetry breaking. From the point of view of the MSSM, however, we can just treat  $\mu$  as an independent parameter.

Finally, the  $\mu$ -term and the Yukawa couplings in the superpotential eq. (2.2) combine to yield (scalar)<sup>3</sup> couplings of the form

$$\begin{aligned} \mathcal{L}_{\text{susy (scalar)}^3} = & \mu^* (\tilde{u}_i^c y_u^{ij} \tilde{u}_j h_d^{0\dagger} + \tilde{d}_i^c y_d^{ij} \tilde{d}_j h_u^{0\dagger} + \tilde{e}_i^c y_e^{ij} \tilde{e}_j h_u^{0\dagger} + \\ & \tilde{u}_i^c y_u^{ij} \tilde{d}_j h_d^{-\dagger} + \tilde{d}_i^c y_d^{ij} \tilde{u}_j h_u^{+\dagger} + \tilde{e}_i^c y_e^{ij} \tilde{\nu}_j h_u^{+\dagger}) + \text{h.c.} \quad (2.7) \end{aligned}$$

## 2.2 Soft supersymmetry breaking

A realistic phenomenological model must contain SUSY breaking. From a theoretical point of view we expect that SUSY is an exact symmetry which is spontaneously broken. Many models have been proposed but we do not discuss them because is beyond our scope. From a practical perspective it is sufficient to parametrize our ignorance introducing extra terms that break explicitly SUSY.

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<sup>1</sup>Some other attractive solutions are proposed in [5]-[7].

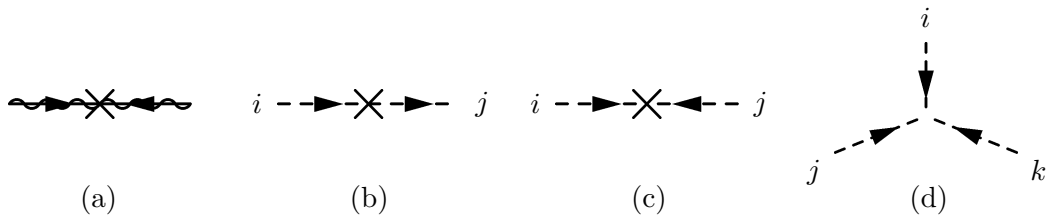


Figure 2.4: Soft supersymmetry-breaking terms: (a) Gaugino mass  $M_a$ ; (b) non-analytic scalar squared mass  $(m^2)_j^i$ ; (c) analytic scalar squared mass  $b^{ij}$ ; and (d) scalar cubic coupling  $a^{ijk}$ . For each of the interactions in a,c,d there is another with all arrows reversed, corresponding to the complex conjugate term in the Lagrangian.

However this breaking should be soft because we do not want to lose the main properties of a SUSY theory (see Chapter 1 and [3]). As illustrated in [4] the one loop divergent contributions to the scalar potential are given by

$$\delta V = \frac{\Lambda^2}{32\pi^2} \text{STr} \mathcal{M}^2 + \frac{1}{64\pi^2} \text{STr} \mathcal{M}^4 \ln \left( \frac{\mathcal{M}^2}{\Lambda^2} \right) \quad (2.8)$$

where  $\Lambda$  is the energy cut-off and  $\mathcal{M}$  is the mass matrix of the theory. As it is well known a SUSY theory is free of quadratic divergences, in fact  $\text{STr} \mathcal{M}^2 = 0$ , so the soft breaking terms must be such that this property still holds. The possible soft supersymmetry breaking of a general renormalizable theory are

$$\mathcal{L}_{\text{soft}} = - \left( \frac{1}{2} M_a \lambda^a \lambda^a + \frac{1}{6} a^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j \right) + \text{h.c.} - \frac{1}{2} (m^2)_j^i \phi^{j\dagger} \phi_i \quad (2.9)$$

They are gaugino masses  $M_a$  for each gauge group, scalar squared mass terms  $b^{ij}$  and  $(m^2)_j^i$  and trilinear scalar coupling  $a^{ijk}$ . There are no mass terms for the chiral supermultiplet fermions, like  $-\frac{1}{2} m^{ij} \psi_i \psi_j + \text{h.c.}$ , because in general they are not soft<sup>2</sup>. The Lagrangian  $\mathcal{L}_{\text{soft}}$  breaks SUSY since it involves only scalars and gauginos and not their respective superpartners. The gaugino masses  $M_a$  are always allowed by gauge symmetry. The  $(m^2)_j^i$  terms are allowed for  $i, j$  such that  $\phi_i, \phi^{j\dagger}$  transform in complex conjugate representations under all gauge groups. This is obviously true when  $i = j$ , so every scalar can acquire mass in this way. The remaining soft terms are restricted by the symmetries. Because the  $a^{ijk}$  and  $b^{ij}$  have the same form as  $y^{ijk}$  and  $M^{ij}$ , they will be allowed if and only if there is a corresponding superpotential term in the Lagrangian. The corresponding Feynman diagrams are given in Fig. 2.4.

<sup>2</sup>In Section 4.1.2 we will see a case in which is possible to have such a term.

Applying what we have shown to the MSSM, we get that

$$\begin{aligned}
 \mathcal{L}_{soft}^{MSSM} = & -\frac{1}{2} \sum_{a=1}^3 (M_a \lambda^{(a)} \lambda^{(a)} + \text{h.c.}) - \left( m_{q_{ij}}^2 \tilde{q}_i \tilde{q}_j^\dagger + m_{u_{ij}}^2 \tilde{u}_i^c \tilde{u}_j^{c\dagger} + m_{d_{ij}}^2 \tilde{d}_i^c \tilde{d}_j^{c\dagger} \right. \\
 & \left. + m_{l_{ij}}^2 \tilde{l}_i \tilde{l}_j^\dagger + m_{e_{ij}}^2 \tilde{e}_i^c \tilde{e}_j^{c\dagger} + m_{h_u}^2 |h_u|^2 + m_{h_d}^2 |h_d|^2 \right) \\
 & - \left( a_u^{ij} \tilde{q}_i \tilde{q}_j h_u - a_d^{ij} \tilde{q}_i \tilde{d}_j h_d - a_e^{ij} \tilde{l}_i \tilde{e}_j h_d + b h_u h_d + \text{h.c.} \right) \quad (2.10)
 \end{aligned}$$

We see soft mass terms for the gluinos  $\lambda^{(3)}$ , the winos  $\lambda^{(2)}$  and the bino  $\lambda^{(1)}$ . Then we have the (scalar)<sup>3</sup> couplings, which are in one to one correspondence with the Yukawa couplings of the superpotential and the sleptons and squarks masses. As usual we suppressed the gauge indices. Finally we have the soft terms that contributes to the Higgs potential,  $m_{h_u}$ ,  $m_{h_d}$  and  $b$ .

The above  $\mathcal{L}_{soft}^{MSSM}$  introduces many new parameters which are absent in the ordinary SM. A careful count [8] shows that there are 105 masses, phases and mixing angles in the MSSM Lagrangian that cannot be rotated away by some redefinition of the basis of quark and lepton supermultiplets. However there are experimental flavor mixing and CP violation data which limit this number [9]. All of these potentially dangerous effects can be avoided assuming that supersymmetry breaking is “universal”. In particular we can suppose that the squark and slepton masses are flavor-blind, then

$$\begin{aligned}
 m_{q_{ij}}^2 &\sim m_q^2 \delta_{ij} & m_{u_{ij}}^2 &\sim m_u^2 \delta_{ij} & m_{d_{ij}}^2 &\sim m_d^2 \delta_{ij} \\
 m_{l_{ij}}^2 &\sim m_l^2 \delta_{ij} & m_{e_{ij}}^2 &\sim m_e^2 \delta_{ij} & &
 \end{aligned} \quad (2.11)$$

If so, then all sfermions mixing angles are trivial. Therefore MSSM contributions to FCNC processes will be very small up to mixing induced by  $a_u$ ,  $a_d$  and  $a_e$ . Making the further assumption that these couplings are proportional to the respective Yukawa coupling,

$$a_u = A_u y_u \quad a_d = A_d y_d \quad a_e = A_e y_e \quad (2.12)$$

will ensure that only the sfermions of the third family can have large (scalar)<sup>3</sup> couplings. Finally, to avoid large CP-violation effects it is usually assumed that soft terms do not introduce new complex phases. This is automatic for the Higgses and the sfermions if eq. (2.11) is true. We can also fix the  $\mu$  and  $b$  parameters to be real, rotating the phase of  $H_{u,d}$  supermultiplets. Assuming also

that the  $A_i$  in eq. (2.12) and the gaugino soft masses are real, then the only CP-violating phase in the theory will be the ordinary CKM phase of the SM.

The conditions (2.11) and (2.12) reflect the assumption of soft-breaking universality invoked in the above. These must be taken as boundary conditions on the running of soft parameters at a very high RG scale  $Q_0$ . Then we must run all the Lagrangian parameters down to the EW scale.

## 2.3 RGE

In this section we list the result for the RGEs in the MSSM. For a method of computation see for instance [4].

The 1 loop RG equations for the gauge couplings are

$$\beta \equiv \frac{d}{dt} \underline{g}_a = \frac{1}{16\pi^2} b_a \underline{g}_a^3, \quad (b_1, b_2, b_3) = \begin{cases} (41/10, -19/6, -7) & \text{SM} \\ (33/5, 1, -3) & \text{MSSM} \end{cases} \quad (2.13)$$

where  $t = \ln(Q/Q_0)$  with  $Q$  the RG scale,  $\underline{g}_1 = \sqrt{5/3} g_1$ ,  $\underline{g}_2 = g_2$  and  $\underline{g}_3 = g_3$ . The normalization of  $\underline{g}_1$  is chosen to agree with Grand Unification Theories (GUTs) like  $SU(5)$  or  $SO(10)$ . The quantities  $\alpha_a = \underline{g}_a^2/4\pi$  turn out to have their reciprocals that run linearly with the RG scale

$$\frac{d}{dt} \alpha_a^{-1} = -\frac{b_a}{2\pi} \quad (2.14)$$

In Fig. 2.5 is plotted the RG evolution of the  $\alpha_a^{-1}$ . Unlike the SM, which has a very rough unification, MSSM has a very ‘‘precise’’ unification at the scale  $M_U \sim 2 \times 10^{16}$  GeV. This can be an accident, but it may also be taken as a hint toward SUSY GUT or superstring models.

Next we consider the 1 loop RGEs of the parameters in the superpotential. Considering the third family approximation (2.3) we get the following equations for the Yukawa couplings

$$\beta_{y_t} \equiv \frac{d}{dt} y_t = \frac{y_t}{16\pi^2} \left[ 6y_t^* y_t + y_b^* y_b - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{13}{15} g_1^2 \right] \quad (2.15)$$

$$\beta_{y_b} \equiv \frac{d}{dt} y_b = \frac{y_b}{16\pi^2} \left[ 6y_b^* y_b + y_t^* y_t + y_\tau^* y_\tau - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{7}{15} g_1^2 \right] \quad (2.16)$$

$$\beta_{y_\tau} \equiv \frac{d}{dt} y_\tau = \frac{y_\tau}{16\pi^2} \left[ 4y_\tau^* y_\tau + 3y_b^* y_b - 3g_2^2 - \frac{9}{5} g_1^2 \right] \quad (2.17)$$

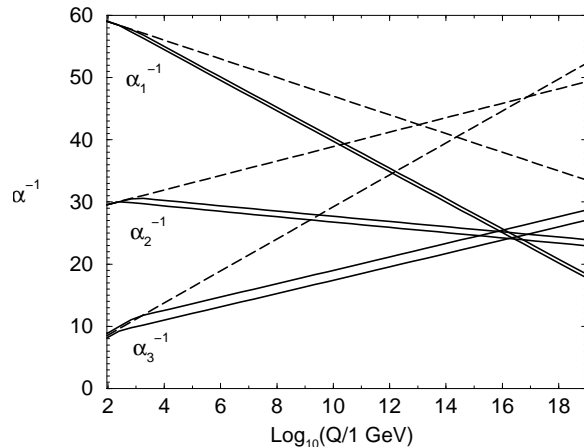


Figure 2.5: RG evolution of the inverse gauge couplings  $\alpha_a^{-1}(Q)$  in the SM (dashed lines) and the MSSM (solid lines). In the MSSM case, the sparticle mass thresholds are varied between 250 GeV and 1 TeV, and  $\alpha_3(M_{Z_0})$  between 0.113 and 0.123. Two-loop effects are included.

while the RGE for the  $\mu$  term is

$$\beta_\mu \equiv \frac{d}{dt}\mu = \frac{\mu}{16\pi^2} \left[ 3y_t^* y_t + 3y_b^* y_b + y_\tau^* y_\tau - 3g_2^2 - \frac{3}{5}g_1^2 \right] \quad (2.18)$$

The one-loop RG equations for the three gaugino mass parameters in the MSSM are determined by the same quantities  $b_a^{\text{MSSM}}$  that appear in the gauge coupling RG eqs. (2.13)

$$\beta_{M_a} \equiv \frac{d}{dt}M_a = \frac{1}{8\pi^2} b_a g_a^2 M_a \quad (b_a = 33/5, 1, -3) \quad (2.19)$$

for  $a = 1, 2, 3$ . It is easy to check that the three ratios  $M_a/g_a^2$  are each constant (RG scale independent) at 1 loop order. Since the gauge couplings unify at  $Q = M_U = 2 \times 10^{16}$  GeV, it is a natural assumption that the gaugino masses also unify near that scale, with a value  $m_{1/2}$ . If so, then it follows that

$$\frac{M_1}{g_1^2} = \frac{M_2}{g_2^2} = \frac{M_3}{g_3^2} = \frac{m_{1/2}}{g_U^2} \quad (2.20)$$

at any RG scale, up to small two-loop effects. Here  $g_U$  is the unified gauge coupling at  $Q = M_U$ .

Then we consider the 1-loop RG equations for the soft parameters connected to the superpotential. Using the third family approximation (2.3) we can write

at any RG scale

$$a_u^{ij} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a_t \end{pmatrix} \quad a_d^{ij} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a_b \end{pmatrix} \quad a_\tau^{ij} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a_\tau \end{pmatrix} \quad (2.21)$$

which defines the running parameters  $a_t$ ,  $a_b$  and  $a_\tau$ . In this approximation the RGEs for the trilinear soft couplings are

$$16\pi^2 \frac{d}{dt} a_t = a_t \left[ 18y_t^* y_t + y_b^* y_b - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{13}{15} g_1^2 \right] + 2a_b y_b^* y_t + y_t \left[ \frac{32}{3} g_3^2 M_3 + 6g_2^2 M_2 + \frac{26}{15} g_1^2 M_1 \right] \quad (2.22)$$

$$16\pi^2 \frac{d}{dt} a_b = a_b \left[ 18y_b^* y_b + y_t^* y_t + y_\tau^* y_\tau - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{7}{15} g_1^2 \right] + 2a_t y_t^* y_b + 2a_\tau y_\tau^* y_b + y_b \left[ \frac{32}{3} g_3^2 M_3 + 6g_2^2 M_2 + \frac{14}{15} g_1^2 M_1 \right] \quad (2.23)$$

$$16\pi^2 \frac{d}{dt} a_\tau = a_\tau \left[ 12y_\tau^* y_\tau + 3y_b^* y_b - 3g_2^2 - \frac{9}{5} g_1^2 \right] + 6a_b y_b^* y_\tau + y_\tau \left[ 6g_2^2 M_2 + \frac{18}{5} g_1^2 M_1 \right] \quad (2.24)$$

while the RGE for the  $b$  parameter is

$$16\pi^2 \frac{d}{dt} b = b \left[ 3y_t^* y_t + 3y_b^* y_b + y_\tau^* y_\tau - 3g_2^2 - \frac{3}{5} g_1^2 \right] + \mu \left[ 6a_t y_t^* + 6a_b y_b^* + 2a_\tau y_\tau^* + 6g_2^2 M_2 + \frac{6}{5} g_1^2 M_1 \right] \quad (2.25)$$

The  $\beta$ -function for each of these soft parameters is not proportional to the parameter itself, because couplings which break supersymmetry are not protected by supersymmetric non-renormalization theorems.

Next let us consider the RG equations for the scalar masses in the MSSM. In the approximation of eqs. (2.3) and (2.21), the scalar masses satisfy boundary conditions like eq. (2.11) at an input RG scale, so when renormalized to any other RG scale, they will stay almost diagonal,

$$m_{q_{ij}}^2 \approx \begin{pmatrix} m_{q_1}^2 & 0 & 0 \\ 0 & m_{q_2}^2 & 0 \\ 0 & 0 & m_{q_3}^2 \end{pmatrix} \quad m_{u_{ij}}^2 \approx \begin{pmatrix} m_{u_1}^2 & 0 & 0 \\ 0 & m_{u_2}^2 & 0 \\ 0 & 0 & m_{u_3}^2 \end{pmatrix} \quad \dots \quad (2.26)$$

The first and second family sfermions remain very nearly degenerate

$$m_{q_2}^2 \approx m_{q_1}^2 \quad m_{u_2}^2 \approx m_{u_1}^2 \quad \dots \quad (2.27)$$

but the third-family sfermions have larger Yukawa couplings and so their squared masses get renormalized differently. The one-loop RGEs for the first and second family squark and slepton squared masses are

$$16\pi^2 \frac{d}{dt} m_{\phi_i}^2 = - \sum_{a=1,2,3} 8g_a^2 C_a(\phi_i) |M_a|^2 + \frac{6}{5} Y_i g_1^2 S \quad (2.28)$$

for each scalar  $\phi_i$ , where  $a$  runs over the three gauge groups  $U(1)_Y$ ,  $SU(2)_L$  and  $SU(3)_C$ ,  $C_a(\phi_i)$  are the quadratic Casimir invariants and  $M_a$  are the corresponding running gaugino masses. Also,

$$S \equiv \text{Tr}[Y_j m_{\phi_j}^2] = m_{H_u}^2 - m_{H_d}^2 + \text{Tr}[m_{q_{ij}}^2 - m_{l_{ij}}^2 - 2m_{u_{ij}}^2 + m_{d_{ij}}^2 + m_{e_{ij}}^2] \quad (2.29)$$

The right-hand side of eq. (2.28) is negative<sup>3</sup>, so the scalar squared-mass parameters grow as they are RG-evolved from the input scale down to the EW scale. The scalar masses will obtain large positive contributions at the electroweak scale, because of the effects of the gaugino masses.

The RG equations for the remaining scalars, Higgs and third family sfermions, also get contributions from the large Yukawa ( $y_{t,b,\tau}$ ) and soft ( $a_{t,b,\tau}$ ) couplings. It is useful to define the following quantities

$$X_t = 2|y_t|^2(m_{h_u}^2 + m_{q_3}^2 + m_{u_3}^2) + 2|a_t|^2 \quad (2.30)$$

$$X_b = 2|y_b|^2(m_{h_d}^2 + m_{q_3}^2 + m_{d_3}^2) + 2|a_b|^2 \quad (2.31)$$

$$X_\tau = 2|y_\tau|^2(m_{h_d}^2 + m_{l_3}^2 + m_{e_3}^2) + 2|a_\tau|^2 \quad (2.32)$$

In terms of these quantities, the RGEs for the soft Higgs squared-masses are

$$16\pi^2 \frac{d}{dt} m_{h_u}^2 = 3X_t - 6g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2 + \frac{3}{5} g_1^2 S \quad (2.33)$$

$$16\pi^2 \frac{d}{dt} m_{h_d}^2 = 3X_b + X_\tau - 6g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2 - \frac{3}{5} g_1^2 S \quad (2.34)$$

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<sup>3</sup>The contributions proportional to  $S$  is zero in mSUGRA models.

while the RG equations for the third family sfermions mass parameters are

$$16\pi^2 \frac{d}{dt} m_{q_3}^2 = X_t + X_b - \frac{32}{3} g_3^2 |M_3|^2 - 6g_2^2 |M_2|^2 - \frac{2}{15} g_1^2 |M_1|^2 + \frac{1}{5} g_1^2 S \quad (2.35)$$

$$16\pi^2 \frac{d}{dt} m_{u_3}^2 = 2X_t - \frac{32}{3} g_3^2 |M_3|^2 - \frac{32}{15} g_1^2 |M_1|^2 - \frac{4}{5} g_1^2 S \quad (2.36)$$

$$16\pi^2 \frac{d}{dt} m_{d_3}^2 = 2X_b - \frac{32}{3} g_3^2 |M_3|^2 - \frac{8}{15} g_1^2 |M_1|^2 + \frac{2}{5} g_1^2 S \quad (2.37)$$

$$16\pi^2 \frac{d}{dt} m_{t_3}^2 = X_\tau - 6g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2 - \frac{3}{5} g_1^2 S \quad (2.38)$$

$$16\pi^2 \frac{d}{dt} m_{e_3}^2 = 2X_\tau - \frac{24}{5} g_1^2 |M_1|^2 + \frac{6}{5} g_1^2 S. \quad (2.39)$$

An examination of eq. (2.22)-(2.25), (2.28), and (2.33)-(2.39) reveals that if the gaugino masses  $M_1$ ,  $M_2$ , and  $M_3$  are non-zero at the input scale, then all of the other soft terms will be generated too. On the other hand, if the gaugino masses vanished at tree-level, then they would not get any contributions to their masses at the one-loop order; in that case the gauginos would be extremely light and the model would not be phenomenologically acceptable.

## 2.4 Mass spectrum

### 2.4.1 Electroweak symmetry breaking

In the MSSM, the description of electroweak symmetry breaking is slightly complicated by the fact that there are two complex Higgs doublets  $h_u = (h_u^+, h_u^0)$  and  $h_d = (h_d^0, h_d^-)$  rather than just one in the ordinary SM. The classical scalar potential for the Higgs sector in the MSSM is given by

$$\begin{aligned} V = & (|\mu|^2 + m_{h_u}^2)(|h_u^0|^2 + |h_u^+|^2) + (|\mu|^2 + m_{h_d}^2)(|h_d^0|^2 + |h_d^-|^2) \\ & + [b(h_u^+ h_d^- - h_u^0 h_d^0) + \text{h.c.}] \\ & + \frac{1}{8}(g_2^2 + g_1^2)(|h_u^0|^2 + |h_u^+|^2 - |h_d^0|^2 - |h_d^-|^2)^2 + \frac{1}{2}g^2 |h_u^+ h_d^{0\dagger} + h_u^0 h_d^{-\dagger}|^2 \end{aligned} \quad (2.40)$$

The terms proportional to  $|\mu|^2$  come from  $F$ -terms while the terms proportional to  $g_1^2$  and  $g_2^2$  are the  $D$ -term contributions. Finally, the terms proportional to  $m_{h_u}^2$ ,  $m_{h_d}^2$  and  $b$  are coming from the soft breaking Lagrangian (2.10). The full scalar potential of the theory also includes many terms involving the sparticle fields which we can ignore here, since they do not get VEVs because they have large positive squared masses.

We now have to demand that the minimum of this potential should give a correct electroweak symmetry breaking  $SU(2)_L \times U(1)_Y \rightarrow U(1)_{\text{EM}}$ . It is easy to see that in the minimum we have  $h_u^+ = h_d^- = 0$ . This is good, because it means that at the minimum of the potential electromagnetism remains unbroken, since the charged components of the  $H_{u,d}$  cannot get VEVs. After setting  $h_u^+ = h_d^- = 0$ , we are left to consider the scalar potential

$$V = (|\mu|^2 + m_{h_u}^2)|h_u^0|^2 + (|\mu|^2 + m_{h_d}^2)|h_d^0|^2 - (b h_u^0 h_d^0 + \text{h.c.}) + \frac{1}{8}(g_2^2 + g_1^2)(|h_u^0|^2 - |h_d^0|^2)^2 \quad (2.41)$$

The only term in this potential that depends on the phases of the fields is the  $b$ -term. Therefore, redefining the phase of  $h_u$  or  $h_d$  we can absorb any phase in  $b$ , so we take  $b$  to be real and positive. To have a bounded from below potential and to avoid the possibility to have  $h_u^0 = h_d^0 = 0$  in the minimum, the parameters in the potential must obey the following constraints

$$2b < 2|\mu|^2 + m_{h_u}^2 + m_{h_d}^2 \quad (2.42)$$

$$b^2 > (|\mu|^2 + m_{h_u}^2)(|\mu|^2 + m_{h_d}^2) \quad (2.43)$$

Having established the conditions necessary for  $h_u^0$  and  $h_d^0$  to get non-zero VEVs, we can now require that they give a correct EW phenomenology. Let us write

$$v_u/\sqrt{2} = \langle h_u^0 \rangle, \quad v_d/\sqrt{2} = \langle h_d^0 \rangle. \quad (2.44)$$

These VEVs are related to the known mass of the  $Z_0$  boson and the gauge couplings (see eq. (2.67))

$$\frac{v_u^2 + v_d^2}{2} = \frac{v^2}{2} = 2M_{Z_0}^2/(g_1^2 + g_2^2) \approx (174 \text{ GeV})^2 \quad (2.45)$$

The ratio of the VEVs is usually written as

$$\tan \beta \equiv v_u/v_d. \quad (2.46)$$

The value of  $\tan \beta$  is not fixed by experiments, but it depends on the Lagrangian parameters of the MSSM in a calculable way. Now we can write down the conditions for minimizing the potential

$$m_{h_u}^2 + |\mu|^2 - b \cot \beta - (M_{Z_0}^2/2) \cos(2\beta) = 0 \quad (2.47)$$

$$m_{h_d}^2 + |\mu|^2 - b \tan \beta + (M_{Z_0}^2/2) \cos(2\beta) = 0 \quad (2.48)$$

Using  $|\mu|^2$ ,  $b$ ,  $m_{h_u}^2$  and  $m_{h_d}^2$  as input parameters, we can solve these two equations to give the values for  $\beta$  and  $M_{Z_0}$

$$\sin(2\beta) = \frac{2b}{m_{h_u}^2 + m_{h_d}^2 + 2|\mu|^2} \quad (2.49)$$

$$M_{Z_0}^2 = \frac{|m_{h_d}^2 - m_{h_u}^2|}{\sqrt{1 - \sin^2(2\beta)}} - m_{h_u}^2 - m_{h_d}^2 - 2|\mu|^2 \quad (2.50)$$

In the following subsections we will treat the mass spectrum for each particle sector of the theory at tree level.

### 2.4.2 Higgs Bosons

The Higgs scalar fields in the MSSM consist of two complex  $SU(2)_L$ -doublet, so eight real, scalar degrees of freedom. When the electroweak symmetry is broken, three of them are the would-be Nambu-Goldstone (NG) bosons  $G^0$ ,  $G^\pm$ , which become the longitudinal modes of the  $Z^0$  and  $W^\pm$  massive vector bosons. The remaining five Higgs scalar mass eigenstates consist of two CP-even neutral scalars  $h^0$  and  $H^0$ , one CP-odd neutral scalar  $A^0$ , and a charge +1 scalar  $H^+$  and its conjugate charge  $-1$  scalar  $H^-$ .<sup>4</sup> The gauge-eigenstate fields can be expressed in terms of the mass eigenstate fields as

$$\begin{pmatrix} h_u^0 \\ h_d^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} v_u \\ v_d \end{pmatrix} + \frac{1}{\sqrt{2}} R_\alpha \begin{pmatrix} h^0 \\ H^0 \end{pmatrix} + \frac{i}{\sqrt{2}} R_\beta \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} \quad (2.51)$$

$$\begin{pmatrix} h_u^+ \\ h_d^{-*} \end{pmatrix} = R_\beta \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} \quad (2.52)$$

where the orthogonal rotation matrices

$$R_\alpha = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \quad R_\beta = \begin{pmatrix} \sin \beta & \cos \beta \\ -\cos \beta & \sin \beta \end{pmatrix} \quad (2.53)$$

are chosen to diagonalize quadratic part of the potential

$$\begin{aligned} V_{\text{Higgs mass}} &= \frac{1}{2} m_{h^0}^2 (h^0)^2 + \frac{1}{2} m_{H^0}^2 (H^0)^2 + \frac{1}{2} m_{G^0}^2 (G^0)^2 + \frac{1}{2} m_{A^0}^2 (A^0)^2 \\ &\quad + m_{G^\pm}^2 |G^\pm|^2 + m_{H^\pm}^2 |H^\pm|^2 \end{aligned} \quad (2.54)$$

<sup>4</sup>Here we define  $G^- = G^{+*}$  and  $H^- = H^{+*}$ . Also, by convention,  $h^0$  is lighter than  $H^0$ .

The mass eigenvalues are given by

$$m_{A^0}^2 = 2|\mu|^2 + m_{h_u}^2 + m_{h_d}^2 \quad (2.55)$$

$$m_{h^0, H^0}^2 = \frac{1}{2} \left[ m_{A^0}^2 + M_{Z_0}^2 \mp \sqrt{(m_{A^0}^2 - M_{Z_0}^2)^2 + 4M_{Z_0}^2 m_{A^0}^2 \sin^2(2\beta)} \right] \quad (2.56)$$

$$m_{H^\pm}^2 = m_{A^0}^2 + M_W^2 = m_{A^0}^2 + g_2^2 \frac{v^2}{4} \quad (2.57)$$

$$m_{G^0, G^\pm}^2 = 0 \quad (2.58)$$

The mixing angle  $\alpha$  is determined by

$$\frac{\sin 2\alpha}{\sin 2\beta} = -\frac{m_{H^0}^2 + m_{h^0}^2}{m_{H^0}^2 - m_{h^0}^2} \quad \tan 2\alpha = \frac{m_{A^0}^2 + M_{Z_0}^2}{m_{A^0}^2 - M_{Z_0}^2} \quad (2.59)$$

and is traditionally chosen to be negative, so  $-\pi/2 < \alpha < 0$  (provided  $m_{A^0} > M_{Z_0}$ ).

The masses of  $A^0$ ,  $H^0$  and  $H^\pm$  can in principle be arbitrarily large since they all grow with  $b/\sin(2\beta)$ . In contrast, the mass of  $h^0$  is bounded above. From eq. (2.56), one finds at tree-level [10]

$$m_{h^0} < M_{Z_0} |\cos(2\beta)| \quad (2.60)$$

If this inequality were robust, we would have discovered the lightest Higgs boson at LEP2. In fact the tree level formula (2.56) is subject to drastic quantum corrections and it is possible to show that  $m_{h^0}^2$  can exceed the LEP bounds. Assuming that none of the sfermions masses exceed 1 TeV and that all the couplings in the model remain perturbative up the GUT scale, we get [11]

$$m_{h^0} \lesssim 150 \text{ GeV}. \quad (2.61)$$

However the previous bound is weakened if we relax one of the previous assumptions.

### 2.4.3 Vector Bosons

The gauge bosons in the MSSM get masses in the same way of the SM with the identification  $v^2 = v_u^2 + v_d^2$ . We recall very briefly the Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{Vect.Mass}} &= g_2^2 \frac{v^2}{8} (A_{1\mu} A_1^\mu + A_{2\mu} A_2^\mu) + \\ &+ \frac{1}{2} (Y_\mu A_{3\mu}) \begin{pmatrix} g_1^2 \frac{v^2}{4} & -g_1 g_2 \frac{v^2}{4} \\ -g_1 g_2 \frac{v^2}{4} & g_2^2 \frac{v^2}{4} \end{pmatrix} \begin{pmatrix} Y^\mu \\ A_3^\mu \end{pmatrix} \\ &= M_W^2 W_\mu^+ W^{-\mu} + M_{Z_0}^2 Z_{0\mu} Z_0^\mu \end{aligned} \quad (2.62)$$

where the eigenstates are (the Weinberg angle  $\theta_W$  is defined by  $\tan \theta_W = g_1/g_2$ )

$$W_\mu^\pm = \frac{A_{1\mu} \mp iA_{2\mu}}{\sqrt{2}} \quad (2.63)$$

$$Z_{0\mu} = \frac{g_2 A_{3\mu} - g_1 Y_\mu}{\sqrt{g_1^2 + g_2^2}} = \cos \theta_W A_{3\mu} - \sin \theta_W Y_\mu \quad (2.64)$$

$$A_\mu = \frac{g_1 A_{3\mu} + g_2 Y_\mu}{\sqrt{g_1^2 + g_2^2}} = \sin \theta_W A_{3\mu} + \cos \theta_W Y_\mu \quad (2.65)$$

and their corresponding eigenvalues are

$$M_W^2 = \frac{1}{4} g_2^2 v^2 \quad (2.66)$$

$$M_{Z_0}^2 = \frac{1}{4} (g_1^2 + g_2^2) v^2 \quad (2.67)$$

$$M_\gamma^2 = 0 \quad (2.68)$$

The corresponding coupling constants are

$$g_W = \frac{g_2}{\sqrt{2}} \quad (2.69)$$

$$g_{Z_0} = \sqrt{g_1^2 + g_2^2} \quad (2.70)$$

$$e = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} \quad (2.71)$$

Exactly the same results of SM.

#### 2.4.4 Neutralinos and Charginos

The higgsinos and electroweak gauginos mix with each other because of the effects of EWSB. The neutral higgsinos ( $\tilde{h}_u^0$  and  $\tilde{h}_d^0$ ) and the neutral gauginos ( $\lambda^{(1)}, \lambda_3^{(2)}$ ) combine to form four mass eigenstates called *neutralinos* ( $\tilde{N}_i$ ,  $i = 1 \dots 4$ ). The charged higgsinos ( $\tilde{h}_u^+$  and  $\tilde{h}_d^-$ ) and winos ( $\tilde{W}^+$  and  $\tilde{W}^-$ ) mix to form two mass eigenstates with charge  $\pm 1$  called *charginos* ( $\tilde{C}_i^\pm$ ,  $i = 1, 2$ ).<sup>5</sup> By convention, these are labelled in ascending order, so that  $m_{\tilde{N}_1} < m_{\tilde{N}_2} < m_{\tilde{N}_3} < m_{\tilde{N}_4}$  and  $m_{\tilde{C}_1} < m_{\tilde{C}_2}$ .

With the basis  $\psi^0 = (\lambda^{(1)}, \lambda^{(2)}, \tilde{h}_d^0, \tilde{h}_u^0)$ , the neutralino mass part of the Lagrangian is

$$\mathcal{L}_{\text{neutralino mass}} = -\frac{1}{2} (\psi^0)^T \mathbf{M}_{\tilde{N}} \psi^0 + \text{h.c.} \quad (2.72)$$

<sup>5</sup> Other common notations use  $\tilde{\chi}_i^0$  or  $\tilde{Z}_i$  for neutralinos, and  $\tilde{\chi}_i^\pm$  or  $\tilde{W}_i^\pm$  for charginos.

where  $\mathbf{M}_{\tilde{N}}$  is the following matrix

$$\mathbf{M}_{\tilde{N}} = \begin{pmatrix} M_1 & 0 & -g_1 v_d/2 & g_1 v_u/2 \\ \dots & M_2 & g_2 v_d/2 & -g_2 v_u/2 \\ \dots & \dots & 0 & -\mu \\ \dots & \dots & \dots & 0 \end{pmatrix} \quad (2.73)$$

where the lower dots denote the obvious terms under symmetrization. The entries  $M_1$  and  $M_2$  are the soft gaugino masses in (2.10), while the entries  $-\mu$  are the supersymmetric higgsino mass terms in (2.2). The terms proportional to  $g_{1,2}$  are the result of Higgs-higgsino-gaugino couplings (see Fig. 2.3 g,h), with the Higgs replaced by their VEVs.

The parameters  $M_1$ ,  $M_2$ , and  $\mu$  in the equations above can have arbitrary complex phases. A redefinition of the phases of  $\lambda^{(1)}$  and  $\lambda^{(2)}$  permit us to choose  $M_1$  and  $M_2$  both real and positive. The phase of  $\mu$  within that convention is then really a physical parameter and cannot be rotated away. However, if  $\mu$  is not real, then there can be potentially disastrous CP-violating effects in low-energy physics, so it is usual to assume that  $\mu$  is real in the same set of phase conventions that make  $M_1$ ,  $M_2$ ,  $b$ ,  $v_u$  and  $v_d$  real and positive. The sign of  $\mu$  is still undetermined by this constraint.

There is a not-unlikely limit in which EWSB is a small perturbation inside  $\mathbf{M}_{\tilde{N}}$

$$M_{Z_0} \ll |\mu \pm M_1|, |\mu \pm M_2| \quad (2.74)$$

In this case the neutralino eigenstates are very nearly a ‘‘bino-like’’  $\tilde{N}_1 \approx \lambda^{(1)}$ , a ‘‘wino-like’’  $\tilde{N}_2 \approx \lambda^{(2)}$  and ‘‘higgsino-like’’  $\tilde{N}_{3,4} \approx (\tilde{h}_u^0 \pm \tilde{h}_d^0)/\sqrt{2}$ , with masses

$$m_{\tilde{N}_1} = M_1 - \frac{M_{Z_0}^2 s_W^2 (M_1 + \mu \sin 2\beta)}{\mu^2 - M_1^2} + \dots \quad (2.75)$$

$$m_{\tilde{N}_2} = M_2 - \frac{M_W^2 (M_2 + \mu \sin 2\beta)}{\mu^2 - M_2^2} + \dots \quad (2.76)$$

$$m_{\tilde{N}_3} = |\mu| + \frac{M_{Z_0}^2 (I - \sin 2\beta) (\mu + M_1 c_W^2 + M_2 s_W^2)}{2(\mu + M_1)(\mu + M_2)} + \dots \quad (2.77)$$

$$m_{\tilde{N}_4} = |\mu| + \frac{M_{Z_0}^2 (I + \sin 2\beta) (\mu - M_1 c_W^2 - M_2 s_W^2)}{2(\mu - M_1)(\mu - M_2)} + \dots \quad (2.78)$$

where  $c_W$  and  $s_W$  are respectively the cosine and sine of the Weinberg angle  $\theta_W$  and  $I = \pm 1$  is the sign of the  $\mu$  parameter. The neutralino labels should be

rearranged depending on the numerical values of the parameters; in particular the above labelling is a common situation in mSUGRA models.

The chargino spectrum can be treated in a similar way. In the gauge-eigenstate basis  $\psi^\pm = (\tilde{W}^+, \tilde{h}_u^+, \tilde{W}^-, \tilde{h}_d^-)$ , the chargino mass terms are

$$\mathcal{L}_{\text{chargino mass}} = -\frac{1}{2}(\psi^\pm)^T \mathbf{M}_{\tilde{C}} \psi^\pm + \text{h.c.} \quad (2.79)$$

where, in  $2 \times 2$  block form,

$$\mathbf{M}_{\tilde{C}} = \begin{pmatrix} 0 & \mathbf{X}^T \\ \mathbf{X} & 0 \end{pmatrix} \quad \text{with} \quad \mathbf{X} = \begin{pmatrix} M_2 & g_2 v_u \\ g_2 v_d & \mu \end{pmatrix} \quad (2.80)$$

Since these are only  $2 \times 2$  matrices, it is easy to find the mass eigenvalues

$$m_{\tilde{C}_{1,2}}^2 = \frac{1}{2} (|M_2|^2 + |\mu|^2 + 2M_W^2) \mp \frac{1}{2} \sqrt{(|M_2|^2 + |\mu|^2 + 2M_W^2)^2 - 4|\mu M_2 - M_W^2 \sin 2\beta|^2} \quad (2.81)$$

In the limit of eq. (2.74), the charginos consist of a wino-like  $\tilde{C}_1^\pm$  and a higgsino-like  $\tilde{C}_2^\pm$ , with masses

$$m_{\tilde{C}_1} = M_2 - \frac{M_W^2 (M_2 + \mu \sin 2\beta)}{\mu^2 - M_2^2} + \dots \quad (2.82)$$

$$m_{\tilde{C}_2} = |\mu| + \frac{M_W^2 I(\mu + M_2 \sin 2\beta)}{\mu^2 - M_2^2} + \dots \quad (2.83)$$

The labelling is given in the same way as the neutralino case. Amusingly,  $\tilde{C}_1$  is nearly degenerate with  $\tilde{N}_2$  in the approximation shown, but it is not an exact result. Similarly the higgsino-like fermions  $\tilde{N}_3$ ,  $\tilde{N}_4$  and  $\tilde{C}_2$  have masses close to  $|\mu|$ .

### 2.4.5 Gluinos

The gluino is a color octet fermion, it cannot mix with any other particle in the MSSM. Because it is not involved in EWSB, the only source of its mass is the soft parameter  $M_3$ . In mSUGRA models this soft mass is related to the other soft gaugino masses, giving the rough prediction

$$M_3 : M_2 : M_1 \approx 6 : 2 : 1 \quad (2.84)$$

near the TeV scale. Then it is reasonable to suppose that the gluino is much heavier than the lighter neutralinos and charginos.

### 2.4.6 Sfermions

A priori, any scalars with the same electric charge,  $R$ -parity<sup>6</sup>, and color quantum numbers can mix with each other. This means that in general, the mass eigenstates of the squarks and sleptons of the MSSM should be obtained by diagonalizing three  $6 \times 6$  squared-mass matrices for up-type squarks ( $\tilde{u}_L, \tilde{c}_L, \tilde{t}_L, \tilde{u}_R, \tilde{c}_R, \tilde{t}_R$ ), down-type squarks ( $\tilde{d}_L, \tilde{s}_L, \tilde{b}_L, \tilde{d}_R, \tilde{s}_R, \tilde{b}_R$ ), and charged sleptons ( $\tilde{e}_L, \tilde{\mu}_L, \tilde{\tau}_L, \tilde{e}_R, \tilde{\mu}_R, \tilde{\tau}_R$ ), and one  $3 \times 3$  matrix for sneutrinos ( $\tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau$ ). However with the assumptions (2.3), (2.21) and the result (2.27) we have that the first and second families are almost eigenstates while we can have the previous mixings only for the third family sfermions. So the masses for the squarks and sleptons of the first two families are

$$m_{\tilde{u}_L}^2 = m_{\tilde{c}_L}^2 = m_{q_1}^2 + \left(\frac{1}{3}g_1^2 - g_2^2\right) \frac{\Delta v^2}{8} \quad (2.85)$$

$$m_{\tilde{u}_R}^2 = m_{\tilde{c}_R}^2 = m_{u_1}^2 - g_1^2 \frac{\Delta v^2}{6} \quad (2.86)$$

$$m_{\tilde{d}_L}^2 = m_{\tilde{s}_L}^2 = m_{q_1}^2 + \left(\frac{1}{3}g_1^2 + g_2^2\right) \frac{\Delta v^2}{8} \quad (2.87)$$

$$m_{\tilde{d}_R}^2 = m_{\tilde{s}_R}^2 = m_{d_1}^2 + g_1^2 \frac{\Delta v^2}{12} \quad (2.88)$$

$$m_{\tilde{\nu}_e}^2 = m_{\tilde{\nu}_\mu}^2 = m_{l_1}^2 - (g_1^2 + g_2^2) \frac{\Delta v^2}{8} \quad (2.89)$$

$$m_{\tilde{e}_L}^2 = m_{\tilde{\mu}_L}^2 = m_{l_1}^2 - (g_1^2 - g_2^2) \frac{\Delta v^2}{8} \quad (2.90)$$

$$m_{\tilde{e}_R}^2 = m_{\tilde{\mu}_R}^2 = m_{e_1}^2 + g_1^2 \frac{\Delta v^2}{4} \quad (2.91)$$

where  $\Delta v^2 = v_u^2 - v_d^2 = -v^2 \cos(2\beta)$ . The first and most important contribution are the soft masses given in (2.11) while the second (and the third) one comes from  $D$  terms. We can see a degeneracy between the sfermions with the same charges. Instead the mass matrix for the third family squarks is

$$\mathcal{L}_{\tilde{t}-\tilde{b} \text{ mass}} = - \begin{pmatrix} \tilde{t}_L^\dagger & \tilde{t}_R^\dagger & \tilde{b}_L^\dagger & \tilde{b}_R^\dagger \end{pmatrix} \begin{pmatrix} m_{\tilde{t}_L \tilde{t}_L}^2 & m_{\tilde{t}_L \tilde{t}_R}^2 & 0 & 0 \\ m_{\tilde{t}_L \tilde{t}_R}^2 & m_{\tilde{t}_R \tilde{t}_R}^2 & 0 & m_{\tilde{t}_R \tilde{b}_R}^2 \\ 0 & 0 & m_{\tilde{b}_L \tilde{b}_L}^2 & m_{\tilde{b}_L \tilde{b}_R}^2 \\ 0 & m_{\tilde{t}_R \tilde{b}_R}^2 & m_{\tilde{b}_L \tilde{b}_R}^2 & m_{\tilde{b}_R \tilde{b}_R}^2 \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \\ \tilde{b}_L \\ \tilde{b}_R \end{pmatrix} \quad (2.92)$$

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<sup>6</sup>See Subsection 2.5.

where

$$\begin{aligned}
 m_{t_L t_L}^2 &= m_{q_3}^2 + \left( \frac{1}{3} g_1^2 - g_2^2 \right) \frac{\Delta v^2}{8} + y_t^2 \frac{v_u^2}{2} \\
 m_{t_R t_R}^2 &= m_{u_3}^2 - g_1^2 \frac{\Delta v^2}{6} + y_t^2 \frac{v_u^2}{2} \\
 m_{b_L b_L}^2 &= m_{q_3}^2 + \left( \frac{1}{3} g_1^2 + g_2^2 \right) \frac{\Delta v^2}{8} + y_b^2 \frac{v_d^2}{2} \\
 m_{b_R b_R}^2 &= m_{d_3}^2 + g_1^2 \frac{\Delta v^2}{12} + y_b^2 \frac{v_d^2}{2} \\
 m_{t_L t_R}^2 &= \mu y_t \frac{v_d}{\sqrt{2}} + a_t \frac{v_u}{\sqrt{2}} \\
 m_{t_R b_R}^2 &= -y_t \frac{v_u v_d}{2} \\
 m_{b_L b_R}^2 &= -\mu y_b \frac{v_u}{\sqrt{2}} + a_b \frac{v_d}{\sqrt{2}}
 \end{aligned} \tag{2.93}$$

We can see that the diagonal entries are similar contributions as the first two families plus the one coming from Yukawa couplings which now are not negligible.

Finally the mass terms for the tau sleptons

$$\mathcal{L}_{\tilde{\tau} \text{ mass}} = - \begin{pmatrix} \tilde{\tau}_L^\dagger & \tilde{\tau}_R^\dagger \end{pmatrix} \begin{pmatrix} m_{l_3}^2 - \frac{(g_1^2 - g_2^2)\Delta v^2}{8} + y_\tau^2 \frac{v_d^2}{2} & -\mu y_\tau \frac{v_u}{\sqrt{2}} + a_\tau \frac{v_d}{\sqrt{2}} \\ -\mu y_\tau \frac{v_u}{\sqrt{2}} + a_\tau \frac{v_d}{\sqrt{2}} & m_{e_3}^2 + g_1^2 \frac{\Delta v^2}{4} + y_\tau^2 \frac{v_d^2}{2} \end{pmatrix} \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix} \tag{2.94}$$

and their corresponding sneutrino

$$m_{\tilde{\nu}_\tau}^2 = m_{l_3}^2 - \frac{(g_1^2 + g_2^2)\Delta v^2}{8} \tag{2.95}$$

As in the previous case the diagonal entries for (2.94) have the similar first(second) family contribution plus the Yukawa correction.

## 2.5 $R$ parity

The superpotential (2.2) is minimal in the sense that it is sufficient to produce a phenomenologically viable model. However we can write other terms (gauge-invariant, renormalizable and analytic in the chiral superfields) which are not included in the MSSM since they do not respect either baryon number (B) or total lepton number (L). These terms are

$$W_{\Delta L=1} = \frac{1}{2} \lambda^{ijk} L_i L_j \bar{e}_k + \lambda'^{ijk} L_i Q_j \bar{d}_k + \mu^i L_i H_u \tag{2.96}$$

$$W_{\Delta B=1} = \frac{1}{2} \lambda''^{ijk} \bar{u}_i \bar{d}_j \bar{d}_k \tag{2.97}$$

where  $i, j, k$  runs over the three families. The baryon and lepton number assignment are the usual ones:  $B = +1/3$  for  $Q_i$ ,  $B = -1/3$  for  $\bar{u}_i, \bar{d}_i$  and  $B = 0$  for all others;  $L = +1$  for  $L_i$ ,  $L = -1$  for  $\bar{e}_i$ , and  $L = 0$  for all others. So the terms in eq. (2.96) violate total lepton number by 1 unit (as well as the individual lepton flavors) and those in eq. (2.97) violate baryon number by 1 unit.

The possible existence of such terms might seem rather disturbing, since corresponding B- and L-violating processes (for example the proton decay) have not been seen experimentally. One could simply try to postulate B and L conservation in the MSSM by adding a new symmetry, which has the effect of eliminating the possibility of B and L violating terms in the renormalizable superpotential, while allowing the good terms in eq. (2.2). This new symmetry is called “ $R$ -parity” [12] and it is defined as

$$P_R = (-1)^{3(B-L)+2s} \quad (2.98)$$

where  $s$  is the spin of the particle. Therefore particles within the same supermultiplet do not have the same  $R$ -parity<sup>7</sup>. The  $R$ -parity assignment is very useful for phenomenology because all of the SM particles and the Higgs bosons have  $P_R = +1$ , while all of the squarks, sleptons, gauginos, and higgsinos have  $P_R = -1$ . The  $R$ -parity odd particles are known as “sparticles” (“supersymmetric particles”), and they are distinguished by a tilde (see Tables 2.1 and 2.2).  $R$ -parity conservation implies that there can be no mixing between the  $P_R = -1$  and the  $P_R = +1$  particles. Furthermore, every interaction vertex in the theory contains an even number of sparticles. This has three extremely important phenomenological consequences:

- i. The lightest  $P_R = -1$  particle, called the LSP (“lightest supersymmetric particle”), must be absolutely stable. If the LSP is electrically neutral, it interacts only weakly with ordinary matter, and it can be an attractive candidate [13] for the non-baryonic dark matter.
- ii. Each sparticle other than the LSP must decay into a state that contains an odd number of sparticles.
- iii. In collider experiments, sparticles can only be produced in even numbers.

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<sup>7</sup>In general, symmetries with the property that fields within the same supermultiplet have different transformations are called  $R$  symmetries; they do not commute with supersymmetry.

We *define* the MSSM to conserve  $R$ -parity. This decision seems to be well-motivated phenomenologically by proton decay constraints, but it might appear somewhat artificial from a theoretical point of view. One attractive solution could occur if  $B-L$  is a continuous gauge symmetry because it forbids the renormalizable terms that violate  $B$  and  $L$  [14, 15]. However, the  $U(1)_{B-L}$  must be spontaneously broken at some very high energy scale, because there is no corresponding massless vector boson. Moreover, if it is only broken by scalar VEVs (or other order parameters) that carry even integer values of  $3(B-L)$ , then  $P_M = (-1)^{3(B-L)}$  will automatically survive as an exactly conserved discrete remnant subgroup [15].  $P_M$  is called “matter-parity” and can be treated in a similar way as  $P_R$ . A variety of extensions of the MSSM in which exact  $R$ -parity conservation is guaranteed in just this way have been proposed (see for example [15, 16]).



## Chapter 3

# Anomalies

A theory is anomalous if at the quantum (loop) level a classical symmetry  $G$  of the Lagrangian is no longer conserved. If  $G$  is an anomalous global group the quantum theory remains consistent (for example  $\pi^0 \rightarrow \gamma\gamma$ ). Instead, if  $G$  is an anomalous gauge group, the quantum theory becomes inconsistent, so the condition of anomaly cancellation may be used as a constraint on the physical gauge theories. The first studies on anomalies emerged in trying to understand the decay rate of the already cited neutral pion, in the form of an anomaly that violates some global symmetry of the strong interactions. In 1969 the source of this anomaly was traced by Bell and Jackiw [17] to the violation of chiral symmetry by the regulator that is needed in order to derive consequences of the conservation of the neutral axial vector current for one loop Feynman diagrams. Their result was confirmed and generalized by Adler [18]. After 10 years, Fujikawa [19] showed that the chiral symmetry breaking anomaly enters only in the measure used to define the path integral over fermion fields. The following treatment of anomalies is based on Chapter 22 of the book by Weinberg [20] and Chapter 6 of the book by Cheng and Li [21]. We will discuss the anomalies only from a Feynman diagram point of view, leaving the Fujikawa argument to the reader (for a review see also Section 22.2 of [20])

### 3.1 Triangular Anomaly

Consider a chiral gauge theory given by a certain number of massless left(right) Weyl fermions charged under two  $U(1)$  gauge fields<sup>1</sup>  $B_\mu$  and  $C_\mu$

$$\mathcal{L}_{\text{chiral}} = -\frac{1}{4}F_{\mu\nu}^B F^{B\mu\nu} - \frac{1}{4}F_{\mu\nu}^C F^{C\mu\nu} + i \sum_f \left( f_L^\dagger \bar{\sigma}^\mu D_\mu f_L + f_R^\dagger \sigma^\mu D_\mu f_R \right) \quad (3.1)$$

where  $f_{L(R)}$  are the left(right) Weyl fermions in the model, and

$$D_\mu = \partial_\mu + iQ_{f_{L(R)}}^B B_\mu + iQ_{f_{L(R)}}^C C_\mu \quad (3.2)$$

is the covariant derivative. We can express the Lagrangian in terms of the Dirac spinors  $\Psi_f = \begin{pmatrix} f_L \\ f_R \end{pmatrix}$  obtaining

$$\begin{aligned} \mathcal{L}_{\text{chiral}} = & -\frac{1}{4}F_{\mu\nu}^B F^{B\mu\nu} - \frac{1}{4}F_{\mu\nu}^C F^{C\mu\nu} + \\ & i \sum_f \bar{\Psi}_f \gamma^\mu \left[ \partial_\mu + \frac{i}{2}(v_f^B - a_f^B \gamma_5) B_\mu + \frac{i}{2}(v_f^C - a_f^C \gamma_5) C_\mu \right] \Psi_f \end{aligned} \quad (3.3)$$

where the vectorial (V) and axial (A) couplings are given by

$$v_f^k = Q_{f_L}^k + Q_{f_R}^k \quad (3.4)$$

$$a_f^k = Q_{f_L}^k - Q_{f_R}^k \quad (3.5)$$

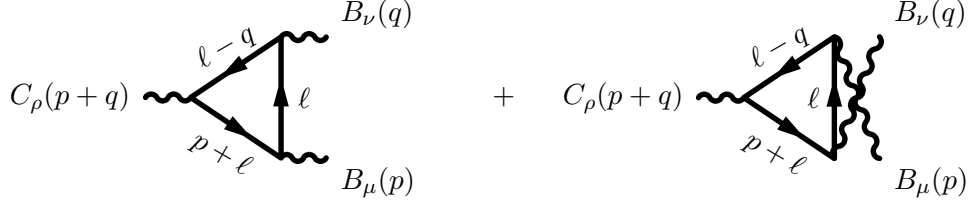
with  $k = B, C$ . For the conventions on the sigma matrices and the properties of gamma matrices see Appendix A.

We may be interested in computing the one loop amplitude and the corresponding Ward Identities (WIs) for an incoming  $C_\mu$  and two outgoing  $B_\mu$ . The Feynman diagrams involved are depicted in Fig. 3.1. The Feynman rules for the interacting vertices are given in Appendix C.1. We can split the amplitude in four contributions: AVV, VAV, VVA and AAA

$$\begin{aligned} \Delta_{\rho\mu\nu}^{CBB} = & -\frac{1}{8} \sum_f \left( a_f^C v_f^B v_f^B \Gamma_{\rho\mu\nu}^{AVV}(p, q; 0) + v_f^C a_f^B v_f^B \Gamma_{\rho\mu\nu}^{VAV}(p, q; 0) + \right. \\ & \left. v_f^C v_f^B a_f^B \Gamma_{\rho\mu\nu}^{VVA}(p, q; 0) + a_f^C a_f^B a_f^B \Gamma_{\rho\mu\nu}^{AAA}(p, q; 0) \right) \end{aligned} \quad (3.6)$$

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<sup>1</sup>The extension to a larger number of gauge fields and to a non abelian case can be treated in a similar way.


 Figure 3.1: Fermionic triangle diagrams for the amplitude  $C_\rho(p+q) \rightarrow B_\mu(p) B_\nu(q)$ .

where

$$\Gamma_{\rho\mu\nu}^{AVV}(p, q; 0) = \int \frac{d^4\ell}{(2\pi)^4} \text{Tr} \left( \gamma_5 \gamma_\rho \frac{1}{\not{\ell} - \not{q}} \gamma_\nu \frac{1}{\not{\ell}} \gamma_\mu \frac{1}{\not{\ell} + \not{p}} \right) + (p \leftrightarrow q, \mu \leftrightarrow \nu) \quad (3.7)$$

$$\Gamma_{\rho\mu\nu}^{VAV}(p, q; 0) = \int \frac{d^4\ell}{(2\pi)^4} \text{Tr} \left( \gamma_\rho \frac{1}{\not{\ell} - \not{q}} \gamma_\nu \frac{1}{\not{\ell}} \gamma_5 \gamma_\mu \frac{1}{\not{\ell} + \not{p}} \right) + (q \leftrightarrow -(p+q), \nu \leftrightarrow \rho) \quad (3.8)$$

$$\Gamma_{\rho\mu\nu}^{VVA}(p, q; 0) = \int \frac{d^4\ell}{(2\pi)^4} \text{Tr} \left( \gamma_\rho \frac{1}{\not{\ell} - \not{q}} \gamma_5 \gamma_\nu \frac{1}{\not{\ell}} \gamma_\mu \frac{1}{\not{\ell} + \not{p}} \right) + (p \leftrightarrow -(p+q), \mu \leftrightarrow \rho) \quad (3.9)$$

$$\Gamma_{\rho\mu\nu}^{AAA}(p, q; 0) = \int \frac{d^4\ell}{(2\pi)^4} \text{Tr} \left( \gamma_5 \gamma_\rho \frac{1}{\not{\ell} - \not{q}} \gamma_5 \gamma_\nu \frac{1}{\not{\ell}} \gamma_5 \gamma_\mu \frac{1}{\not{\ell} + \not{p}} \right) + (p \leftrightarrow q, \mu \leftrightarrow \nu) \quad (3.10)$$

The notation  $\Gamma_{\rho\mu\nu}^{AVV}$  means that we have an axial vertex ( $a_f^C$ ) on the  $\rho$  line, a vectorial vertex ( $v_f^B$ ) on the  $\mu$  line and a vectorial vertex ( $v_f^B$ ) on the  $\nu$  line. Analogously for the other  $\Gamma$ 's.

Consider the first contribution. We have

$$\begin{aligned} \Delta_{\rho\mu\nu}^{CBB} |_{AVV} &= -\frac{1}{8} \sum_f a_f^C v_f^B v_f^B \Gamma_{\rho\mu\nu}^{AVV}(p, q; 0) = \\ &= -\frac{1}{8} \sum_f a_f^C v_f^B v_f^B \int \frac{d^4\ell}{(2\pi)^4} \text{Tr} \left( \gamma_5 \gamma_\rho \frac{1}{\not{\ell} - \not{q}} \gamma_\nu \frac{1}{\not{\ell}} \gamma_\mu \frac{1}{\not{\ell} + \not{p}} \right) + (p_\mu \leftrightarrow q_\nu) \end{aligned} \quad (3.11)$$

The integral  $\Gamma_{\rho\mu\nu}^{AVV}(p, q; 0)$  is superficially divergent so it is not uniquely defined (see Appendix D.1). Eq. 3.11 implies a particular choice for the assignment of the internal momentum  $\ell$ : the fermion line between the two  $B$  lines carries momentum  $\ell$ . We could have chosen a different assignment of it so that this fermion line carries  $\ell + a$ , where  $a$  is some arbitrary combination of  $p$  and  $q$

$$a = \alpha p + (\alpha - \beta) q \quad (3.12)$$

## Anomalies

The fact that the integral is linearly divergent implies that  $\Gamma_{\rho\mu\nu}^{CBB}|_{AVV}$  has an ambiguity in its definition by an amount

$$\begin{aligned}
D_{\rho\mu\nu}(a) &= \Gamma_{\rho\mu\nu}^{AVV}(a) - \Gamma_{\rho\mu\nu}^{AVV} = \\
&= \left[ \int \frac{d^4\ell}{(2\pi)^4} \text{Tr} \left( \gamma_5 \gamma_\rho \frac{1}{\not{\ell} - \not{q} + \not{d}} \gamma_\nu \frac{1}{\not{\ell} + \not{d}} \gamma_\mu \frac{1}{\not{\ell} + \not{p} + \not{d}} \right) \right. \\
&\quad \left. - \int \frac{d^4\ell}{(2\pi)^4} \text{Tr} \left( \gamma_5 \gamma_\rho \frac{1}{\not{\ell} - \not{q}} \gamma_\nu \frac{1}{\not{\ell}} \gamma_\mu \frac{1}{\not{\ell} + \not{p}} \right) \right] + (p \leftrightarrow q, \mu \leftrightarrow \nu) \\
&= D_{\rho\mu\nu}^{(1)} + D_{\rho\mu\nu}^{(2)} \tag{3.13}
\end{aligned}$$

Applying the result (D.6), we have

$$\begin{aligned}
D_{\rho\mu\nu}^{(1)} &= \int \frac{d^4\ell}{(2\pi)^4} a^\rho \frac{\partial}{\partial \ell_\rho} \text{Tr} \left( \gamma_5 \gamma_\rho \frac{1}{\not{\ell} - \not{q}} \gamma_\nu \frac{1}{\not{\ell}} \gamma_\mu \frac{1}{\not{\ell} + \not{p}} \right) \\
&= \frac{2i\pi^2}{(2\pi)^4} a^\rho \lim_{\ell \rightarrow \infty} \ell^2 \ell_\rho \text{Tr}[\gamma_5 \gamma_\rho \not{\ell} \gamma_\nu \not{\ell} \gamma_\mu \not{\ell}] \frac{1}{\ell^6} = \frac{2i\pi^2}{(2\pi)^4} a^\rho \lim_{\ell \rightarrow \infty} \frac{\ell^\rho \ell^\delta}{\ell^2} (-4i) \epsilon_{\rho\delta\nu\mu} \\
&= \frac{1}{(8\pi)^2} \epsilon_{\delta\mu\nu\rho} a^\delta \tag{3.14}
\end{aligned}$$

where we used

$$\lim_{\ell \rightarrow \infty} \frac{\ell^\rho \ell^\delta}{\ell^2} = \frac{\eta^{\rho\delta}}{4} \tag{3.15}$$

Since  $D_{\rho\mu\nu}^{(2)}$  is related to  $D_{\rho\mu\nu}^{(1)}$  by the exchanges  $p \leftrightarrow q$  and  $\mu \leftrightarrow \nu$ , we have from eq. (3.12), (3.13) and (3.14)

$$D_{\rho\mu\nu}(a) = D_{\rho\mu\nu}^{(1)} + D_{\rho\mu\nu}^{(2)} = -\frac{\beta}{8\pi^2} \epsilon_{\rho\mu\nu\delta} (p - q)^\delta \tag{3.16}$$

Thus the definition of  $\Gamma_{\rho\mu\nu}^{AVV}$  has an ambiguity parametrized by

$$\Gamma_{\rho\mu\nu}^{AVV}(p, q, \beta; 0) = \Gamma_{\rho\mu\nu}^{AVV}(p, q; 0) - \frac{\beta}{8\pi^2} \epsilon_{\rho\mu\nu\delta} (p - q)^\delta \tag{3.17}$$

Now we are going to show what is the relation between the  $\beta$  ambiguity in the amplitude and the WIs. In quantum field theory, the WIs are identities between correlation functions that follows from the global or gauged symmetries of the theory, and which remains valid after renormalization. The WIs are a quantum version of the classical Noether's theorem, and any symmetries in a quantum field theory can lead to an equation of motion for correlation functions. We can express them using Feynman amplitude in momentum space or by the path integral formulation. We will usually adopt the first formulation. The WIs for

$$-ik_\mu \left( V^\mu(k) \text{---} \text{blob} \right) = 0$$

Figure 3.2: Ward Identity for the massless gauge field  $V^\mu$ . The blob denotes all the 1PI diagrams.

massless gauge fields can be written in the diagrammatic form showed in Fig. 3.2. We start with the incoming momentum  $(p+q)_\rho$ . Using eq. (3.17), we have

$$(p+q)^\rho \Gamma_{\rho\mu\nu}^{AVV}(p, q, \beta; 0) = (p+q)^\rho \Gamma_{\rho\mu\nu}^{AVV}(p, q; 0) - (p+q)^\rho \frac{\beta}{8\pi^2} \epsilon_{\rho\mu\nu\delta} (p-q)^\delta \quad (3.18)$$

and we focus on the first contribution

$$(p+q)^\rho \Gamma_{\rho\mu\nu}^{AVV}(p, q; 0) = D_{\mu\nu}^{(A)} + D_{\mu\nu}^{(B)} \quad (3.19)$$

where the first term is

$$\begin{aligned} D_{\mu\nu}^{(A)} &= (p+q)^\rho \int \frac{d^4\ell}{(2\pi)^4} \text{Tr} \left[ \gamma_5 \gamma_\rho \frac{1}{\not{\ell} - \not{q}} \gamma_\nu \frac{1}{\not{\ell}} \gamma_\mu \frac{1}{\not{\ell} + \not{p}} \right] \\ &= \int \frac{d^4\ell}{(2\pi)^4} \text{Tr} \left[ \gamma_5 (\not{p} + \not{q}) \frac{1}{\not{\ell} - \not{q}} \gamma_\nu \frac{1}{\not{\ell}} \gamma_\mu \frac{1}{\not{\ell} + \not{p}} \right] \\ &= - \int \frac{d^4\ell}{(2\pi)^4} \text{Tr} \left\{ [\gamma_5(\not{\ell} - \not{q}) + (\not{\ell} + \not{p})\gamma_5] \left[ \frac{1}{\not{\ell} - \not{q}} \gamma_\nu \frac{1}{\not{\ell}} \gamma_\mu \frac{1}{\not{\ell} + \not{p}} \right] \right\} \\ &= - \int \frac{d^4\ell}{(2\pi)^4} \text{Tr} \left[ \frac{\gamma_5 \gamma_\nu \not{\ell} \gamma_\mu (\not{\ell} + \not{p})}{\ell^2 (l+p)^2} - \frac{(\not{\ell} - \not{q}) \gamma_5 \gamma_\nu \not{\ell} \gamma_\mu}{\ell^2 (l-q)^2} \right] \end{aligned} \quad (3.20)$$

while the second one is  $D_{\mu\nu}^{(B)}$  with the exchange  $(p \leftrightarrow q, \mu \leftrightarrow \nu)$

$$D_{\mu\nu}^{(B)} = - \int \frac{d^4\ell}{(2\pi)^4} \text{Tr} \left[ \frac{\gamma_5 \gamma_\mu \not{\ell} \gamma_\nu (\not{\ell} + \not{q})}{\ell^2 (l+q)^2} - \frac{(\not{\ell} - \not{p}) \gamma_5 \gamma_\mu \not{\ell} \gamma_\nu}{\ell^2 (l-p)^2} \right] \quad (3.21)$$

We rearrange the computation grouping the integrals with only a  $p$  dependence or only a  $q$  dependence. So

$$D_{\mu\nu}^{(A)} + D_{\mu\nu}^{(B)} = D_{\mu\nu}^{(1)} + D_{\mu\nu}^{(2)} \quad (3.22)$$

where

$$D_{\mu\nu}^{(1)} = - \int \frac{d^4\ell}{(2\pi)^4} \text{Tr} \left[ \frac{\gamma_5 \gamma_\mu \not{\ell} \gamma_\nu (\not{\ell} + \not{p})}{\ell^2 (l+p)^2} - \frac{(\not{\ell} - \not{p}) \gamma_5 \gamma_\mu \not{\ell} \gamma_\nu}{\ell^2 (l-p)^2} \right] \quad (3.23)$$

$$D_{\mu\nu}^{(2)} = D_{\mu\nu}^{(1)} \Big|_{(p \leftrightarrow q, \mu \leftrightarrow \nu)} \quad (3.24)$$

Using the result (D.6) we get

$$\begin{aligned}
D_{\mu\nu}^{(1)} &= - \int \frac{d^4\ell}{(2\pi)^4} p^\rho \frac{\partial}{\partial \ell_\rho} \text{Tr} \left[ \frac{\gamma_5 \gamma_\nu (\not{\ell} - \not{p}) \gamma_\mu \not{\ell}}{\ell^2 (\ell - p)^2} \right] \\
&= - \frac{2i\pi^2}{(2\pi)^4} p^\rho \lim_{\ell \rightarrow \infty} \ell^2 \ell_\rho \text{Tr} [\gamma_5 \gamma_\nu (\not{\ell} - \not{p}) \gamma_\mu \not{\ell}] \frac{1}{\ell^4} = - \frac{1}{8\pi^2} \lim_{\ell \rightarrow \infty} \frac{\ell^\rho \ell^\beta}{\ell^2} \epsilon_{\nu\alpha\mu\beta} p_\rho p^\alpha \\
&= - \frac{1}{(32\pi)^2} \epsilon_{\nu\alpha\mu\beta} p^\beta p^\alpha = 0 = D_{\mu\nu}^{(2)}
\end{aligned} \tag{3.25}$$

In the last line we used eq. (3.15) and the last identity follows simply by the definition (3.24). Then using eq. (3.18), (3.19), (3.22) and (3.25) we find

$$(p + q)^\rho \Gamma_{\rho\mu\nu}^{AVV}(p, q, \beta; 0) = \frac{\beta}{4\pi^2} \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta \tag{3.26}$$

In a similar way we can compute the WIs for the  $p^\mu$  and  $q^\nu$  momenta, obtaining

$$p^\mu \Gamma_{\rho\mu\nu}^{AVV}(p, q, \beta; 0) = - \frac{2 + \beta}{8\pi^2} \epsilon_{\nu\rho\alpha\beta} q^\alpha p^\beta \tag{3.27}$$

$$q^\nu \Gamma_{\rho\mu\nu}^{AVV}(p, q, \beta; 0) = - \frac{2 + \beta}{8\pi^2} \epsilon_{\rho\mu\alpha\beta} p^\alpha q^\beta \tag{3.28}$$

A similar analysis can be done for the other VAV, VVA and AAA amplitudes in eq. (3.8)-(3.10). So the amplitude  $\Delta_{\rho\mu\nu}^{CBB}$  has the following WIs

$$(p + q)^\rho \Delta_{\rho\mu\nu}^{CBB}(\beta) = - \sum_f t_f^{CBB} \frac{\beta}{32\pi^2} \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta \tag{3.29}$$

$$p^\mu \Delta_{\rho\mu\nu}^{CBB}(\beta) = \sum_f t_f^{CBB} \frac{2 + \beta}{64\pi^2} \epsilon_{\nu\rho\alpha\beta} q^\alpha p^\beta \tag{3.30}$$

$$q^\nu \Delta_{\rho\mu\nu}^{CBB}(\beta) = \sum_f t_f^{CBB} \frac{2 + \beta}{64\pi^2} \epsilon_{\rho\mu\alpha\beta} p^\alpha q^\beta \tag{3.31}$$

with

$$\begin{aligned}
t_f^{CBB} &= a_f^C v_f^B v_f^B + 2v_f^C v_f^B a_f^B + a_f^C a_f^B a_f^B \\
&= 4 \left[ Q_{fL}^C (Q_{fL}^B)^2 - Q_{fR}^C (Q_{fR}^B)^2 \right]
\end{aligned} \tag{3.32}$$

where we used eq. (3.4) and (3.5). As we can see from eq. (3.29), (3.30) and (3.31), there is no choice of  $\beta$  that cancels simultaneously all the WIs and preserves gauge invariance, recovering the result depicted in Fig. 3.2. The same problem will arise in the computation of the  $CCC$ ,  $CCB$ , and  $BBB$  triangles. This is the “triangular (or chiral) anomaly”. The classical gauge invariance is broken by

the triangular fermionic loop making the theory anomalous. Obviously, to have a physical gauge theory we must cancel this anomaly and the standard (and simplest) way is to cancel the overall sum of charges in the WIs. In other words the theory will be anomaly free if the charges satisfy the following constraints

$$\mathcal{A}^{CCC} = \sum_f \left[ (Q_{fL}^C)^3 - (Q_{fR}^C)^3 \right] = 0 \quad (3.33)$$

$$\mathcal{A}^{CCB} = \sum_f \left[ (Q_{fL}^C)^2 Q_{fL}^B - (Q_{fR}^C)^2 Q_{fR}^B \right] = 0 \quad (3.34)$$

$$\mathcal{A}^{CBB} = \sum_f \left[ Q_{fL}^C (Q_{fL}^B)^2 - Q_{fR}^C (Q_{fR}^B)^2 \right] = 0 \quad (3.35)$$

$$\mathcal{A}^{BBB} = \sum_f \left[ (Q_{fL}^B)^3 - (Q_{fR}^B)^3 \right] = 0 \quad (3.36)$$

Depending on the symmetries of the problem, there are cases in which there is a preferential choice for the  $\beta$  parameter. The most common values are (see eq. (3.29), (3.30) and (3.31)):

- $\beta = -2$ , for which all the anomaly is concentrated in the  $\rho$  vertex .
- $\beta = -2/3$ , for which the anomaly is equally distributed among the three vertices (to be used when we have three identical external vectors).

The extension to non abelian theories goes in the same way. The overall charge factor will become

$$\mathcal{A}^{ijk} = \frac{1}{2} \text{Tr} [\{T_i^L, T_j^L\} T_k^L] - \frac{1}{2} \text{Tr} [\{T_i^R, T_j^R\} T_k^R] \quad (3.37)$$

where  $T_{i,j,k}$  are the generators of the three external gauge fields. In some theories the condition of anomaly cancellation will fix completely the charge content of the model. This is the case for the SM (see for instance Section 22.4 of [20]).

## 3.2 Anomaly cancellation. SM and MSSM

The SM gauge group is  $SU(3)_c \times SU(2)_L \times U(1)_Y$  and its fermionic content is displayed in Table 3.1

Now we check that  $\mathcal{A}^{ijk}$  vanishes for all the generators of the SM gauge group. We need only to consider those combinations of generators for which the product

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$(u_i d_i)$	<b>3</b>	<b>2</b>	1/6
$u_i^c$	$\bar{\mathbf{3}}$	<b>1</b>	-2/3
$d_i^c$	$\bar{\mathbf{3}}$	<b>1</b>	1/3
$(\nu_i e_i)$	<b>1</b>	<b>2</b>	-1/2
$e_i^c$	<b>1</b>	<b>1</b>	1

Table 3.1: Fermionic charges in the Standard Model. The index  $i$  runs over the three families.

of  $T_i$ ,  $T_j$  and  $T_k$  is neutral under  $SU(3)_c \times SU(2)_L \times U(1)_Y$ , since  $\mathcal{A}^{ijk}$  obviously vanishes for all the others. We can make invariants out of either zero, two or three  $SU(3)_c$  or  $SU(2)_L$  generators and any number of  $U(1)_Y$  generators. We will use the following notation.  $Y_f$  is the hypercharge and  $T_{k_a}^{(a)}$ ,  $a = 2, 3$ ;  $k_a = 1, \dots, \dim G^{(a)}$  are the generators of the  $G^{(2)} = SU(2)$  and  $G^{(3)} = SU(3)$  algebras respectively. In our notation  $\text{Tr}[T_j^{(a)} T_k^{(a)}] = \frac{1}{2} \delta_{jk}$ . So we need to check the following cases

- $SU(3)_c - SU(3)_c - SU(3)_c$ . Here  $\mathcal{A}^{ijk}$  vanishes because the left-handed fermions furnish a representation  $\mathbf{3} + \mathbf{3} + \bar{\mathbf{3}} + \bar{\mathbf{3}} + \mathbf{1} + \mathbf{1} + \mathbf{1}$  of  $SU(3)_c$  which is real. In other words, for each quark, the  $q$  contribution is cancelled exactly by the corresponding  $q^c$  contribution.

- $U(1)_Y - SU(3)_c - SU(3)_c$ . Here the anomaly is

$$\sum_{f \in \mathbf{3}, \bar{\mathbf{3}}} Y_f \text{Tr}[T_{k_3}^{(3)} T_{k_3}^{(3)}] = \frac{1}{2} \sum_{f \in \mathbf{3}, \bar{\mathbf{3}}} Y_f = \frac{3}{2} \left[ 2 \left( \frac{1}{6} \right) - \frac{2}{3} + \frac{1}{3} \right] = 0 \quad (3.38)$$

- $SU(2)_L - SU(2)_L - SU(2)_L$ . There is no anomaly here because  $SU(2)_L$  only has real or pseudoreal representations.

- $U(1)_Y - SU(2)_L - SU(2)_L$ . There the anomaly is

$$\sum_{f \in \mathbf{2}} Y_f \text{Tr}[T_{k_2}^{(2)} T_{k_2}^{(2)}] = \frac{1}{2} \sum_{f \in \mathbf{2}} Y_f = \frac{3}{2} \left[ 3 \left( \frac{1}{6} \right) - \frac{1}{2} \right] = 0 \quad (3.39)$$

- $U(1)_Y - U(1)_Y - U(1)_Y$ . There the anomaly is

$$\sum_f (Y_f)^3 = 3 \left[ 6 \left( \frac{1}{6} \right)^3 + 3 \left( -\frac{2}{3} \right)^3 + 3 \left( \frac{1}{3} \right)^3 + 2 \left( -\frac{1}{2} \right)^3 + (1)^3 \right] = 0 \quad (3.40)$$

### 3.3 Anomaly cancellation. Green-Schwarz mechanism

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There is one more anomaly that needs to be evaluated. All the fermions interact also with gravity. We can have anomalies different from zero generated by the fermionic with an incoming gauge vector a two outgoing gravitons. This anomaly is proportional to the trace of the gauge generator involved in the diagram. For  $SU(3)_c$  and  $SU(2)_L$  there is no problem because the generator are traceless, so we need to check only the hypercharge. We have

$$\sum_f Y_f = 3 \left[ 6 \left( \frac{1}{6} \right) + 3 \left( -\frac{2}{3} \right) + 3 \left( \frac{1}{3} \right) + 2 \left( -\frac{1}{2} \right) + 1 \right] = 0 \quad (3.41)$$

so there are no gravitational anomalies in the SM.

The same happens for the MSSM. With respect to the SM now we have two new fermions that are involved in the anomalies, the higgsinos. As we have just seen, the SM fermions contribution is zero, so the only relevant contribution comes from the supersymmetric sector. However  $\tilde{h}_u$  and  $\tilde{h}_d$  have opposite charge assignment (see Table 2.1) so that the higgsino contribution is zero. The final result is an anomaly free model.

## 3.3 Anomaly cancellation. Green-Schwarz mechanism

### 3.3.1 Introduction

Before introducing the GS mechanism, we give a generalization of the chiral anomalies in even dimensions  $d = 2\omega$ . In any even dimensions (and only in even dimensions) one can define a chirality matrix  $\gamma_{2\omega+1}$  that anticommutes with all matrices  $\gamma_\mu$ ,  $\mu = 0, \dots, 2\omega - 1$  (compare with eq. (A.6)). One can then have massless fermions that are chiral, i.e. either left-handed or right-handed. As in Section 3.1, the anomalies can again be traced to the presence of the chirality matrix  $\gamma_{2\omega+1}$  (i.e. axial coupling) in the interaction vertices between fermions and vectors. In  $2\omega$  dimensions the traces involving  $\gamma_{2\omega+1}$  will lead to an  $\epsilon^{\mu_1 \dots \mu_{2\omega}}$ , and again the WIs on the amplitude with  $k + 1$  external vector,  $\Gamma^{\rho\mu_1 \dots \mu_k}(p_1, \dots, p_k)$ , must involve this  $\epsilon^{\mu_1 \dots \mu_{2\omega}}$  tensor. Since this  $\epsilon$ -tensor is completely antisymmetric in all its  $2\omega$  indices, to get a non vanishing expression we need at least  $\omega$  indices  $\mu_i$  and  $\omega$  different independent momenta. Hence the minimal value for  $k$  is  $\omega$ . It follows that in  $2\omega$  dimensions the anomaly first manifests itself in the loop amplitude  $\Gamma^{\rho\mu_1 \dots \mu_\omega}$  with  $(\omega + 1)$  external vectors.

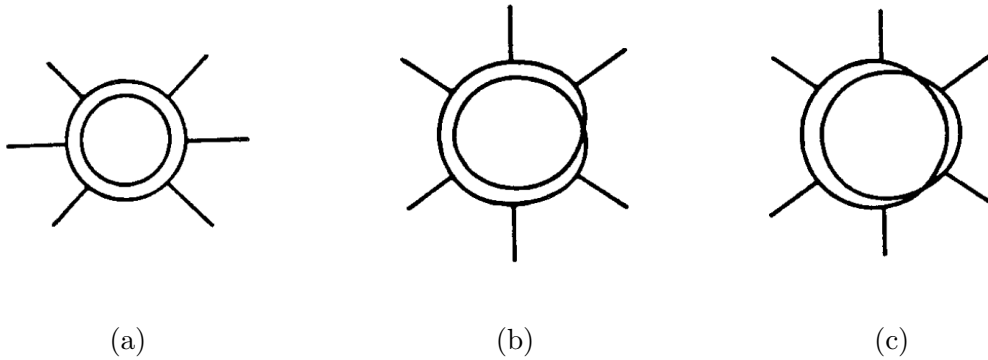


Figure 3.3: Hexagon string diagrams: planar (a), non orientable (b) and non planar (c).

In four dimensions this corresponds to the fermionic triangle loop. For further details and general properties of the anomalies, we remand for instance to [22].

In 1984, M. Green and J. H. Schwarz realized the cancellation of anomalies in type I superstring theory with the gauge group  $SO(32)$ . Now we consider only the effective field theory point of view. For further details and the string formalism, we remand to the original papers [23]. In ten-dimensional theories gauge anomalies, mixed anomalies, and gravitational anomalies were expected to arise from a hexagon loop diagram ( $\omega = 10/2 = 5$  then  $k = 6$ ). The anomaly cancels because of an extra contribution from a 2-form field,  $B_{\mu\nu}$ , which is one of the new states predicted by string theory (if we can think of a gauge vector  $V_\mu$  as column, we can think of  $B_{\mu\nu}$  as a matrix). For the special choice of the gauge group  $SO(32)$  or  $E_8 \times E_8$ , however, the anomaly factorizes and may be cancelled by a tree diagram. The tree diagram describes the exchange of a virtual  $B$ . It is somewhat counterintuitive to see that a tree diagram cancels a one-loop diagram, but in reality, both of these diagrams arise as one-loop diagrams in superstring theory, in which the anomaly cancellation is more transparent. In Fig. 3.3 and Fig. 3.4 are depicted respectively the diagrams involved in the string theory mechanism and in the effective field theory mechanism. The diagram (a) and (b) of Fig. 3.3 are generated by planar open string loops and we can naively say that they produce the effective fermionic hexagon diagram, while the diagram (c) is a non planar open string loop. However the latter can also be viewed as the tree level propagation of a closed string. One of the closed string states is the

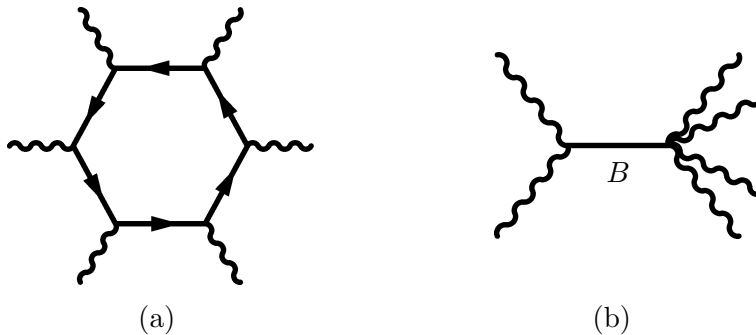


Figure 3.4: Feynman diagrams with six external gauge fields, involved in the GS mechanism in 10-D theories: (a) the hexagon fermionic loop; (b) the tree level diagram with the 2-form  $B$  exchanged.

$B$  field, so we can naively say that this diagram, apart from contributing to the diagram of Fig. 3.4(a), produces the effective tree level diagram of Fig. 3.4(b).

Let us see how the mechanism works for the  $SO(32)$  gauge group. The low energy effective field theory is  $N = 1$   $D = 10$  supergravity coupled to supersymmetric Yang-Mills theory. The bosonic action is given by

$$S_0 = - \int d^{10}x e \left\{ \frac{1}{2k^2} R + \frac{1}{k^2} \varphi^{-2} \partial_\mu \varphi \partial^\mu \varphi + \frac{1}{4g^2} \varphi^{-1} F_{\mu\nu}^a F^{\mu\nu a} + \frac{3k^2}{2g^4} \varphi^{-2} H_{\mu\nu\rho} H^{\mu\nu\rho} \right\} \quad (3.42)$$

where  $g$  and  $k$  are respectively the Yang-Mills and the gravitational coupling,  $e$  is the determinant of the zehnbain  $e_\mu^m$  and  $\varphi$  is a scalar field. The Yang-Mills field strength is defined (in the language of forms) by

$$F = \frac{1}{2} F_{\mu\nu} dx^\mu \wedge dx^\nu = dA + A^2 \quad (3.43)$$

where  $A^2 = A \wedge A$ . The field strength  $H$  associated with the two-form potential  $B = B_{\mu\nu} dx^\mu \wedge dx^\nu$  of the supergravity multiplet is defined by

$$H = dB - \omega_{3Y}^0 - \omega_{3L}^0 \quad (3.44)$$

where  $\omega_{3Y}^0$  is the Yang-Mills Chern-Simons three-form

$$\omega_{3Y}^0 = \text{Tr}(AF - \frac{1}{3} A^3) \quad (3.45)$$

## Anomalies

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and  $\omega_{3L}^0$  is the Lorentz group Chern-Simons three-form

$$\omega_{3L}^0 = \text{Tr}(\omega R - \frac{1}{3}\omega^3) \quad (3.46)$$

Here  $\omega = \omega_\mu dx^\mu$  is the local Lorentz connection and  $R = d\omega + \omega^2$  is the Lorentz curvature two-form. The Chern-Simons forms (3.45) and (3.46) satisfy

$$d\omega_{3Y}^0 = \text{Tr}F^2 \quad (3.47)$$

$$d\omega_{3L}^0 = \text{Tr}R^2 \quad (3.48)$$

Under infinitesimal local Yang-Mills transformation (with parameter  $\Lambda$ ) and local Lorentz transformation (with parameter  $\Theta$ ), the fields transform as follows

$$\delta A = d\Lambda + [A, \Lambda] \quad \delta F = [F, \Lambda] \quad (3.49)$$

$$\delta\omega = d\Theta + [\omega, \Theta] \quad \delta R = [R, \Theta] \quad (3.50)$$

$$\delta B = \text{Tr}(A d\Lambda) + \text{Tr}(\omega d\Theta) \quad \delta H = 0 \quad (3.51)$$

Consider the Yang-Mills hexagon anomaly. The analysis of [24] relates the anomalies to a formal expression in 12 dimensions, namely the gauge invariant two-form  $\Omega_{12}$ , where

$$\Omega_{12} = \text{Tr}F^6 = 15\text{Tr}F^2\text{Tr}F^4 \quad (3.52)$$

The last equality is true only for the  $SO(32)$  gauge group. This can be expressed as

$$\text{Tr}F^2\text{Tr}F^4 = d(\omega_{3Y}^0\text{Tr}F^4) = d(\omega_{7Y}^0\text{Tr}F^2) \quad (3.53)$$

where  $\omega_{2n+1Y}^0$  is defined by

$$d\omega_{2n+1Y}^0 = \text{Tr}F^{n+1} \quad (3.54)$$

Similarly, applying an infinitesimal gauge transformation, one defines  $\omega_{2nY}^1$  by

$$\delta\omega_{2n+1Y}^0 = -d\omega_{2nY}^1 \quad (3.55)$$

In the case of Yang-Mills hexagon, the consistent anomaly is

$$G = \mathcal{A} \int \left( \frac{1}{3}\omega_{2Y}^1\text{Tr}F^4 + \frac{2}{3}\omega_{6Y}^1\text{Tr}F^2 \right) \quad (3.56)$$

where the anomaly factor  $\mathcal{A}$  can be deduced from [24]. This anomaly can be cancelled by adding to the effective action

$$S_1 = \mathcal{A} \int \left( B\text{Tr}F^4 + \frac{2}{3}\omega_{3Y}^0\omega_{7Y}^0 \right) \quad (3.57)$$

The non trivial  $\Lambda$  transformation of  $B$  in eq. (3.51) is the key of the cancellation

$$G + \delta_\Lambda S_1 = 0 \tag{3.58}$$

The diagram (b) in Fig. 3.4 is built by connecting the  $B\text{Tr}F^4$  vertex in eq. (3.57) to the  $\omega_{3Y}^0 dB$  vertex (contained in the  $H^2$  term) in eq. (3.42) by a  $B$  propagator.

This is the cancellation of the Yang-Mills anomaly for the gauge group  $SO(32)$ . The cancellation of the gravitational and mixed anomaly goes in a similar way and we remand to the original paper [23] for details.

In the following section we will see that something similar happens in four-dimensional theories. The anomaly, as shown in the previous section, is related to a triangle loop diagram and it can be cancelled by the contribution of an extra field, the *axion*<sup>2</sup>.

### 3.3.2 Triangular Anomaly

Let us go back to the Lagrangian (3.1). Let  $B_\mu$  be anomaly free i.e. eq. (3.36) holds, but let  $C_\mu$  be anomalous i.e.  $\mathcal{A}^{CCC}, \mathcal{A}^{CCB}, \mathcal{A}^{CBB} \neq 0$ . We use the GS mechanism to restore gauge invariance. As stated before, we need to modify the Lagrangian inserting a new field, the axion  $\phi$ . So we get

$$\mathcal{L} = \mathcal{L}_{\text{chiral}} + \mathcal{L}_{\text{axion}} + \mathcal{L}_{\text{GCS}} \tag{3.59}$$

$\mathcal{L}_{\text{axion}}$  contains the couplings of  $\phi$

$$\begin{aligned} \mathcal{L}_{\text{axion}} = & \frac{1}{2} (\partial_\mu \phi + 2b_3 C_\mu)^2 \\ & - \frac{1}{4} b_2^{CCC} \phi \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^C F_{\rho\sigma}^C - \frac{1}{4} b_2^{CCB} \phi \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^B F_{\rho\sigma}^C - \frac{1}{4} b_2^{CBB} \phi \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^B F_{\rho\sigma}^B \end{aligned} \tag{3.60}$$

The first row is the Stückelberg Lagrangian [26]. The axion is uncharged under  $B_\mu$  and  $C_\mu$  and it transforms as

$$\begin{aligned} C_\mu & \rightarrow C_\mu - \partial_\mu \epsilon \\ \phi & \rightarrow \phi + 2b_3 \epsilon \end{aligned} \tag{3.61}$$

so that the Stückelberg Lagrangian is gauge invariant. The reason to have these terms is to give a mass  $2b_3$  to the gauge vector  $C_\mu$  without using a proper scalar

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<sup>2</sup>In four dimensions the axion is dual to a 2-form. See for instance [25] for the GS-mechanism with this duality manifest.

potential breaking some symmetries . The axion is the third degree of freedom of the massive  $C_\mu$ , so it is an unphysical particle, a Goldstone boson. The second row of terms contains the GS couplings<sup>3</sup> ( $\phi F \tilde{F}$ ) which are not gauge invariant because of the  $\phi$  transformation. These terms are directly involved in the anomaly cancellation procedure and their form is dictated by the form of the WIs for the triangle amplitudes.

$\mathcal{L}_{\text{GCS}}$  contains the so called Generalized Chern-Simons couplings

$$\mathcal{L}_{\text{GCS}} = d^{CCB} \epsilon^{\mu\nu\rho\sigma} C_\mu B_\nu F_{\rho\sigma}^C - d^{CBB} \epsilon^{\mu\nu\rho\sigma} C_\mu B_\nu F_{\rho\sigma}^B \quad (3.62)$$

which are antisymmetric trilinear interactions of gauge vectors. Also these terms are not gauge invariant and they will play some role in the GS mechanism.

The cancellation of the anomalies will fix the GS and GCS couplings. In the following we will show two different approaches to the GS mechanism: using WIs or using the effective Lagrangian. Obviously we will get the same result.

### 3.3.3 Ward Identities for massive gauge fields

As stated before, the WIs on a Feynman amplitude reflect the gauge invariance of a quantum field theory. They can be written in the diagrammatic form showed in Fig. 3.2. However, it is well known that gauge invariance can be achieved only with massless vector fields. On the other side we experience the existence of not only physical massless vectors, with two degrees of freedom, such as the photon, but also of physical massive vectors, with three degrees of freedom, such as the  $Z_0$ . The Proca Lagrangian describes the last one, but it is not gauge invariant since an explicit mass term for the vector breaks the gauge invariance. Fortunately, it is well known that we can avoid such a problem with the Higgs or with the Stückelberg mechanism. We add in a proper way (which depends on the mechanism used) a scalar field (Goldstone boson) and vector mass terms to the massless vector Lagrangian. This scalar field will become the third degree of freedom of the massive vector. However this will become manifest after a unitary gauge fixing. In this gauge the Goldstone bosons are set equal to zero, we turn back to the Proca Lagrangian and the physical degrees of freedom are manifest. In this way we can achieve a gauge theory with a massive vector and since we have the gauge invariance we expect that the WIs hold. But now we have to take

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<sup>3</sup>We use the notation  $\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$ .

$$-ik^\mu \left( V^\mu(k) \text{---} \text{blob} \text{---} \right) + m_V \left( G_V(k) \text{---} \text{blob} \text{---} \right) = 0$$

Figure 3.5: Ward Identity for the massive gauge field  $V^\mu$ .  $G_V$  is the corresponding Goldstone boson and  $m_V$  is the mass of  $V_\mu$ . The blob denotes all the 1PI diagrams.

$$-i(p+q)^\mu \left( (Z_0)_\mu(p+q) \text{---} \text{blob} \text{---} \begin{array}{l} e^-(q) \\ e^+(p) \end{array} \right) + M_{Z_0} \left( G_0(p+q) \text{---} \text{blob} \text{---} \begin{array}{l} e^-(q) \\ e^+(p) \end{array} \right) = 0$$

Figure 3.6: Ward Identities for the decay  $Z_0 \rightarrow e^+e^-$ . The first term is the usual one in WI while the second one is the correction coming from the Goldstone boson  $G_0$ .

into account that we have a new degree of freedom coming from the Goldstone boson, which is directly related to the gauge sector. This modifies the WIs, which get the diagrammatic form showed in Fig. 3.5 [27]. As we can see, now we have a contribution coming from the Goldstone boson<sup>4</sup>. To clarify the point we consider a specific example: the decay  $Z_0 \rightarrow e^+e^-$  in the SM. The Feynman amplitude of the process is given by (for the Feynman rules see Appendices C.2 and C.3)

$$\Gamma_\mu(Z_0 \rightarrow e^+e^-) = -\frac{i}{2} g_{Z_0} \bar{u}(q) \gamma_\mu (v_e^{Z_0} - a_e^{Z_0} \gamma_5) v(p) \quad (3.63)$$

where  $v_e = -1/2 + 2 \sin^2 \theta_W$ ,  $a_e = -1/2$  and  $g_{Z_0} = \sqrt{g_1^2 + g_2^2}$ . The WI for the process is given in Fig. 3.6. The first term is

$$\begin{aligned} -i(p+q)^\mu \Gamma_\mu(Z_0 \rightarrow e^+e^-) &= -\frac{1}{2} g_{Z_0} \bar{u}(q) (\not{p} + \not{q}) (v_e^{Z_0} - a_e^{Z_0} \gamma_5) v(p) \\ &= +\frac{1}{2} g_{Z_0} m_e \bar{u}(q) (v_e^{Z_0} + a_e^{Z_0} \gamma_5) v(p) \\ &\quad -\frac{1}{2} g_{Z_0} m_e \bar{u}(q) (v_e^{Z_0} - a_e^{Z_0} \gamma_5) v(p) \\ &= g_{Z_0} m_e a_e \bar{u}(q) \gamma_5 v(p) \\ &= -\frac{1}{2} m_e \sqrt{g_1^2 + g_2^2} \bar{u}(q) \gamma_5 v(p) \end{aligned} \quad (3.64)$$

<sup>4</sup>For a brief demonstration see Appendix D.2.

where  $m_e$  is the electron mass and we used the equation of motion of the spinors

$$\bar{u}(\not{q} - m_e) = 0 \quad (3.65)$$

$$(\not{p} + m_e)v = 0 \quad (3.66)$$

The second term is

$$\begin{aligned} M_{Z_0}\Gamma(G_0 \rightarrow e^+e^-) &= M_{Z_0}\frac{y_e}{\sqrt{2}}\bar{u}(q)\gamma_5v(p) \\ &= \frac{1}{2}v\sqrt{g_1^2 + g_2^2}\frac{m_e}{v}\bar{u}(q)\gamma_5v(p) \\ &= \frac{1}{2}m_e\sqrt{g_1^2 + g_2^2}\bar{u}(q)\gamma_5v(p) \end{aligned} \quad (3.67)$$

where we used  $y_e/\sqrt{2} = m_e/v$  and eq. (2.67). Summing the eq. (3.64) and (3.67) we get

$$-i(p+q)^\mu\Gamma_\mu(Z_0 \rightarrow e^+e^-) + M_{Z_0}\Gamma(G_0 \rightarrow e^+e^-) = 0 \quad (3.68)$$

in agreement with the result showed in Fig. 3.5.

### 3.3.4 Ward Identities and Anomaly Cancellation

Let us go back to the anomaly cancellation procedure. We focus on the  $CBB$  anomaly. The WIs (3.29), (3.30) and (3.31) modified because of the presence of GS and GCS couplings. Using Fig. 3.5, we get the WIs for the  $CBB$  amplitude expressed in diagrammatic form in Fig. 3.7. The Feynman rules for the GS mechanism are given in Appendix C.1. We denote by  $(\text{GS})_{\mu\nu}^{BB}$  the contribution coming from the GS coupling  $\phi F^B \tilde{F}^B$  in Lagrangian (3.60)

$$(\text{GS})_{\mu\nu}^{BB} = -2ib_2^{CBB}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta \quad (3.69)$$

and we denote by  $(\text{GCS})_{\rho\mu\nu}^{CBB}$  the contribution from the GCS coupling  $CB\tilde{F}^B$  in Lagrangian (3.62)

$$(\text{GCS})_{\rho\mu\nu}^{CBB} = 2d^{CBB}\epsilon_{\rho\nu\mu\alpha}(p-q)^\alpha \quad (3.70)$$

The WIs in Fig. 3.7 correspond to

$$\begin{aligned} (p+q)^\rho\left(\Delta_{\rho\mu\nu}^{CBB}(p,q;0) + (\text{GCS})_{\rho\mu\nu}^{CBB}\right) + 2ib_3(\text{GS})_{\mu\nu}^{BB} &= 0 \\ p^\mu\left(\Delta_{\rho\mu\nu}^{CBB}(p,q;0) + (\text{GCS})_{\rho\mu\nu}^{CBB}\right) &= 0 \\ q^\nu\left(\Delta_{\rho\mu\nu}^{CBB}(p,q;0) + (\text{GCS})_{\rho\mu\nu}^{CBB}\right) &= 0 \end{aligned} \quad (3.71)$$

$$\begin{aligned}
 (p+q)^\rho & \left( C_\rho(p+q) \begin{array}{c} \text{triangle with } \psi \text{ and } B \text{ lines} \\ B_\nu(q) \text{ top, } B_\mu(p) \text{ bottom} \end{array} + C \begin{array}{c} \text{blob with } B \text{ lines} \\ B \text{ top, } B \text{ bottom} \end{array} \right) + 2ib_3 \left( \phi \text{ --- } \begin{array}{c} \text{blob with } B \text{ lines} \\ B \text{ top, } B \text{ bottom} \end{array} \right) = 0 \\
 p^\mu & \left( C \begin{array}{c} \text{triangle with } \psi \text{ and } B \text{ lines} \\ B \text{ top, } B \text{ bottom} \end{array} + C \begin{array}{c} \text{blob with } B \text{ lines} \\ B \text{ top, } B \text{ bottom} \end{array} \right) = 0 \\
 q^\nu & \left( C \begin{array}{c} \text{triangle with } \psi \text{ and } B \text{ lines} \\ B \text{ top, } B \text{ bottom} \end{array} + C \begin{array}{c} \text{blob with } B \text{ lines} \\ B \text{ top, } B \text{ bottom} \end{array} \right) = 0
 \end{aligned}$$

Figure 3.7: The Ward Identities for the amplitude  $C_\rho(p+q) \rightarrow B_\mu(p) B_\nu(q)$  include the GCS as well as the axionic (GS) couplings. The GS coupling arises from Lagrangian (3.60) while the GCS coupling comes from Lagrangian (3.62). Each depicted diagram also contains the exchange  $(\mu, p) \leftrightarrow (\nu, q)$ .

Using eq. (3.29), (3.30) and (3.31), the last two identities imply

$$(2 + \beta) \frac{\mathcal{A}^{CBB}}{16\pi^2} - 2d^{CBB} = 0 \quad \Rightarrow \quad d^{CBB} = \frac{2 + \beta}{2} \frac{\mathcal{A}^{CBB}}{16\pi^2} \quad (3.72)$$

and the first identity becomes

$$-\beta \frac{\mathcal{A}^{CBB}}{8\pi^2} + 4 \frac{2 + \beta}{2} \frac{\mathcal{A}^{CBB}}{16\pi^2} + 4b_2^{CBB} b_3 = 0 \quad \Rightarrow \quad b_2^{CBB} b_3 = -\frac{\mathcal{A}^{CBB}}{16\pi^2} \quad (3.73)$$

In the same way<sup>5</sup>, the cancellation of the remaining anomalies gives

$$b_2^{CCC} b_3 = -\frac{\mathcal{A}^{CCC}}{48\pi^2} \quad (3.74)$$

$$b_2^{CCB} b_3 = -\frac{\mathcal{A}^{CBB}}{16\pi^2} \quad d^{CCB} = \frac{\beta'}{4} \frac{\mathcal{A}^{CCB}}{8\pi^2} \quad (3.75)$$

where we used different shift parameters  $\beta'$  and  $\beta''$  respectively for the new triangles  $CCB$  and  $CCC$ . However as stated in the end of Section 3.1 and done

<sup>5</sup>The momenta of the  $CCB$  triangle are so that on the incoming  $(p+q)_\rho$  line we have the  $B$  vector.

in eq. (3.74),  $\beta''$  must be fixed to  $\beta'' = -2/3$  because we have three equal external vectors. It is worth noting that the GCS coefficients are determined in terms of the  $\mathcal{A}$ 's by the WIs, while the  $b_2$ 's depend only on the free parameter  $b_3$ , which is related to the mass of the anomalous  $U(1)$ . Moreover the GCS couplings depends on the scheme of the anomaly distribution so they do not contain any physical information (see Section 3.5). We can always cancel them with a particular choice of the shift parameters, in fact they are also called GCS counterterms because they parametrize the arbitrariness of the scheme of the anomaly distribution in the anomaly cancellation procedure. On the contrary, the GS coupling does not depend to any shift parameters and so they have some physical relevance as we will show in the computation of the amplitudes in Section 3.5.

### 3.3.5 Effective Lagrangian

Now we show the anomaly cancellation using the variation of the effective Lagrangian. As before we focus on the  $CBB$  amplitude. The WIs (3.29), (3.30) and (3.31) induce the following variations on the 1 loop effective Lagrangian. For

$$C_\mu \rightarrow C_\mu - \partial_\mu \epsilon^C \quad (3.76)$$

$$B_\mu \rightarrow B_\mu - \partial_\mu \epsilon^B \quad (3.77)$$

we have

$$\delta_C \mathcal{L}_{\text{chiral}}^{CBB} = \epsilon_C \frac{\beta}{2} \frac{1}{32\pi^2} \mathcal{A}^{CBB} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^B F_{\rho\sigma}^B \quad (3.78)$$

$$\delta_B \mathcal{L}_{\text{chiral}}^{CBB} = -2\epsilon_B \frac{2+\beta}{4} \frac{1}{32\pi^2} \mathcal{A}^{CBB} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^B F_{\rho\sigma}^C \quad (3.79)$$

The requirement of gauge invariance implies

$$\delta_C \mathcal{L}_{1\text{loop}}^{CBB} = \delta_C \mathcal{L}_{\text{chiral}}^{CBB} + \delta_C \mathcal{L}_{\text{axion}}^{BB} + \delta_C \mathcal{L}_{\text{GCS}}^{CBB} = 0 \quad (3.80)$$

$$\delta_B \mathcal{L}_{1\text{loop}}^{CBB} = \delta_B \mathcal{L}_{\text{chiral}}^{CBB} + \delta_B \mathcal{L}_{\text{axion}}^{BB} + \delta_B \mathcal{L}_{\text{GCS}}^{CBB} = 0 \quad (3.81)$$

The gauge variations for the axion Lagrangian are

$$\delta_C \mathcal{L}_{\text{axion}}^{BB} = -\epsilon_C \frac{1}{2} b_2^{CBB} b_3 \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^B F_{\rho\sigma}^B \quad (3.82)$$

$$\delta_B \mathcal{L}_{\text{axion}}^{BB} = 0 \quad (3.83)$$

### 3.4 Supersymmetric Green-Schwarz mechanism

and for the GCS terms are

$$\begin{aligned}\delta_C \mathcal{L}_{\text{GCS}}^{CBB} &= +d^{CBB} \epsilon^{\mu\nu\rho\sigma} \partial_\mu \epsilon_C B_\nu F_{\rho\sigma}^B = -\epsilon_C d^{CBB} \epsilon^{\mu\nu\rho\sigma} \partial_\mu (B_\nu F_{\rho\sigma}^B) \\ &= -\frac{1}{2} d^{CBB} \epsilon_C \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^B F_{\rho\sigma}^B\end{aligned}\quad (3.84)$$

$$\begin{aligned}\delta_B \mathcal{L}_{\text{GCS}}^{CBB} &= +d^{CBB} \epsilon^{\mu\nu\rho\sigma} C_\mu \partial_\nu \epsilon_B F_{\rho\sigma}^B = -\epsilon_B d^{CBB} \epsilon^{\mu\nu\rho\sigma} \partial_\nu (C_\mu F_{\rho\sigma}^B) \\ &= +\frac{1}{2} d^{CBB} \epsilon_B \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^B F_{\rho\sigma}^C\end{aligned}\quad (3.85)$$

where we used  $\partial_\mu F_{\rho\sigma}^{B,C} = 0$ . Then eq. (3.81) implies

$$-\frac{2+\beta}{2} \frac{1}{32\pi^2} \mathcal{A}^{CBB} + \frac{1}{2} d^{CBB} = 0 \quad \Rightarrow \quad d^{CBB} = \frac{2+\beta}{32\pi^2} \mathcal{A}^{CBB}\quad (3.86)$$

Substituting this result into eq. (3.80), we get

$$\begin{aligned}\frac{\beta}{2} \frac{1}{32\pi^2} \mathcal{A}^{CBB} - \frac{1}{2} b_2^{CBB} b_3 - \frac{1}{2} \frac{2+\beta}{32\pi^2} \mathcal{A}^{CBB} &= -\frac{1}{2} \frac{2}{32\pi^2} \mathcal{A}^{CBB} - \frac{1}{2} b_2^{CBB} b_3 \\ \Rightarrow b_2^{CBB} b_3 &= -\frac{1}{16\pi^2} \mathcal{A}^{CBB}\end{aligned}\quad (3.87)$$

obtaining the same relations as eq. (3.72) and (3.73).

We showed the anomaly cancellation procedure in two different ways, the WI method or the effective Lagrangian one. Dependently on what we need to compute we will choose the most convenient procedure.

### 3.4 Supersymmetric Green-Schwarz mechanism

In this section we illustrate how to supersymmetrize what we did in the previous section. The supersymmetric version of eq. (3.60) and (3.62) are (see [25] and [28])

$$\begin{aligned}\mathcal{L}_{\text{axion}} &= \frac{1}{4} (S + S^\dagger + 4b_3 V^C)^2 \Big|_{\theta^2 \bar{\theta}^2} - \frac{1}{2} \left\{ [b_2^{CCC} S \text{Tr}(W^C W^C) \right. \\ &\quad \left. + b_2^{CBB} S \text{Tr}(W^B W^B) + b_2^{CCB} S W^B W^C]_{\theta^2} + h.c. \right\}\end{aligned}\quad (3.88)$$

$$\begin{aligned}\mathcal{L}_{\text{GCS}} &= +d^{CCB} [(V^B D^\alpha V^C - V^C D^\alpha V^B) W_\alpha^C + h.c.]_{\theta^2 \bar{\theta}^2} + \\ &\quad -d^{CBB} [(V^B D^\alpha V^C - V^C D^\alpha V^B) W_\alpha^B + h.c.]_{\theta^2 \bar{\theta}^2}\end{aligned}\quad (3.89)$$

where the  $V$ 's and the  $W$ 's are the gauge superfields and the corresponding superfield strength, and  $S$  is the Stückelberg multiplet

$$S = s + i\sqrt{2}\theta\psi_S + \theta^2 F_S - i\theta\sigma^\mu\bar{\theta}\partial_\mu s + \frac{\sqrt{2}}{2}\theta^2\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi_S - \frac{1}{4}\theta^2\bar{\theta}^2\Box s\quad (3.90)$$

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which transforms under the  $U(1)_C$  as

$$\begin{aligned} V^C &\rightarrow V^C + i(\Lambda^C - \Lambda^{C\dagger}) \\ S &\rightarrow S - 4i b_3 \Lambda^C \end{aligned} \quad (3.91)$$

The component fields of  $S$  are the complex scalar  $s = \alpha + i\phi$ , where  $\alpha$  is the *saxion* and  $\phi$  is the axion, and the spinorial field  $\psi_S$  which is called *axino*. To fix the  $b$ 's and  $d$ 's parameter we act on the bosonic part as we did in Section 3.3, since the supersymmetric part will be fixed by supersymmetry, or we act directly on the superfields. We show the last method for the  $CBB$  case. Under the gauge transformations

$$\begin{aligned} V^C &\rightarrow V^C + i(\Lambda^C - \Lambda^{C\dagger}) \\ V^B &\rightarrow V^B + i(\Lambda^B - \Lambda^{B\dagger}) \end{aligned} \quad (3.92)$$

we have

$$\delta_C \mathcal{L}_{\text{chiral}}^{CBB} = -i \frac{\beta}{2} \frac{1}{32\pi^2} \mathcal{A}^{CBB} [\Lambda^C \text{Tr}(W^B W^B)]_{\theta^2} + h.c. \quad (3.93)$$

$$\delta_B \mathcal{L}_{\text{chiral}}^{CBB} = 2i \frac{2+\beta}{4} \frac{1}{32\pi^2} \mathcal{A}^{CBB} [\Lambda^B \text{Tr}(W^B W^C)]_{\theta^2} + h.c. \quad (3.94)$$

$$\delta_C \mathcal{L}_{\text{axion}}^{BB} = 2ib_2^{CBB} b_3 [\Lambda^C \text{Tr}(W^B W^B)]_{\theta^2} + h.c. \quad (3.95)$$

$$\delta_B \mathcal{L}_{\text{axion}}^{BB} = 0 \quad (3.96)$$

$$\delta_C \mathcal{L}_{\text{GCS}}^{CBB} = +\frac{i}{2} d^{CBB} [\Lambda^C \text{Tr}(W^B W^B)]_{\theta^2} + h.c. \quad (3.97)$$

$$\delta_B \mathcal{L}_{\text{GCS}}^{CBB} = -\frac{i}{2} d^{CBB} [\Lambda^B \text{Tr}(W^B W^C)]_{\theta^2} + h.c. \quad (3.98)$$

The requirement of gauge invariance will impose the following conditions

$$\begin{aligned} \delta_C \mathcal{L}_{\text{1loop}}^{CBB} &= \delta_C \mathcal{L}_{\text{chiral}}^{CBB} + \delta_C \mathcal{L}_{\text{axion}}^{BB} + \delta_C \mathcal{L}_{\text{GCS}}^{CBB} \\ &= \left[ -i \frac{\beta}{2} \frac{1}{32\pi^2} \mathcal{A}^{CBB} + 2ib_2^{CBB} b_3 + \frac{i}{2} d^{CBB} \right] [\Lambda^C \text{Tr}(W^B W^B)]_{\theta^2} + h.c. = 0 \end{aligned} \quad (3.99)$$

$$\begin{aligned} \delta_B \mathcal{L}_{\text{1loop}}^{CBB} &= \delta_B \mathcal{L}_{\text{chiral}}^{CBB} + \delta_B \mathcal{L}_{\text{axion}}^{BB} + \delta_B \mathcal{L}_{\text{GCS}}^{CBB} \\ &= \left[ 2i \frac{2+\beta}{4} \frac{1}{32\pi^2} \mathcal{A}^{CBB} - \frac{i}{2} d^{CBB} \right] [\Lambda^B \text{Tr}(W^B W^C)]_{\theta^2} + h.c. = 0 \end{aligned} \quad (3.100)$$

whose solutions are the eq. (3.72) and (3.73). Analogously for the other anomalies.

In this section we do not give the component fields expansion for eq. (3.88) and (3.89), leaving it for the details of our model in Section 4.1.1.

### 3.5 Computation of the amplitudes

In this section we show how we compute the 1 loop amplitudes<sup>6</sup> involving three external vectors. As usual we focus on the  $CBB$  case since the generalization to the other cases is straightforward.

Before going on we recall the Rosenberg parametrization<sup>7</sup> [29]. The amplitudes (3.7), (3.8), (3.9) and (3.10) can be written as

$$\begin{aligned} \Gamma_{\rho\mu\nu}(p, q; 0) = & \\ & \frac{1}{\pi^2} \left( I_1(p, q; 0) \epsilon[p, \mu, \nu, \rho] + I_2(p, q; 0) \epsilon[q, \mu, \nu, \rho] + I_3(p, q; 0) \epsilon[p, q, \mu, \rho] p_\nu \right. \\ & \left. + I_4(p, q; 0) \epsilon[p, q, \mu, \rho] q_\nu + I_5(p, q; 0) \epsilon[p, q, \nu, \rho] p_\mu + I_6(p, q; 0) \epsilon[p, q, \nu, \rho] q_\mu \right) \end{aligned} \quad (3.101)$$

with  $\epsilon[p, q, \rho, \sigma] = \epsilon_{\mu\nu\rho\sigma} p^\mu q^\nu$  and where

$$\begin{aligned} I_3(p, q; 0) &= - \int_0^1 dx \int_0^{1-x} dy \frac{xy}{y(1-y)p^2 + x(1-x)q^2 + 2xy p \cdot q} \\ I_4(p, q; 0) &= \int_0^1 dx \int_0^{1-x} dy \frac{x(x-1)}{y(1-y)p^2 + x(1-x)q^2 + 2xy p \cdot q} \\ I_5(p, q; 0) &= -I_4(q, p; 0) \\ I_6(p, q; 0) &= -I_3(p, q; 0) \end{aligned} \quad (3.102)$$

In terms of the Rosenberg parametrization the  $\beta$  dependence of (3.17) is contained only in  $I_1$  and  $I_2$  (which are superficially divergent). However, using the WIs (3.26), (3.27) and (3.28) it is possible to show that they can be expressed in terms of  $I_3 \dots I_6$  as

$$\begin{aligned} I_1^{AVV}(p, q, \beta; 0) &= p \cdot q I_3(p, q; 0) + q^2 I_4(p, q; 0) + \frac{2 + \beta}{8} \\ I_2^{AVV}(p, q, \beta; 0) &= -I_1^{AVV}(q, p, \beta; 0) \end{aligned} \quad (3.103)$$

Analogously for the VAV, VVA and AAA triangles.

We can act in the same way for the  $CBB$  amplitude. The total fermionic triangle can be written as

$$\Delta_{\rho\mu\nu}^{CBB} = -\frac{\mathcal{A}^{CBB}}{2} \Gamma_{\rho\mu\nu}(p, q; 0) \quad (3.104)$$

<sup>6</sup>For a massive fermionic triangle see Appendix D.3.

<sup>7</sup>For a demonstration of the Rosenberg parametrization see Appendix D.3.

where  $\Gamma_{\rho\mu\nu}(p, q; 0)$  is the parametrization (3.101). Imposing the eq. (3.29), (3.30) and (3.31) we find that

$$\begin{aligned} I_1^{CBB}(p, q, \beta; 0) &= p \cdot q I_3(p, q; 0) + q^2 I_4(p, q; 0) + \frac{2 + \beta}{8} \\ I_2^{CBB}(p, q, \beta; 0) &= -I_1^{CBB}(q, p, \beta; 0) \end{aligned} \quad (3.105)$$

and the amplitude, as we know, is not uniquely defined. The anomaly cancellation will solve this problem. In the standard case we have  $\mathcal{A}^{CBB} = 0$  so the amplitude is uniquely defined but identically zero. Instead with the GS mechanism, the final amplitude will not be zero. In this case we have

$$A_{\rho\mu\nu}^{CBB} = \Delta_{\rho\mu\nu}^{CBB} + (\text{GCS})_{\rho\mu\nu}^{CBB} = \Delta_{\rho\mu\nu}^{CBB} + 2d^{CBB} \epsilon_{\rho\nu\mu\alpha} (p - q)^\alpha \quad (3.106)$$

The GCS terms can be reabsorbed by the following redefinitions

$$A_{\rho\mu\nu}^{CBB} = \Delta_{\rho\mu\nu}^{CBB} + (\text{GCS})_{\rho\mu\nu}^{CBB} = -\frac{\mathcal{A}^{CBB}}{2} \tilde{\Gamma}_{\rho\mu\nu} \quad (3.107)$$

where

$$\begin{aligned} \tilde{\Gamma}_{\rho\mu\nu} &= \frac{1}{\pi^2} \left( \tilde{I}_1 \epsilon[p, \mu, \nu, \rho] + \tilde{I}_2 \epsilon[q, \mu, \nu, \rho] + I_3 \epsilon[p, q, \mu, \rho] p^\nu \right. \\ &\quad \left. + I_4 \epsilon[p, q, \mu, \rho] q^\nu + I_5 \epsilon[p, q, \nu, \rho] p^\mu + I_6 (\epsilon[p, q, \nu, \rho] q^\mu) \right) \end{aligned} \quad (3.108)$$

with

$$\begin{aligned} \left( \frac{\mathcal{A}^{CBB}}{2\pi^2} \right) \tilde{I}_1(p, q) &= \left( \frac{\mathcal{A}^{CBB}}{2\pi^2} \right) I_1(p, q) - 2d^{CBB} \\ \left( \frac{\mathcal{A}^{CBB}}{2\pi^2} \right) \tilde{I}_2(p, q) &= \left( \frac{\mathcal{A}^{CBB}}{2\pi^2} \right) I_2(p, q) + 2d^{CBB} \end{aligned} \quad (3.109)$$

Using eq. (3.72) and (3.105) we get

$$\begin{aligned} \tilde{I}_1(p, q) &= p \cdot q I_3(p, q) + q^2 I_4(p, q) \\ \tilde{I}_2(p, q) &= -\tilde{I}_1(q, p) \end{aligned} \quad (3.110)$$

that relate  $\tilde{I}_1$  and  $\tilde{I}_2$  to the other  $I_i$ 's. So we have removed the  $\beta$  dependence of the fermionic triangle, thanks to the GCS coupling, and the amplitude is uniquely defined and it obeys the following WIs

$$\begin{aligned} (p + q)^\rho A_{\rho\mu\nu}^{CBB} &= \frac{\mathcal{A}^{CBB}}{32\pi^2} \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta = -2ib_3 (\text{GS})_{\mu\nu}^{BB} \\ p^\mu A_{\rho\mu\nu}^{CBB} &= 0 \\ q^\nu A_{\rho\mu\nu}^{CBB} &= 0 \end{aligned} \quad (3.111)$$

### 3.5 Computation of the amplitudes

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Finally the theory is gauge invariant, but in this case the amplitude is different from zero. This result does not depend on the scheme of the anomaly distribution.



## Chapter 4

# MiAUMSSM. Model building

One of the most attractive scenario for physics beyond the SM is the existence of additional massive neutral gauge bosons [30]-[38]. They could be one of the first discoveries at LHC if their mass is in the range of a few TeV. Many different models have been developed in the past in order to investigate this possibility. The mass could be acquired in a variety of ways: from Kaluza-Klein modes to a standard Higgs mechanism or even the Stückelberg mechanism [39, 40]. The latter is common to low energy effective field theories which appear anomalous. The anomaly cancellation is achieved by the GS mechanism which ensure the consistency of these models [41, 42].

In string theory anomalous  $U(1)$ 's are very common. D-brane models contain several abelian factors, living on each stack of branes, and they are typically anomalous [41]-[62]. In the presence of these anomalous  $U(1)$ 's, the Stückelberg mixing with the axions cancels mixed anomalies, and renders the “anomalous” gauge fields massive. The masses depend non-trivially on the internal volumes and on other moduli, allowing the physical masses of the anomalous  $U(1)$  gauge bosons to be much smaller than the string scale (even at a few TeV range).

This chapter is entirely based on [1], where we presented for the first time our model: the Minimal Anomalous  $U(1)'$  Extension of the MSSM (MiAUMSSM).

## 4.1 Preliminaries

We study an extension of the MSSM<sup>1</sup> by the addition of an abelian vector multiplet  $V^{(0)}$ . We assume that all the MSSM fields are charged under the additional vector multiplet  $V^{(0)}$ , with charges that are given in Table 4.1, where  $Q_i, L_i$  are the left handed quarks and leptons respectively while  $U_i^c, D_i^c, E_i^c$  are the right handed up and down quarks and the electrically charged leptons. The superscript  $c$  stands for charge conjugation. The index  $i = 1, 2, 3$  denotes the three different families.  $H_{u,d}$  are the two Higgs scalars. In order to gain in flexibility,

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)'$
$Q_i$	<b>3</b>	<b>2</b>	1/6	$Q_Q$
$U_i^c$	$\bar{\mathbf{3}}$	<b>1</b>	-2/3	$Q_{U^c}$
$D_i^c$	$\bar{\mathbf{3}}$	<b>1</b>	1/3	$Q_{D^c}$
$L_i$	<b>1</b>	<b>2</b>	-1/2	$Q_L$
$E_i^c$	<b>1</b>	<b>1</b>	1	$Q_{E^c}$
$H_u$	<b>1</b>	<b>2</b>	1/2	$Q_{H_u}$
$H_d$	<b>1</b>	<b>2</b>	-1/2	$Q_{H_d}$

Table 4.1: Charge assignment.

our model is only string inspired: we do not commit to a specific brane model and this is why the charges are not fixed, even if the effective cut-off is related to the mass of the  $Z'$ . Since our model is an extension of the MSSM, the gauge invariance of the superpotential (2.2), that contains the Yukawa couplings and a  $\mu$ -term, put constraints on the above charges

$$\begin{aligned}
 Q_{U^c} &= -Q_Q - Q_{H_u} \\
 Q_{D^c} &= -Q_Q + Q_{H_u} \\
 Q_{E^c} &= -Q_L + Q_{H_u} \\
 Q_{H_d} &= -Q_{H_u}
 \end{aligned} \tag{4.1}$$

Thus,  $Q_Q, Q_L$  and  $Q_{H_u}$  are free parameters of the model.

The extra vector multiplet generically is anomalous and consistency of the

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<sup>1</sup>Since we do not discuss phenomenological consequences, the treatment of all this section is before the insertion of the coupling constant using  $V \rightarrow 2gV$ .

model requires an additional Stückelberg multiplet  $S$  with the proper couplings as well as GCS terms. For the complete Lagrangian of the model see Appendix B.2.

#### 4.1.1 Anomalies and GS mechanism

As showed in Section 3.2, the MSSM is anomaly free. All the anomalies that involve only the  $SU(3)$ ,  $SU(2)$  and  $U(1)_Y$  factors vanish identically. However, triangles with  $U(1)'$  in the external legs in general are potentially anomalous. These anomalies are<sup>2</sup>

$$U(1)' - U(1)' - U(1)' \quad : \quad \mathcal{A}^{(0)} = \sum_f Q_f^3 \quad (4.2)$$

$$U(1)' - U(1)_Y - U(1)_Y \quad : \quad \mathcal{A}^{(1)} = \sum_f Q_f Y_f^2 \quad (4.3)$$

$$U(1)' - SU(2) - SU(2) \quad : \quad \mathcal{A}^{(2)} = \sum_f Q_f \text{Tr}[T_{k_2}^{(2)} T_{k_2}^{(2)}] \quad (4.4)$$

$$U(1)' - SU(3) - SU(3) \quad : \quad \mathcal{A}^{(3)} = \sum_f Q_f \text{Tr}[T_{k_3}^{(3)} T_{k_3}^{(3)}] \quad (4.5)$$

$$U(1)' - U(1)' - U(1)_Y \quad : \quad \mathcal{A}^{(4)} = \sum_f Q_f^2 Y_f \quad (4.6)$$

where  $f$  runs over the fermions in Table 4.1,  $Q_f$  is the corresponding  $U(1)'$  charge,  $Y_f$  is the hypercharge and  $T_{k_a}^{(a)}$ ,  $a = 2, 3$ ;  $k_a = 1, \dots, \dim G^{(a)}$  are the generators of the  $G^{(2)} = SU(2)$  and  $G^{(3)} = SU(3)$  algebras respectively. In our notation  $\text{Tr}[T_j^{(a)} T_k^{(a)}] = \frac{1}{2} \delta_{jk}$ . All the remaining anomalies that involve  $U(1)'$ s vanish identically due to group theoretical arguments (see Chapter 22 of [20]). Using the charge constraints (4.1) we get

$$\mathcal{A}^{(0)} = 3 \left\{ Q_{H_u}^3 + 3Q_{H_u} Q_L^2 + Q_L^3 - 3Q_{H_u}^2 (Q_L + 6Q_Q) \right\} \quad (4.7)$$

$$\mathcal{A}^{(1)} = -\frac{3}{2} (3Q_Q + Q_L) \quad (4.8)$$

$$\mathcal{A}^{(2)} = \frac{3}{2} (3Q_Q + Q_L) \quad (4.9)$$

$$\mathcal{A}^{(3)} = 0 \quad (4.10)$$

$$\mathcal{A}^{(4)} = -6Q_{H_u} (3Q_Q + Q_L) \quad (4.11)$$

<sup>2</sup>We are working in an effective field theory framework and we ignore throughout the Thesis all the gravitational effects. In particular, we do not consider the gravitational anomalies which, however, could be cancelled by the GS mechanism.

Notice that the mixed anomaly between the anomalous  $U(1)$  and the  $SU(3)$  nonabelian factors  $\mathcal{A}^{(3)}$  vanishes identically.

Many models have been developed in the past where all the anomalies (4.7-4.11) vanish by constraining the charges  $Q_f$  (see [30, 31] and references therein). On the contrary, in this Thesis we assume that the  $U(1)'$  is anomalous, i.e. (4.7)-(4.11) do not vanish. Consistency of the model is achieved by the GS mechanism showed in Sections 3.3 and 3.4. The axionic Lagrangian now reads

$$\begin{aligned} \mathcal{L}_{\text{axion}} = & \frac{1}{4} (S + S^\dagger + 4b_3 V^{(0)})^2 \Big|_{\theta^2 \bar{\theta}^2} \\ & - \frac{1}{2} \left\{ \left[ \sum_{a=0}^2 b_2^{(a)} S \text{Tr} (W^{(a)} W^{(a)}) + b_2^{(4)} S W^{(1)} W^{(0)} \right]_{\theta^2} + h.c. \right\} \end{aligned} \quad (4.12)$$

where the index  $a = 0, \dots, 3$  runs over the  $U(1)'$ ,  $U(1)_Y$ ,  $SU(2)$  and  $SU(3)$  gauge groups respectively. The Stückelberg multiplet is given in (3.90) and transforms under the  $U(1)'$  as

$$\begin{aligned} V^{(0)} & \rightarrow V^{(0)} + i (\Lambda - \Lambda^\dagger) \\ S & \rightarrow S - 4i b_3 \Lambda \end{aligned} \quad (4.13)$$

where  $b_3$  is a constant. The lowest component of  $S$  is a complex scalar field  $s = \alpha + i\phi$ . We assume that the real part  $\alpha$  gets an expectation value by an effective potential of stringy or different origin and contributes to the coupling constants as

$$\frac{1}{16g_a^2\tau_a} = \frac{1}{16\tilde{g}_a^2\tau_a} - \frac{1}{2} b_2^{(a)} \langle \alpha \rangle \quad (4.14)$$

where  $g_a$  is the redefined coupling constant and the gauge factors  $\tau_a$  take the values 1, 1, 1/2, 1/2. The first line in (4.12) is the Stückelberg Lagrangian [26] of the  $U(1)'$  which is gauge invariant and provides the kinetic terms and the axion- $U(1)'$  mixing. The second line is not gauge invariant and provides the GS couplings that participate in the anomaly cancellation procedure. Notice that in (4.12) the sum over  $a$  omits the  $a = 3$  case since there is no mixed anomaly between the  $U(1)'$  and the  $SU(3)$  factors as from eq.(4.10), i.e.  $b_2^{(3)} = 0$ . The values of the other constants,  $b_2^{(a)}$ , are fixed by the anomalies.

At first sight our Lagrangian (see Appendix B.2) may not look like the most general possible one. In particular, an explicit Fayet-Iliopoulos term  $\xi V^{(0)}$  could be added. It is well known that in certain string-inspired models (see, e.g. [63]),

an one-loop FI term is absent, even if  $Tr(Q) \neq 0$ . This is in apparent conflict with the observation [64] that in field theory a quadratically divergent FI term is always generated at one loop. The solution to this paradox is that in the low-energy Lagrangian there should be a counterterm, which compensates precisely, i.e. both the divergent and the finite part of, the one-loop contribution. We do not write explicitly this counterterm, since its exact expression is model and regularization dependent, but we implicitly assume that such a cancellation occurs. As mentioned before, also the terms responsible for the cancellation of gravitational anomalies are omitted.

Expanding  $\mathcal{L}_{\text{axion}}$  in component fields, using the Wess-Zumino gauge and substituting  $\alpha$  by its VEV we get

$$\begin{aligned}
 \mathcal{L}_{\text{axion}} = & \frac{1}{2} (\partial_\mu \phi + 2b_3 V_\mu^{(0)})^2 + \frac{i}{4} \psi_S \sigma^\mu \partial_\mu \bar{\psi}_S + \frac{i}{4} \bar{\psi}_S \bar{\sigma}^\mu \partial_\mu \psi_S \quad (4.15) \\
 & + \frac{1}{2} F_S \bar{F}_S + 2b_3 \langle \alpha \rangle D^{(0)} - \sqrt{2} b_3 (\psi_S \lambda^{(0)} + h.c.) \\
 & - \frac{1}{4} \phi \epsilon^{\mu\nu\rho\sigma} \sum_{a=0}^2 b_2^{(a)} \text{Tr} (F_{\mu\nu}^{(a)} F_{\rho\sigma}^{(a)}) - \frac{1}{4} b_2^{(4)} \epsilon^{\mu\nu\rho\sigma} \phi F_{\mu\nu}^{(1)} F_{\rho\sigma}^{(0)} \\
 & + \frac{1}{2} b_2^{(4)} \langle \alpha \rangle F_{\mu\nu}^{(1)} F_{\mu\nu}^{(0)} - b_2^{(4)} \langle \alpha \rangle D^{(1)} D^{(0)} \\
 & - \frac{1}{2} \left\{ \sum_{a=0}^2 b_2^{(a)} \left[ -2\phi \text{Tr} (\lambda^{(a)} \sigma^\mu D_\mu \bar{\lambda}^{(a)}) + \frac{i}{\sqrt{2}} \text{Tr} (\lambda^{(a)} \sigma^\mu \bar{\sigma}^\nu F_{\mu\nu}^{(a)}) \psi_S \right. \right. \\
 & \left. \left. - F_S \text{Tr} (\lambda^{(a)} \lambda^{(a)}) - \sqrt{2} \psi_S \text{Tr} (\lambda^{(a)} D^{(a)}) \right] \right. \\
 & \left. + b_2^{(4)} \left[ \left( -\phi \lambda^{(1)} \sigma^\mu \partial_\mu \bar{\lambda}^{(0)} + i \langle \alpha \rangle \lambda^{(1)} \sigma^\mu \partial_\mu \bar{\lambda}^{(0)} - \frac{1}{2} F_S \lambda^{(1)} \lambda^{(0)} \right. \right. \right. \\
 & \left. \left. \left. - \frac{1}{\sqrt{2}} \psi_S \lambda^{(1)} D^{(0)} + \frac{i}{2\sqrt{2}} \lambda^{(1)} \sigma^\mu \bar{\sigma}^\nu F_{\mu\nu}^{(0)} \psi_S \right) + (0 \leftrightarrow 1) \right] + h.c. \right\}
 \end{aligned}$$

where we omit terms which are coming from  $\langle \alpha \rangle W^{(a)} W^{(a)}$ , since they are absorbed in the coupling constant redefinition (4.14). As stressed in the previous chapter, this mechanism cancels some mixed anomalies and in addition provides a mass term to the anomalous  $U(1)$ . Therefore, the anomalous  $U(1)$  behaves *almost* like the usual  $Z'$  extensively studied in the past.

For what concern the GCS terms we have

$$\begin{aligned}
 \mathcal{L}_{\text{GCS}} = & +d_4 [(V^{(1)} D^\alpha V^{(0)} - V^{(0)} D^\alpha V^{(1)}) W_\alpha^{(0)} + h.c.]_{\theta^2 \bar{\theta}^2} + \\
 & -d_5 [(V^{(1)} D^\alpha V^{(0)} - V^{(0)} D^\alpha V^{(1)}) W_\alpha^{(1)} + h.c.]_{\theta^2 \bar{\theta}^2} + \\
 & -d_6 \text{Tr} \left[ (V^{(2)} D^\alpha V^{(0)} - V^{(0)} D^\alpha V^{(2)}) W_\alpha^{(2)} + \right. \\
 & \left. + \frac{1}{6} V^{(2)} D^\alpha V^{(0)} \bar{D}^2 ([D_\alpha V^{(2)}, V^{(2)}]) + h.c. \right]_{\theta^2 \bar{\theta}^2} \quad (4.16)
 \end{aligned}$$

and their expression in component fields

$$\begin{aligned}
 \mathcal{L}_{\text{GCS}} = & +d_4 \epsilon^{\mu\nu\rho\sigma} V_\mu^{(0)} V_\nu^{(1)} F_{\rho\sigma}^{(0)} - d_5 \epsilon^{\mu\nu\rho\sigma} V_\mu^{(0)} V_\nu^{(1)} F_{\rho\sigma}^{(1)} \\
 & -d_6 \epsilon^{\mu\nu\rho\sigma} V_\mu^{(0)} \text{Tr} \left[ V_\nu^{(2)} F_{\rho\sigma}^{(2)} - \frac{i}{3} V_\nu^{(2)} [V_\rho^{(2)}, V_\sigma^{(2)}] \right] \\
 & +d_4 (\lambda^{(0)} \sigma^\mu \bar{\lambda}^{(0)} V_\mu^{(1)} - \lambda^{(0)} \sigma^\mu \bar{\lambda}^{(1)} V_\mu^{(0)} + h.c.) \\
 & -d_5 (\lambda^{(1)} \sigma^\mu \bar{\lambda}^{(1)} V_\mu^{(0)} - \lambda^{(1)} \sigma^\mu \bar{\lambda}^{(0)} V_\mu^{(1)} + h.c.) \\
 & -d_6 \text{Tr} [\lambda^{(2)} \sigma^\mu \bar{\lambda}^{(2)} V_\mu^{(0)} - \lambda^{(2)} \sigma^\mu \bar{\lambda}^{(0)} V_\mu^{(2)} + h.c.] \quad (4.17)
 \end{aligned}$$

The constants  $d_4$ ,  $d_5$  and  $d_6$  are fixed by the cancellation of the mixed anomalies.

The cancellation of anomalies goes in the same way as in Sections 3.3 and 3.4 so we give only the results

$$\begin{aligned}
 b_2^{(1)} b_3 &= -\frac{\mathcal{A}^{(1)}}{128\pi^2} & b_2^{(2)} b_3 &= -\frac{\mathcal{A}^{(2)}}{64\pi^2} \\
 b_2^{(0)} b_3 &= -\frac{\mathcal{A}^{(0)}}{384\pi^2} & b_2^{(4)} b_3 &= -\frac{\mathcal{A}^{(4)}}{128\pi^2} \\
 d_4 &= -\frac{\mathcal{A}^{(4)}}{384\pi^2} & d_5 &= \frac{\mathcal{A}^{(1)}}{192\pi^2} & d_6 &= \frac{\mathcal{A}^{(2)}}{96\pi^2} \quad (4.18)
 \end{aligned}$$

For simplicity we chose a symmetric distribution for all the anomalies (see the discussion at the end of Section 3.1). For the anomaly cancellation in the broken phase we remand to a specific case in Section 5.2.1 and to our paper [1].

Before going on we want to spend other few words on these anomalies. In our model there are two extra states in the neutral fermionic sector, namely the axino and the primeino (see Section 4.2.7) which do not contribute to the fermionic loop. The remaining MSSM fermionic states are a bino, a wino and the two higgsinos. Both  $U(1)_Y$  and  $SU(2)$  gauginos do not contribute to the fermionic loop due to group theoretical arguments (see Section 28.1 of [65]). The higgsino eigenstates do not participate because the  $\tilde{h}_u$  contribution is cancelled by the  $\tilde{h}_d$

one. This is due to the fact that each diagram is proportional to an odd product of charges and the two higgsinos have opposite charges (see Table 4.1 and the constraints (4.1)). Therefore only SM fermions give the relevant contribution to the anomalies (4.7)-(4.11).

### 4.1.2 Soft breaking terms

Since we added new matter content to the MSSM Lagrangian we expect that new soft breaking terms can be added to the theory. The total soft breaking Lagrangian can be written as

$$\mathcal{L}_{soft} = \mathcal{L}_{soft}^{MSSM} + \mathcal{L}_{soft}^{new} \quad (4.19)$$

where  $\mathcal{L}_{soft}^{MSSM}$  is the MSSM soft breaking Lagrangian given in eq. (2.10) and

$$\mathcal{L}_{soft}^{new} = -\frac{1}{2} (M_0 \lambda^{(0)} \lambda^{(0)} + h.c.) - \frac{1}{2} \left( \frac{M_S}{2} \psi_S \psi_S + h.c. \right) \quad (4.20)$$

where  $\lambda^{(0)}$  is the gaugino of the added  $U(1)'$  and  $\psi_S$  is the axino. We allow a soft mass term for the axino since it couples only through GS interactions and not through Yukawa interactions [66]. Notice also that a mass term for the axion  $\phi$  is not allowed since it transforms non trivially under the anomalous  $U(1)'$  gauge transformation (4.13).

## 4.2 Model setup

In this section we analyze the effects of the additional terms on the rest of the Lagrangian. From now on we work with the Lagrangian (B.8) after substitution  $V^{(a)} \rightarrow g_a V^{(a)}$ .

### 4.2.1 Kinetic diagonalization of $U(1)$ 's

As we mentioned before, the Stückelberg multiplet contains a complex scalar field whose real part gets an expectation value that modifies the coupling constant (4.14). Therefore, the second line in (4.12) contributes to the kinetic terms for the gauge fields and the term  $\langle \alpha \rangle W^{(1)} W^{(0)}$  gives a kinetic mixing between the  $V^{(1)}$  and  $V^{(0)}$  gauge bosons. We have

$$\left( \frac{1}{4} W^{(0)} W^{(0)} + \frac{1}{4} W^{(1)} W^{(1)} + \frac{\delta}{2} W^{(1)} W^{(0)} \right) \Big|_{\theta^2} \quad (4.21)$$

with  $\delta = -4b_2^{(4)}g_0g_1\langle\alpha\rangle$ . In order to diagonalize the kinetic terms, we use the matrix

$$\begin{pmatrix} V^{(0)} \\ V^{(1)} \end{pmatrix} = \begin{pmatrix} C_\delta & 0 \\ -S_\delta & 1 \end{pmatrix} \begin{pmatrix} V_C \\ V_B \end{pmatrix} \quad (4.22)$$

where  $C_\delta = 1/\sqrt{1-\delta^2}$  and  $S_\delta = \delta C_\delta$ . Let us stress that in this case the mixing is a consequence of the anomaly cancellation procedure. Note that, since  $b_2^{(4)} \sim b_3^{-1} \sim M_{V^{(0)}}^{-1}$  (see eq. (4.18)), where  $M_{V^{(0)}}$  is the mass of the anomalous  $U(1)$  that we assume to be in the TeV range, this mixing is tiny and can be ignored for our purposes.

#### 4.2.2 $D$ and $F$ terms

The additional fields give rise also to  $D$  and  $F$  terms. More precisely,  $D$  term contributions come from: (i) the kinetic terms of chiral multiplets and (ii) the axionic Lagrangian (4.12), providing

$$\begin{aligned} \mathcal{L}_D = & \frac{1}{2} \sum_{a=0}^3 D_{k_a}^{(a)} D_{k_a}^{(a)} + \sum_{a=0}^3 g_a D_{k_a}^{(a)} z_i^\dagger (T_{k_a}^{(a)})_j^i z^j + 4g_0 b_3 \langle\alpha\rangle D^{(0)} + \delta D^{(1)} D^{(0)} + \\ & + 2 \left[ \sum_{a=0}^2 g_a^2 b_2^{(a)} \sqrt{2} \psi_S \text{Tr} (\lambda^{(a)} D^{(a)}) + g_0 g_1 \frac{b_2^{(4)}}{\sqrt{2}} \psi_S (\lambda^{(1)} D^{(0)} + \lambda^{(0)} D^{(1)}) + h.c. \right] \end{aligned} \quad (4.23)$$

where  $a = 0, 1, 2, 3$  denotes, as usual, the gauge group factors,  $z_i$  are the lowest components of the  $i$ -th chiral multiplet (except the multiplet which contains the axion) and  $T_{k_a}^{(a)}$ ,  $k_a = 1, \dots, \dim G^{(a)}$ , are the generators of the corresponding gauge groups,  $G^{(a)}$ . Solving the equations of motion for the  $D$ 's and substituting back we obtain

$$\begin{aligned} \mathcal{L}_{D_C} = & -\frac{1}{2} \left\{ \left[ C_\delta g_0 \sum_f Q_f |z_f|^2 - S_\delta g_1 \sum_f Y_f |z_f|^2 \right] + C_\delta 4g_0 b_3 \langle\alpha\rangle \right. \\ & + 2\sqrt{2} b_2^{(0)} g_0^2 [\psi_S (C_\delta^2 \lambda_C) + h.c.] + 2\sqrt{2} b_2^{(1)} g_1^2 [\psi_S (S_\delta^2 \lambda_C - S_\delta \lambda_B) + h.c.] \\ & \left. + \sqrt{2} b_2^{(4)} g_0 g_1 [\psi_S (C_\delta \lambda_B - 2C_\delta S_\delta \lambda_C) + h.c.] \right\}^2 \end{aligned} \quad (4.24)$$

$$\mathcal{L}_{D_B} = -\frac{1}{2} \left\{ g_1 \sum_f Y_f |z_f|^2 + 2\sqrt{2}b_2^{(1)} g_1^2 [\psi_S (\lambda_B - S_\delta \lambda_C) + h.c.] + \sqrt{2}b_2^{(4)} g_0 g_1 [\psi_S C_\delta \lambda_C + h.c.] \right\}^2 \quad (4.25)$$

$$\mathcal{L}_{D^{(2)}} = -\frac{1}{2} \sum_k \left\{ g_2 z_i^\dagger (T_k^{(2)})^i_j z^j + b_2^{(2)} g_2^2 [\sqrt{2}\psi_S \lambda_k^{(2)} + h.c.] \right\}^2 \quad (4.26)$$

$$\mathcal{L}_{D^{(3)}} = -\frac{1}{2} \sum_k \left\{ g_3 z_i^\dagger (T_k^{(3)})^i_j z^j \right\}^2 \quad (4.27)$$

Similarly, the  $F$  term contributions are

$$\mathcal{L}_F = \sum_{f \in MSSM} \left( F^f F_f^\dagger - \frac{\partial W}{\partial z^f} F^f - \frac{\partial W^\dagger}{\partial z_f^\dagger} F_f^\dagger \right) + \frac{1}{2} F_S F_S^\dagger + \frac{1}{2} \left\{ 4F_S \left[ \sum_{a=0}^2 g_a^2 b_2^{(a)} \text{Tr} (\lambda^{(a)} \lambda^{(a)}) + g_0 g_1 b_2^{(4)} \lambda^{(1)} \lambda^{(0)} \right] + h.c. \right\} \quad (4.28)$$

where the first line is the standard MSSM  $F$  term contribution while the second line contains the new Stückelberg terms. Solving the equation of motion

$$\mathcal{L}_{F_S} = -8 \left[ \sum_a b_2^{(a)} g_a^2 \text{Tr} (\lambda^{(a)} \lambda^{(a)}) + g_1 g_0 b_2^{(4)} \lambda^{(1)} \lambda^{(0)} \right] \times \left[ \sum_a b_2^{(a)} g_a^2 \text{Tr} (\bar{\lambda}^{(a)} \bar{\lambda}^{(a)}) + g_1 g_0 b_2^{(4)} \bar{\lambda}^{(1)} \bar{\lambda}^{(0)} \right] \quad (4.29)$$

Eq. (4.29) can also be written in the basis (4.22), but we will not need this term in the following.

We would like to mention that no  $D$  and  $F$  terms are coming from the GCS since they include only vector multiplets in an antisymmetric form. Our results are in accordance with [62].

### 4.2.3 Scalar potential

As we have seen in the previous section, the additional  $F$  terms (4.29) do not give any contribution to the scalar potential. The  $D_B$ ,  $D^{(2)}$  and  $D^{(3)}$  terms (see eq. (4.25), (4.26) and (4.27)) provide the usual contributions to the MSSM potential.

The only new contribution comes from the first line of (4.24). Thus the scalar potential can be written as

$$V = V_{MSSM} + V_{DC} \quad (4.30)$$

$$V_{DC} = \frac{1}{2} \left\{ \left[ C_\delta g_0 \sum_f Q_f |z_f|^2 - S_\delta g_1 \sum_f Y_f |z_f|^2 \right] + C_\delta 4g_0 b_3 \langle \alpha \rangle \right\}^2 \quad (4.31)$$

Solving the equations for the minima of the potential

$$\frac{\partial V}{\partial z_f} = 0 \quad (4.32)$$

we get  $\langle z_f \rangle = 0$  for all the sfermions as in the MSSM case. Inserting back these VEVs into (4.30) we get the following Higgs scalar potential

$$\begin{aligned} V_h = & \left\{ |\mu|^2 + m_{h_u}^2 + 4g_0^2 b_3 \langle \alpha \rangle C_\delta X_\delta \right\} \left( |h_u^0|^2 + |h_u^+|^2 \right) \\ & + \left\{ |\mu|^2 + m_{h_d}^2 - 4g_0^2 b_3 \langle \alpha \rangle C_\delta X_\delta \right\} \left( |h_d^0|^2 + |h_d^-|^2 \right) \\ & + \left\{ \frac{1}{2} (g_0 X_\delta)^2 + \frac{1}{8} (g_1^2 + g_2^2) \right\} \left( |h_u^0|^2 + |h_u^+|^2 - |h_d^0|^2 - |h_d^-|^2 \right)^2 \\ & + \left\{ b (h_u^+ h_d^- - h_u^0 h_d^0) + h.c. \right\} + \frac{1}{2} g_2^2 |h_u^+ h_d^{0\dagger} + h_u^0 h_d^{-\dagger}|^2 \end{aligned} \quad (4.33)$$

which can be brought to the same form of the MSSM potential (see eq. (2.40)), after the following redefinitions

$$\begin{aligned} m_{h_u}^2 + 4g_0^2 b_3 \langle \alpha \rangle C_\delta X_\delta & \rightarrow \tilde{m}_{h_u}^2 \\ m_{h_d}^2 - 4g_0^2 b_3 \langle \alpha \rangle C_\delta X_\delta & \rightarrow \tilde{m}_{h_d}^2 \\ \left( (g_0 X_\delta)^2 + \frac{1}{4} (g_1^2 + g_2^2) \right) v^2 & \rightarrow \tilde{M}_{Z_0}^2 \end{aligned} \quad (4.34)$$

where

$$g_0 X_\delta = C_\delta g_0 Q_{H_u} - \frac{1}{2} S_\delta g_1 \quad (4.35)$$

At the minimum, we recover the MSSM result  $\langle h_u^+ \rangle = \langle h_d^- \rangle = 0$  for the Higgs charged components. Defining as usual  $\langle h_i^0 \rangle = v_i / \sqrt{2}$ ,  $v_u^2 + v_d^2 = v^2$  and  $v_u / v_d = \tan \beta$  we can still write the tree level conditions for the electroweak symmetry breaking as

$$b^2 > (|\mu|^2 + \tilde{m}_{h_u}^2) (|\mu|^2 + \tilde{m}_{h_d}^2) \quad (4.36)$$

$$2b < 2|\mu|^2 + \tilde{m}_{h_u}^2 + \tilde{m}_{h_d}^2 \quad (4.37)$$

in complete analogy with the MSSM case (using  $\tilde{m}$ 's).

#### 4.2.4 Higgs sector

It is worth noting that in our model there is no axi-Higgs mixing. This is due to the fact that we do not consider scalar potential terms for the axion (on the contrary to [42]).

After the electroweak symmetry breaking we have four gauge generators that are broken, so we have four longitudinal degrees of freedom. One of them is the axion, while the other three are the usual NG bosons coming from the Higgs sector.

As it was mentioned above, the potential has the standard MSSM form, upon the redefinitions (4.34). Since the discussion goes in the same way as Section 2.4.2, here we give only the main ideas. Higgs scalar fields consist of eight real scalar degrees of freedom: three of them are the NG bosons  $G^0$ ,  $G^\pm$ . The remaining five Higgs scalar mass eigenstates consist of two CP-even neutral scalars  $h^0$  and  $H^0$ , one CP-odd neutral scalar  $A^0$  and a charge +1 scalar  $H^+$  as well as its charge conjugate  $H^-$  with charge  $-1$ . The gauge-eigenstate fields can be expressed in terms of the mass eigenstate fields as

$$\begin{pmatrix} h_u^0 \\ h_d^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} v_u \\ v_d \end{pmatrix} + \frac{1}{\sqrt{2}} R_\alpha \begin{pmatrix} h^0 \\ H^0 \end{pmatrix} + \frac{i}{\sqrt{2}} R_\beta \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} \quad (4.38)$$

$$\begin{pmatrix} h_u^+ \\ h_d^{-*} \end{pmatrix} = R_\beta \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} \quad (4.39)$$

where the orthogonal rotation matrices  $R_\alpha, R_\beta$  are the same as in (2.53). Acting with these matrices on the gauge eigenstate fields we obtain the diagonal mass terms. Replacing the tilde parameters (4.34) we obtain the masses (compare with eq. (2.55), (2.56) and (2.57))

$$m_{A^0}^2 = 2|\mu|^2 + m_{h_u}^2 + m_{h_d}^2 \quad (4.40)$$

$$\begin{aligned} m_{h^0, H^0}^2 &= \frac{1}{2} \left\{ m_{A^0}^2 + \left( (g_0 X_\delta)^2 + \frac{1}{4}(g_1^2 + g_2^2) \right) v^2 \right. \\ &\quad \mp \left[ \left( m_{A^0}^2 - \left( (g_0 X_\delta)^2 + \frac{1}{4}(g_1^2 + g_2^2) \right) v^2 \right)^2 \right. \\ &\quad \left. \left. + 4 \left( (g_0 X_\delta)^2 + \frac{1}{4}(g_1^2 + g_2^2) \right) v^2 m_{A^0}^2 \sin^2(2\beta) \right]^{\frac{1}{2}} \right\} \quad (4.41) \end{aligned}$$

$$m_{H^\pm}^2 = m_{A^0}^2 + m_W^2 = m_{A^0}^2 + g_2^2 \frac{v^2}{4} \quad (4.42)$$

and the mixing angles (compare with eq. (2.59))

$$\begin{aligned}\frac{\sin 2\alpha}{\sin 2\beta} &= -\frac{m_{H^0}^2 + m_{h^0}^2}{m_{H^0}^2 - m_{h^0}^2} \\ \frac{\tan 2\alpha}{\tan 2\beta} &= \frac{m_{A^0}^2 + ((g_0 X_\delta)^2 + \frac{1}{4}(g_1^2 + g_2^2)) v^2}{m_{A^0}^2 - ((g_0 X_\delta)^2 + \frac{1}{4}(g_1^2 + g_2^2)) v^2}\end{aligned}\quad (4.43)$$

Notice that only the  $h^0$  and  $H^0$  masses modified with respect to the MSSM, due to the additional anomalous  $U(1)'$ .

#### 4.2.5 Neutral Vectors

There are two mass-sources for the gauge bosons: (i) the Stückelberg mechanism and (ii) the Higgs mechanism. In this extension of the MSSM, the mass terms for the gauge fields are given by

$$\mathcal{L}_M = \frac{1}{2} \begin{pmatrix} C_\mu & B_\mu & V_{3\mu}^{(2)} \end{pmatrix} M^2 \begin{pmatrix} C^\mu \\ B^\mu \\ V_3^{(2)\mu} \end{pmatrix}\quad (4.44)$$

$C_\mu, B_\mu$  are the spin 1 components of the vector multiplets  $V_C, V_B$ . The gauge boson mass matrix is

$$M^2 = \begin{pmatrix} M_C^2 & g_0 g_1 \frac{v^2}{2} X_\delta & -g_0 g_2 \frac{v^2}{2} X_\delta \\ \dots & g_1^2 \frac{v^2}{4} & -g_1 g_2 \frac{v^2}{4} \\ \dots & \dots & g_2^2 \frac{v^2}{4} \end{pmatrix}\quad (4.45)$$

where  $M_C^2 = 16g_0^2 b_3^2 C_\delta^2 + g_0^2 (v^2) X_\delta^2$  and the lower dots denote the obvious terms under symmetrization. After diagonalization, we obtain the eigenstates

$$A_\mu = \frac{g_2 B_\mu + g_1 V_{3\mu}^{(2)}}{\sqrt{g_1^2 + g_2^2}}\quad (4.46)$$

$$Z_{0\mu} = \frac{g_2 V_{3\mu}^{(2)} - g_1 B_\mu}{\sqrt{g_1^2 + g_2^2}} + g_0 Q_{H_u} \frac{\sqrt{g_1^2 + g_2^2} v^2}{2M_{V^{(0)}}^2} C_\mu + \mathcal{O}[g_0^3, M_{V^{(0)}}^{-3}]\quad (4.47)$$

$$Z'_\mu = C_\mu + \frac{g_0 Q_{H_u} v^2}{2M_{V^{(0)}}^2} \left( g_1 B_\mu - g_2 V_{3\mu}^{(2)} \right) + \mathcal{O}[g_0^3, M_{V^{(0)}}^{-3}]\quad (4.48)$$

and the corresponding masses

$$M_\gamma^2 = 0 \quad (4.49)$$

$$M_{Z_0}^2 = \frac{1}{4} (g_1^2 + g_2^2) v^2 - (Q_{H_u})^2 \frac{(g_1^2 + g_2^2) g_0^2 v^4}{4M_{V^{(0)}}^2} + \mathcal{O}[g_0^3, M_{V^{(0)}}^{-3}] \quad (4.50)$$

$$M_{Z'}^2 = M_{V^{(0)}}^2 + g_0^2 \left[ (Q_{H_u})^2 \left( 1 + \frac{g_1^2 v^2 + g_2^2 v^2}{4M_{V^{(0)}}^2} \right) - \frac{\langle \alpha \rangle g_1^3 \mathcal{A}^{(4)}}{64\pi^2 M_{V^{(0)}}} \right] v^2 + \mathcal{O}[g_0^3, M_{V^{(0)}}^{-3}] \quad (4.51)$$

where  $M_{V^{(0)}} = 4b_3 g_0$  is the mass parameter for the anomalous  $U(1)$  and it is assumed to be in the TeV range. Due to their complicated form, the eigenstates and eigenvalues of  $M^2$  (4.45) are expressed as power expansions in  $g_0$  and  $1/M_{V^{(0)}}$  keeping only the leading terms. Higher terms are denoted by  $\mathcal{O}[g_0^3, M_{V^{(0)}}^{-3}]$ .

The first eigenstate (4.46) corresponds to the photon and it is exact to all orders. It slightly differs from the usual MSSM expression (2.65) due to the kinetic mixing between  $V^{(0)}$  and  $V^{(1)}$ .

For the rest of the Thesis, we neglect the kinetic mixing contribution since they are higher loop effects which go beyond the scope of the Thesis. Then the rotation matrix from the hypercharge to the photon basis, up to  $\mathcal{O}[g_0^3, M_{V^{(0)}}^{-3}]$  is

$$\begin{aligned} \begin{pmatrix} Z'_\mu \\ Z_{0\mu} \\ A_\mu \end{pmatrix} &= O_{ij} \begin{pmatrix} V_\mu^{(0)} \\ V_\mu^{(1)} \\ V_{3\mu}^{(2)} \end{pmatrix} \\ &= \begin{pmatrix} 1 & g_1 \frac{g_0 Q_{H_u} v^2}{2M_{V^{(0)}}^2} & -g_2 \frac{g_0 Q_{H_u} v^2}{2M_{V^{(0)}}^2} \\ g_0 Q_{H_u} \frac{\sqrt{g_1^2 + g_2^2} v^2}{2M_{V^{(0)}}^2} & -\frac{g_1}{\sqrt{g_1^2 + g_2^2}} & \frac{g_2}{\sqrt{g_1^2 + g_2^2}} \\ 0 & \frac{g_2}{\sqrt{g_1^2 + g_2^2}} & \frac{g_1}{\sqrt{g_1^2 + g_2^2}} \end{pmatrix} \begin{pmatrix} V_\mu^{(0)} \\ V_\mu^{(1)} \\ V_{3\mu}^{(2)} \end{pmatrix} \end{aligned} \quad (4.52)$$

where  $i, j = 0, 1, 2$ .

#### 4.2.6 Sfermions

In general, the contributions to the sfermion masses are coming from (i) the  $D$  and  $F$  terms in the superpotential and (ii) the soft-terms. However, in our case,

the new contribution comes only from the  $D_C$  terms

$$V_{\text{mass}}^{D_C} = \left\{ \left( C_\delta g_0 Q_{H_u} + \frac{1}{2} S_\delta g_1 \right) \left( \frac{v_u^2 - v_d^2}{2} \right) + 4C_\delta g_0 b_3 \langle \alpha \rangle \right\} \times \left\{ \sum_f (C_\delta g_0 Q_f - S_\delta g_1 Y_f) |y_f|^2 \right\} \quad (4.53)$$

where the  $y_f$  stand for all possible sfermions.

#### 4.2.7 Neutralinos

With respect to the MSSM, now we have two new fields:  $\psi_S$  and  $\lambda^{(0)}$ . Thus, we have

$$\mathcal{L}_{\text{neutralino mass}} = -\frac{1}{2} (\psi^0)^T \mathbf{M}_{\tilde{N}} \psi^0 + h.c. \quad (4.54)$$

where

$$(\psi^0)^T = (\psi_S, \lambda_C, \lambda_B, \lambda^{(2)}, \tilde{h}_d^0, \tilde{h}_u^0) \quad (4.55)$$

The neutralino mass matrix  $\mathbf{M}_{\tilde{N}}$  gets contributions from (i) the MSSM terms, (ii) the  $h - \tilde{h} - \lambda^{(0)}$  couplings, (iii) the new soft-breaking terms  $\mathcal{L}_{\text{soft}}^{\text{new}}$ , (iv) the Stückelberg action and (v) the  $D$  terms. Finally, we obtain the symmetric matrix

$$\mathbf{M}_{\tilde{N}} = \begin{pmatrix} \frac{M_S}{2} & m_{SC} & m_{SB} & \frac{2g_2^3 b_2^{(2)}}{\sqrt{2}} \Delta v^2 & 0 & 0 \\ \dots & M_0 C_\delta^2 + M_1 S_\delta^2 & -M_1 S_\delta & 0 & -g_0 v_d X_\delta & g_0 v_u X_\delta \\ \dots & \dots & M_1 & 0 & -\frac{g_1 v_d}{2} & \frac{g_1 v_u}{2} \\ \dots & \dots & \dots & M_2 & \frac{g_2 v_d}{2} & -\frac{g_2 v_u}{2} \\ \dots & \dots & \dots & \dots & 0 & -\mu \\ \dots & \dots & \dots & \dots & \dots & 0 \end{pmatrix} \quad (4.56)$$

where  $M_0$ ,  $M_1$ ,  $M_2$  and  $M_S$  are the masses coming from the soft breaking terms (4.19), and

$$m_{SC} = \sqrt{2} \left\{ 2 \left( C_\delta^2 g_0^2 b_2^{(0)} + S_\delta^2 g_1^2 b_2^{(1)} - C_\delta S_\delta g_0 g_1 b_2^{(4)} \right) (g_0 X_\delta \Delta v^2 + C_\delta M_{V^{(0)}} \langle \alpha \rangle) + \frac{1}{2} \left( -2S_\delta g_1^2 b_2^{(1)} + C_\delta g_0 g_1 b_2^{(4)} \right) g_1 \Delta v^2 + \frac{C_\delta}{2} M_{V^{(0)}} \right\} \quad (4.57)$$

$$m_{SB} = \sqrt{2} \left\{ \left( C_\delta g_0 g_1 b_2^{(4)} - 2S_\delta g_1^2 b_2^{(1)} \right) (g_0 X_\delta \Delta v^2 + C_\delta M_{V^{(0)}} \langle \alpha \rangle) + b_2^{(1)} g_1^3 \Delta v^2 \right\}$$

with  $\Delta v^2 = v_u^2 - v_d^2$ . It is worth noting that the  $D$  terms and kinetic mixing terms are only higher order corrections and they can be neglected in the computations of the eigenvalues and eigenstates.



## Chapter 5

# MiAUMSSM. Phenomenology

In this chapter we discuss some phenomenology consequences of the model showed in Chapter 4. We focus on the simple but still interesting case  $Q_{H_u} = 0$ . In this case there is no mixing between the  $V^{(0)}$  and the other SM gauge fields therefore  $Z' = V^{(0)}$  (see (4.52)). Then the  $Z'$  couples only to SM fermions and their corresponding superpartners. There is no tree level interaction between the  $Z'$  and the  $W$ 's or the neutralinos or the charginos since the corresponding couplings are proportional to  $Q_{H_u}$ . Moreover, as we will see in the following, this simple choice implies interesting properties of the  $Z'$  involved in fermionic triangles.

First of all we will study the tree level decays of the  $Z'$  into fermions and the anomalous decays into neutral gauge bosons. We will check the anomaly cancellation in the broken phase and we will also give some prediction about these processes in LHC.

Then we will pass to the supersymmetric sector. We will consider the case in which the axino is the LSP and we will check if the prediction for the relic density is in agreement or not with the current experimental bounds.

This chapter is entirely based on our papers [1] and [2].

## 5.1 Tree level decays

First of all we discuss the tree level decays of the  $Z'$ . As suggested in the introduction, the tree level final states can only be fermions or sfermions. Since this is a standard part we do not spend much time in details. We remember that we are in the case  $Q_{H_u} = 0$ .

### 5.1.1 Fermions

The Lagrangian describing the interaction of the  $Z'$  with SM fermions is given by

$$\mathcal{L}_{Z'}^{int} = -\frac{1}{2} g_{Z'} \bar{\Psi}_f \gamma^\mu \left( v_f^{Z'} - a_f^{Z'} \gamma_5 \right) \Psi_f Z'_\mu \quad (5.1)$$

where  $g_{Z'} = g_0$ ,  $\Psi_f$  is the usual Dirac spinor (see eq. (3.3)) and the vectorial and axial couplings are defined as

$$\begin{aligned} v_f^{Z'} &= Q_{fL}^{Z'} + Q_{fR}^{Z'} \\ a_f^{Z'} &= Q_{fL}^{Z'} - Q_{fR}^{Z'} \end{aligned} \quad (5.2)$$

and listed in Table 5.1.

	$v_f^{Z'}$	$a_f^{Z'}$
$\nu_e, \nu_\mu, \nu_\tau$	$Q_L$	$Q_L$
$e, \mu, \tau$	$2Q_L$	0
$u, c, t$	$2Q_Q$	0
$d, s, b$	$2Q_Q$	0

Table 5.1: Couplings of the SM fermions with the  $Z'$  gauge boson.

The decay rate is given by

$$\Gamma(Z' \rightarrow f\bar{f}) = C_f \frac{g_{Z'}^2}{48\pi} M_{Z'} \sqrt{1 - 4 \frac{m_f^2}{M_{Z'}^2}} \left[ \left( v_f^{Z'} \right)^2 R_v^f + \left( a_f^{Z'} \right)^2 R_a^f \right] \quad (5.3)$$

where the colour factor  $C_f = 1(3)$  for leptons (quarks) and the kinematic factors

$$\begin{aligned} R_v^f &= 1 + 2 \frac{m_f^2}{M_{Z'}^2} \\ R_a^f &= 1 - 4 \frac{m_f^2}{M_{Z'}^2} \end{aligned} \quad (5.4)$$

take into account the fermion mass  $m_f$ .

### 5.1.2 Sfermions

The Lagrangian describing the interaction of the  $Z'$  with sfermions is given by

$$\tilde{\mathcal{L}}_{Z'}^{int} = -\frac{i}{2}g_{Z'} \left( v_f^{Z'} \pm a_f^{Z'} \right) \tilde{f}_{L,R}^\dagger \partial_\mu \tilde{f}_{L,R} Z'_\mu \quad (5.5)$$

where the  $+$ ( $-$ ) is for left (right) handed sfermions respectively and with  $v_f^{Z'}$  and  $a_f^{Z'}$  given in eq. (5.2). The decay rate for  $Z'$  decay to sfermions is calculated to be

$$\Gamma(Z' \rightarrow \tilde{f}_{L,R} \tilde{f}_{L,R}^\dagger) = C_f \frac{g_{Z'}^2}{48\pi} \frac{(v_f^{Z'} \pm a_f^{Z'})^2}{4} M_{Z'} \left( 1 - 4 \frac{m_{\tilde{f}_{L,R}}^2}{M_{Z'}^2} \right)^{3/2} \quad (5.6)$$

where the colour factor  $C_f$  is defined as in the fermion case and  $m_{\tilde{f}_{L,R}}$  is the  $\tilde{f}_{L,R}$  sfermion mass.

In the case of non-negligible sfermion mixing (such as the top squark) we must work in the mass eigenstate basis. The decay width is similar to (5.6) with the appropriate couplings, except that the phase space factor in the case of different mass decay products is  $[1 - 2(m_1^2 + m_2^2)/M_{Z'}^2 + (m_1^2 - m_2^2)^2/M_{Z'}^4]^{3/2}$ . For details see for instance [33].

### 5.1.3 Numerical results

The tree level decay width for a  $Z'$ , is given by the sum of the decay rates given in the previous sections

$$\Gamma_{Z'} = \sum_{f \in SM} \Gamma(Z' \rightarrow f \bar{f}) + \sum_{\tilde{f}} \Gamma(Z' \rightarrow \tilde{f} \tilde{f}^\dagger) \quad (5.7)$$

We focus on the case  $M_{Z'} = 1$  TeV. The reason will be clear in Section 5.3. Since we suppose that the LSP has mass of around 500 GeV (see Section 5.4), the sfermion masses are heavier than  $M_{Z'}/2$ , so they cannot be produced by an on-shell  $Z'$  decay. We have

$$\begin{aligned} \Gamma_{Z'} &= \sum_{f \in SM} \Gamma(Z' \rightarrow f \bar{f}) \\ &= \sum_{f \in SM} C_f \frac{g_{Z'}^2}{48\pi} M_{Z'} \sqrt{1 - 4 \frac{m_f^2}{M_{Z'}^2}} \left[ (v_f^{Z'})^2 R_v^f + (a_f^{Z'})^2 R_a^f \right] \\ &\cong [119.37 (g_0 Q_L)^2 + 477.02 (g_0 Q_Q)^2] \text{ GeV} \end{aligned} \quad (5.8)$$

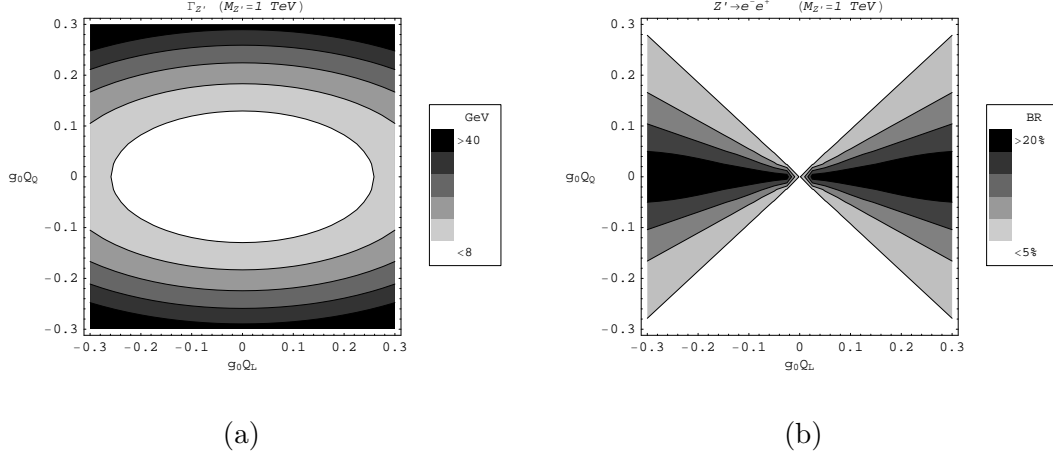


Figure 5.1: (a): Tree level decay width for a  $Z'$  with mass equal to 1 TeV. The darker shaded regions correspond to larger decay widths. (b): Tree level branching ratio for  $Z' \rightarrow e^- e^+$  with mass equal to 1 TeV. The darker shaded regions correspond to larger values for the branching ratio.

where we used eq. (5.3). Since the  $Z'$  will be detected at LHC through its decays into electric charged leptons we give also the branching ratio (BR) for the decay into an electron-positron pair

$$\begin{aligned} \text{BR}(Z' \rightarrow e^- e^+) &= \Gamma(Z' \rightarrow e^- e^+) / \Gamma_{Z'} \\ &\cong \frac{26.52 (g_0 Q_L)^2}{119.37 (g_0 Q_L)^2 + 477.02 (g_0 Q_Q)^2} \end{aligned} \quad (5.9)$$

The results are plotted in Fig. 5.1 in the form of contour plots in the plane  $g_0 Q_L, g_0 Q_Q$ . Our choices for  $g_0, Q_Q, Q_L$  and  $M_{Z'}$  are in agreement with the current experimental bounds [67]. The tree level decay width has elliptic contours since eq. (5.8) is the equation of an ellipse in the plane  $g_0 Q_L, g_0 Q_Q$ . Concerning the  $\text{BR}(Z' \rightarrow e^- e^+)$  let us observe that if the quarks and the leptons have the same mass and the same couplings, then the branching ratio will be simply the inverse of the total number of fermions which is the number of leptons (6) plus the number of quarks (6) times the color factor (3)

$$\text{BR}(Z' \rightarrow e^- e^+) = \frac{1}{6 + 3 \times 6} = \frac{1}{24} \simeq 0.04 \quad (5.10)$$

This is the reason why  $\text{BR}(Z' \rightarrow e^- e^+)$  has a lot of values less than 5%. In a region of parameters the most important quantity is the number of the possible final states.

## 5.2 Anomalous Decays

In this section we compute the amplitudes for the on-shell decays of the  $Z'$  into neutral gauge bosons. The SM fermion interaction terms with SM neutral gauge bosons are

$$\begin{aligned}
 \mathcal{L}_{Z'}^{int} &= J_{Z'}^\mu Z'_\mu = -\frac{1}{2} g_{Z'} \bar{\Psi}_f \gamma^\mu \left( v_f^{Z'} - a_f^{Z'} \gamma_5 \right) \Psi_f Z'_\mu \\
 \mathcal{L}_{Z_0}^{int} &= J_{Z_0}^\mu Z_{0\mu} = -\frac{1}{2} g_{Z_0} \bar{\Psi}_f \gamma^\mu \left( v_f^{Z_0} - a_f^{Z_0} \gamma_5 \right) \Psi_f Z_{0\mu} \\
 \mathcal{L}_\gamma^{int} &= J_\gamma^\mu A_\mu = -e q_f \bar{\Psi}_f \gamma^\mu \Psi_f A_\mu
 \end{aligned} \tag{5.11}$$

where  $q_f$  denote the electric charges,  $v_f^{Z_0}$  and  $a_f^{Z_0}$  are the vectorial and axial couplings with  $Z_0$  and  $v_f^{Z'}$  and  $a_f^{Z'}$  are the vectorial and axial couplings with  $Z'$ , respectively. They are listed in Table 5.2.

	$q_f$	$v_f^{Z_0}$	$a_f^{Z_0}$	$v_f^{Z'}$	$a_f^{Z'}$
$\nu_e, \nu_\mu, \nu_\tau$	0	1/2	1/2	$Q_L$	$Q_L$
$e, \mu, \tau$	-1	$-1/2 + 2 \sin^2 \theta_W$	-1/2	$2Q_L$	0
$u, c, t$	2/3	$1/2 - 4/3 \sin^2 \theta_W$	1/2	$2Q_Q$	0
$d, s, b$	-1/3	$-1/2 + 2/3 \sin^2 \theta_W$	-1/2	$2Q_Q$	0

Table 5.2: Couplings of the SM fermions with the neutral gauge bosons.

We remind that, for  $Q_{H_u} = 0$ , the  $Z'$  is decoupled for the SM neutral gauge sector, so  $g_{Z'} = g_0$  and the SM coupling constants are given in eq. (2.70) and (2.71). For the corresponding Feynman rules see Appendices C.2 and C.4.

A priori we could have three possibilities:  $Z' \rightarrow \gamma\gamma$ ,  $Z' \rightarrow Z_0\gamma$  and  $Z' \rightarrow Z_0 Z_0$ . But the on-shell decay  $Z' \rightarrow \gamma\gamma$  is prohibited by the Landau-Yang theorem [68], for which we give a brief demonstration in Appendix D.4. This decay is permitted only if at least one of the two photons is off-shell. Moreover with the choice  $Q_{H_u} = 0$ , we have that the electric charged fermions have only vectorial couplings with the  $Z'$  (see Table 5.2), so the fermionic triangle loop for the decay  $Z' \rightarrow \gamma\gamma$  has three vectorial vertices, so the corresponding amplitude is zero because of the Furry theorem [69]. So in our case, the only possible decays in the neutral gauge sector (on and off-shell) are  $Z' \rightarrow Z_0\gamma$  and  $Z' \rightarrow Z_0 Z_0$ . However we remind that we are interested only in on-shell decays.

Since there is no tree level interaction between the  $Z'$  and neutralinos or

charginos, no contribution to the fermionic triangles is given by the SUSY sector so, we get the same results, for what the decays of interest are concerned, of non-SUSY models.

### 5.2.1 Anomaly cancellation

Before going on in our computations we check that the anomaly cancellation still holds in the broken phase. We focus on the process  $Z' \rightarrow Z_0\gamma$  since the extension to the other cases is straightforward. In the broken phase, additional contributions coming from the NG boson ( $G^0$ ) exchange must be added. The corresponding WI's, given in diagrammatic form in Fig. 5.2, are

$$(p+q)^\rho \left( \Delta_{\rho\mu\nu}^{Z'Z_0\gamma} + (\text{GCS})_{\rho\mu\nu}^{Z'Z_0\gamma} \right) + iM_{Z'}(\text{GS})_{\mu\nu}^{Z_0\gamma} = 0 \quad (5.12)$$

$$p^\mu \left( \Delta_{\rho\mu\nu}^{Z'Z_0\gamma} + (\text{GCS})_{\rho\mu\nu}^{Z'Z_0\gamma} \right) + iM_{Z_0}(\text{NG})_{\rho\nu}^{Z'\gamma} = 0 \quad (5.13)$$

$$q^\nu \left( \Delta_{\rho\mu\nu}^{Z'Z_0\gamma} + (\text{GCS})_{\rho\mu\nu}^{Z'Z_0\gamma} \right) = 0 \quad (5.14)$$

where  $\Delta_{\rho\mu\nu}^{Z'Z_0\gamma}$  is the fermionic loop with external  $Z'$ ,  $Z_0$  and  $\gamma$ ,  $(\text{GCS})_{\rho\mu\nu}^{Z'Z_0\gamma}$  is the GCS coupling for the vectors cited above,  $(\text{GS})_{\mu\nu}^{Z_0\gamma}$  is the GS coupling between the axion, the  $Z_0$  and the photon and  $(\text{NG})_{\rho\nu}^{Z'\gamma}$  is the fermionic loop with external  $Z'$ ,  $G^0$  and  $\gamma$ . The fermionic triangle  $\Delta_{\rho\mu\nu}^{Z'Z_0\gamma}$  is given by

$$\Delta_{\rho\mu\nu}^{Z'Z_0\gamma} = -\frac{1}{4}g_0g_{Z_0}e \sum_f t_f^{Z'Z_0\gamma} \Gamma_{\rho\mu\nu}^{VAV}(p, q; m_f) \quad (5.15)$$

where  $\Gamma_{\rho\mu\nu}^{VAV}(p, q; m_f)$  is given by (D.19) and

$$t_f^{Z'Z_0\gamma} = v_f^{Z'} a_f^{Z_0} q_f \quad (5.16)$$

So we have (compare with eq. (D.40))

$$(p+q)^\rho \Delta_{\rho\mu\nu}^{Z'Z_0\gamma} = \frac{1}{4}g_0g_{Z_0}e \sum_f t_f^{Z'Z_0\gamma} \left[ \frac{1}{6\pi^2} \right] \epsilon[p, q, \mu, \nu] \quad (5.17)$$

$$p^\mu \Delta_{\rho\mu\nu}^{Z'Z_0\gamma} = \frac{1}{4}g_0g_{Z_0}e \sum_f t_f^{Z'Z_0\gamma} \frac{1}{\pi^2} \left[ \frac{1}{6} - m_f^2 I_0(p, q; m_f) \right] \epsilon[q, p, \nu, \rho] \quad (5.18)$$

$$q^\nu \Delta_{\rho\mu\nu}^{Z'Z_0\gamma} = \frac{1}{4}g_0g_{Z_0}e \sum_f t_f^{Z'Z_0\gamma} \left[ \frac{1}{6\pi^2} \right] \epsilon[q, p, \rho, \mu] \quad (5.19)$$

since we choose a symmetric distribution of the anomaly. The integral  $I_0$  is given in eq. (D.42) and  $\epsilon[p, q, \rho, \sigma]$  is defined after (3.101). Using eq. (2.70) and (2.71)

$$\begin{aligned}
 (p+q)^\rho & \left( Z'_\rho(p+q) \text{ [triangle loop with } \psi, Z_0, Z' \text{]} + Z' \text{ [tree-level]} \right) + iM_{Z'} \left( \phi \text{ [tree-level]} \right) = 0 \\
 p^\mu & \left( Z' \text{ [triangle loop with } \psi, Z_0, Z' \text{]} + Z' \text{ [tree-level]} \right) + iM_{Z_0} \left( Z' \text{ [triangle loop with } \psi, Z_0, G^0 \text{]} \right) = 0 \\
 q^\nu & \left( Z' \text{ [triangle loop with } \psi, Z_0, Z' \text{]} + Z' \text{ [tree-level]} \right) = 0
 \end{aligned}$$

Figure 5.2: The Ward identities for the amplitude  $Z' \rightarrow Z_0 \gamma$  in the broken phase.

and computing  $\sum_f t_f^{Z'Z_0\gamma}$  we get

$$(p+q)^\rho \Delta_{\rho\mu\nu}^{Z'Z_0\gamma} = \frac{1}{8\pi^2} g_0 g_1 g_2 (3Q_Q + Q_L) \epsilon[p, q, \mu, \nu] \quad (5.20)$$

$$p^\mu \Delta_{\rho\mu\nu}^{Z'Z_0\gamma} = \frac{1}{4\pi^2} g_0 g_1 g_2 \left[ \frac{3Q_Q + Q_L}{2} - \sum_f t_f^{Z'Z_0\gamma} m_f^2 I_0 \right] \epsilon[q, p, \nu, \rho] \quad (5.21)$$

$$q^\nu \Delta_{\rho\mu\nu}^{Z'Z_0\gamma} = \frac{1}{8\pi^2} g_0 g_1 g_2 (3Q_Q + Q_L) \epsilon[q, p, \rho, \mu] \quad (5.22)$$

The corresponding GCS coupling is given by the proper combination of the GCS couplings in the hypercharge basis (see Fig. 5.3a)

$$(\text{GCS})_{\rho\mu\nu}^{Z'Z_0\gamma} = \left( 16g_0 g_1^2 R_{011}^{Z'Z_0\gamma} d_5 + 8g_0 g_2^2 R_{022}^{Z'Z_0\gamma} d_6 \right) \epsilon[\rho, \nu, \mu, p - q] \quad (5.23)$$

where the rotation factors are

$$\begin{aligned}
 R_{011}^{Z'Z_0\gamma} &= -\frac{g_1 g_2}{g_1^2 + g_2^2} \\
 R_{022}^{Z'Z_0\gamma} &= \frac{g_1 g_2}{g_1^2 + g_2^2}
 \end{aligned} \quad (5.24)$$

$$\begin{aligned}
 \text{(a)} \quad Z' \text{---} \text{wavy} \text{---} \gamma &= R_{011}^{Z'Z_0\gamma} \left( V^{(0)} \text{---} \text{wavy} \text{---} V^{(1)} \right) + R_{022}^{Z'Z_0\gamma} \left( V^{(0)} \text{---} \text{wavy} \text{---} V^{(2)} \right) \\
 \text{(b)} \quad \phi \text{---} \text{dashed} \text{---} \text{wavy} \text{---} Z_0 &= R_{011}^{Z'Z_0\gamma} \left( \phi \text{---} \text{dashed} \text{---} V^{(1)} \right) + R_{022}^{Z'Z_0\gamma} \left( \phi \text{---} \text{dashed} \text{---} V^{(2)} \right)
 \end{aligned}$$

Figure 5.3: Decomposition of the (GCS) $^{Z'Z_0\gamma}$  and (GS) $^{Z_0\gamma}$  diagrams in terms of the diagrams in the hypercharge basis. As usual each depicted diagram also contains the exchange  $(\mu, p) \leftrightarrow (\nu, q)$ . The corresponding Feynman rules are given in Appendix C.4. The rotation factors are given in eq. (5.24).

So we get

$$\begin{aligned}
 (\text{GCS})_{\rho\mu\nu}^{Z'Z_0\gamma} &= \frac{16g_0g_1g_2}{g_1^2 + g_2^2} \left( -g_1^2 d_5 + g_2^2 \frac{d_6}{2} \right) \epsilon[\rho, \nu, \mu, p - q] \\
 &= \frac{16g_0g_1g_2}{192(g_1^2 + g_2^2)\pi^2} \left( -g_1^2 \mathcal{A}^{(1)} + g_2^2 \mathcal{A}^{(2)} \right) \epsilon[\rho, \nu, \mu, p - q] \\
 &= \frac{g_0g_1g_2}{8\pi^2} (3Q_Q + Q_L) \epsilon[\rho, \nu, \mu, p - q] \quad (5.25)
 \end{aligned}$$

where we used eq. (4.18), (4.8) and (4.9). Then using eq. (5.22) we have

$$\begin{aligned}
 q^\nu \left( \Delta_{\rho\mu\nu}^{Z'Z_0\gamma} + (\text{GCS})_{\rho\mu\nu}^{Z'Z_0\gamma} \right) &= \frac{g_0g_1g_2}{8\pi^2} (3Q_Q + Q_L) (\epsilon[q, p, \rho, \mu] + \epsilon[\rho, q, \mu, p - q]) \\
 &= \frac{g_0g_1g_2}{8\pi^2} (3Q_Q + Q_L) (\epsilon[q, p, \rho, \mu] + \epsilon[\rho, q, \mu, p]) \\
 &= 0 \quad (5.26)
 \end{aligned}$$

and eq. (5.14) is verified. From eq. (5.20) and (5.25) we easily obtain

$$(p + q)^\rho \left( \Delta_{\rho\mu\nu}^{Z'Z_0\gamma} + (\text{GCS})_{\rho\mu\nu}^{Z'Z_0\gamma} \right) = \frac{3g_0g_1g_2}{8\pi^2} (3Q_Q + Q_L) \epsilon[p, q, \mu, \nu] \quad (5.27)$$

The corresponding GS coupling is given by the proper combination of the GS couplings in the gauge interaction basis (see Fig. 5.3b)

$$\begin{aligned}
 (\text{GS})_{\mu\nu}^{Z_0\gamma} &= \left( 8ig_1^2 R_{011}^{Z'Z_0\gamma} b_2^{(1)} - 4ig_2^2 R_{022}^{Z'Z_0\gamma} b_2^{(2)} \right) \epsilon[\mu, \nu, p, q] \\
 &= \frac{i}{b_3} \frac{3g_1g_2}{32\pi^2} (3Q_Q + Q_L) \epsilon[p, q, \mu, \nu] \quad (5.28)
 \end{aligned}$$

where we used eq. (4.18), (4.8), (4.9) and (5.24). We remember that since  $Q_{H_u} = 0$ ,  $M_{Z'} = 4g_0b_3$  (see eq. (4.51)). Then using eq. (5.27) we have

$$\begin{aligned}
 (p+q)^\rho \left( \Delta_{\rho\mu\nu}^{Z'Z_0\gamma} + (\text{GCS})_{\rho\mu\nu}^{Z'Z_0\gamma} \right) + iM_{Z'}(\text{GS})_{\mu\nu}^{Z_0\gamma} &= \\
 &= \frac{3g_1g_2}{8\pi^2} (3Q_Q + Q_L) \left( g_0 + iM_{Z'} \frac{i}{4b_3} \right) \epsilon[p, q, \mu, \nu] \\
 &= \frac{3g_1g_2}{8\pi^2} (3Q_Q + Q_L) \left( g_0 - \frac{4g_0b_3}{4b_3} \right) \epsilon[p, q, \mu, \nu] \\
 &= 0
 \end{aligned} \tag{5.29}$$

and eq. (5.12) is verified. From eq. (5.21) and (5.25) we easily obtain

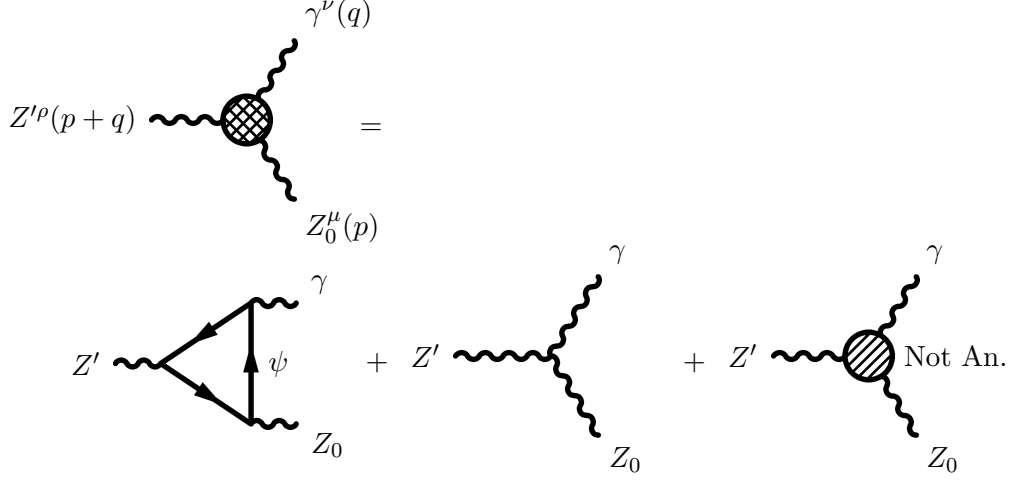
$$p^\mu \left( \Delta_{\rho\mu\nu}^{Z'Z_0\gamma} + (\text{GCS})_{\rho\mu\nu}^{Z'Z_0\gamma} \right) = -\frac{1}{4\pi^2} g_0g_1g_2 \sum_f \left[ t_f^{Z'Z_0\gamma} m_f^2 I_0 \right] \epsilon[q, p, \nu, \rho] \tag{5.30}$$

Computing the Feynman amplitude for (NG) $_{\rho\nu}^{Z'\gamma}$  diagram we find

$$\begin{aligned}
 (\text{NG})_{\rho\nu}^{Z'\gamma} &= -\frac{i}{4\pi^2} g_0 e \left[ \sum_{e,\mu,\tau,d,s,b} \left( \frac{y_{fd}}{\sqrt{2}} \cos \beta v_{fd}^{Z'} q_{fd} m_{fd} I_0 \right) + \right. \\
 &\quad \left. + \sum_{u,c,t} \left( \frac{y_{fu}}{\sqrt{2}} \sin \beta v_{fu}^{Z'} q_{fu} m_{fu} I_0 \right) \right] \epsilon[q, p, \nu, \rho] \\
 &= -\frac{i}{4\pi^2} g_0 \frac{g_1g_2}{\frac{1}{2}\sqrt{g_1^2 + g_2^2}v} \left[ - \sum_{e,\mu,\tau,d,s,b} \left( v_{fd}^{Z'} \frac{1}{2} q_{fd} m_{fd}^2 I_0 \right) + \right. \\
 &\quad \left. + \sum_{u,c,t} \left( v_{fu}^{Z'} \frac{1}{2} q_{fu} m_{fu}^2 I_0 \right) \right] \epsilon[q, p, \nu, \rho] \\
 &= -\frac{i}{4\pi^2} \frac{g_0g_1g_2}{M_{Z_0}} \sum_f \left[ v_f^{Z'} a_f^{Z_0} q_f m_f^2 I_0 \right] \epsilon[q, p, \nu, \rho] \\
 &= -\frac{i}{4\pi^2} \frac{g_0g_1g_2}{M_{Z_0}} \sum_f \left[ t_f^{Z'Z_0\gamma} m_f^2 I_0 \right] \epsilon[q, p, \nu, \rho]
 \end{aligned} \tag{5.31}$$

where we used  $y_{fu} \sin \beta / \sqrt{2} = m_{fu}/v$ ,  $-y_{fd} \cos \beta / \sqrt{2} = m_{fd}/v$ , eq. (2.71) and (4.50) for  $Q_{H_u} = 0$ , the values for  $a_f^{Z_0}$  given in Table 5.2 and the definition of  $t_f^{Z'Z_0\gamma}$  given in eq. (5.16). Then summing eq. (5.30) and (5.31) we get

$$\begin{aligned}
 p^\mu \left( \Delta_{\rho\mu\nu}^{Z'Z_0\gamma} + (\text{GCS})_{\rho\mu\nu}^{Z'Z_0\gamma} \right) + iM_{Z_0}(\text{NG})_{\rho\nu}^{Z'\gamma} &= \\
 &= \frac{1}{4\pi^2} g_0g_1g_2 \sum_f \left[ t_f^{Z'Z_0\gamma} m_f^2 I_0 \right] \left( -1 + iM_{Z_0} \frac{-i}{M_{Z_0}} \right) \epsilon[q, p, \nu, \rho] \\
 &= 0
 \end{aligned} \tag{5.32}$$


 Figure 5.4: Diagrams for  $Z' \rightarrow Z_0 \gamma$ .

and also eq. (5.13) is verified. So we checked the consistence of the anomaly cancellation in the broken phase.

### 5.2.2 $Z' \rightarrow Z_0 \gamma$

We calculate the decay rate for  $Z' \rightarrow Z_0 \gamma$ . We compute all the relevant diagrams in the  $R_\xi$  gauge, thus removing the interaction vertex  $V^\mu \partial_\mu G_V$  that involves the massive gauge bosons and the Stückelberg or NG boson. Therefore, the only diagrams that remain are the fermionic loop, the GCS vertex and a not anomalous remnant contribution (Fig. 5.4). It is possible to show that the last blob-diagram, that involves several diagrams, is equal to zero. For the interested reader we give further details in Appendix D.5. The decay rate for the process is given by

$$\Gamma(Z' \rightarrow Z_0 \gamma) = \frac{p_F}{32\pi^2 M_{Z'}^2} \int |A^{Z'Z_0\gamma}|^2 d\Omega \quad (5.33)$$

where  $A^{Z'Z_0\gamma}$  is the total scalar amplitude and  $p_F$  is the momentum of the outgoing vectors in the CM frame

$$p_F = \frac{M_{Z'}}{2} \left( 1 - \frac{M_{Z_0}^2}{M_{Z'}^2} \right) \quad (5.34)$$

The square of the total scalar amplitude is given by

$$|A^{Z'Z_0\gamma}|^2 = \frac{1}{3} \sum_{\lambda'} \epsilon_{(\lambda')}^{\rho_1} \epsilon_{(\lambda')}^{*\rho_2} \sum_{\lambda^0} \epsilon_{(\lambda^0)}^{\mu_1} \epsilon_{(\lambda^0)}^{*\mu_2} \sum_{\lambda_\gamma} \epsilon_{(\lambda_\gamma)}^{\nu_1} \epsilon_{(\lambda_\gamma)}^{*\nu_2} A_{\rho_1 \mu_1 \nu_1}^{Z'Z_0\gamma} A_{\rho_2 \mu_2 \nu_2}^{*Z'Z_0\gamma} \quad (5.35)$$

where  $\epsilon$  are the polarizations of the gauge bosons, and  $A_{\rho\mu\nu}^{Z'Z_0\gamma}$  is the Feynman amplitude of the process. The factor  $1/3$  comes from the average over the  $Z'$  helicity states. The polarizations obey to the following completeness relations

$$\sum_{\lambda'} \epsilon_{(\lambda')}^{\rho_1} \epsilon_{(\lambda')}^{*\rho_2} = -\eta^{\rho_1\rho_2} + \frac{k_{(\lambda')}^{\rho_1} k_{(\lambda')}^{\rho_2}}{M_{Z'}^2} \quad (5.36)$$

$$\sum_{\lambda^0} \epsilon_{(\lambda^0)}^{\mu_1} \epsilon_{(\lambda^0)}^{*\mu_2} = -\eta^{\mu_1\mu_2} + \frac{k_{(\lambda^0)}^{\mu_1} k_{(\lambda^0)}^{\mu_2}}{M_{Z_0}^2} \quad (5.37)$$

$$\sum_{\lambda_\gamma} \epsilon_{(\lambda_\gamma)}^{\nu_1} \epsilon_{(\lambda_\gamma)}^{*\nu_2} \rightarrow -\eta^{\nu_1\nu_2} \quad (5.38)$$

where (5.38) gives only the relevant part of the sum over helicities. Other terms are omitted since they give vanishing contributions to the decay. The amplitude is given by the sum of the fermionic triangle  $\Delta_{\rho\mu\nu}^{Z'Z_0\gamma}$  plus the proper GCS vertex

$$A_{\rho\mu\nu}^{Z'Z_0\gamma} = \Delta_{\rho\mu\nu}^{Z'Z_0\gamma} + (\text{GCS})_{\rho\mu\nu}^{Z'Z_0\gamma} \quad (5.39)$$

where  $\Delta_{\rho\mu\nu}^{Z'Z_0\gamma}$  and  $(\text{GCS})_{\rho\mu\nu}^{Z'Z_0\gamma}$  are given in eq. (5.15) and (5.25). It is convenient to express the triangle amplitude by using the Rosenberg parametrization [29] (see also Appendix D.3)

$$\begin{aligned} \Delta_{\rho\mu\nu}^{Z'Z_0\gamma} = & -\frac{1}{4\pi^2} g_0 g_{Z_0} e \left( A_1 \epsilon[p, \mu, \nu, \rho] + A_2 \epsilon[q, \mu, \nu, \rho] + A_3 \epsilon[p, q, \mu, \rho] p_\nu \right. \\ & \left. + A_4 \epsilon[p, q, \mu, \rho] q_\nu + A_5 \epsilon[p, q, \nu, \rho] p_\mu + A_6 \epsilon[p, q, \nu, \rho] q_\mu \right) \end{aligned} \quad (5.40)$$

where

$$A_i = \sum_f t_f^{Z'Z_0\gamma} I_i \quad \text{for } i = 3, \dots, 6 \quad (5.41)$$

$I_3$ ,  $I_4$ ,  $I_5$  and  $I_6$  are finite integrals (their explicit forms are given in (D.25)),  $\epsilon[p, q, \rho, \sigma]$  is defined after (D.24) and  $t_f^{Z'Z_0\gamma}$  is given in eq. (5.16).  $A_1$  and  $A_2$  are naively divergent by power counting and so they must be regularized. We compute them by using the Ward identities. In this way it is possible to express  $A_1$  and  $A_2$  in terms of the finite integrals  $I_3$ ,  $I_4$ ,  $I_5$  and  $I_6$ . The GCS term has the following tensorial structure

$$d^{Z'Z_0\gamma} \left( \epsilon[p, \mu, \nu, \rho] - \epsilon[q, \mu, \nu, \rho] \right) \quad (5.42)$$

so it can be absorbed by shifting the first two coefficients of the Rosenberg parametrization for the triangle (see Section 3.5). The resulting amplitude can

be written as

$$\begin{aligned} \Delta_{\rho\mu\nu}^{Z'Z_0\gamma} = & -\frac{1}{4\pi^2}g_0g_{Z_0}e\left(\tilde{A}_1\epsilon[p,\mu,\nu,\rho] + \tilde{A}_2\epsilon[q,\mu,\nu,\rho] + A_3\epsilon[p,q,\mu,\rho]p_\nu \right. \\ & \left. + A_4\epsilon[p,q,\mu,\rho]q_\nu + A_5\epsilon[p,q,\nu,\rho]p_\mu + A_6\epsilon[p,q,\nu,\rho]q_\mu\right) \end{aligned} \quad (5.43)$$

The WIs (5.12), (5.13) and (5.14) now read

$$(p+q)^\rho A_{\rho\mu\nu}^{Z'Z_0\gamma} + iM_{Z'}(\text{GS})_{\mu\nu}^{Z_0\gamma} = 0 \quad (5.44)$$

$$p^\mu A_{\rho\mu\nu}^{Z'Z_0\gamma} + iM_{Z_0}(\text{NG})_{\rho\nu}^{Z'\gamma} = 0 \quad (5.45)$$

$$q^\nu A_{\rho\mu\nu}^{Z'Z_0\gamma} = 0 \quad (5.46)$$

where  $M_{Z'} = 4b_3g_0$  and  $M_{Z_0}$  are the  $Z'$  and  $Z_0$  masses respectively. Using the results of the previous section we obtain

$$(p+q)^\rho A_{\rho\mu\nu}^{Z'Z_0\gamma} = \frac{1}{4\pi^2}g_0g_{Z_0}e\frac{1}{2}\sum_f t_f^{Z'Z_0\gamma} \epsilon[p,q,\mu,\nu] \quad (5.47)$$

$$p^\mu A_{\rho\mu\nu}^{Z'Z_0\gamma} = -\frac{1}{4\pi^2}g_0g_{Z_0}e\sum_f t_f^{Z'Z_0\gamma} m_f^2 I_0 \epsilon[q,p,\nu,\rho] \quad (5.48)$$

$$q^\nu A_{\rho\mu\nu}^{Z'Z_0\gamma} = 0 \quad (5.49)$$

and inserting (5.43) into the above identities we get

$$\begin{aligned} \tilde{A}_1 &= (q^2 A_4 + p \cdot q A_3) \\ \tilde{A}_2 &= (p^2 A_5 + p \cdot q A_6 + (\text{NG})^{Z'\gamma}) \end{aligned} \quad (5.50)$$

with

$$(\text{NG})^{Z'\gamma} = \sum_f t_f^{Z'Z_0\gamma} m_f^2 I_0 \quad (5.51)$$

where  $I_0$  is the integral given in (D.42). Substituting  $\tilde{A}_1$ ,  $\tilde{A}_2$  from (5.50) into the amplitude (5.43) and performing all the contractions we finally obtain

$$\begin{aligned} |A^{Z'Z_0\gamma}|^2 = & g_0^2 g_{Z_0}^2 e^2 \frac{(M_{Z'}^2 - M_{Z_0}^2)^2 (M_{Z'}^2 + M_{Z_0}^2)}{96 M_{Z_0}^2 M_{Z'}^2 \pi^4} \times \\ & \left[ \sum_f t_f^{Z'Z_0\gamma} \left( (I_3 + I_5) M_{Z_0}^2 + m_f^2 I_0 \right) \right]^2 \end{aligned} \quad (5.52)$$

5.2.3  $Z' \rightarrow Z_0 Z_0$ 

The computations are similar to the previous case so we point out only the differences with the other decay. Mutatis mutandis, the decay rate for the process is given in (5.33) with the proper amplitude and

$$p_F = \frac{M_{Z'}}{2} \sqrt{1 - \frac{4M_{Z_0}^2}{M_{Z'}^2}} \quad (5.53)$$

The square of the total scalar amplitude is given by

$$|A^{Z'Z_0Z_0}|^2 = \frac{1}{3} \sum_{\lambda'} \epsilon_{(\lambda')}^{\rho_1} \epsilon_{(\lambda')}^{*\rho_2} \sum_{\lambda^0} \epsilon_{(\lambda^0)}^{\nu_1} \epsilon_{(\lambda^0)}^{*\nu_2} \sum_{\lambda^0} \epsilon_{(\lambda^0)}^{\mu_1} \epsilon_{(\lambda^0)}^{*\mu_2} A_{\rho_1\mu_1\nu_1}^{Z'Z_0Z_0} A_{\rho_2\mu_2\nu_2}^{*Z'Z_0Z_0} \quad (5.54)$$

where the amplitude  $A_{\rho\mu\nu}^{Z'Z_0Z_0}$  is always the sum of the fermionic triangle and the GCS term. The contribution to the fermionic triangle is

$$\begin{aligned} \Delta_{\rho\mu\nu}^{Z'Z_0Z_0} = & -\frac{1}{8} g_0 g_{Z_0}^2 \left[ \sum_f \left( v_f^{Z'} a_f^{Z_0} v_f^{Z_0} \Gamma_{\rho\mu\nu}^{VAV} + v_f^{Z'} v_f^{Z_0} a_f^{Z_0} \Gamma_{\rho\mu\nu}^{VVA} \right) + \right. \\ & \left. \sum_n \left( a_n^{Z'} v_n^{Z_0} v_n^{Z_0} \Gamma_{\rho\mu\nu}^{AVV} + a_n^{Z'} a_n^{Z_0} a_n^{Z_0} \Gamma_{\rho\mu\nu}^{AAA} \right) \right] \quad (5.55) \end{aligned}$$

where  $n$  runs over all the neutrinos while the  $\Gamma_{\rho\mu\nu}$ 's are given by (D.18), (D.19), (D.20), (D.21). Using the fact that for the three neutrino families we have  $v_n^{Z'} = a_n^{Z'}$  and  $v_n^{Z_0} = a_n^{Z_0}$  we write the total amplitude (the sum of the triangles plus GCS terms) as

$$\begin{aligned} A_{\rho\mu\nu}^{Z'Z_0Z_0} = & -\frac{1}{8\pi^2} g_0 g_{Z_0}^2 \left( \tilde{A}_1 \epsilon[p, \mu, \nu, \rho] + \tilde{A}_2 \epsilon[q, \mu, \nu, \rho] + A_3 \epsilon[p, q, \mu, \rho] p^\nu \right. \\ & \left. + A_4 \epsilon[p, q, \mu, \rho] q^\nu + A_5 \epsilon[p, q, \nu, \rho] p^\mu + A_6 \epsilon[p, q, \nu, \rho] q^\mu \right) \quad (5.56) \end{aligned}$$

with

$$A_i = 2 \sum_f t_f^{Z'Z_0Z_0} I_i \quad \text{for } i = 3, \dots, 6 \quad (5.57)$$

where

$$t_f^{Z'Z_0Z_0} = \tilde{v}_f^{Z'} a_f^{Z_0} v_f^{Z_0} \quad (5.58)$$

with  $\tilde{v}_n^{Z'} = 2v_n^{Z'}$  for neutrinos and  $\tilde{v}_f^{Z'} = v_f^{Z'}$  for the other fermions. The WIs now read

$$(p+q)^\rho A_{\rho\mu\nu}^{Z'Z_0Z_0} + iM_{Z'}(\text{GS})_{\mu\nu}^{Z_0Z_0} = 0 \quad (5.59)$$

$$p^\mu A_{\rho\mu\nu}^{Z'Z_0Z_0} + iM_{Z_0}(\text{NG})_{\rho\nu}^{Z'Z_0} = 0 \quad (5.60)$$

$$q^\nu A_{\rho\mu\nu}^{Z'Z_0Z_0} + iM_{Z_0}(\text{NG})_{\mu\rho}^{Z_0Z'} = 0 \quad (5.61)$$

leading to

$$(p + q)^\rho A_{\rho\mu\nu}^{Z'Z_0Z_0} = \frac{1}{8\pi^2} g_0 g_{Z_0}^2 \sum_f t_f^{Z'Z_0Z_0} \epsilon[p, q, \mu, \nu] \quad (5.62)$$

$$p^\mu A_{\rho\mu\nu}^{Z'Z_0Z_0} = -\frac{1}{8\pi^2} g_0 g_{Z_0}^2 \sum_f t_f^{Z'Z_0Z_0} m_f^2 I_0 \epsilon[q, p, \nu, \rho] \quad (5.63)$$

$$q^\nu A_{\rho\mu\nu}^{Z'Z_0Z_0} = -\frac{1}{8\pi^2} g_0 g_{Z_0}^2 \sum_f t_f^{Z'Z_0Z_0} m_f^2 I_0 \epsilon[q, p, \rho, \mu] \quad (5.64)$$

From these equations we find the following values for  $\tilde{A}_1$  and  $\tilde{A}_2$

$$\tilde{A}_1 = \left( q^2 A_4 + p \cdot q A_3 - (\text{NG})^{Z'Z_0} \right) \quad (5.65)$$

$$\tilde{A}_2 = \left( p^2 A_5 + p \cdot q A_6 + (\text{NG})^{Z'Z_0} \right) \quad (5.66)$$

with

$$(\text{NG})^{Z'Z_0} = \sum_f t_f^{Z'Z_0Z_0} m_f^2 I_0 \quad (5.67)$$

where  $I_0$  is the integral given in (D.42). Substituting back into the amplitude and performing all the contractions we finally obtain

$$|A^{Z'Z_0Z_0}|^2 = g_0^2 g_{Z_0}^4 \frac{(M_{Z'}^2 - 4M_{Z_0}^2)^2}{192M_{Z_0}^2 \pi^4} \left[ \sum_f t_f^{Z'Z_0Z_0} \left( 2(I_3 + I_5)M_{Z_0}^2 + m_f^2 I_0 \right) \right]^2 \quad (5.68)$$

#### 5.2.4 Numerical Results

We show some numerical computations for the two decay rates  $\Gamma(Z' \rightarrow Z_0\gamma)$  and  $\Gamma(Z' \rightarrow Z_0Z_0)$ . They depend on the free parameters of the model, i.e. the couplings  $g_0 Q_Q$ ,  $g_0 Q_L$  and the mass of the  $Z'$ , since  $Q_{H_u} = 0$ . We show our results in Fig. 5.5-5.7 in the form of contour plots in the plane  $g_0 Q_L, g_0 Q_Q$  for  $M_{Z'} = 1, 2$  and 4 TeV. Our choices for  $g_0$ ,  $Q_Q$ ,  $Q_L$  and  $M_{Z'}$  are in agreement with the current experimental bounds [67].

The darker shaded regions correspond to larger decay rates. The white region corresponds to the value  $10^{-6}$  GeV that can be considered as a rough lower limit for the detection of the corresponding process. It is worth noting that increasing  $M_{Z'}$  the mean value of the decay rate of  $Z' \rightarrow Z_0\gamma$  grows while the one of  $Z' \rightarrow Z_0Z_0$  decreases. We would also like to mention that increasing  $M_{Z'}$  the iso-decay rate contours in the plot rotate clockwise getting more and more parallel to

## 5.2 Anomalous Decays

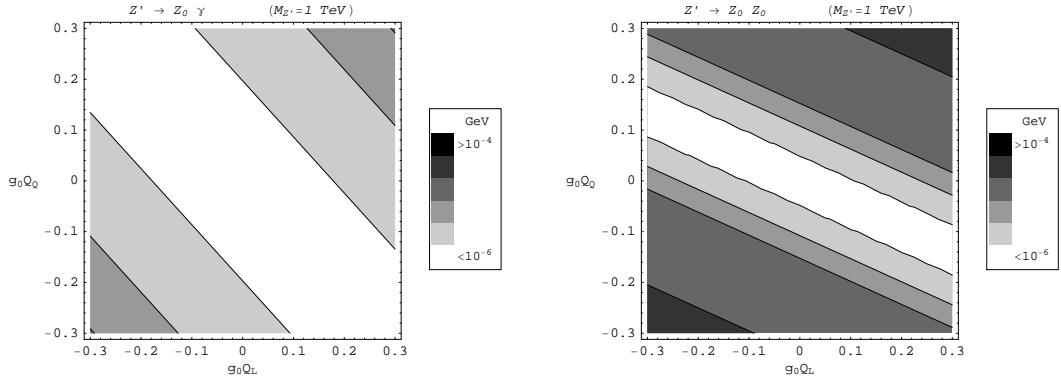


Figure 5.5:  $M_{Z'} = 1 \text{ TeV}$ .

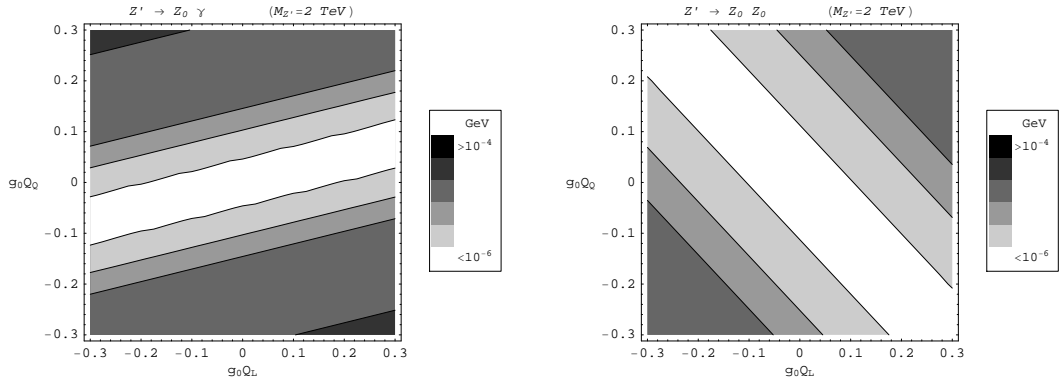


Figure 5.6:  $M_{Z'} = 2 \text{ TeV}$ .

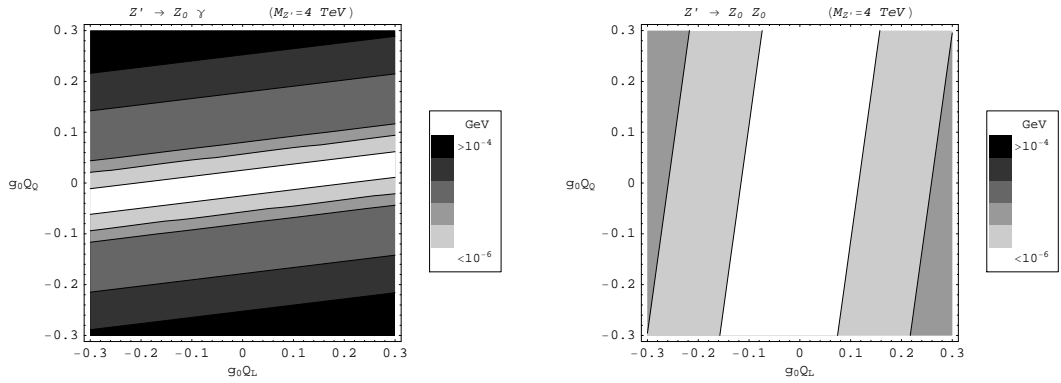


Figure 5.7:  $M_{Z'} = 4 \text{ TeV}$ .

the  $g_0 Q_L$ -axis. This effect is due to the fact that the contribution of the triangle diagram with the top quark circulating inside the loop becomes the dominant contribution for high  $M_{Z'}$ . In this case the decays strongly depend on the top quark coupling  $g_0 Q_Q$  while the lepton couplings  $g_0 Q_L$  become irrelevant. Finally, we find that the region that gives the largest values (of order of  $10^{-4}$  GeV) of the decay  $Z' \rightarrow Z_0 \gamma$  is for  $M_{Z'} \sim 4$  TeV and for  $g_0 Q_Q \sim 0.3$ ,  $g_0 Q_L \sim -0.2$ .

### 5.3 LHC prediction

To estimate the number of the decays that can be observed at LHC we shall use the narrow width approximation,

$$N_{Z' \rightarrow \text{particles}} = N_{Z'} \text{BR}(Z' \rightarrow \text{particles}) \quad (5.69)$$

where  $N_{Z'}$  is the total number of  $Z'$  produced and  $\text{BR}(Z' \rightarrow \text{particles})$  is the branching ratio of the decay of interest. We remind that we can use such approximation if  $(\Gamma_{Z'}/M_{Z'})^2 \ll 1$ . For our choice of parameters, the maximum value will be for  $g_0 Q_L = g_0 Q_Q = 0.3$  and  $M_{Z'} = 1$  TeV, so  $(\Gamma_{Z'}/M_{Z'})^2 \simeq 3 \times 10^{-3}$  and the latter inequality holds.

The total number of  $Z'$  is  $N_{Z'} = \sigma_{Z'} \mathcal{L} t$  where  $\mathcal{L} = 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$  the luminosity and  $t = 1$  year. Finally  $\sigma_{Z'}$  is the  $Z'$  production cross section [34]

$$\frac{d\sigma_{Z'}}{dy} = \frac{4\pi^2 x_1 x_2}{3M_{Z'}^3} \sum_i [f_{q_i}(x_1) f_{\bar{q}_i}(x_2) + f_{\bar{q}_i}(x_1) f_{q_i}(x_2)] \Gamma(Z' \rightarrow q_i \bar{q}_i) \quad (5.70)$$

where  $f_{q_i, \bar{q}_i}$  are the quark  $q_i$  (or antiquark  $\bar{q}_i$ ) structure functions in the proton, and the momentum fractions are

$$x_{1,2} = (M_{Z'}/\sqrt{s}) e^{\pm y} \quad (5.71)$$

The formula (5.70) does not take into account the gluon contribution inside the proton. The gluons can only contribute with the fermionic loop triangle. There is no GCS contribution since there is no anomaly (see eq. (4.10)). However the  $q \bar{q} g$  vertex is vectorial like, as the corresponding vertex with the  $Z'$  for  $Q_{H_u} = 0$  (see Table 5.1), so we have a  $VVV$  triangle which is zero because of the Furry theorem [69]. Then, in this case, we do not have any gluon contribution.

We integrate numerically the PDFs using a Mathematica package [70]. In Fig. 5.8 we show the result for  $N_{Z'}$  at  $\sqrt{s} = 14$  TeV. We can see that the number

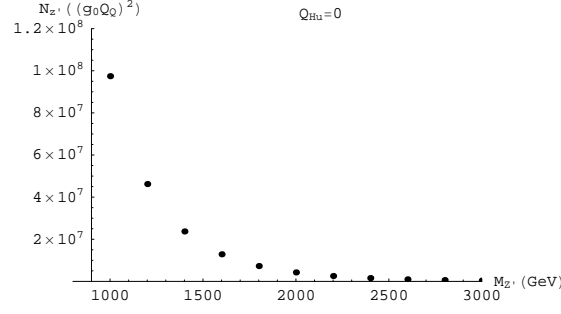


Figure 5.8: Number of  $Z'$  produced at LHC in 1 year for  $\mathcal{L} = 10^{34} \text{cm}^{-2} \text{s}^{-1}$  and  $\sqrt{s} = 14$  TeV, in units of  $(g_0 Q_Q)^2$ , in function of the mass of the  $Z'$ .

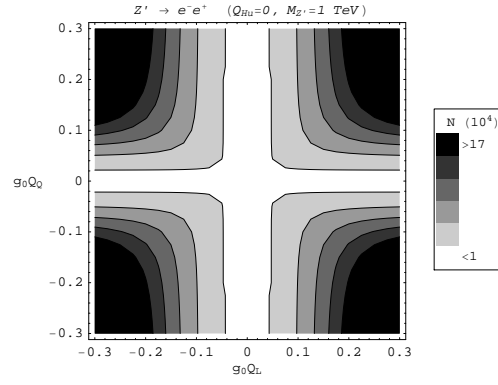


Figure 5.9: Number of  $Z' \rightarrow e^-e^+$  at LHC in 1 year for  $\mathcal{L} = 10^{34} \text{cm}^{-2} \text{s}^{-1}$ ,  $\sqrt{s} = 14$  TeV and  $M_{Z'} = 1$  TeV.

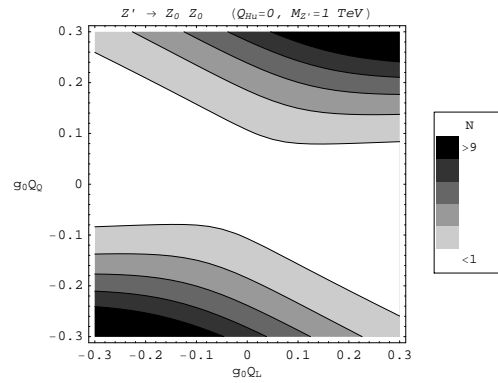


Figure 5.10: Number of  $Z' \rightarrow Z_0 Z_0$  at LHC in 1 year for  $\mathcal{L} = 10^{34} \text{cm}^{-2} \text{s}^{-1}$ ,  $\sqrt{s} = 14$  TeV and  $M_{Z'} = 1$  TeV.

of the  $Z'$  produced falls off exponentially with  $M_{Z'}$ , so we shall focus on the case  $M_{Z'} \sim 1$  TeV. As said before, we assume that the LSP has a mass around 500 GeV (see Section 5.4), so that we can ignore sparticles contribution in the computation of the branching ratios.

In Fig. 5.9, we estimate the number of decays  $Z' \rightarrow e^-e^+$  for 1 year of integrated luminosity. We can see that in a wide region of parameter the number of events is larger than ten thousand. This gives a lot of chance to discover a  $Z'$  in LHC.

In the case  $M_{Z'} = 1$  TeV the most favorite anomalous decay is  $Z' \rightarrow Z_0Z_0$ . In Fig. 5.10, we estimate the number of decays for 1 year of integrated luminosity which turns out to be  $N_{Z' \rightarrow Z_0Z_0} \sim 10$  for large values of the couplings  $g_0Q_L$  and  $g_0Q_Q$ .

## 5.4 Axino Dark Matter

Assuming the conservation of R-parity the LSP is a good weak interacting massive particle (WIMP) dark matter candidate. As in the MSSM the LSP is given by a linear combination of fields in the neutralino sector. The general form of the neutralino mass matrix is given in (4.56). This is a six-by-six matrix. From the point of view of the strength of the interactions the two extra states are not on the same footing with respect to the standard ones. The axino and the primeino are in fact extremely weak interacting massive particle (XWIMP). Thus we are interested in situations in which the extremely weak sector is decoupled from the standard one and the LSP belongs to this sector. This can be achieved at tree level with the choice  $Q_{H_u} = 0$ . The neutralino mass matrix  $\mathbf{M}_{\tilde{N}}$  becomes

$$\mathbf{M}_{\tilde{N}} = \begin{pmatrix} \frac{M_S}{2} & \frac{M_{V(0)}}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ \dots & M_0 & 0 & 0 & 0 & 0 \\ \dots & \dots & M_1 & 0 & -\frac{g_1 v_d}{2} & \frac{g_1 v_u}{2} \\ \dots & \dots & \dots & M_2 & \frac{g_2 v_d}{2} & -\frac{g_2 v_u}{2} \\ \dots & \dots & \dots & \dots & 0 & -\mu \\ \dots & \dots & \dots & \dots & \dots & 0 \end{pmatrix} \quad (5.72)$$

where  $M_S$ ,  $M_0$ ,  $M_1$ ,  $M_2$  are the soft masses coming from the soft breaking terms (4.19) while  $M_{V(0)}$  is the  $U(1)'$  Stückelberg mass given after eq. (4.51). It

is worth noting that the  $D$  terms and kinetic mixing terms can be neglected in the tree-level computations of the eigenvalues and eigenstates.

Moreover we make the assumption that  $M_0 \gg M_S, M_{V^{(0)}}$  and  $g_{1,2}v_{u,d} \ll M_{1,2}$ , so the eigenstates are nearly pure axino, gauginos and higgsinos. Finally we suppose that the axino  $\psi_S$  is the LSP with a mass of around 500 GeV while the bino  $\lambda^{(1)}$  is the NLSP.

The decoupling in the neutralino sector implies also a decoupling in the gauge bosons sector as showed in the previous sections, since  $Q_{H_u} = 0$  (see eq. (4.52)).

In the following sections we present the axino relic density computation following the lines of our paper [2]. For a detailed discussion we remand to [71].

#### 5.4.1 Axino Interactions

The axino interactions can be read off from the interaction Lagrangian (4.15). The relevant term, written in terms of four components Majorana spinors, is given by

$$\mathcal{L} = i\sqrt{2}g_1^2 b_2^{(1)} \bar{\Lambda}^{(1)} \gamma_5 [\gamma^\mu, \gamma^\nu] (\partial_\mu V_\nu^{(1)}) \Psi_S + i\frac{\sqrt{2}}{2} g_2^2 b_2^{(2)} \bar{\Lambda}_3^{(2)} \gamma_5 [\gamma^\mu, \gamma^\nu] (\partial_\mu V_{3\nu}^{(2)}) \Psi_S \quad (5.73)$$

where the  $b_2^{(a)}$  coefficients are given in (4.18). The related interaction vertex is

$$C[\gamma^\mu, \gamma^\nu] i k_\mu \quad (5.74)$$

where  $C$  is a factor which contains the coupling constants

$$C = \sqrt{2} g_{(a)}^2 b_2^{(a)} R(\theta_W) f \quad a = 1, 2 \quad (5.75)$$

where  $R(\theta_W) = \cos \theta_W$ ,  $\sin \theta_W$  ( $\theta_W$  is the Weinberg angle) while  $f$  is the gaugino coefficient in the mass eigenstate basis. The factor (5.75) contains the parameters  $b_2^{(a)}$  which are related to the anomalous  $U(1)$  (see eq. (4.18)). Therefore  $C \ll g_{(a)}$  and the axino interactions will be extremely weak, being suppressed by an order of magnitude factor with respect to the weak interactions. At tree level there is only one type of annihilation diagram, represented in Fig. 5.11. We denoted with  $p_1$  and  $p_2$  the incoming momenta of the axinos while  $k_1$  and  $k_2$  are the two outgoing momenta of the gauge bosons in the final state. We will concentrate on the case with two photons in the final state. In this case the result for the

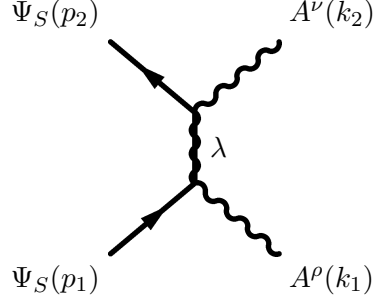


Figure 5.11: Annihilation of two axinos into two gauge vectors via the exchange of a gaugino.

differential cross section is given by

$$\frac{d\sigma}{d\Omega} = \frac{4M_S^2\omega_1}{16\pi^2(\omega_1 + \omega_2)^2(\sqrt{M_S^2 - E_2^2})} \sum_{i,j=1}^2 \mathcal{M}_i \mathcal{M}_j^* \quad (5.76)$$

where  $\omega_1$  and  $\omega_2$  are the energies of the two outgoing photons. Each amplitude  $\mathcal{M}_i$  is proportional to the related coefficient  $C$  whose generic form is given in (5.75). The cross section (5.76), being extremely weak, cannot give a relic density in the WMAP preferred range. Thus we are forced to consider a scenario in which coannihilations between the axino and the NLSP became sizable. We will assume a NLSP, coming from the “usual” MSSM, with a mass comparable to the axino mass. Without loss of generality the NLSP can be identified with the bino. The allowed coannihilation processes are those which involve an exchange of a photon or a  $Z_0$  in the intermediate state and with a SM fermion-antifermion pair, Higgses and  $W$ 's in the final state. The differential cross section in the CM frame has the following general form

$$\frac{d\sigma}{d\Omega} \propto \frac{1}{s} \frac{p_f}{p_i} |\mathcal{M}|^2 \quad (5.77)$$

where  $s$  is the usual Mandelstam variable and  $p_{f,i}$  is the spatial momentum of the outgoing (incoming) particles. On dimensional ground  $|\mathcal{M}|^2$  has at least a linear dependence on  $p_f$  and this implies that the dominant contribution comes from the diagram with the SM fermion-antifermion pair  $f$  and  $\bar{f}$  in the final state:

$$\Psi_S \lambda^{(a)} \rightarrow f \bar{f} \quad (5.78)$$

The two possible diagrams associated to this process are sketched in Fig. 5.12. The resulting differential cross section, computed in the CM frame, is

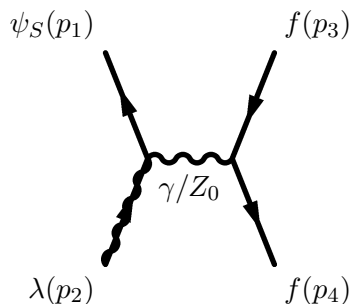


Figure 5.12: Coannihilation of an axino and a bino into a  $f\bar{f}$  pair via the exchange of a photon or a  $Z_0$ .

$$\frac{d\sigma}{d\Omega} = \sum_f C_f \frac{\sqrt{(E_3 - m_f)^2}}{64\pi^2(E_1 + E_2)^2 \sqrt{(E_1^2 - M_S^2)}} (\mathcal{M}_\gamma^2 + \mathcal{M}_{Z_0}^2 + \mathcal{M}_\gamma^* \mathcal{M}_{Z_0} + \mathcal{M}_\gamma \mathcal{M}_{Z_0}^*) \quad (5.79)$$

where the sum is extended to all the SM fermions (with mass  $m_f$ ) while  $C_f$  is a color factor. In Appendix D.7 we give the Feynman amplitude for the decay into two neutrinos.

#### 5.4.2 Axino Relic Density

In this section we compute the relic density of the axino. The case of the axino as a cold dark matter candidate has been studied for the first time in [72]. As we said in the previous section we assumed a LSP which coannihilates with a NLSP degenerate in mass. Without loss of generality the NLSP is assumed to be a bino. The relic density can be evaluated solving the Boltzmann equation for two particle species:

$$\frac{dn}{dt} = -3Hn - \sum_{i,j=1}^2 \langle \sigma_{ij} v_{ij} \rangle (n_i n_j - n_i^{eq} n_j^{eq}) \quad (5.80)$$

where  $n_i$  denotes the number density per unit of comoving volume of the species  $i$  ( $i = 1$  refers to the LSP while  $i = 2$  refers to the NLSP),  $n = \sum_i n_i$ ,  $H$  is the Hubble constant,  $\sigma_{ij}$  is the annihilation cross section between a species  $i$  and a species  $j$ ,  $v_{ij}$  is the modulus of the relative velocity while  $n_i^{eq}$  is the equilibrium number density of the species  $i$ . Having computed the number density, the relic density is simply given by

$$\Omega h^2 \cong \frac{\rho}{\rho_c} = \frac{M_S n}{\rho_c} \quad (5.81)$$

since the two species are nearly degenerate in mass, where  $\rho_c$  is the critical mass density of the Universe.

We rewrite (5.80) by defining the thermal average of the effective cross section

$$\langle \sigma_{eff} v \rangle \equiv \sum_{i,j=1}^2 \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{eq} n_j^{eq}}{n^{eq} n^{eq}} \quad (5.82)$$

obtaining

$$\frac{dn}{dt} = -3Hn - \langle \sigma_{eff} v \rangle (n^2 - (n^{eq})^2) \quad (5.83)$$

We parametrize the effective cross section as in [73]:

$$\langle \sigma_{eff} v \rangle = \frac{A}{n_{eq}^2} \quad (5.84)$$

by introducing the following quantity

$$A = \frac{g_1^2 T}{4\pi^4} \int dp_{eff} p_{eff}^2 W_{eff} K_1 \left( \frac{\sqrt{s}}{T} \right) \quad (5.85)$$

where

$$p_{eff} = \frac{1}{2} \sqrt{s - 4M_S^2} \quad (5.86)$$

and  $K_1$  is the modified Bessel function of the first type while

$$W_{eff} = \frac{2(s(s - 2M_1^2)\sigma_{22} + 2(s - (M_S + M_1)^2)(s - (M_S - M_1)^2)\sigma_{12})}{\sqrt{s(s - 2M_S^2)}} \quad (5.87)$$

Notice that in the above expression we have neglected the term  $\sigma_{11}$  due to the suppression factor with respect to the other cross sections. The leading term in (5.87) is given by the term proportional to the NLSP (a bino in our case) annihilation cross section  $\sigma_{22}$ . This implies that  $\sigma_{22}$  must be of the right order of magnitude to satisfy the upper bound coming from the WMAP data [74]. This constraints can always be satisfied with a suitable choice of the soft breaking parameters.

The equilibrium number density is given by

$$n_{eq} = \sum_i \frac{g_i m_i^2 T}{2\pi^2} K_2 \left( \frac{m_i}{T} \right) \quad (5.88)$$

where  $K_2$  is the modified Bessel function of the second type. In our notation we have  $m_1 = M_S$  and  $m_2 = M_1$ . Substituting (5.85) and (5.88) into (5.84) we get:

$$\langle \sigma_{eff} v \rangle (T) = \frac{\int dp_{eff} p_{eff}^2 W_{eff} K_1 \left( \frac{\sqrt{s}}{T} \right)}{M_S^4 T [\sum_i \frac{g_i}{g_1} \frac{m_i^2}{M_S^2} K_2 \left( \frac{m_i}{T} \right)]^2} \quad (5.89)$$

In order to solve the Boltzmann equation (5.83) it is convenient to introduce a new variable  $Y = n/S$  where  $S$  is the entropy density. The Boltzmann equation written in terms of this quantity reads

$$\frac{dY}{dT} = \frac{1}{3H} \frac{dS}{dT} \langle \sigma_{eff} v \rangle (Y^2 - Y_{eq}^2) \quad (5.90)$$

By introducing the adimensional quantity  $x = T/M_S$  we can further simplify the Boltzmann equation

$$\frac{dY}{dx} = M_S \sqrt{\frac{\pi g_*}{45G}} \langle \sigma_{eff} v \rangle (Y^2 - Y_{eq}^2) \quad (5.91)$$

where we have introduced the number of effective degrees of freedom  $g_*$ . It can be shown that  $g_*^{1/2} \simeq 9$ .

The final result for the relic density is

$$\Omega h^2 = \frac{1.07 \times 10^9 \text{GeV}^{-1}}{M_{Pl} \sqrt{g_*}} \frac{1}{J(x_f)} \quad (5.92)$$

where  $M_{Pl}$  is the four-dimensional Planck mass while

$$J(x_f) = \int_0^{x_f} dx \langle \sigma_{eff} v \rangle (x) \quad (5.93)$$

The quantity  $x_f$  is related to the freeze-out temperature. It turns out that its value weakly depend on  $\langle \sigma_{eff} v \rangle$ . A numerical computation gives  $x_f \simeq 1/20$  [75]. In the general case the integral in (5.93) can not be solved analytically. Therefore we solved it numerically by sampling the function  $\langle \sigma_{eff} v \rangle (x)$  at different value of  $x$ .

Our aim is to find regions of the parameter space, determined by  $Q_Q$ ,  $Q_L$  and the coupling constant  $g_0$  of the extra  $U(1)$ , that fulfills the WMAP constraints. This task can be performed by imposing suitable conditions on the  $C^2$ . In the limit in which we can neglect the electroweak symmetry breaking effects since the soft mass are much heavier than the EW scale and the NLSP is essentially a bino, which implies  $f = 1$  in (5.75), we have

$$C_\gamma^2 = 2(b_2^{(1)})^2 g_1^4 \cos^2 \theta_W \quad (5.94)$$

for the contribution  $\mathcal{M}_\gamma^2$  in (5.79). From the formula (4.8) of the mixed  $U(1)' - U(1)_Y - U(1)_Y$  anomaly and from the eq. (4.18) we have the following relation

$$b_2^{(1)} = \frac{3(3Q_Q + Q_L)}{128\pi^2 b_3} \quad (5.95)$$

In Section 5.3 we supposed  $M_{V(0)} \simeq 1$  TeV, then we have  $b_3 \simeq 1/4g_0$  TeV and we finally get

$$\frac{C_\gamma^2}{e^2} = 0.94 \times 10^{-8} (3g_0Q_Q + g_0Q_L)^2 \text{ GeV}^{-2} \quad (5.96)$$

where  $e$  is the electric charge. We get similar expressions for the other three terms in (5.79). Collecting all these terms we obtain an upper bound on  $(3g_0Q_Q + g_0Q_L)^2$  for which the cross section gives a relic density in the  $3\sigma$  WMAP range [74]:

$$0.0913 < \Omega h^2 < 0.1285 \quad (5.97)$$

The lower bound is  $(3g_0Q_Q + g_0Q_L)^2 = 0$  and in this case we have no coannihilations. In (5.87) the only terms that survives is that proportional to  $\sigma_{22}$  and therefore we recover the standard result. The upper bound is

$$(3g_0Q_Q + g_0Q_L)^2 < 16 \quad (5.98)$$

which is valid for  $M_S$  ranging from 500 GeV up to 2 TeV and for a mass gap  $\Delta m/m = (M_1 - M_S)/M_S$  ranging from 1% up to 5%. The bound (5.98) corresponds to the perturbative constraints  $g_0Q_Q < 1$  and  $g_0Q_L < 1$ . Our result is in agreement with the XWIMP relic density estimate given in [76] which is obtained without an actual computation of the coannihilation cross section.

## Chapter 6

# Conclusions

In this Thesis, we have reviewed an extension of the MSSM by the addition of an anomalous abelian vector multiplet and showed some original results concerning the phenomenology of an anomalous  $Z'$ .

In Chapter 2 we introduced the MSSM and its main properties such as the gauge coupling unification.

In Chapter 3 we presented the chiral anomalies in quantum field theory and their cancellation by the standard procedure or by a string inspired mechanism firstly discovered by Green and Schwarz. Then we discussed the Green-Schwarz mechanism in a standard and in a supersymmetric framework showing which are its main features and physical consequences.

In Chapter 4 we introduced our model, the MiAUMSSM. We showed how the MSSM modified by the addition of an anomalous  $U(1)$  gauge field, a Stückelberg multiplet and Chern-Simons terms. We checked the correct electroweak symmetry breaking and we found new contributions to the mass matrices for almost all the particle in the spectrum with tree level or 1-loop intensity.

In Chapter 5 we presented the first phenomenological results of our model for the choice  $Q_{H_u} = 0$ . We computed the  $Z'$  production at LHC for variable  $M_{Z'}$ , noticing that the number of the  $Z'$  produced falls off exponentially with its mass, so the most favoured case is  $M_{Z'} \sim 1$  TeV. Then we estimated the number of leptonic decays and anomalous decays for a  $Z'$  with  $M_{Z'} \sim 1$  TeV for 1 year of integrated luminosity in LHC. We saw that in a wide region of parameter we have more than  $10^4$   $Z' \rightarrow e^-e^+$  decays, while the maximum number of  $Z' \rightarrow Z_0Z_0$  is of the order of 10. Then we studied a possible dark matter candidate in

## Conclusions

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the framework of our model. In the decoupling limit  $Q_{H_u} = 0$  and under the assumptions  $M_0 \gg M_S$  and  $g_{1,2}v_{u,d} \ll M_{1,2}$  the axino turns out to be the LSP. Being an XWIMP the axino annihilation cross section is suppressed with respect to the typical weak interaction cross sections. This implies that in order to satisfy the WMAP constraints on the relic density we must have a bino-like NLSP almost degenerate in mass with the axino. We computed the LSP-NLSP coannihilation cross section and we found that in the parameter space where our model is perturbative the WMAP constraints are always satisfied. This result holds for  $M_S$  ranging from 500 GeV up to 2 TeV and for a mass gap  $\Delta m/m$  of few percents.

The eventual discover of extra gauge bosons and dark matter particles are some of the main issues in today high energy physics. In the next months, the new colliders and satellite experiments, such as LHC and PAMELA will give us clues on the fundamental theory underlying the Standard Model.

# Appendix A

## Gammology

### A.1 Notations and conventions

We use the space-time metric  $\eta_{\mu\nu} = \text{diag}(+, -, -, -)$  and the spinorial conventions

$$\epsilon_{21} = \epsilon^{12} = 1 \quad \epsilon_{12} = \epsilon^{21} = -1 \quad \epsilon_{11} = \epsilon^{11} = \epsilon_{22} = \epsilon^{22} = 0 \quad (\text{A.1})$$

$$\psi^\alpha = \epsilon^{\alpha\beta} \psi_\beta \quad \psi_\alpha = \epsilon_{\alpha\beta} \psi^\beta \quad \bar{\psi}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}} \bar{\psi}_{\dot{\beta}} \quad \bar{\psi}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}} \bar{\psi}^{\dot{\beta}} \quad (\text{A.2})$$

$$\psi\chi = \psi^\alpha \chi_\alpha \quad \bar{\psi}\bar{\chi} = \bar{\psi}_{\dot{\alpha}} \bar{\chi}^{\dot{\alpha}} \quad (\text{A.3})$$

The Dirac matrices are

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \quad \text{where} \quad \begin{cases} \sigma^\mu & = (1, -\vec{\sigma}) \\ \bar{\sigma}^\mu & = (1, \vec{\sigma}) \end{cases} \quad (\text{A.4})$$

and we define

$$\gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad (\text{A.5})$$

### A.2 Properties of $\gamma$ matrices and trace theorems

The basic properties of  $\gamma$  matrices are

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \quad \text{and} \quad \{\gamma^\mu, \gamma_5\} = 0 \quad (\text{A.6})$$

The trace theorems are

$$\begin{aligned} \text{Tr}1 &= 4 \\ \text{Tr}(\text{odd number of } \gamma\text{'s matrices}) &= 0 \\ \text{Tr}(\gamma^\mu \gamma^\nu) &= 4\eta^{\mu\nu} \end{aligned} \tag{A.7}$$

$$\begin{aligned} \text{Tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma) &= 4[\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}] \\ \text{Tr}\gamma_5 &= 0 \end{aligned} \tag{A.8}$$

$$\begin{aligned} \text{Tr}[\gamma_5(\text{odd number of } \gamma\text{'s matrices})] &= 0 \\ \text{Tr}(\gamma_5 \gamma^\mu \gamma^\nu) &= 0 \\ \text{Tr}(\gamma_5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma) &= 4i\epsilon^{\mu\nu\rho\sigma} \\ \text{Tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \dots) &= \text{Tr}(\dots \gamma^\sigma \gamma^\rho \gamma^\mu \gamma^\nu) \end{aligned} \tag{A.9}$$

where  $\epsilon^{\mu\nu\rho\sigma} = 1(-1)$  for  $\mu, \nu, \rho, \sigma$  an even (odd) permutation of 0,1,2,3; and 0 if two indices are the same.

Other useful results for simplifying trace calculations are ( $\not{a} = a_\mu \gamma^\mu$ )

$$\begin{aligned} \gamma_\mu \gamma^\mu &= 4 \\ \gamma_\mu \not{a} \gamma^\mu &= -2\not{a} \\ \gamma_\mu \not{a} \not{b} \gamma^\mu &= 2a \cdot b \\ \gamma_\mu \not{a} \not{b} \not{c} \gamma^\mu &= -2\not{c} \not{b} \not{a} \end{aligned} \tag{A.10}$$

# Appendix B

## Lagrangians

### B.1 MSSM Lagrangian

The Lagrangian of the MSSM contains several terms

$$\mathcal{L}^{MSSM} = \mathcal{L}_Q^{MSSM} + \mathcal{L}_L^{MSSM} + \mathcal{L}_{gauge}^{MSSM} + \mathcal{L}_H^{MSSM} + \mathcal{L}_W^{MSSM} + \mathcal{L}_{Soft}^{MSSM} \quad (\text{B.1})$$

where

$$\begin{aligned} \mathcal{L}_Q^{MSSM} = & \left( Q_i^\dagger e^{V(3)} e^{V(2)} e^{\frac{1}{6}V(1)} Q_i \right. \\ & \left. + (U_i^c)^\dagger e^{-V(3)} e^{-\frac{2}{3}V(1)} U_i^c + (D_i^c)^\dagger e^{-V(3)} e^{\frac{1}{3}V(1)} D_i^c \right)_{\theta^2 \bar{\theta}^2} \end{aligned} \quad (\text{B.2})$$

$$\mathcal{L}_L^{MSSM} = \left( L_i^\dagger e^{V(2)} e^{-\frac{1}{2}V(1)} L_i + (E_i^c)^\dagger e^{V(1)} E_i^c \right)_{\theta^2 \bar{\theta}^2} \quad (\text{B.3})$$

$$\mathcal{L}_H^{MSSM} = \left( H_u^\dagger e^{V(2)} e^{\frac{1}{2}V(1)} H_u + H_d^\dagger e^{V(2)} e^{-\frac{1}{2}V(1)} H_d \right)_{\theta^2 \bar{\theta}^2} \quad (\text{B.4})$$

$$\begin{aligned} \mathcal{L}_{gauge}^{MSSM} = & \left( \frac{1}{8g_3^2} \text{Tr} (W^{(3)} W^{(3)}) + \frac{1}{8g_2^2} \text{Tr} (W^{(2)} W^{(2)}) \right. \\ & \left. + \frac{1}{16g_1^2} \text{Tr} (W^{(1)} W^{(1)}) \right)_{\theta^2} + h.c. \end{aligned} \quad (\text{B.5})$$

$$\mathcal{L}_W^{MSSM} = (y_u^{ij} Q_i U_j^c H_u - y_d^{ij} Q_i D_j^c H_d - y_e^{ij} L_i E_j^c H_d + \mu H_u H_d)_{\theta^2} + h.c. \quad (\text{B.6})$$

$$\begin{aligned}
\mathcal{L}_{soft}^{MSSM} = & -\frac{1}{2} \sum_{a=1}^3 (M_a \lambda^{(a)} \lambda^{(a)} + \text{h.c.}) - \left( m_{\tilde{q}_{ij}}^2 \tilde{q}_i \tilde{q}_j^\dagger + m_{\tilde{u}_{ij}}^2 \tilde{u}_i^c \tilde{u}_j^{c\dagger} + m_{\tilde{d}_{ij}}^2 \tilde{d}_i^c \tilde{d}_j^{c\dagger} \right. \\
& + m_{\tilde{l}_{ij}}^2 \tilde{l}_i \tilde{l}_j^\dagger + m_{\tilde{e}_{ij}}^2 \tilde{e}_i^c \tilde{e}_j^{c\dagger} + m_{h_u}^2 |h_u|^2 + m_{h_d}^2 |h_d|^2 \left. \right) \\
& - \left( a_u^{ij} \tilde{q}_i \tilde{q}_j h_u - a_d^{ij} \tilde{q}_i \tilde{d}_j h_d - a_e^{ij} \tilde{l}_i \tilde{e}_j h_d + b h_u h_d + \text{h.c.} \right) \quad (\text{B.7})
\end{aligned}$$

Notice that in order to include the coupling constants in the gauge interactions we need to substitute  $V \rightarrow 2gV$ .

## B.2 MiAUMSSM Lagrangian

The Lagrangian of the model contains several terms

$$\mathcal{L} = \mathcal{L}_Q + \mathcal{L}_L + \mathcal{L}_{gauge} + \mathcal{L}_H + \mathcal{L}_W + \mathcal{L}_{\text{axion}} + \mathcal{L}_{\text{GCS}} + \mathcal{L}_{\text{Soft}} \quad (\text{B.8})$$

where

$$\begin{aligned}
\mathcal{L}_Q = & \left( Q_i^\dagger e^{V^{(3)}} e^{V^{(2)}} e^{\frac{1}{6}V^{(1)}} e^{Q_Q V^{(0)}} Q_i \right. \\
& \left. + (U_i^c)^\dagger e^{-V^{(3)}} e^{-\frac{2}{3}V^{(1)}} e^{Q_{U^c} V^{(0)}} U_i^c + (D_i^c)^\dagger e^{-V^{(3)}} e^{\frac{1}{3}V^{(1)}} e^{Q_{D^c} V^{(0)}} D_i^c \right)_{\theta^2 \bar{\theta}^2} \quad (\text{B.9})
\end{aligned}$$

$$\mathcal{L}_L = \left( L_i^\dagger e^{V^{(2)}} e^{-\frac{1}{2}V^{(1)}} e^{Q_L V^{(0)}} L_i + (E_i^c)^\dagger e^{V^{(1)}} e^{Q_{E^c} V^{(0)}} E_i^c \right)_{\theta^2 \bar{\theta}^2} \quad (\text{B.10})$$

$$\mathcal{L}_H = \left( H_u^\dagger e^{V^{(2)}} e^{\frac{1}{2}V^{(1)}} e^{Q_{H_u} V^{(0)}} H_u + H_d^\dagger e^{V^{(2)}} e^{-\frac{1}{2}V^{(1)}} e^{Q_{H_d} V^{(0)}} H_d \right)_{\theta^2 \bar{\theta}^2} \quad (\text{B.11})$$

$$\begin{aligned}
\mathcal{L}_{gauge} = & \left( \frac{1}{8g_3^2} \text{Tr} (W^{(3)} W^{(3)}) + \frac{1}{8\tilde{g}_2^2} \text{Tr} (W^{(2)} W^{(2)}) \right. \\
& \left. + \frac{1}{16\tilde{g}_1^2} \text{Tr} (W^{(1)} W^{(1)}) + \frac{1}{16\tilde{g}_0^2} \text{Tr} (W^{(0)} W^{(0)}) \right)_{\theta^2} + \text{h.c.} \quad (\text{B.12})
\end{aligned}$$

$$\mathcal{L}_W = (y_u^{ij} Q_i U_j^c H_u - y_d^{ij} Q_i D_j^c H_d - y_e^{ij} L_i E_j^c H_d + \mu H_u H_d)_{\theta^2} + \text{h.c.} \quad (\text{B.13})$$

$$\begin{aligned}
\mathcal{L}_{\text{axion}} = & \frac{1}{4} (S + \bar{S} + 4b_3 V^{(0)})^2 \Big|_{\theta^2 \bar{\theta}^2} \\
& - \frac{1}{2} \left\{ \left[ \sum_{a=0}^2 b_2^{(a)} S \text{Tr} (W^{(a)} W^{(a)}) + b_2^{(4)} S W^{(1)} W^{(0)} \right]_{\theta^2} + \text{h.c.} \right\} \quad (\text{B.14})
\end{aligned}$$

$$\begin{aligned}
 \mathcal{L}_{\text{GCS}} = & +d_4 [(V^{(1)}D^\alpha V^{(0)} - V^{(0)}D^\alpha V^{(1)})W_\alpha^{(0)} + h.c.]_{\theta^2\bar{\theta}^2} + \\
 & -d_5 [(V^{(1)}D^\alpha V^{(0)} - V^{(0)}D^\alpha V^{(1)})W_\alpha^{(1)} + h.c.]_{\theta^2\bar{\theta}^2} + \\
 & -d_6 \text{Tr} \left[ (V^{(2)}D^\alpha V^{(0)} - V^{(0)}D^\alpha V^{(2)})W_\alpha^{(2)} + \right. \\
 & \left. + \frac{1}{6}V^{(2)}D^\alpha V^{(0)}\bar{D}^2 ([D_\alpha V^{(2)}, V^{(2)}]) + h.c. \right]_{\theta^2\bar{\theta}^2} \quad (\text{B.15})
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{L}_{\text{soft}} = & -\frac{1}{2}\sum_{a=0}^3 (M_a\lambda^{(a)}\lambda^{(a)} + \text{h.c.}) - \frac{1}{2}\left(\frac{M_S}{2}\psi_S\psi_S + h.c.\right) \\
 & - \left(m_{q_{ij}}^2\tilde{q}_i\tilde{q}_j^\dagger + m_{u_{ij}}^2\tilde{u}_i^c\tilde{u}_j^{c\dagger} + m_{d_{ij}}^2\tilde{d}_i^c\tilde{d}_j^{c\dagger}\right) \\
 & + m_{l_{ij}}^2\tilde{l}_i\tilde{l}_j^\dagger + m_{e_{ij}}^2\tilde{e}_i^c\tilde{e}_j^{c\dagger} + m_{h_u}^2|h_u|^2 + m_{h_d}^2|h_d|^2 \\
 & - \left(a_u^{ij}\tilde{q}_i\tilde{q}_j h_u - a_d^{ij}\tilde{q}_i\tilde{d}_j h_d - a_e^{ij}\tilde{l}_i\tilde{e}_j h_d + bh_u h_d + \text{h.c.}\right) \quad (\text{B.16})
 \end{aligned}$$

where  $\mathcal{L}_Q$ ,  $\mathcal{L}_L$  and  $\mathcal{L}_H$  provide the kinetic terms and the gauge interactions of the matter particles such as (s)quarks, (s)leptons, Higgs(ino)s;  $\mathcal{L}_{\text{gauge}}$  contains the kinetic terms for the gauge supermultiplet;  $\mathcal{L}_W$  is the usual MSSM superpotential;  $\mathcal{L}_{\text{axion}}$  provides the kinetic term of the Stückelberg multiplet and its Green-Schwarz interactions used in the anomaly cancellation procedure;  $\mathcal{L}_{\text{GCS}}$  contains the Generalized Chern Simons interactions giving trilinear gauge boson couplings needed to complete the anomaly cancellation procedure; finally,  $\mathcal{L}_{\text{Soft}}$  contains the usual soft breaking terms of the MSSM as well as the new terms for the primeino and the axino.

Notice that in order to include the coupling constants in the gauge interactions we need to redefine them as shown in equation (4.14) and to substitute  $V \rightarrow 2gV$ .







$Z_0$  interactions

$$\text{fermionic vertex: } \begin{array}{c} Z_{0\mu} \\ \text{wavy line} \\ \swarrow \quad \searrow \\ f \quad \quad f \end{array} = -\frac{i}{2}g_{Z_0}\gamma_\mu (v_f^{Z_0} - a_f^{Z_0}\gamma_5) \quad (\text{C.7})$$

The coupling constant  $g_{Z_0}$  and the vectorial and axial couplings  $v_f^{Z_0}$  and  $a_f^{Z_0}$  are given respectively in eq. (2.70) and in Table 5.2.

$W^\pm$  interactions

$$\text{fermionic vertex: } \begin{array}{c} W_\mu \\ \text{wavy line} \\ \swarrow \quad \searrow \\ l(q_u) \quad \nu_l(q_d) \end{array} = -\frac{i}{2}g_W \gamma_\mu (1 - \gamma_5) \quad (\text{C.8})$$

$$\begin{array}{c} Z_0 \text{ vertex: } \\ \begin{array}{c} Z_{0\rho}(p+q) \\ \text{wavy line} \\ \downarrow \\ \begin{array}{c} \swarrow \quad \searrow \\ W_\mu(p) \quad W_\nu(q) \end{array} \end{array} \end{array} = i e \cot \theta_W [\eta_{\rho\mu}(2p+q)_\nu + \eta_{\mu\nu}(q-p)_\rho + \eta_{\rho\nu}(p+2q)_\mu] \quad (\text{C.9})$$

where  $g_W$  and  $\theta_W$  are given in Section 2.4.3.

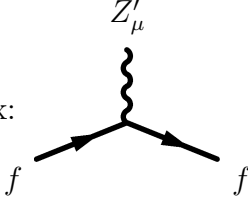
C.3 Yukawa's interactions

$$\text{NG-Yukawa vertex: } \begin{array}{c} G^0 \\ \text{dashed line} \\ \swarrow \quad \searrow \\ f \quad \quad f \end{array} = \begin{cases} SM : & \frac{y_f}{\sqrt{2}} \gamma_5 \\ MSSM f_u : & \frac{y_f}{\sqrt{2}} \sin \beta \gamma_5 \\ MSSM f_d : & -\frac{y_f}{\sqrt{2}} \cos \beta \gamma_5 \end{cases} \quad (\text{C.10})$$

where  $y_f$  is the Yukawa coupling of the fermion, and  $\beta$  is the angle in eq. (2.46).

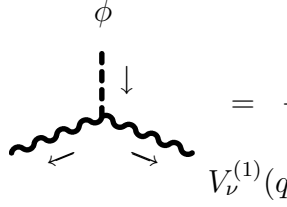
## C.4 $Z'$ interactions

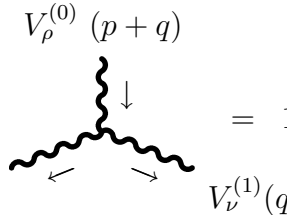
Tree level

fermionic vertex:  =  $-\frac{i}{2}g_{Z'}\gamma_\mu\left(v_f^{Z'} - a_f^{Z'}\gamma_5\right)$  (C.11)

The coupling constant  $g_{Z'} = g_0$  for  $Q_{H_u} = 0$  and the vectorial and axial couplings  $v_f^{Z'}$  and  $a_f^{Z'}$  are given in Table 5.2.

GS mechanism

GS vertex:  =  $-4ig_1^2 b_2^{(1)} \epsilon_{\mu\nu\rho\sigma} p^\rho q^\sigma$  (C.12)

GCS vertex:  =  $16g_0g_1^2 d_5 \epsilon_{\rho\nu\mu\alpha} p^\alpha$  (C.13)

(C.14)

The GS coupling  $b_2^{(1)}$  and the GCS coupling  $d_5$  are given in eq. (4.18).

## Appendix D

# More on amplitudes

### D.1 Linearly divergent integrals

Consider a shift of integration variable in a one dimensional integral

$$\Delta(a) = \int_{-\infty}^{\infty} dx [f(x+a) - f(x)] \quad (\text{D.1})$$

It can be easily shown that the shift may be illegitimate if the integral is divergent. To see this, we expand

$$\begin{aligned} \Delta(a) &= \int_{-\infty}^{\infty} dx \left[ a f'(x) + \frac{a^2}{2} f''(x) + \dots \right] \\ &= a[f(\infty) - f(-\infty)] + \frac{a^2}{2}[f'(\infty) - f'(-\infty)] + \dots \end{aligned} \quad (\text{D.2})$$

where the primes mean differentiation. When the integral  $\int_{-\infty}^{\infty} dx f(x)$  converges (or at most diverges logarithmically) we have  $0 = f(\pm\infty) = f'(\pm\infty) = \dots$  so  $\Delta(a) = 0$ . Instead, for a linearly divergent integral,  $0 \neq f(\pm\infty)$  and  $0 = f'(\pm\infty) = \dots$  so we get

$$\Delta(a) = a[f(\infty) - f(-\infty)] \neq 0 \quad (\text{D.3})$$

which corresponds to a “surface term”. The generalization to  $n$  dimensions is straightforward

$$\begin{aligned} \Delta(a) &= \int d^n r [f(r+a) - f(r)] \\ &= \int d^n r \left[ a^\tau \frac{\partial}{\partial r_\tau} f(r) + \frac{1}{2} a^\tau a^\sigma \frac{\partial^2}{\partial r_\tau \partial r_\sigma} f(r) + \dots \right] \end{aligned} \quad (\text{D.4})$$

Applying Gauss's theorem, all terms but the first vanish upon integrating over the surface at  $r \rightarrow \infty$

$$\Delta(a) = a^\tau \frac{r^\tau}{r} f(r) S_n(r) \quad (\text{D.5})$$

where  $S_n(r)$  is the surface area of the hypersphere with radius  $r$ . For the case of a four-dimensional Minkowski space-time, we have

$$\Delta(a) = a^\tau \int d^4r \partial_\tau f(r) = 2i\pi^2 a^\tau \lim_{r \rightarrow \infty} r^2 r_\tau f(r) \quad (\text{D.6})$$

## D.2 Ward Identities and Goldstones

In this section we give an intuitive demonstration of the WI used in Fig. 3.6. For a more rigorous and general demonstration we remand to [27].

An external  $Z_0$  is created by the Lagrangian term

$$\mathcal{L}_{Z_0}^{int} = J_{Z_0}^\mu Z_{0\mu} = -\frac{1}{2} g_{Z_0} \bar{\Psi}_e \gamma^\mu (v_f^{Z_0} - a_f^{Z_0} \gamma_5) \Psi_e Z_{0\mu} \quad (\text{D.7})$$

Therefore we expect the amplitude  $\Gamma_\mu$  (involving an external incoming  $Z_0$ ) to be given by the matrix element of  $J_{Z_0}^\mu$

$$\Gamma^\mu(Z_0) = \int d^4x e^{-ik \cdot x} \langle f | J_{Z_0}^\mu | i \rangle \quad (\text{D.8})$$

where  $f(i)$  represents the final (initial) state. From the equation of motion we know that

$$\begin{aligned} \partial_\mu J_{Z_0}^\mu &= \frac{1}{2} g_{Z_0} a_e^{Z_0} \partial_\mu (\bar{\Psi}_e \gamma^\mu \gamma_5 \Psi_e) \\ &= i g_{Z_0} a_e^{Z_0} m_e \bar{\Psi}_e \gamma_5 \Psi_e \end{aligned} \quad (\text{D.9})$$

since, as it is well known, the vectorial current is conserved while the axial one has the divergence proportional to the fermion mass. We can show that the result is proportional to the axial Goldstone current coming from the Yukawa coupling between the Higgs scalar and the electron. In the SM (the extension to MSSM is straightforward) we give mass to the electron with the Yukawa Lagrangian

$$\mathcal{L}_{y_e} = -y_e \bar{L}_e H e_R + h.c. \quad (\text{D.10})$$

where  $y_e$  is the Yukawa coupling (taken to be real),  $e_R$  is the right handed electron, and the leptonic and Higgs doublets are respectively

$$L_e = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} \quad H = \begin{pmatrix} iG^+ \\ \frac{(h^0 + v) + iG^0}{\sqrt{2}} \end{pmatrix} \quad (\text{D.11})$$

with  $e_L$  the left handed electron,  $\nu_e$  the corresponding neutrino,  $h_0$  the real scalar Higgs field,  $G^0$  the real Goldstone scalar of the  $Z_0$ ,  $G^+$  the complex Goldstone scalar of  $W^+$  and  $v$  the Higgs VEV which we take to be real. So we have

$$\begin{aligned}
 \mathcal{L}_{y_e} &= -\frac{y_e}{\sqrt{2}} [\bar{e}_L (h^0 + v + iG^0) e_R + \bar{e}_R (h^0 + v - iG^0) e_L] + \dots \\
 &= -\frac{y_e}{\sqrt{2}} [(h^0 + v) (\bar{e}_R e_L + \bar{e}_L e_R) + iG^0 (-\bar{e}_R e_L + \bar{e}_L e_R)] + \dots \\
 &= -\frac{y_e}{\sqrt{2}} [(h^0 + v) \bar{\Psi}_e \Psi_e + iG^0 \bar{\Psi}_e \gamma_5 \Psi_e] + \dots \\
 &= -\frac{y_e}{\sqrt{2}} v \bar{\Psi}_e \Psi_e - \frac{y_e}{\sqrt{2}} h^0 \bar{\Psi}_e \Psi_e + J_{G^0}^5 G^0 + \dots
 \end{aligned} \tag{D.12}$$

where we omitted the terms in  $G^+$ , since we are not interested on them. The Dirac spinor  $\Psi_e = \begin{pmatrix} e_L \\ e_R \end{pmatrix}$  and the Goldstone current is

$$J_{G^0}^5 = -i \frac{y_e}{\sqrt{2}} \bar{\Psi}_e \gamma_5 \Psi_e \tag{D.13}$$

We can identify the electron mass  $m_e = \frac{y_e}{\sqrt{2}} v$ . Inserting these results into eq. (D.9), we find

$$\begin{aligned}
 \partial_\mu J_{Z_0}^\mu &= i g_{Z_0} a_e^{Z_0} m_e \bar{\Psi}_e \gamma_5 \Psi_e = i M_{Z_0} \frac{y_e}{\sqrt{2}} \bar{\Psi}_e \gamma_5 \Psi_e \\
 &= -M_{Z_0} J_{G^0}^5
 \end{aligned} \tag{D.14}$$

where we used the definitions of  $g_{Z_0}$ ,  $a_e^{Z_0}$ ,  $M_{Z_0}$  given after eq. (3.63) and after eq. (3.67). Applying the result (D.14), we get

$$\begin{aligned}
 0 &= \int d^4x \partial_\mu (e^{ik \cdot x} \langle f | J_{Z_0}^\mu | i \rangle) \\
 &= ik_\mu \int d^4x (e^{ik \cdot x} \langle f | J_{Z_0}^\mu | i \rangle) + \int d^4x e^{ik \cdot x} \partial_\mu \langle f | J_{Z_0}^\mu | i \rangle \\
 &= ik_\mu \int d^4x (e^{ik \cdot x} \langle f | J_{Z_0}^\mu | i \rangle) - M_{Z_0} \int d^4x e^{ik \cdot x} \langle f | J_{G^0}^5 | i \rangle
 \end{aligned} \tag{D.15}$$

which implies the WI

$$-ik_\mu \Gamma^\mu(Z_0) + M_{Z_0} \Gamma(G^0) = 0 \tag{D.16}$$

where we used eq. (D.8) and

$$\Gamma(G^0) = \int d^4x e^{-ik \cdot x} \langle f | J_{G^0}^5 | i \rangle \tag{D.17}$$

is the amplitude for an external incoming  $G^0$  Goldstone boson. This result also includes the massless vector case which is recovered putting the vector mass equal to zero.

### D.3 Fermionic loop diagram

In this section we give some general properties of the massive fermionic triangle diagram of Fig. D.1. To make the discussion self-consistent we will repeat some equations seen in the previous chapters.

Consider a case in which only a single fermion circulates in the loop and each coupling is either axial (A) or vectorial (V) with charge equal to minus one. The fermionic triangles containing an odd number of axial couplings, denoted by AVV, VAV, VVA and AAA are

$$\Gamma_{\rho\mu\nu}^{AVV}(p, q; m_f) = \int \frac{d^4\ell}{(2\pi)^4} \text{Tr} \left( \gamma_5 \gamma_\rho \frac{1}{\not{\ell} - \not{q} - m_f} \gamma_\nu \frac{1}{\not{\ell} - m_f} \gamma_\mu \frac{1}{\not{\ell} + \not{p} - m_f} \right) + (p \leftrightarrow q, \mu \leftrightarrow \nu) \quad (\text{D.18})$$

$$\Gamma_{\rho\mu\nu}^{VAV}(p, q; m_f) = \int \frac{d^4\ell}{(2\pi)^4} \text{Tr} \left( \gamma_\rho \frac{1}{\not{\ell} - \not{q} - m_f} \gamma_\nu \frac{1}{\not{\ell} - m_f} \gamma_5 \gamma_\mu \frac{1}{\not{\ell} + \not{p} - m_f} \right) + (q \leftrightarrow -(p+q), \nu \leftrightarrow \rho) \quad (\text{D.19})$$

$$\Gamma_{\rho\mu\nu}^{VV A}(p, q; m_f) = \int \frac{d^4\ell}{(2\pi)^4} \text{Tr} \left( \gamma_\rho \frac{1}{\not{\ell} - \not{q} - m_f} \gamma_5 \gamma_\nu \frac{1}{\not{\ell} - m_f} \gamma_\mu \frac{1}{\not{\ell} + \not{p} - m_f} \right) + (p \leftrightarrow -(p+q), \mu \leftrightarrow \rho) \quad (\text{D.20})$$

$$\Gamma_{\rho\mu\nu}^{AAA}(p, q; m_f) = \int \frac{d^4\ell}{(2\pi)^4} \text{Tr} \left( \gamma_5 \gamma_\rho \frac{1}{\not{\ell} - \not{q} - m_f} \gamma_5 \gamma_\nu \frac{1}{\not{\ell} - m_f} \gamma_5 \gamma_\mu \frac{1}{\not{\ell} + \not{p} - m_f} \right) + (p \leftrightarrow q, \mu \leftrightarrow \nu) \quad (\text{D.21})$$

These integrals are superficially divergent (by power counting) and thus there is an ambiguity in their definition. The internal momentum  $\ell$  can, in fact, be arbitrarily shifted (see Section 6.2 of [21])

$$\ell_\sigma \rightarrow \ell_\sigma + \alpha p_\sigma + (\alpha - \beta) q_\sigma \quad (\text{D.22})$$

leading to

$$\Gamma_{\rho\mu\nu}^{AVV}(p, q, \beta; m_f) = \Gamma_{\rho\mu\nu}^{AVV}(p, q; m_f) - \frac{\beta}{8\pi^2} \epsilon_{\rho\mu\nu\sigma} (p - q)^\sigma \quad (\text{D.23})$$

The amplitudes (D.18), (D.19), (D.20) and (D.21) can be written using the Rosenberg parametrization [29] as

$$\begin{aligned} \Gamma_{\rho\mu\nu}(p, q; m_f) = & \\ & \frac{1}{\pi^2} \left( I_1(p, q; m_f) \epsilon[p, \mu, \nu, \rho] + I_2(p, q; m_f) \epsilon[q, \mu, \nu, \rho] + I_3(p, q; m_f) \epsilon[p, q, \mu, \rho] p_\nu \right. \\ & \left. + I_4(p, q; m_f) \epsilon[p, q, \mu, \rho] q_\nu + I_5(p, q; m_f) \epsilon[p, q, \nu, \rho] p_\mu + I_6(p, q; m_f) \epsilon[p, q, \nu, \rho] q_\mu \right) \end{aligned} \quad (\text{D.24})$$

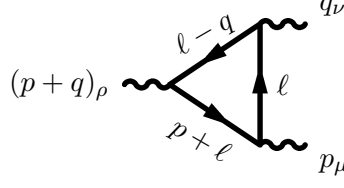


Figure D.1: The anomalous triangle diagram.

with  $\epsilon[p, q, \rho, \sigma] = \epsilon_{\mu\nu\rho\sigma} p^\mu q^\nu$  and where

$$\begin{aligned}
 I_3(p, q; m_f) &= - \int_0^1 dx \int_0^{1-x} dy \frac{xy}{y(1-y)p^2 + x(1-x)q^2 + 2xy p \cdot q - m_f^2} \\
 I_4(p, q; m_f) &= \int_0^1 dx \int_0^{1-x} dy \frac{x(x-1)}{y(1-y)p^2 + x(1-x)q^2 + 2xy p \cdot q - m_f^2} \\
 I_5(p, q; m_f) &= -I_4(q, p; m_f) \\
 I_6(p, q; m_f) &= -I_3(p, q; m_f)
 \end{aligned} \tag{D.25}$$

are finite integrals in the Feynman parameters  $x, y$  while  $I_1$  and  $I_2$  contain all the divergent contribution of the 1 loop integration. We give a demonstration of the Rosenberg parametrization for the AVV triangle. The extension to the other triangles is straightforward. Our starting point is eq. (D.18). Lorentz invariance and momentum conservation imply the following expansion

$$\begin{aligned}
 \Gamma_{\rho\mu\nu}(p, q; m_f) &= B_1(p, q; m_f) \epsilon[p, \mu, \nu, \rho] + B_2(p, q; m_f) \epsilon[q, \mu, \nu, \rho] + \\
 &B_3(p, q; m_f) \epsilon[p, q, \mu, \rho] p_\nu + B_4(p, q; m_f) \epsilon[p, q, \mu, \rho] q_\nu + \\
 &B_5(p, q; m_f) \epsilon[p, q, \nu, \rho] p_\mu + B_6(p, q; m_f) \epsilon[p, q, \nu, \rho] q_\mu + \\
 &B_7(p, q; m_f) \epsilon[p, q, \mu, \nu] p_\rho + B_8(p, q; m_f) \epsilon[p, q, \mu, \nu] q_\rho
 \end{aligned} \tag{D.26}$$

where the  $B$ 's are functions of the vector momenta  $p, q$  and of the fermion mass  $m_f$ . Applying the Schouten identity

$$\begin{aligned}
 k^\rho \epsilon[\alpha, \beta, \mu, \nu] + k^\alpha \epsilon[\beta, \mu, \nu, \rho] + k^\beta \epsilon[\mu, \nu, \rho, \alpha] + \\
 k^\mu \epsilon[\nu, \rho, \alpha, \beta] + k^\nu \epsilon[\rho, \alpha, \beta, \mu] = 0
 \end{aligned} \tag{D.27}$$

## More on amplitudes

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on  $p$  and  $q$ , we get

$$\begin{aligned}
\Gamma_{\rho\mu\nu}(p, q; m_f) = & [B_1 + p \cdot q B_7 + q^2 B_8] \epsilon[p, \mu, \nu, \rho] + \\
& [B_2 - p^2 B_7 - p \cdot q B_8] \epsilon[q, \mu, \nu, \rho] + \\
& [B_3 + B_7] \epsilon[p, q, \mu, \rho] p_\nu + [B_4 + B_8] \epsilon[p, q, \mu, \rho] q_\nu + \\
& [B_5 - B_7] \epsilon[p, q, \nu, \rho] p_\mu + [B_6 - B_8] \epsilon[p, q, \nu, \rho] q_\mu \quad (D.28)
\end{aligned}$$

If we identify

$$\begin{aligned}
I_1 &= \pi^2 [B_1 + p \cdot q B_7 + q^2 B_8] \\
I_2 &= \pi^2 [B_2 - p^2 B_7 - p \cdot q B_8] \\
I_3 &= \pi^2 [B_3 + B_7] \\
I_4 &= \pi^2 [B_4 + B_8] \\
I_5 &= \pi^2 [B_5 - B_7] \\
I_6 &= \pi^2 [B_6 - B_8] \quad (D.29)
\end{aligned}$$

we get the Rosenberg parametrization (D.24).

Now we want to compute the finite part of the parametrization i.e. integrals  $I_3 \dots I_6$ . Let us go back to eq. (D.18). It is easy to check that the exchanged contribution has the only the effect of doubling the amplitude. So we get

$$\Gamma_{\rho\mu\nu} = 2 \int \frac{d^4\ell}{(2\pi)^4} \frac{\text{Tr} [\gamma_5 \gamma_\rho (\ell - \not{q} + m_f) \gamma_\nu (\ell + m_f) \gamma_\mu (\ell + \not{p} + m_f)]}{[(\ell - q)^2 - m_f^2] [\ell^2 - m_f^2] [(\ell + p)^2 - m_f^2]} \quad (D.30)$$

Introducing the Feynman parameters  $x, y, z$  we obtain

$$\begin{aligned}
\Gamma_{\rho\mu\nu} &= \int \frac{d^4\ell}{(2\pi)^4} \int \frac{4N_{\rho\mu\nu} \delta(1 - x - y - z) dx dy dz}{\{[(\ell - q)^2 - m_f^2] x + [(\ell + p)^2 - m_f^2] y + [\ell^2 - m_f^2] z\}^3} \\
&= 4 \int \frac{d^4\ell}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} dy \frac{N_{\rho\mu\nu}}{[\ell^2 + 2y\ell \cdot p - 2x\ell \cdot q + yp^2 + q^2 - m_f^2]^3} \quad (D.31)
\end{aligned}$$

with

$$\begin{aligned}
N_{\rho\mu\nu} &= \text{Tr} [\gamma_5 \gamma_\rho (\ell - \not{q} + m_f) \gamma_\nu (\ell + m_f) \gamma_\mu (\ell + \not{p} + m_f)] \quad (D.32) \\
&= \text{Tr} [\gamma_5 \gamma_\rho (\ell - \not{q}) \gamma_\nu (\ell) \gamma_\mu (\ell + \not{p})] + m_f^2 \text{Tr} [\gamma_5 \gamma_\rho \gamma_\nu \gamma_\mu (\ell + \not{p} - \not{q})]
\end{aligned}$$

The numerator  $N_{\rho\mu\nu}$  has terms with three momenta and terms with one momenta. The latter contribute to  $I_1$  and  $I_2$  in eq. (D.24). At this point we are

not interested in an explicit computation of such integrals, so we leave them indicated as dots in the following computations. Using the change of variable<sup>1</sup>  $\ell = \tilde{\ell} + k$  with  $k = -yp + xq$  we get

$$\Gamma_{\rho\mu\nu} = 4 \int \frac{d^4\tilde{\ell}}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} dy \frac{\tilde{N}_{\rho\mu\nu}}{[\tilde{\ell}^2 - \Delta]^3} \quad (\text{D.33})$$

where

$$\Delta = -[y(1-y)p^2 + x(1-x)q^2 + 2xyp \cdot q - m_f^2] \quad (\text{D.34})$$

and

$$\begin{aligned} \tilde{N}_{\rho\mu\nu} &= \text{Tr} \left[ \gamma_5 \gamma_\rho (\tilde{\ell} + \not{k} - \not{q}) \gamma_\nu (\tilde{\ell} + \not{k}) \gamma_\mu (\tilde{\ell} + \not{k} + \not{p}) \right] + \dots \\ &= \text{Tr} [\gamma_5 \gamma_\rho (\not{k} - \not{q}) \gamma_\nu \not{k} \gamma_\mu (\not{k} + \not{p})] + \dots \\ &= -2[x(x-1)q_\nu - xyp_\nu] q^\alpha p^\beta \text{Tr} [\gamma_5 \gamma_\rho \gamma_\mu \gamma_\alpha \gamma_\beta] + \\ &\quad -2[xyq_\mu - y(y-1)q_\mu] q^\alpha p^\beta \text{Tr} [\gamma_5 \gamma_\rho \gamma_\nu \gamma_\alpha \gamma_\beta] + \dots \end{aligned} \quad (\text{D.35})$$

where we omitted terms with odd powers of  $\tilde{\ell}$  because their contribution is identically zero since we integrate over all the space a function which is odd in  $\tilde{\ell}$ . Then we find

$$\begin{aligned} \Gamma_{\rho\mu\nu} &= -8 \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4\tilde{\ell}}{(2\pi)^4} \frac{1}{[\tilde{\ell}^2 - \Delta]^3} \times \\ &\quad \times \left\{ [x(x-1)q_\nu - xyp_\nu] q^\alpha p^\beta \text{Tr} [\gamma_5 \gamma_\rho \gamma_\mu \gamma_\alpha \gamma_\beta] + \right. \\ &\quad \left. [xyq_\mu - y(y-1)q_\mu] q^\alpha p^\beta \text{Tr} [\gamma_5 \gamma_\rho \gamma_\nu \gamma_\alpha \gamma_\beta] \right\} + \dots \end{aligned} \quad (\text{D.36})$$

Having isolated the finite part, we can compute the integral over the momentum and take the trace over the Dirac matrices. Using the result

$$\int \frac{d^4\tilde{\ell}}{(2\pi)^4} \frac{1}{[\tilde{\ell}^2 - \Delta]^3} = -\frac{i}{2} \frac{1}{(4\pi)^2} \frac{1}{\Delta} \quad (\text{D.37})$$

we obtain

$$\begin{aligned} \Gamma_{\rho\mu\nu} &= \frac{1}{\pi^2} \int_0^1 dx \int_0^{1-x} dy \left\{ \frac{[x(x-1)q_\nu - xyp_\nu] \epsilon[p, q, \mu, \rho]}{y(1-y)p^2 + x(1-x)q^2 + 2xyp \cdot q - m_f^2} + \right. \\ &\quad \left. \frac{[xyq_\mu - y(y-1)q_\mu] \epsilon[p, q, \nu, \rho]}{y(1-y)p^2 + x(1-x)q^2 + 2xyp \cdot q - m_f^2} \right\} + \dots \end{aligned} \quad (\text{D.38})$$

---

<sup>1</sup>The change is allowed since we are interested only on the finite part of the integral.

Comparing with eq. (D.24) we identify

$$\begin{aligned}
I_3(p, q; m_f) &= - \int_0^1 dx \int_0^{1-x} dy \frac{xy}{y(1-y)p^2 + x(1-x)q^2 + 2xy p \cdot q - m_f^2} \\
I_4(p, q; m_f) &= \int_0^1 dx \int_0^{1-x} dy \frac{x(x-1)}{y(1-y)p^2 + x(1-x)q^2 + 2xy p \cdot q - m_f^2} \\
I_5(p, q; m_f) &= - \int_0^1 dx \int_0^{1-x} dy \frac{y(y-1)}{y(1-y)p^2 + x(1-x)q^2 + 2xy p \cdot q - m_f^2} \\
I_6(p, q; m_f) &= \int_0^1 dx \int_0^{1-x} dy \frac{xy}{y(1-y)p^2 + x(1-x)q^2 + 2xy p \cdot q - m_f^2} \quad (\text{D.39})
\end{aligned}$$

Is it easy to check that  $I_5(p, q; m_f) = -I_4(q, p; m_f)$  and  $I_6(p, q; m_f) = -I_3(p, q; m_f)$ , reproducing exactly the definitions in eq. (D.25).

In terms of the Rosenberg parametrization the  $\beta$  dependence of (D.23) is contained only in  $I_1$  and  $I_2$  (which are superficially divergent). However, using the WIs,

$$\begin{aligned}
(p+q)^\rho \Gamma_{\rho\mu\nu}^{AVV}(p, q, \beta; m_f) &= \frac{1}{\pi^2} \left[ \frac{\beta}{4} + m_f^2 I_0(p, q; m_f) \right] \epsilon[p, q, \mu, \nu] \\
p^\mu \Gamma_{\rho\mu\nu}^{AVV}(p, q, \beta; m_f) &= -\frac{2+\beta}{8\pi^2} \epsilon[q, p, \nu, \rho] \\
q^\nu \Gamma_{\rho\mu\nu}^{AVV}(p, q, \beta; m_f) &= -\frac{2+\beta}{8\pi^2} \epsilon[q, p, \rho, \mu] \quad (\text{D.40})
\end{aligned}$$

it is possible to show that they can be expressed in terms of  $I_3 \dots I_6$  as

$$\begin{aligned}
I_1^{AVV}(p, q, \beta; m_f) &= p \cdot q I_3(p, q) + q^2 I_4(p, q) + \frac{2+\beta}{8} \\
I_2^{AVV}(p, q, \beta; m_f) &= -I_1^{AVV}(q, p, \beta; m_f) \quad (\text{D.41})
\end{aligned}$$

where  $I_0$  is defined as

$$I_0(p, q; m_f) = - \int_0^1 dx \int_0^{1-x} dy \frac{1}{y(1-y)p^2 + x(1-x)q^2 + 2xy p \cdot q - m_f^2} \quad (\text{D.42})$$

The  $I_0$  contribution is only present in a massive fermionic triangle. We encounter such contribution in theories with chiral massive fermions such as SM, MSSM and MiAUMSSM. Finally we remark that the  $I_0$  contribution in eq. (D.40) is not  $\beta$  dependent so it is fixed on the axial vertex. The same will happen for the VAV, VVA and AAA cases.

## D.4 Landau-Yang theorem

The Landau-Yang theorem states that two massless spin-one objects cannot combine to form a spin-one object. From a phenomenological point of view this means that a  $Z'$  cannot decay into two on-shell photons. Now we give a partial demonstration of this fact using the Rosenberg parametrization (D.24). We put the  $Z'$  on the  $\rho$  leg and the photons on the  $\mu, \nu$  legs. The amplitude of the process is given by

$$A_{\rho\mu\nu}^{Z'\gamma\gamma} = -\frac{1}{2\pi^2}g_{Z'}e^2\left(\tilde{A}_1\epsilon[p, \mu, \nu, \rho] + \tilde{A}_2\epsilon[q, \mu, \nu, \rho] + A_3\epsilon[p, q, \mu, \rho]p_\nu + A_4\epsilon[p, q, \mu, \rho]q_\nu + A_5\epsilon[p, q, \nu, \rho]p_\mu + A_6\epsilon[p, q, \nu, \rho]q_\mu\right) \quad (\text{D.43})$$

where  $\tilde{A}_1$  and  $\tilde{A}_2$  considered the GCS absorption (see Section 3.5), and  $A_3$ ,  $A_4$ ,  $A_5$  and  $A_6$  give the finite contribution of the amplitude. The WIs on the photon lines are

$$p^\mu A_{\rho\mu\nu}^{Z'\gamma\gamma}(p, q; m_f) = 0 \quad (\text{D.44})$$

$$q^\nu A_{\rho\mu\nu}^{Z'\gamma\gamma}(p, q; m_f) = 0 \quad (\text{D.45})$$

which imply that

$$\tilde{A}_1 = p \cdot q A_3 + q^2 A_4 = p \cdot q A_3 \quad (\text{D.46})$$

$$\tilde{A}_2 = p \cdot q A_6 + p^2 A_5 = p \cdot q A_6 \quad (\text{D.47})$$

since the photons are massless. Using this result we get

$$\begin{aligned} \left|A^{Z'\gamma\gamma}\right|^2 &\propto \sum_{\lambda'} \epsilon_{(\lambda')}^{\rho_1} \epsilon_{(\lambda')}^{*\rho_2} \sum_{\lambda_\gamma} \epsilon_{(\lambda_\gamma)}^{\nu_1} \epsilon_{(\lambda_\gamma)}^{*\nu_2} \sum_{\lambda_\gamma} \epsilon_{(\lambda_\gamma)}^{\mu_1} \epsilon_{(\lambda_\gamma)}^{*\mu_2} A_{\rho_1\mu_1\nu_1}^{Z'\gamma\gamma} A_{\rho_2\mu_2\nu_2}^{*Z'\gamma\gamma} = \\ &\propto M_{Z'}^6 (A_3 + A_6)^2 \end{aligned} \quad (\text{D.48})$$

where we used the completeness relations

$$\sum_{\lambda'} \epsilon_{(\lambda')}^{\rho_1} \epsilon_{(\lambda')}^{*\rho_2} = -\eta^{\rho_1\rho_2} + \frac{k_{(\lambda')}^{\rho_1} k_{(\lambda')}^{\rho_2}}{M_{Z'}^2} \quad (\text{D.49})$$

$$\sum_{\lambda_\gamma} \epsilon_{(\lambda_\gamma)}^{\nu_1} \epsilon_{(\lambda_\gamma)}^{*\nu_2} \rightarrow -\eta^{\nu_1\nu_2} \quad (\text{D.50})$$

$$\sum_{\lambda_\gamma} \epsilon_{(\lambda_\gamma)}^{\mu_1} \epsilon_{(\lambda_\gamma)}^{*\mu_2} \rightarrow -\eta^{\mu_1\mu_2} \quad (\text{D.51})$$

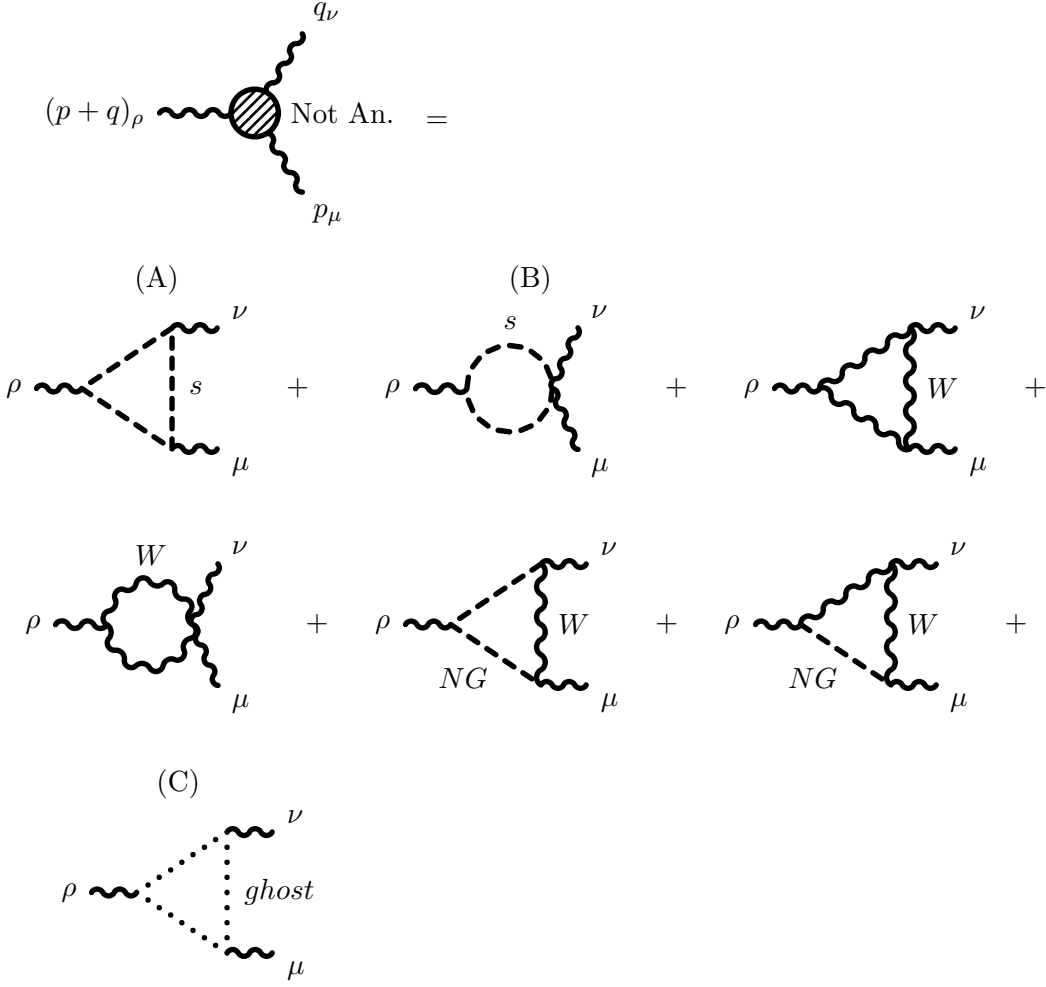


Figure D.2: Non Anomalous diagrams for trilinear neutral gauge boson amplitudes.

From eq. (D.25) we have  $A_6 = -A_3$ , so

$$\Gamma(Z' \rightarrow \gamma\gamma) \propto |A^{Z'\gamma\gamma}|^2 = 0 \quad (\text{D.52})$$

in agreement with the Landau-Yang theorem.

## D.5 Treatment of non anomalous diagrams

In this section we show that the non anomalous diagrams in Fig. 5.4 vanish. The diagrams we consider, reported in Fig. D.2, have no specific assignment for the external legs, to keep the discussion as general as possible. All the factors which are not relevant for our aim are omitted and all the possible leg exchanges are

understood. Finally, we use dimensional regularization and the  $R_\xi$  gauge with  $\xi = 1$ , in such a way that each diagram vanishes separately.

A) The Scalar triangle loop is given by

$$\begin{aligned}
 D_{\mu\nu\rho}^A(p, q) &= \int \frac{d^{2\omega}l}{(2\pi)^{2\omega}} \frac{(2l+p-q)_\rho(2l-q)_\nu(2l+p)_\mu}{[(l-q)^2-m^2][l^2-m^2][(l+p)^2-m^2]} + \\
 &\quad + (p \leftrightarrow q, \mu \leftrightarrow \nu) \\
 &= \int \frac{d^{2\omega}l}{(2\pi)^{2\omega}} \frac{(2l+p-q)_\rho(2l-q)_\nu(2l+p)_\mu}{[(l-q)^2-m^2][l^2-m^2][(l+p)^2-m^2]} \\
 &\quad + \int \frac{d^{2\omega}l}{(2\pi)^{2\omega}} \frac{(2l+q-p)_\rho(2l-p)_\mu(2l+q)_\nu}{[(l-p)^2-m^2][l^2-m^2][(l+q)^2-m^2]} \quad (D.53)
 \end{aligned}$$

Performing the change of variable  $l_\mu \rightarrow -l_\mu$  in the second integral, one gets

$$\begin{aligned}
 D_{\mu\nu\rho}^A(p, q) &= \int \frac{d^{2\omega}l}{(2\pi)^{2\omega}} \frac{(2l+p-q)_\rho(2l-q)_\nu(2l+p)_\mu}{[(l-q)^2-m^2][l^2-m^2][(l+p)^2-m^2]} \\
 &\quad - \int \frac{d^{2\omega}l}{(2\pi)^{2\omega}} \frac{(2l+p-q)_\rho(2l+p)_\mu(2l-q)_\nu}{[(l-q)^2-m^2][l^2-m^2][(l+p)^2-m^2]} = 0 \quad (D.54)
 \end{aligned}$$

B) The ‘‘Scalar bubble loop’’ is given by

$$\begin{aligned}
 D_{\mu\nu\rho}^B(p, q) &= -2 \int \frac{d^{2\omega}l}{(2\pi)^{2\omega}} \frac{(2l+p+q)_\rho \eta_{\mu\nu}}{[l^2-m^2][(l+p+q)^2-m^2]} \\
 &= -2 \int \frac{d^{2\omega}l}{(2\pi)^{2\omega}} \frac{(l+p+q)_\rho \eta_{\mu\nu}}{[l^2-m^2][(l+p+q)^2-m^2]} \\
 &\quad - 2 \int \frac{d^{2\omega}l}{(2\pi)^{2\omega}} \frac{(l)_\rho \eta_{\mu\nu}}{[l^2-m^2][(l+p+q)^2-m^2]} \quad (D.55)
 \end{aligned}$$

Performing the change of variable  $l \rightarrow -l-p-q$  in the second integral one gets

$$\begin{aligned}
 D_{\mu\nu\rho}^B(p, q) &= -2 \int \frac{d^{2\omega}l}{(2\pi)^{2\omega}} \frac{(l+p+q)_\rho \eta_{\mu\nu}}{[l^2-m^2][(l+p+q)^2-m^2]} \\
 &\quad + 2 \int \frac{d^{2\omega}l}{(2\pi)^{2\omega}} \frac{(l+p+q)_\rho \eta_{\mu\nu}}{[(l+p+q)^2-m^2][l^2-m^2]} = 0 \quad (D.56)
 \end{aligned}$$

C) Since the ghost interact with neutral vectors only through the third component of  $SU(2)$ , the Ghost triangle loop is proportional to

$$\epsilon_{3bc}\epsilon_{3cd}\epsilon_{3db} = -\delta_{bd}\epsilon_{3db} = 0 \quad (D.57)$$

The other diagrams in Fig. D.2 can also be shown to vanish after manipulations similar to the ones used in (D.54), (D.56), (D.57).

## D.6 Decay rates. General case

In this section we compute the amplitudes for the decays  $Z' \rightarrow Z_0 \gamma$  and  $Z' \rightarrow Z_0 Z_0$  in the general case  $Q_{H_u} \neq 0$ , still neglecting the effects coming from the kinetic mixing. We work in the limit

$$g_a v_{u,d} \ll \mu, M_0, M_1, M_2, M_S, M_{V^{(0)}} \quad (\text{D.58})$$

in which  $m_{SC} \approx M_{V^{(0)}}$ ,  $m_{SB} \approx 0$  (see (4.57), (4.22), (4.35)). Hence, (4.56) takes the same form as in the symmetric phase in which neutralinos and charginos do not contribute to the anomaly (see Section 4.1.1). In the limit (D.58) an extension of the SM by an extra  $U(1)$  and our SUSY model give the same results for what the decays of interest are concerned.

The SM fermion interaction terms with the neutral gauge bosons are given in eq. (5.11) where now

$$\begin{aligned} v_f^{Z'} &= Q_{f_L}^{Z'} + Q_{f_R}^{Z'} & a_f^{Z'} &= Q_{f_L}^{Z'} - Q_{f_R}^{Z'} \\ v_f^{Z_0} &= Q_{f_L}^{Z_0} + Q_{f_R}^{Z_0} & a_f^{Z_0} &= Q_{f_L}^{Z_0} - Q_{f_R}^{Z_0} \\ q_f &= Q_{f_L} = Q_{f_R} \end{aligned} \quad (\text{D.59})$$

The left and right fermions and the Dirac spinors are related using the usual definition  $\Psi_f = \begin{pmatrix} f_L \\ f_R \end{pmatrix}$ . The left and right charges are defined in the following way

$$g_{Z'} Q_{f_L}^{Z'} = g_2 T_3 O_{02} + g_1 Y_{f_L} O_{01} + g_0 Q_{f_L} \quad (\text{D.60})$$

$$g_{Z'} Q_{f_R}^{Z'} = g_1 Y_{f_R} O_{01} + g_0 Q_{f_R} \quad (\text{D.61})$$

$$g_{Z_0} Q_{f_L}^{Z_0} = g_2 T_3 O_{12} + g_1 Y_{f_L} O_{11} + g_0 Q_{f_L} O_{10} \quad (\text{D.62})$$

$$g_{Z_0} Q_{f_R}^{Z_0} = g_1 Y_{f_R} O_{11} + g_0 Q_{f_R} O_{10} \quad (\text{D.63})$$

$$e Q_{f_L} = g_2 T_3 O_{22} + g_1 Y_{f_L} O_{21} = g_1 Y_{f_R} O_{21} = e Q_{f_R} \quad (\text{D.64})$$

where  $O_{ij}$  is given in (4.52) and  $T_3$  is the eigenvalue of  $T_3^{(2)}$ .

D.6.1  $Z' \rightarrow Z_0 \gamma$ 

The amplitude is given by the sum of the fermionic triangle  $\Delta_{\rho\mu\nu}^{Z'Z_0\gamma}$  plus the proper GCS vertex

$$\begin{aligned} A_{\rho\mu\nu}^{Z'Z_0\gamma} &= \Delta_{\rho\mu\nu}^{Z'Z_0\gamma} + (\text{GCS})_{\rho\mu\nu}^{Z'Z_0\gamma} \\ \Delta_{\rho\mu\nu}^{Z'Z_0\gamma} &= -\frac{1}{4}g_{Z'}g_{Z_0}e \sum_f \left( v_f^{Z'} a_f^{Z_0} q_f \Gamma_{\rho\mu\nu}^{VAV} + a_f^{Z'} v_f^{Z_0} q_f \Gamma_{\rho\mu\nu}^{AVV} \right) \end{aligned} \quad (\text{D.65})$$

The resulting amplitude can be written as

$$\begin{aligned} A_{\rho\mu\nu}^{Z'Z_0\gamma} &= -\frac{1}{4\pi^2}g_{Z'}g_{Z_0}e \left( \tilde{A}_1 \epsilon[p, \mu, \nu, \rho] + \tilde{A}_2 \epsilon[q, \mu, \nu, \rho] + A_3 \epsilon[p, q, \mu, \rho] p^\nu \right. \\ &\quad \left. + A_4 \epsilon[p, q, \mu, \rho] q^\nu + A_5 \epsilon[p, q, \nu, \rho] p^\mu + A_6 \epsilon[p, q, \nu, \rho] q^\mu \right) \end{aligned} \quad (\text{D.66})$$

with

$$A_i = \sum_f t_f^{Z'Z_0\gamma} I_i \quad \text{for } i = 3, \dots, 6 \quad (\text{D.67})$$

where

$$t_f^{Z'Z_0\gamma} = \left( v_f^{Z'} a_f^{Z_0} + a_f^{Z'} v_f^{Z_0} \right) q_f \quad (\text{D.68})$$

and the integrals  $I_i$  given in (D.25).  $\tilde{A}_1$  and  $\tilde{A}_2$  are the new coefficients with the GCS absorbed similarly to (3.109).

The Ward identities in Fig. 3.5 for the amplitude now read

$$(p+q)^\rho A_{\rho\mu\nu}^{Z'Z_0\gamma} + iM_{Z'} \left[ (\text{GS})_{\mu\nu}^{Z_0\gamma} + (\text{NG})_{\mu\nu}^{Z_0\gamma} \right] = 0 \quad (\text{D.69})$$

$$p^\mu A_{\rho\mu\nu}^{Z'Z_0\gamma} + iM_{Z_0} \left[ (\text{GS})_{\rho\nu}^{Z'\gamma} + (\text{NG})_{\rho\nu}^{Z'\gamma} \right] = 0 \quad (\text{D.70})$$

$$q^\nu A_{\rho\mu\nu}^{Z'Z_0\gamma} = 0 \quad (\text{D.71})$$

where  $M_{Z'}$  and  $M_{Z_0}$  are the  $Z'$  and  $Z_0$  masses respectively. In both (D.69) and (D.70) we have a (GS) and a (NG) contribution due to the two Goldstone bosons which are a linear combination of the axion and  $G^0$ . We use (D.70) and (D.71) to fix  $\tilde{A}_1$  and  $\tilde{A}_2$  while (D.69) is automatically satisfied. Contracting with  $p^\mu$  we get

$$\begin{aligned} p^\mu A_{\rho\mu\nu}^{Z'Z_0\gamma} &= - \left\{ 8 \left[ 4g_0g_1^2 R_{101}^{Z'Z_0\gamma} b_2^{(1)} b_3 + 2g_0g_2^2 R_{202}^{Z'Z_0\gamma} b_2^{(2)} b_3 + 2g_0^2g_1 R_{001}^{Z'Z_0\gamma} b_2^{(4)} b_3 \right] \right. \\ &\quad \left. + \frac{1}{4\pi^2}g_{Z'}g_{Z_0}e \sum_f v_f^{Z'} a_f^{Z_0} q_f m_f^2 I_0 \right\} \epsilon[q, p, \nu, \rho] \end{aligned} \quad (\text{D.72})$$

## More on amplitudes

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where  $I_0$  is the integral given in (D.42). The solution for  $\tilde{A}_1$  and  $\tilde{A}_2$  is

$$\tilde{A}_1 = (q^2 A_4 + p \cdot q A_3) \quad (\text{D.73})$$

$$\tilde{A}_2 = (p^2 A_5 + p \cdot q A_6) + (\text{GS})^{Z'\gamma} + (\text{NG})^{Z'\gamma} \quad (\text{D.74})$$

with

$$(\text{NG})^{Z'\gamma} = \sum_f v_f^{Z'} a_f^{Z_0} q_f m_f^2 I_0 \quad (\text{D.75})$$

$$(\text{GS})^{Z'\gamma} = \frac{64\pi^2 g_0}{g_{Z'} g_{Z_0} e} \left[ 2g_1^2 R_{101}^{Z'Z_0\gamma} b_2^{(1)} b_3 + g_2^2 R_{202}^{Z'Z_0\gamma} b_2^{(2)} b_3 + g_0 g_1 R_{001}^{Z'Z_0\gamma} b_2^{(4)} b_3 \right] \quad (\text{D.76})$$

The rotation factors are

$$\begin{aligned} R_{101}^{Z'Z_0\gamma} &= O_{01} O_{10} O_{21} \\ R_{202}^{Z'Z_0\gamma} &= O_{02} O_{10} O_{22} \\ R_{001}^{Z'Z_0\gamma} &= O_{10} O_{21} \end{aligned} \quad (\text{D.77})$$

with  $O_{ij}$  given by (4.52). Substituting  $\tilde{A}_1$ ,  $\tilde{A}_2$  into the amplitude (D.66) and performing all the contractions we finally obtain

$$\begin{aligned} |A^{Z'Z_0\gamma}|^2 &= g_{Z'}^2 g_{Z_0}^2 e^2 \frac{(M_{Z'}^2 - M_{Z_0}^2)^2 (M_{Z'}^2 + M_{Z_0}^2)}{96 M_{Z_0}^2 M_{Z'}^2 \pi^4} \times \\ &\quad \left[ \sum_f t_f^{Z'Z_0\gamma} (I_3 + I_5) M_{Z_0}^2 + (\text{GS})^{Z'\gamma} + (\text{NG})^{Z'\gamma} \right]^2 \end{aligned} \quad (\text{D.78})$$

### D.6.2 $Z' \rightarrow Z_0 Z_0$

The contribution to the fermionic triangle is

$$\begin{aligned} \Delta_{\rho\mu\nu}^{Z'Z_0Z_0} &= -\frac{1}{8} g_{Z'} g_{Z_0}^2 \left[ \sum_f \left( v_f^{Z'} a_f^{Z_0} v_f^{Z_0} \Gamma_{\rho\mu\nu}^{VAV} + v_f^{Z'} v_f^{Z_0} a_f^{Z_0} \Gamma_{\rho\mu\nu}^{VVA} + \right. \right. \\ &\quad \left. \left. a_f^{Z'} v_f^{Z_0} v_f^{Z_0} \Gamma_{\rho\mu\nu}^{AVV} + a_f^{Z'} a_f^{Z_0} a_f^{Z_0} \Gamma_{\rho\mu\nu}^{AAA} \right) \right] \end{aligned} \quad (\text{D.79})$$

We write the total amplitude (the sum of the triangles plus GCS terms) as

$$\begin{aligned} A_{\rho\mu\nu}^{Z'Z_0Z_0} &= -\frac{1}{8\pi^2} g_{Z'} g_{Z_0}^2 \left[ \tilde{A}_1 \epsilon[p, \mu, \nu, \rho] + \tilde{A}_2 \epsilon[q, \mu, \nu, \rho] + A_3 \epsilon[p, q, \mu, \rho] p_\nu \right. \\ &\quad \left. + A_4 \epsilon[p, q, \mu, \rho] q_\nu + A_5 \epsilon[p, q, \nu, \rho] p_\mu + A_6 \epsilon[p, q, \nu, \rho] q_\mu \right] \end{aligned} \quad (\text{D.80})$$

with

$$A_i = \sum_f t_f^{Z'Z_0Z_0} I_i \quad \text{for } i = 3, \dots, 6 \quad (\text{D.81})$$

where

$$t_f^{Z'Z_0Z_0} = \left( a_f^{Z'} v_f^{Z_0} v_f^{Z_0} + 2v_f^{Z'} a_f^{Z_0} v_f^{Z_0} + a_f^{Z'} a_f^{Z_0} a_f^{Z_0} \right) \quad (\text{D.82})$$

and the integrals  $I_i$  are given in (D.25). The Ward identities now read

$$(p+q)^\rho A_{\rho\mu\nu}^{Z'Z_0Z_0} + iM_{Z'} \left[ (\text{GS})_{\mu\nu}^{Z_0Z_0} + (\text{NG})_{\mu\nu}^{Z_0Z_0} \right] = 0 \quad (\text{D.83})$$

$$p^\mu A_{\rho\mu\nu}^{Z'Z_0Z_0} + iM_{Z_0} \left[ (\text{GS})_{\rho\nu}^{Z'Z_0} + (\text{NG})_{\rho\nu}^{Z'Z_0} \right] = 0 \quad (\text{D.84})$$

$$q^\nu A_{\rho\mu\nu}^{Z'Z_0Z_0} + iM_{Z_0} \left[ (\text{GS})_{\mu\rho}^{Z_0Z'} + (\text{NG})_{\mu\rho}^{Z_0Z'} \right] = 0 \quad (\text{D.85})$$

where  $M_{Z'}$  and  $M_{Z_0}$  are the  $Z'$  and  $Z_0$  masses respectively. In (D.83)-(D.85) the (GS) and (NG) terms are present for the same reason as in the preceding subsection. We use (D.84) and (D.85) to fix  $\tilde{A}_1$  and  $\tilde{A}_2$  while (D.83) is automatically satisfied. Contracting with  $p^\mu$  and  $q^\nu$  we get

$$\begin{aligned} p^\mu A_{\rho\mu\nu}^{Z'Z_0Z_0} = & - \left\{ 8 \left[ 4g_0^3 R_{000}^{Z'Z_0Z_0} b_2^{(0)} b_3 + 4g_0g_1^2 R_{101}^{Z'Z_0Z_0} b_2^{(1)} b_3 + \right. \right. \\ & \left. \left. 2g_0g_2^2 R_{202}^{Z'Z_0Z_0} b_2^{(2)} b_3 + 2g_0^2g_1 R_{001}^{Z'Z_0Z_0} b_2^{(4)} b_3 \right] + \right. \\ & \left. + \frac{1}{8\pi^2} g_{Z'} g_{Z_0}^2 \sum_f \left( v_f^{Z'} a_f^{Z_0} v_f^{Z_0} + \frac{1}{3} a_f^{Z'} a_f^{Z_0} a_f^{Z_0} \right) m_f^2 I_0 \right\} \epsilon[q, p, \nu, \rho] \end{aligned} \quad (\text{D.86})$$

$$\begin{aligned} q^\nu A_{\rho\mu\nu}^{Z'Z_0Z_0} = & - \left\{ 8 \left[ 4g_0^3 R_{000}^{Z'Z_0Z_0} b_2^{(0)} b_3 + 4g_0g_1^2 R_{101}^{Z'Z_0Z_0} b_2^{(1)} b_3 + \right. \right. \\ & \left. \left. 2g_0g_2^2 R_{202}^{Z'Z_0Z_0} b_2^{(2)} b_3 + 2g_0^2g_1 R_{001}^{Z'Z_0Z_0} b_2^{(4)} b_3 \right] + \right. \\ & \left. + \frac{1}{8\pi^2} g_{Z'} g_{Z_0}^2 \sum_f \left( v_f^{Z'} a_f^{Z_0} v_f^{Z_0} + \frac{1}{3} a_f^{Z'} a_f^{Z_0} a_f^{Z_0} \right) m_f^2 I_0 \right\} \epsilon[q, p, \rho, \mu] \end{aligned} \quad (\text{D.87})$$

where  $I_0$  is the integral given in (D.42). The solution for  $\tilde{A}_1$  and  $\tilde{A}_2$  is

$$\tilde{A}_1 = (q^2 A_4 + p \cdot q A_3) - \left[ (\text{GS})^{Z'Z_0} + (\text{NG})^{Z'Z_0} \right] \quad (\text{D.88})$$

$$\tilde{A}_2 = (p^2 A_5 + p \cdot q A_6) + (\text{GS})^{Z'Z_0} + (\text{NG})^{Z'Z_0} \quad (\text{D.89})$$

with

$$(\text{NG})^{Z'Z_0} = \sum_f \left( v_f^{Z'} a_f^{Z_0} v_f^{Z_0} + \frac{1}{3} a_f^{Z'} a_f^{Z_0} a_f^{Z_0} \right) m_f^2 I_0 \quad (\text{D.90})$$

$$(\text{GS})^{Z'Z_0} = \frac{64\pi^2}{g_{Z'} g_{Z_0}^2} \left[ 4g_0^3 R_{000}^{Z'Z_0Z_0} b_2^{(0)} b_3 + 4g_0 g_1^2 R_{101}^{Z'Z_0Z_0} b_2^{(1)} b_3 + \right. \\ \left. + 2g_0 g_2^2 R_{202}^{Z'Z_0Z_0} b_2^{(2)} b_3 + 2g_0^2 g_1 R_{001}^{Z'Z_0Z_0} b_2^{(4)} b_3 \right] \quad (\text{D.91})$$

The rotation factors are

$$\begin{aligned} R_{000}^{Z'Z_0Z_0} &= O_{10} O_{10} \\ R_{101}^{Z'Z_0Z_0} &= O_{01} O_{10} O_{11} \\ R_{202}^{Z'Z_0Z_0} &= O_{02} O_{10} O_{12} \\ R_{001}^{Z'Z_0Z_0} &= O_{10} O_{11} + O_{01} O_{10} O_{10} \end{aligned} \quad (\text{D.92})$$

with  $O_{ij}$  given by (4.52). Substituting back into the amplitude and performing all the contractions we finally obtain

$$|A^{Z'Z_0Z_0}|^2 = g_{Z'}^2 g_{Z_0}^4 \frac{(M_{Z'}^2 - 4M_{Z_0}^2)^2}{192M_{Z_0}^2 \pi^4} \times \\ \left[ \sum_f t_f^{Z'Z_0Z_0} (I_3 + I_5) M_{Z_0}^2 + (\text{GS})^{Z'Z_0} + (\text{NG})^{Z'Z_0} \right]^2 \quad (\text{D.93})$$

## D.7 Amplitude for $\psi_S \lambda^{(1)} \rightarrow \nu\nu$

In this appendix we give the amplitude for the process  $\psi_S \lambda^{(1)} \rightarrow \nu\nu$ , which is the simplest case over all the possible decays  $\psi_S \lambda^{(1)} \rightarrow f\bar{f}$ , since the neutrinos are massless and are coupled only to the  $Z_0$

$$\mathcal{M} = -\frac{i}{2} C_{Z_0} g_{Z_0} \frac{k_\mu}{k^2 - M_{Z_0}^2} \bar{v}_S \gamma_5 [\gamma^\mu, \gamma^\nu] u_{\lambda_1} \bar{u}_{\nu_1} \gamma_\nu (v_\nu - a_\nu \gamma_5) v_{\nu_2} \quad (\text{D.94})$$

The corresponding square modulus is

$$\begin{aligned}
 |\mathcal{M}|^2 &= \sum_{\text{spin } \lambda_1} \sum_{\text{spin } \psi_S} \sum_{\text{spin } \nu_1} \sum_{\text{spin } \nu_2} \mathcal{M} \mathcal{M}^* & (D.95) \\
 &= 64 C_{Z_0}^2 g_{Z_0}^2 \frac{a_\nu^2 + v_\nu^2}{(k^2 - M_{Z_0}^2)^2} (p_{\nu_1} \cdot p_{\nu_2}) \times \\
 &\quad \times \left[ (p_{\lambda_1} \cdot p_{\nu_1})(p_{\nu_1} \cdot p_S) + (p_{\lambda_1} \cdot p_{\nu_2})(p_{\nu_2} \cdot p_S) - (p_{\nu_1} \cdot p_{\nu_2}) M_1 M_S \right]
 \end{aligned}$$

where  $p_{\lambda_1}$ ,  $p_S$ ,  $p_{\nu_1}$  and  $p_{\nu_2}$  are the bino, axino and neutrinos 4-momenta respectively and  $k^2 = s$  is the momentum of the intermediate  $Z_0$ . The result (D.95) is valid only for one family. If we consider all the families the above amplitude must be multiplied by 3.



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