UNIVERSITÀ DEGLI STUDI DI ROMA "TOR VERGATA"

FACOLTÀ DI SCIENZE MATEMATICHE FISICHE E NATURALI

DOTTORATO DI RICERCA IN FISICA

DOTTORATO **XX** CICLO

New experimental limit on the Pauli Exclusion Principle violation by electrons from theVIP experiment

LAURA SPERANDIO

Relatore: **Dott. ssa Catalina Curceanu (Petrascu)**

Coordinatore: **Prof. Piergiorgio Picozza**

Contents

Introduction

The Pauli Exclusion Principle (PEP) represents one of the fundamental principles of the modern physics and is at the very basis of our understanding of matter: thus it is of foremost importance to test the limits of its validity.

The PEP is a manifestation of the Spin-Statistics relation: the half-integer spin particles (fermions) follow Fermi statistics, while the integer-spin particles (bosons) the Bose –Einstein one, and is intimately related with the notion of "identity" of particles. In its original form the PEP was an adjunct to Bohr's old quantum theory, which stated that an electron in an atom could only move to one of a discrete set of orbits; on the basis of experimental findings (X-ray atomic spectra for example) Pauli added the postulate that only one electron in the atom can fill each of the allowed orbits (including the spin in the orbit's definition). When Bohr's theory gave way to modern quantum mechanics, in which the traditional planet-like orbits of electrons in atoms are replaced by the more abstract notion of the electron states, the PEP survived as the statement that no two electrons could be in the same state (i.e. they cannot have the same quantum numbers). More generally, there is a strong connection between spin and symmetry class, as Pauli stressed in his Nobel Lecture "*...we want to stress here a law of Nature which is generally valid, namely, the connection between spin and symmetry class. A half-integer value of the spin quantum number is always connected with antisymmetrical states (exclusion principle), an integer spin with symmetrical states*" [1].

In the same Nobel lecture however, Pauli himself says *"Already in my original paper I stressed the circumstance that I was unable to give a logical reason for the exclusion principle or to deduce it from more general assumption. I had the feeling and I still have it today, that this is a deficiency. The impression that the shadow of some incompleteness fell here on the bright light of success of the new quantum mechanics seems to me unavoidable".* This statement, more than 60 years after Pauli formulated it, can be repeated practically unchanged – not much progress has been achieved in the deep understanding of the physical origin of the spin-statistics relation. As a consequence,

even if today there are no compelling reasons to doubt the validity of the Pauli Exclusion Principle, it still spurs a lively debate on its limits, as testified by the abundant contributions found in the literature and in topical conferences.

Here I describe a method to observe possible small violations of the PEP for electrons, through the search for anomalous X-ray transitions in copper atoms, produced by new electrons in a copper block (introduced by a circulating current), which can be captured in a Pauli-forbidden transition to the 1S level, already occupied by two electrons. In 1990, an upper limit on the PEP violation was found using this method by Ramberg and Snow (RS) with a dedicated experiment [2]. The search method is implemented in the VIP (Violation of the Pauli Exclusion Principle) experiment, an international collaboration among 6 Institutions from 4 countries, that has the scientific goal to improve by three-four orders of magnitude the RS's limit on the probability of PEP violation for electrons, bringing it into the 10^{-29}–10^{-30} region, which may be of particular interest for all those theories related to possible PEP violations, and that would come from new physics. Reaching such limits opens up some very interesting scenarios, as stressed by Duck and Sudarshan:

"… recently… membrane theorists have been speculating on a large compactification radius for one of their eleven dimensions, which could give a ratio (for PEP violation) of 10^{-30}*"* (quoted from ref. [3]).

This thesis is structured as follows: in Chapter 1 I present the origin of the spin-statistics relation, with a brief historical excursus, followed by a brief outline of some of the proofs found in the literature. Chapter 2 is an introduction to the theories of small violations of the Pauli Exclusion Principle. The experimental method used in VIP is discussed in detail in Chapter 3, which includes a short presentation of the Ramberg and Snow experimental results; Chapter 4 presents the feasibility study of VIP, while Chapter 5 describes the VIP apparatus, and includes a description of the VIP X-ray detector: the Charge-Coupled Device (CCD). The results of a first measurement, performed at the Laboratori Nazionali di Frascati dell'INFN in December 2005, are the subject of Chapter 6. Since Spring 2006 VIP is taking data at the Gran Sasso underground laboratories; the data collected in the first year were analyzed and the preliminary results are presented in Chapter 7. The work ends with the Conclusions.

VIP has performed the most precise measurement on the validity of PEP for electrons, establishing the limit on the probability of its violation at 5.7×10^{-29}. This result, presented here for the first time, improves on RS measurement by almost three orders of magnitude, and represents the reference value for all those theories that deal with possible violations of spin-statistics relation.

Chapter 1

The Spin-Statistics Theorem, Symmetrization and the Pauli Exclusion Principle

1.1 Historical introduction

Pauli established his principle before the establishment of the "new" Quantum Mechanics (QM) in the years 1925–1927, while trying to explain the regularities in the classification of atomic spectral terms in a strong magnetic field. During his initial study of the atomic shell structure Pauli was influenced first by Sommerfeld and then by Bohr. In 1922 he made a serious effort to explain the anomalous Zeeman effect and in 1924 he published some arguments about a new quantum theoretic property of the electron, which he called a "*two-valuedness not describable classically*". In 1925, Pauli formulated his principle as follows [1]: "In an atom there cannot be two or more equivalent electrons for which the values of all four quantum numbers coincide. If an electron exists in an atom for which all of these numbers have definite values, then this state is '*occupied.*' " The fundamental idea can be stated in the following way: "the complicated numbers of electrons in closed subgroups are reduced to the simple number one if the division of the groups by giving the values of the four quantum numbers of an electron is carried so far that every degeneracy is removed. An entirely non-degenerate energy level is already « closed », if it is occupied by a single electron; states in contradiction with this postulate have to be excluded" [1]. Since that time, the exclusion principle has been closely connected with the idea of spin, which cannot be measured by classically describable

experiments but must therefore be considered as an essentially quantum-mechanical property of the electron. The subsequent developments were determined by the occurrence of the new QM. In 1925, the same year in which Pauli published his paper on the exclusion principle, De Broglie formulated his idea of matter-waves and Heisenberg the new matrix-mechanics, after which, in the next year, Schrödinger's wave mechanics quickly followed.

After the first studies devoted to applying the new QM to many-particle systems by Heisenberg and Dirac, the Pauli principle, formulated as the preclusion for two electrons to occupy the same quantum state, was derived as a consequence of the antisymmetry of the wave function of the system of electrons. Thus, with the creation of new QM, the preclusion on the occupation numbers of electron system states was supplemented by the prohibition of all types of permutation symmetry of electron wave functions except for antisymmetrical ones. Later, analysis of experimental data allowed the formulation of the Pauli exclusion principle for all known elementary particles in this way:

> *The only possible states of a system of identical particles possessing spin s are those for which the total wave function transforms upon interchange of any two particles as*
>
> $$P_{ij}\Psi(1,...,i,...,j,...,N) = (-1)^{2s}\Psi(1,...,i,...,j,...,N)$$
>
> *that is, it is symmetric for integer values of s and antisymmetric for half-integer s.*

As it was first shown by Heisenberg, as a consequence of the impossibility to distinguish one of several identical particles, the wave functions describing an ensemble of a given number of identical particles in the configuration space are sharply separated into different classes of symmetry which can never be transformed into each other by external perturbations. Three different hypotheses turned out to be logically possible concerning the actual ensemble of several identical particles in Nature:

I. The ensemble is a mixture of all symmetry classes.

II. Only the symmetrical class occurs.

III. Only the antisymmetrical class occurs.

As everybody knows, the first assumption is never realized in nature. Moreover, it is only the third assumption that is in accordance with the exclusion principle, since an

antisymmetrical function containing two particles in the same state is identically zero. The assumption III can therefore be considered as the correct and general wave mechanical formulation of the exclusion principle.

1.2 Symmetrization postulate in non-relativistic QM

The symmetrization postulate (SP) can be expressed in this way: states containing several identical elementary particles are, according to the species, either symmetric (bosons) or antisymmetric (fermions). This postulate has very important experimental consequences, which can be expressed as a selection rule (SP selection rule): *states which cannot be represented by wave functions of the allowed symmetry type are absolutely forbidden.* This is an extremely strong condition, very much stronger than what is implied by the indistinguishability of identical particles [4].

Historically, the SP has played a very important role in the understanding of atomic phenomena, mainly as a consistent way of inserting the PEP in the formalism of QM. Following the QM approach without resorting to field theory (FT), it is possible to formulate precisely the requirement of indistinguishability of identical particles, in terms of invariance of physical properties of states in the permutation operations, for system which conserve the number of particles and then in the general case. In QM it is possible that particles not obeying SP don't violate any basic principle, pending some modification to the current formulation of QM, for example to replace the usual ray by a many-dimensional "generalized ray" as the representative of the physical state[1]. Thus, the requirement that identical particles be indistinguishable does not imply SP.

For interactions conserving the number of particles there is a superselection rule, which absolutely forbids transitions between states transforming under inequivalent representations of the permutation group. It follows that the SP can be inserted into the

[1] *State vectors in the usual Hilbert space of QM are determined up to an arbitrary phase factor: the set of all these physically equivalent state vectors is called "ray". Similarly, a state of many identical particles can only be determined up to an arbitrary permutation of the particle labels, and the set of all physically equivalent many-particle state vectors is called "generalized ray".*

formalism although it is not implied by the axioms of the theory. But there is a much stronger result: under quite broad assumptions the SP selection rule is verified in all states produced from the initial states which are at present available experimentally.

The first important point regards the indistinguishability of identical particles, for a system with an arbitrary number of particles. Without the assumption of the SP this concept can be expressed by the following: *dynamical states represented by vectors which differ only by a permutation of identical particles cannot be distinguished by any observation at any instant of time*. This requirement has consequences both on the properties of physical observables and on the time evolution of states. We consider now the case of systems with a fixed number *N of* particles of the same species, in the associated Hilbert space $\mathcal{E}^{(N)}$. In ordinary QM the result of a measurement is expressed as the expectation value of a suitably defined Hermitian operator A. Then the fact that dynamical states represented by $|u\rangle$ and by the permuted vector $P|u\rangle$ cannot be distinguished in a measurement is expressed by the equality:

$$\langle u|A|u\rangle = \langle u|P^{-1}A\,P|u\rangle \tag{1.1}$$

for any $|u\rangle$. For two superpositions $|u\rangle + \alpha|v\rangle$ and $|u\rangle - i\alpha|v\rangle$ it becomes:

$$\langle u|A|v\rangle = \langle u|P^{-1}A\,P|u\rangle \tag{1.2}$$

or equivalently:

$$[P, A] = 0\,. \tag{1.3}$$

Thus, all physical observables must be permutation-invariant. Furthermore, dynamical states represented by $|u\rangle$ and $P|u\rangle$ at time 0 should not exhibit any observable difference at any later time t. If $U(t)$ is the evolution operator, the above condition becomes:

$$\left[P, U^{*}(t)\,A\,U(t)\right] = 0\,. \tag{1.4}$$

and $U^* A U$ must be permutation-invariant, for any physical observable A and for any value of t. Obviously $U(t)$ must be itself permutation-invariant.

The permutation-invariance is a rigorous invariance property in ordinary QM. But in general the state-vector space $\mathcal{E}^{(N)}$ is not irreducible with respect to the algebra of physical observables; this situation leads to a superselection rule. It is essential to prepare a *maximal* system (it gives the maximum amount of information compatible with the indistinguishability of identical particles), which is permutation-invariant and, in general, reducible. While in the ordinary QM a system is represented by a ray in a Hilbert space, now for the maximal system, in analogy, the state of a system of identical particles corresponds to a "generalized ray". In this way we can obtain the same results as in QM. Particular subspace of interest are $\mathcal{E}_S^{(N)}$ and $\mathcal{E}_A^{(N)}$ spanned by symmetric and antisymmetric vectors, respectively. Maximal observation leads to state vectors of definite state of symmetry type. Therefore, it is natural to advance the postulate that only one type occurs in nature. SP is a somewhat restricted form of this postulate, in that it is assumed that the allowed component has to be either $\mathcal{E}_S^{(N)}$ or $\mathcal{E}_A^{(N)}$. That the postulate could be consistently inserted in the QM framework is evident by proving that *a superselection rule operates between vectors of different symmetry type*. Also, due to the permutation invariance of $U(t)$, vectors in an irreducible representation remain in an equivalent one in the course of time. Therefore vectors of a definite symmetry type keep this symmetry type in the course of time. Finally, since physical observables are permutation invariant, they cannot have non vanishing matrix elements between subspaces of different symmetry type, as this would be incompatible with the assumed particle indistinguishability.

There are many papers that attempt to prove that in ordinary QM only Bose and Fermi particles can occur. These arguments, when formulated correctly, usually imply an additional assumption. In a nutshell, the most common argument is based on the requirement that:

$$P|u\rangle = c|u\rangle \quad , \quad |c| = 1 \ , \tag{1.5}$$

so that permuting the particles in a state vector changes it only by a numerical phase factor, and this is the same as requiring that the representation be symmetric or antisymmetric. The last relation complies with the requirement of indistinguishability, but it is stronger. It cannot be deduced from this requirement alone. One has to assume that the dynamical states which cannot be distinguished by an observation are represented by the same vector to within a phase factor.

Another common argument starts from the consideration of "one-body" states. Here one assumes that a complete knowledge of the single-particle states entails a complete knowledge of the system, i.e. one assumes that a complete set of one-body measurement is maximal. Clearly, it is necessary to postulate that the state vectors must have a definite symmetry type (symmetric or antisymmetric). Hence the SP.

In summary, for system with a fixed number of particles, there is a superselection rule between symmetry types which allows one to insert SP in the quantum theory in a consistent way. However the postulate does not appear as a necessary feature of the QM description of nature. Whether it is followed by nature or not has to be decided by experiment.

All these considerations can be extended to systems with a variable number of particles. Moreover the validity of SP is experimentally well established for electrons and many other fundamental particles.

1.3 Elementary considerations on the Pauli Exclusion Principle

The Pauli Exclusion Principle is one of the fundamental principles of modern physics and is discussed at an elementary level in any good book on QM. Here we summarize the discussion of PEP in non-relativistic quantum mechanics (QM) following the textbook by Mandl [5].

Consider, for example, the helium atom Hamiltonian:

$$H_0(r_1, r_2) = \sum_{i=1,2}\left(-\frac{\hbar^2}{2m}\nabla_i^2 - \frac{2e^2}{4\pi\varepsilon_0 r_1} \right) + \frac{e^2}{4\pi\varepsilon_0 r_{12}} \tag{1.6}$$

which is symmetric with respect to the coordinate exchange of the two electrons. We have already remarked above that requiring the Hamiltonian to be symmetric corresponds to the assumption that the two electrons are indistinguishable. This holds true both for approximate and for exact Hamiltonians. For example, if one includes the spin as well as the spatial coordinate, the Hamiltonian should still be symmetric with respect to the 1-2 exchange operation.

Generally, the Hamiltonian of a system of N electrons must be invariant with respect to any electron exchange operation:

$$H(1,...,i,...,j,...,N) = H(1,...,j,...,i,...,N) \tag{1.7}$$

In particular, we notice that for a 2-particle system the symmetry of the Hamiltonian induces a degeneration of the eigenvalues, namely:

$$H(1,2)\Psi(1,2) = H(1,2)\Psi(2,1) = E\Psi(1,2) = E\Psi(2,1). \tag{1.8}$$

One can therefore define the following linear combinations:

$$\Psi(1,2) \pm \Psi(2,1) \tag{1.9}$$

which are the eigenfunctions for the same eigenvalue having a defined parity (±1) with respect to the 1-2 permutation.

Also, we observe that the time evolution dictated by the Schröedinger equation:

$$i\hbar\frac{\partial}{\partial t}\Psi(1,2,t) = H(1,2)\Psi(1,2,t) \tag{1.10}$$

cannot modify the symmetry of the wave-functions, since the Hamiltonian commutes with the permutations of particle labels.

In the case of 3 or more particles the situation is a bit more complex. Let's consider, for example, the linear combination:

$$\Psi(1,2,3)+\Psi(1,3,2)-\Psi(3,2,1) \tag{1.11}$$

which for the 1-2 exchange becomes the new linear combination:

$$\Psi(2,1,3)+\Psi(2,3,1)-\Psi(3,1,2)\ , \tag{1.12}$$

which has no common term with the initial combination, or the linear combination

$$\Psi(1,2,3)+\Psi(1,3,2)-\Psi(2,1,3)+\Psi(2,3,1)+\Psi(3,1,2)-\Psi(3,2,1) \tag{1.13}$$

which contains terms with all possible permutations, but, when the 1-2 exchange is done, becomes:

$$-\Psi(1,2,3)+\Psi(1,3,2)+\Psi(2,1,3)+\Psi(2,3,1)-\Psi(3,1,2)+\Psi(3,2,1) \tag{1.14}$$

and obviously has no defined parity. Generalizing these considerations, one can see that the group of the linear combinations:

$$\pm\Psi(1,2,3)\pm\Psi(1,3,2)\pm\Psi(2,1,3)\pm\Psi(2,3,1)\pm\Psi(3,1,2)\pm\Psi(3,2,1) \tag{1.15}$$

is closed with respect to the group of the 1,2,3 symbol permutations, and, moreover, there exists a subgroup constituted by only one linear completely symmetric combination:

$$\Psi(1,2,3)+\Psi(1,3,2)+\Psi(2,1,3)+\Psi(2,3,1)+\Psi(3,1,2)+\Psi(3,2,1) \tag{1.16}$$

which transforms in itself. The same happens to the completely antisymmetric linear combination:

$$\Psi(1,2,3)-\Psi(1,3,2)-\Psi(2,1,3)+\Psi(2,3,1)+\Psi(3,1,2)-\Psi(3,2,1) \tag{1.17}$$

Here the symmetrization postulate states that the only acceptable ("physical") linear combinations are either completely symmetric or antisymmetric, a condition which adds to the symmetry of the Hamiltonian (which is dictated by the indistinguishability of particles).

This is as far as one can go with ordinary QM. Moving now to relativistic QM, it is rather easy to demonstrate that the second quantization of the Dirac's equation retains physical meaning only if the corresponding fields do anticommute, while the Klein-Gordon equation is valid for commuting fields. It is however less easy to prove that, generally, a quantum theory is coherent only if the half-integer spin particles are described by anticommutators while the integer spin ones by commutators.

1.4 The spin-statistics theorem

The exclusion principle, first discovered by Pauli for electrons and later extended to all fermions, was based on the analysis of experimental data. Pauli himself was never satisfied by this. In his Nobel Prize lecture [1], Pauli said: "*Already in my initial paper, I especially emphasized the fact that I could not find a logical substantiation for the exclusion principle nor derive it from more general assumptions. I always had a feeling, which remains until this day, that this is the fault of some flaw in the theory*." This was said in 1945, after the Pauli famous theorem [6] of the relation between spin and statistics.

In 1940, Pauli published the first proof of the spin-statistics theorem. It is based on a classification of the spinor representations of the proper Lorentz group into four classes different under the strong-reflection transformation $x_\mu \rightarrow -x_\mu$. From this very general beginning, Pauli obtains the Spin-Statistics connection for non-interacting particles of arbitrary spin which are not necessarily represented by irreducible spinors or subject to canonical field theory.

After 1940, numerous proofs of the spin-statistics theorem were published. All these proofs contain some explicit or implicit assumption. It seems appropriate here to

mention Schwinger's proof. It is based on the postulate of invariance under strong-reflection, following Pauli's original observation. Schwinger requires an anticommutation between Dirac field operators and an interchange of initial and final states, to maintain the invariance.

This logic is reversed by Lüders and Zumino. In their approach the spin-statistics theorem is proved on the basis of "common sense" postulates and the TCP theorem also follows.

Below I summarize these three different approaches to the connection between spin and statistics.

1.4.1 Pauli

In his original paper [6,7], Pauli did not give a direct proof. He showed that due to some physical contradictions the second quantization operators for particles with integral spins cannot obey the fermion commutation relations, while for particles with half-integral spins they cannot obey the boson commutation relations. From this, Pauli concluded that particles with integral spin have to obey the Bose–Einstein statistics, while those with half-integral spin have to obey the Fermi– Dirac statistics.

Pauli first considers classical fields, which, like scalars, vectors, and tensors transform with respect to rotations in the ordinary space according to a one-valued representation of the rotation group and assume that all field components will satisfy a second-order wave equation, permitting a superposition of plane waves as a general solution. Pauli wanted to give a proof of the indefinite character of the charge in case of integral and of the energy in case of half-integral spin. The field components can be assumed to be either real or complex. For a complex field, in addition to energy and momentum of the field, a four-vector can be defined which satisfies the continuity equation and can be interpreted as the four-vector of the electric current. Its fourth component determines the electric charge density and can assume both positive and negative values (in the particular case of real fields this four-vector of current vanishes

identically). Pauli denotes the (vector or tensor) wave functions with the symbols U_i and U_{ik}, T_{ik} is the energy-momentum tensor and S_i the current vector, with $i = 1,\ldots, 4$ and $k = 1, \ldots, 3$.

Treating at first particles with integral spin, he considers a homogeneous and linear equation for the quantities U (not necessarily of the first order) and assumes a plane wave (and thus free particles) with wave vector $k_l = -i\partial / \partial x_l$. Because of the invariance with respect to the proper Lorentz group, the field equations must have the form:

$$\sum kU^+ = \sum U^- \quad , \qquad \sum kU^- = \sum U^+ \quad . \tag{1.18}$$

This form means that several terms of the same type may be present. Also, among the U^+ both U^+ as well as the $(U^+)^*$ may occur. These equations remain invariant under the substitutions:

$$\begin{cases} k_i \to -k_i \\ U^+ \to U^+, \qquad \left[\left(U^+\right)^* \to \left(U^+\right)^* \right] \\ U^- \to -U^-, \qquad \left[\left(U^-\right)^* \to -\left(U^-\right)^* \right] \end{cases} . \tag{1.19}$$

Then Pauli considers tensors T of even rank, which are bilinear functions of the U's, with the typical form:

$$T \approx \sum U^+ U^+ + \sum U^- U^- + \sum U^+ k U^- \tag{1.20}$$

These tensors remains unchanged under the strong reflection transformation.
The situation is different for tensors of odd rank S, which are characterized by the typical form:

$$S \approx \sum U^+ k U^+ + \sum U^- k U^- + \sum U^- U^+ \tag{1.21}$$

and which change sign under the strong reflection transformation.

The indefinite character of the current density and the total charge for integer spin follows, since to every solution of the field equation belongs another solution for which the same components change their sign. So, the definition of a definite particle density for integer spin which transforms like the 4-component of a vector is impossible.

The case of half-integer spin is less simple and both T and S are characterized by a more complex form, but now the opposite situation occurs: the tensors T change their sign while the tensors S remain unchanged: $T \to -T$; $S \to S$.

The result is that a positive defined energy density, as well as a positive defined total energy, is impossible. It follows from the fact that the energy density in every space-time point changes its sign because the total energy also changes its sign.

At this point, to define the connection between spin and statistics, Pauli assumes that fields are quantized in the absence of interactions. He supposes that all fields U satisfy the second order wave equation:

$$\Box U - m^2 U = 0 \tag{1.22}$$

where $\Box$ is the D'Alembertian operator and m the rest mass of the particles in units $\hbar / c$. Then he takes the invariant function D, which satisfies the wave equation (1.22) and is given within a volume V with periodic boundary conditions by:

$$D(\mathbf{x}, x_0) = \frac{1}{V} \sum \exp(i\mathbf{kx}) \frac{\sin k_0 x_0}{k_0} \tag{1.23}$$

or in the limit $V \to \infty$:

$$D(\mathbf{x}, x_0) = \frac{1}{(2\pi)^3} \int d^3k \, \exp(i\mathbf{kx}) \frac{\sin k_0 x_0}{k_0} \tag{1.24}$$

with the positive root for: $k_0 = +\sqrt{k^2 + m^2}$. The D function is uniquely determined by the conditions:

$$\Box D - m^2 D = 0\,, \quad D(\mathbf{x},0) = 0\,, \quad \left(\frac{\partial D}{\partial x_0}\right)_{x_0=0} = \delta(\mathbf{x}) \tag{1.25}$$

For $m = 0$ this is simply:

$$D(\mathbf{x},x_o) = \{\delta(r - x_0) - \delta(r + x_0)\}/\,4\pi r \tag{1.26}$$

while for $m \neq 0$ we have a singularity of $D(\mathbf{x},x_0)$ on the light cone. In this case D is different from zero in the inner part of the cone and its value is:

$$D(\mathbf{x},x_0) = -\frac{1}{4\pi r}\frac{\partial}{\partial r}F(r,x_0)\,, \tag{1.27}$$

with

$$F(r,x_0) = \begin{cases} J_0\left[m\sqrt{x_0{}^2 - r^2}\right] & x_0 > r \\ 0 & r > x_0 > -r \\ -J_0\left[m\sqrt{x_0{}^2 - r^2}\right] & r > x_0 \end{cases} \tag{1.28}$$

The discontinuity of the function F on the light cone is due to the δ singularity of D on the cone. And in the exterior of the cone D vanishes.

But there exists another function, apart from D, which also satisfies the wave equation (1.22):

$$D_1(\mathbf{x},x_0) = \frac{1}{(2\pi)^3}\int d^3k\, \exp(i\mathbf{kx})\frac{\cos k_0 x_0}{k_0} \quad . \tag{1.29}$$

For $m = 0$ one finds:

$$D_1(\mathbf{x},x_0)=\frac{1}{2\pi^2}\,\frac{1}{r^2-{x_0}^2}\quad , \tag{1.30}$$

(as shown in Fig. 1.1), but the general solution is:

$$D_1(\mathbf{x},x_0)=\frac{1}{4\pi}\,\frac{1}{r}\,\frac{\partial}{\partial r}F_1(r,x_0)\quad , \tag{1.31}$$

with:

$$F_1(r,x_0)=\begin{cases} N_0\left[m\sqrt{{x_0}^2-r^2}\right] & x_0>r \\ -iH_0^{(1)}[im]\sqrt{r^2-{x_0}^2} & r>x_0>-r \\ N_0\left[m\sqrt{{x_0}^2-r^2}\right] & -r>x_0 \end{cases} \tag{1.32}$$

where N_0 is the Neumann function and $H_0^{(1)}$ the first Hankel cylinder function. Here the form (1.29) determines the singularity of D on the surface of the light cone.

The behavior of D_1 thus violates the "locality postulate": *all physical operators outside the light cone commute*. It follows that D (and its derivatives) is the only physically possible function which satisfies the force-free wave equation.

At this point it is necessary to determine the number of derivatives of the D function, taking into account the invariance of the Lorentz group and the class division of the tensors. In particular, each field component is composed only of quantities of the same class. Pauli considers the commutation relation of a field component $U^{(r)}$ with its own complex conjugate $\left[U^{(r)}(x',x_0'),U^{*(r)}(x'',x_0'')\right]$. Then he distinguishes the two cases of half-integral and integral spin, for which the last expression transforms according Lorentz transformations as a tensor of odd or even rank, respectively. Hence:

$$\textit{half-integral spin} \rightarrow \left[U^{(r)}(\mathbf{x}',x_0'),U^{*(r)}(\mathbf{x}'',x_0'')\right]=\partial^{2n+1}D(\mathbf{x}'-\mathbf{x}'',x_0'-x_0'') \tag{1.33}$$

$$\textit{integral spin} \rightarrow \left[U^{(r)}(\mathbf{x}',x_0'),U^{*(r)}(\mathbf{x}'',x_0'')\right]=\partial^{2n+1}D(\mathbf{x}'-\mathbf{x}'',x_0'-x_0'') \tag{1.34}$$

Considering the symmetry of the two-point expression:

$$X \equiv \left[U^{(r)}(\mathbf{x}', x'), U^{*(r)}(\mathbf{x}'', x_0'') \right] + \left[U^{(r)}(\mathbf{x}'', x_0''), U^{*(r)}(\mathbf{x}', x') \right] \tag{1.35}$$

it follows from its symmetry that $X \equiv$ even number of space-like times odd numbers of time-like derivatives of $D(\mathbf{x}' - \mathbf{x}'', x_0' - x_0'')$. This is consistent with the postulate (1.33) for half-integral spin, but in contradiction with (1.34) for integral spin unless X vanishes.

This lead us to the final result: *for integral spin the quantization according to the exclusion principle is not possible*. In the same way it is possible to quantize the theory of half-integral spins according to Einstein-Bose statistics, but *the energy of the system cannot be positive definite*. So, for physical reasons we must apply the exclusion principle in connection with Dirac's hole theory.

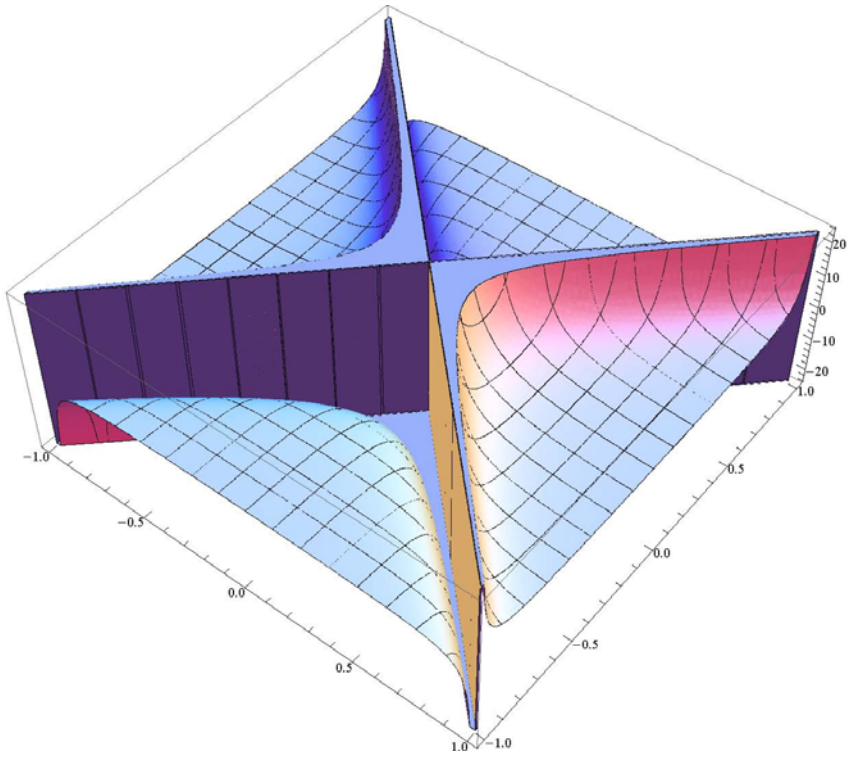


Fig. 1.1: The D_1 function: notice that the function is nonzero also outside the light-cone and therefore it is nonlocal (noncausal).

1.4.2 Schwinger

Schwinger produced more than one proof of the spin-statistics theorem [7,8] and here I discuss only the proof that he published in 1958 [8] and which places the spin-statistics connection in the broader context of TCP invariance. The proof is based on the hypotheses of Lorentz-invariance and of the existence of a lowest-energy state, and on the general dynamical structure of quantum field theory.

The dynamical principle of the theory of fields, governing the development of the system between two space-like surfaces σ, is the differential action principle:

$$\delta\langle\sigma_1|\sigma_2\rangle = \left\langle \sigma_1 \middle| \delta \left\{ \int_{\sigma_1}^{\sigma_2} (dx) L[\chi] \right\} \sigma_2 \right\rangle \tag{1.36}$$

where L is the Lagrange Hermitian operator:

$$L[\chi] = \frac{1}{4}\left(\chi^T A^\mu \partial_\mu \chi - \partial_\mu \chi^T A^\mu \chi\right) - H(\chi) \tag{1.37}$$

which is a function of of the multicomponent Hermitian field χ, and where the finite dimensional matrices A can be reduced into two classes of real antisymmetric matrices and imaginary symmetrical matrices. This decomposition reflects the separation of χ into the fundamental classes of bosons (fields ϕ) and fermions (fields ψ). Hermiticity of the Lagrangian then requires that:

$$L[\chi] = L[\phi,\psi] = L^{T*}[\phi,\psi] = L^*[\phi,i\psi] \ . \tag{1.38}$$

The last form of the Lagrangian leads to a modified form of the action principle. The action principle is further manipulated after the remark that the Lagrangian function must

be Lorentz-invariant, and therefore must be invariant with respect to simple rotations after performing a Wick-rotation of the time axis. The rotated version of the action principle is:

$$R:\delta\langle\sigma_1|\sigma_2\rangle=\left\langle\sigma_1\left|\delta\left\{\int_{\sigma_1}^{\sigma_2}(dx)L\left[\phi_{\text{int}},i\phi_{1/2\,\text{int}},i\psi_{\text{int}},\psi_{1/2\,\text{int}}\right]\right\}\right|\sigma_2\right\rangle \tag{1.39}$$

where the int and 1/2int subscript denote the integer of half-integer spin components. This must be the same as the original action principle:

$$\delta\langle\sigma_1|\sigma_2\rangle=\left\langle\sigma_1\left|\delta\left\{\int_{\sigma_1}^{\sigma_2}(dx)L\left[\phi_{\text{int}},\phi_{1/2\,\text{int}},\psi_{\text{int}},\psi_{1/2\,\text{int}}\right]\right\}\right|\sigma_2\right\rangle \tag{1.40}$$

and we see that the two forms are actually the same only if $\phi_{1/2\,\text{int}}$ and ψ_{int} both vanish (and this is the spin-statistics connection). The remaining equivalence between variations is a statement of the TCP theorem, which, in this context, is deeply linked to the spin-statistics connection.

In his proof Schwinger equivocated on the question of the spin-statistics relation in the presence of interactions. He noticed that at sufficiently high energies the kinematic term in the Lagrangian dominates the interaction term, and that they have the same exchange symmetry because of consistency. Similarly he assumed that higher spin particles to be composite or to have wave function which where, and to have exchange properties which are simply the product of the constituents. This assumption is supported by the conclusion of Johnson and Sudarshan [9] that only spin-1/2 fields can be regarded as fundamental, while higher half-integral spin fields cannot be represented by a local action principle. Their results support the view that the spin-statistics connection need be demonstrated only for spin-0 and spin-1/2.

1.4.3 Lüders and Zumino

Lüders and Zumino proved the spin-statistics theorem starting from the basic postulates of QFT, including invariance under the proper Lorentz group with no reflections. The analytic continuation in the separation of the two fields in a vacuum expectation value of their product as well as their ability to treat interacting fields, are the key ingredients which characterize their proof [7,10].

It is important to remark that the proof of Lüders and Zumino is independent from the TCP theorem (while previous proofs of both spin-statistics and TCP theorem seemed to support one another) and hold for interacting fields.

The Lüders and Zumino proof is based on five basic postulates plus gauge invariance. The postulates are:

I. Invariance under the proper inhomogeneous Lorentz group (not containing any reflections).
II. Locality. Two operators of the same field at points separated by a space-like interval either commute or anticommute.
III. The vacuum is the unique state of lowest energy.
IV. The metric of the Hilbert space of states is positive definite.
V. The vacuum state is not identically annihilated by a field.

The proof treats separately the Hermitian and the non-Hermitian cases, and each case is further split in the spin-0 and spin-1/2 subcases.

A. Hermitian spin-0 case. From I the expectation value with respect to the physical vacuum is an invariant function of the difference four-vector $\xi = x - y$:

$$\langle 0|\phi(x)\phi(y)|0\rangle = f(\xi) \tag{1.41}$$

for ξ space-like $f(\xi)$ depends only on the length of the four-vector ξ^2 and not on its direction, therefore, assuming the Lorentz-invariance of the vacuum state, in the spacelike case

$$\langle 0|\phi(x)\phi(y)|0\rangle = f(\xi) = f(-\xi) = \langle 0|\phi(y)\phi(x)|0\rangle \ . \tag{1.42}$$

so that:

$$\langle 0|\phi(x)\phi(y)|0\rangle - \langle 0|\phi(y)\phi(x)|0\rangle = \langle 0|\left[\phi(x),\phi(y)\right]_-|0\rangle_{space-like} = 0 \tag{1.43}$$

Postulate II asserts that fields may commute or anticommute: if we assume that these spin-0 fields anticommute for spacelike separation we have:

$$\langle 0|\left[\phi(x),\phi(y)\right]_+|0\rangle = 0 \tag{1.44}$$

Then, adding the commutator and anticommutator for space-like ξ:

$$\langle 0|\phi(x)\phi(y)|0\rangle_{space-like} = 0 \, . \tag{1.45}$$

By analytic continuation and using postulate III, this equality must hold for all ξ including zero separation. Then, using the postulate on the positive-definiteness of the metric and the last result one finds

$$\phi(x)|0\rangle = 0 \tag{1.46}$$

which contradicts postulate V ,leading to the conclusion that the assumption that spin-0 fields anticommute is untenable.

B. Hermitian spin-1/2 case. In the case of Hermitian – or more generally Majorana – spin-1/2 fields the following relation holds

$$\overline{\psi} = \psi C \tag{1.47}$$

where *C* is the 4-by-4 charge conjugation matrix. The vacuum expectation value is:

$$\langle 0|\psi_\beta^*(x)\psi_\beta(y)|0\rangle = \eta_{\alpha\beta}\langle 0|\psi_\alpha(x)\psi_\beta(y)|0\rangle \tag{1.48}$$

where $\eta_{\alpha\beta} = C_{\alpha\delta}\gamma^0_{\delta\beta} = \eta_{\beta\alpha}$ (a symmetric matrix, which contains γ_0). Expression (1.48) is the time-component of a four-vector, and from Lorentz-invariance (Postulate I) we find:

$$\eta_{\alpha\beta}\langle 0|\psi_\alpha(x)\psi_\beta(y)|0\rangle = \xi_0 g(\xi) \quad , \quad \xi = x - y\,. \tag{1.49}$$

For ξ space-like $g(\xi)$ depends only on ξ^2, so that:

$$\eta_{\alpha\beta}\langle 0|\left[\psi_\alpha(x),\psi_\beta(y)\right]_+|0\rangle_{space-like} = 0\,. \tag{1.50}$$

If we assume that the fields commute, i.e.

$$\eta_{\alpha\beta}\langle 0|\left[\psi_\alpha(x),\psi_\beta(y)\right]_-|0\rangle = 0 \tag{1.51}$$

then

$$\eta_{\alpha\beta}\langle 0|\psi_\alpha(x)\psi_\beta(y)|0\rangle_{space-like} = 0\,. \tag{1.52}$$

The analytic continuation gives:

$$\langle 0|\psi^*(x)\psi(x)|0\rangle = 0 \tag{1.53}$$

and from postulate IV we find $\psi(x)|0\rangle = 0$ which violates postulate V. The conclusion is that Hermitian spin-1/2 fields cannot commute.

The case of non-Hermitian fields requires the postulate of gauge invariance, i.e., in the case of spin-0 fields

$$\begin{aligned} \phi(x) &\to \phi(x)e^{i\alpha} \\ \phi^*(x) &\to \phi^*(x)e^{-i\alpha} \end{aligned} \tag{1.54}$$

from which it follows that:

$$\langle 0|\phi(x)\phi(y)|0\rangle = \langle 0|\phi^*(x)\phi^*(y)|0\rangle = 0\,. \tag{1.55}$$

Assuming that the fields anticommute for space-like separation, i.e., that:

$$\langle 0|\left[\phi^*(x)\phi(y)\right]_+|0\rangle = 0\,. \tag{1.56}$$

it is easy to see that the following formula holds:

$$\langle 0|\left(\phi(x)\pm\phi^*(x)\right)\left(\phi(y)\pm\phi^*(y)\right)|0\rangle = 0\,. \tag{1.57}$$

Then the above proof for Hermitian fields is applied to the Hermitian fields:

$$\phi_1 \to \phi + \phi^*\ ,\ \phi_2 \to i\left(\phi - \phi^*\right) \tag{1.58}$$

and one finds again that assuming anticommutation in the case of spin-0 fields is incompatible with postulate V.

The same proof can be repeated with small modifications in the case of non-Hermitian spin-1/2 fields and in that case one finds that the assumption that fields commute is incompatible with postulate V; the Lüders-Zumino proof is thus complete.

Chapter 2

Models of violation of the Pauli Exclusion Principle

2.1 Types of experiment to detect PEP violation

Atomic spectroscopy is the first place to search for violations of the exclusion principle since that is where Pauli discovered it. One looks for funny lines which do not correspond to lines in the normal theory of atomic spectra.

There exist several experimental tests, for electrons and nucleons, which are already provided different upper limit on the probability of PEP violation. Generally speaking, tests of the PEP fall into one of four classes: searches for PEP-forbidden electronic [11,12,13,14] or nuclear [15] states (non-Paulian state), or for PEP-forbidden electronic [2,16,17] or nuclear [18,19] transitions (non-Paulian transitions).

An insightful experiment by Maurice and Trudy Goldhaber [20] was designed to answer the question "Are the electrons emitted in nuclear β-decay quantum mechanically identical to the electrons in atoms?" We know that the β-decay electrons have the same spin, charge and mass as electrons in atoms; however they realized that if the β-decay electrons were not quantum mechanically identical to those in atoms, then the β-decay electrons would not see the K shell of a heavy atom as filled and would fall into the K shell and emit an x-ray. The Goldhabers looked for such X-rays by letting β-decay electrons from a natural source fall on a block of lead. No such X-rays were found. They were able to confirm that electrons from the two sources are indeed quantum

mechanically identical. At the same time, they found that any violation of the exclusion principle for electrons must be less than 5%. The Goldhabers also pointed out that the same experimental data which were used to set a limit on the lifetime of the electron can be used to test the validity of the PEP for atomic electrons [16].

A pionieering analysis was performed by Reines and Sobel on X-rays emitted in the transition of an L-shell electron to the filled K-shell in an atom [17].

Novikov et al. [12] investigated PEP violating by using accelerator mass spectrometry to search for anomalous atoms with three electrons in the K shell and obtained a limit of $\beta^2/2 < 2 \cdot 10^{-21}$.

A simple method to detect PEP violation (for electrons) consisting in the observation of the anomalous X-rays coming from Pauli forbidden transitions was suggested by Ramberg and Snow [2]. They developed an experiment into one which yields a high precision bound on violations of the exclusion principle. Their idea was to replace the natural source, which provides relatively few electrons, by an electric current, in which case Avogadro's number is on our side. The possible violation of the exclusion principle is that a given collection of electrons can, with different probabilities, be in different permutation symmetry states. The probability to be in the "normal" totally antisymmetric state would presumably be close to one, the next largest probability would occur for the state with its Young tableau having one row with two boxes, etc. The idea of the experiment is that each collection of electrons has a possibility of being in an "abnormal" permutation state. If the density matrix for a conduction electron together with the electrons in an atom has a projection onto such an "abnormal" state, then the conduction electron will not see the K shell of that atom as filled. Then a transition into the K shell with x-ray emission is allowed. Each conduction electron which comes sufficiently close to a given atom has an independent chance to make such an x-ray-emitting transition, and thus the probability of seeing such an x-ray is proportional to the number of conduction electrons which traverse the sample and the number of atoms which the electrons visit, as well as the probability that a collection of electrons can be in the anomalous state. Ramberg and Snow chose to run 30 amperes through a thin copper strip for about a month. They estimated the energy of the X-rays which would be emitted

due to the transition to the K shell. No excess of X-rays above background was found in this energy region. Ramberg and Snow set the limit $\beta^2/2 \le 1.7 \cdot 10^{-26}$.

A more stringent limit on $\beta^2/2$ can be found for the PEP- forbidden transition of nucleons in nuclei [18]. Logan and Ljubičić searched for γ-quanta emitted in a nucleons in nuclei. Violation of PEP in the nucleon system has been studied by searching for the non-Paulian transitions with γ- [18,21,22], p-[23,24] and n-[19] emission, non-Paulian β^+, β^-- decays [22,25] and in nuclear (p, p),(p, α)-reactions on ^{12}C [26]. The sensitivity of the forbidden transitions method is defined by the mass of the detector and by the background level of detector.

Kekez et al. [25] investigated the non-Paulian electron-capture decay of ^{71}Ge and deduced a limit of $\beta^2/2 \le 3 \cdot 10^{-12}$ for neutrons. They also consider the possibility of non-Paulian decays in stable nuclei through β^- (β^+) decays of higher-shell neutrons (protons) directly into the proton (neutron) shell: in particular the non-paulian decays of ^{127}I into the non-Paulian nuclei ^{127}Xe or ^{127}Te. They searched for β^- (β^+) particles with a large NaI(Tl) detector and obtained a limit of $\beta^2/2 \le 2 \cdot 10^{-26}$.

A much more severe limit on $\beta^2/2$ can be deduced using the existing data from underground laboratories. The liquid scintillation detector (LSD) of the Mont Blanc collaboration is particularly suitable. It has been in operation since 1984 in a cavity in a tunnel linking France and Italy, about 1,700 m below the surface. The LSD contains 90 tons of liquid scintillator. The most probable non-Paulian transition is expected to be the first forbidden $2p \rightarrow 1s$ β^- (β^+) transition from ^{12}C into ^{12}N(^{12}B). In this case $\beta^2/2 \le 6.5 \cdot 10^{-34}$.

2.2 Comments on testing the Pauli Principle

The Pauli Exclusion Principle shows a surprising rigidity: it is very difficult to produce theories that incorporate a small violation of the principle (see below) and it is correspondingly difficult to set up experiments that test the principle. As we have seen

above, the archetypal experiments to test the PEP are the searches for non-Paulian isotopes and the search for "forbidden" *K* X-rays from a block of material. Setting a limit on the number of non-Paulian isotopes or on the intensity of the *K* X-rays seems to be equivalent to putting a limit on PEP violation. This interpretation has been criticized: Amado and Primakoff [27] assert that the interpretation is not correct and that such experiments do test particle stability only. More generally Amado and Primakoff challenge the sheer possibility of performing such tests, and they arguments can be summarized as follows:

1. if the Hamiltonian is symmetric, then transitions to a state with a different permutation symmetry are not allowed;

2. if the electrons can have a symmetric component, then, because of the indistinguishability of electrons, all electrons should be "a bit" symmetric. On the other hand, this "bit" should not be necessarily small, and then one can wonder why, in reality, should it be so "extremely small";

3. if different symmetry states do exist, than the components having different symmetry should be degenerate in energy and this is close to a "miracle", since it is exceedingly unlikely that such a coincidence takes place;

4. a way out could be to give up to the perfect indistinguishability and to assume that particles that are very similar, but not identical, do exist. But – if all electrons could be, even very slightly different, the exclusion principle would not hold in many cases and this is in contrast with the experimental evidence (no doubling of the Bhabha cross-section is observed);

5. another possibility is that only some electrons are different, but in such a case the radiative transitions to the K-shell should have occurred long time ago, and then it is not very likely that one can presently observe X-rays from this process. Moreover, this hypothesis would give rise to an increase (not observed) of the experimental cross-section in the e^+e^- colliders.

These arguments can all be somehow circumvented, however they must be kept in mind when interpreting experimental results.

One way to escape Amado and Primakoff's arguments was suggested by Akama, Terazawa and Yasuè [28] who put forward a model of composite electron. They discuss about the ensuing "superficial" violation of the PEP due to this possible substructure of electrons in composite models of quarks and leptons. The proposed composite electron consists of a fermion *w* and a boson *C*. Neither the PEP nor the causality is violated at the level of constituent fields of *w* and *C*, if *w* and *C* do obey Fermi and Bose statistics respectively. Also, the anticommutator of *w*'s and the commutator of *C*'s do respect causality. However, the composite electron field may exhibit a statistics that is neither purely fermionic nor purely bosonic when two electrons are very close to and overlap each other at a distance of the order of their size r_0.

Akama, Terazawa and Yasuè have estimated the ratio of the Pauli forbidden atomic transition to the allowed one, to be about $10^{-50} \div 10^{-44}$ for heavy atoms ($Z = 10 \div 10^2$) if the size of the electrons is of order 10^{-17} cm (an estimate extremely small which includes the mutual Coulomb repulsion of electrons as well).

2.3 The Ignatiev and Kuzmin model

In 1987 the Russian physicists A.Yu. Ignatiev and V.A. Kuzmin described a quantum-mechanical model of particles which included a small violation of the PEP; there was a small probability that two particles might be in the same state. This is the simplest algebra of the creation and annihilation operators with a parameter β which incorporates the small violations of the PEP (for $\beta = 0$ the PEP holds absolutely true). The analysis, to some extent a development of some ideas due to Green [29], involves trilinear commutation relations that include this small parameter $\beta << 1$; $\beta^2/2$ is a measure of the violation of the PEP and is defined as the relative probability of finding

two fermions in the same state. Thus, the Hamiltonian of this model describing the PEP violating transition, shows transitions of two identical particle into the same state with the probability suppressed by a factor of β^2. Ignatiev and Kuzmin follow a direct method: instead of exploring possible operator algebras, they construct a suitable operator algebra, possessing the necessary properties, and after that they find the commutation relations. They find it more convenient to work with the creation and annihilation operators than with the field operators. They also simplify the problem by discarding the momentum and spin variables and considering only electron, but not positron, operators (thus the model is not a true field theory).

In the Ignatiev and Kuzmin (IK) model [30], creation and annihilation operators connect 3 states, the vacuum state $|0\rangle$, the single occupancy state $|1\rangle$ and a nonstandard double occupancy state $|2\rangle$, through the following relations:

$$\begin{cases} a^+|0\rangle = |1\rangle; & a|0\rangle = 0; \\ a^+|1\rangle = \beta|2\rangle; & a|1\rangle = |0\rangle; \\ a^+|2\rangle = 0; & a|2\rangle = \beta|1\rangle; \end{cases} \tag{2.1}$$

where the parameter β is supposed to be real. Then the matrices of these operators in the chosen basis take the form:

$$a = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \beta \\ 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad a^+ = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & \beta & 0 \end{pmatrix}. \tag{2.2}$$

The Hilbert state space H can be decomposed into the direct sum of the subspaces H_2 (spanned by the vectors $|0\rangle$ and $|1\rangle$), and H_1 (spanned by the vector $|2\rangle$). Clearly, if $\beta = 0$ the transitions between the states in H_2 and H_1 become forbidden so that the space H_1 gets completely decoupled from H_2.

To construct the commutation relation satisfied by the operator a and a^+, it is convenient to choose 9 basic matrices M_{ij} (i,j = 1,2,3) as follows:

$M_{11} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $M_{12} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $M_{13} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, ….. and so on. They are not Hermitean: $M_{ij}^{+} \neq M_{ji}$. Calculating various products of the operators a and a^{+} and decomposing them with respect to M_{ij}, we find:

$$
\begin{aligned}
& a = M_{12} + \beta M_{23} && a^3 = 0 \\
& a^2 = \beta M_{13} && a^2 a^{+} = \beta^2 M_{12} \\
& a^{+} a = M_{22} + \beta^2 M_{33} && a a^{+} a = M_{12} + \beta^3 M_{23} \\
& a a^{+} = M_{11} + \beta^2 M_{22} && a^2 a^{+} = \beta M_{23}
\end{aligned}
\tag{2.3}
$$

and also the Hermitean conjugate relations (not listed here). From that Ignatiev and Kuzmin found four independent linear relations between the operators:

$$
\begin{cases}
a^2 a^{+} + \beta^2 a^{+} a^2 = \beta^2 a. \\
a^2 a^{+} + \beta^4 a^{+} a^2 = \beta^2 a a^{+} a \\
a^3 = 0 \\
\left(a^{+}\right)^3 = 0
\end{cases}
\tag{2.4}
$$

plus their Hermitean conjugate relations. These relations define the desired algebra. Now, the particle number operator N must be constructed. In the chosen representation it has the form:

$$
N = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}
\tag{2.5}
$$

which obeys the well known commutation relations:

$$
\left[N, a\right] = -a; \qquad \left[N, a^{+}\right] = a^{+};
\tag{2.6}
$$

and one finds that this operator can be expressed as a function of the creation and destruction operators as:

$$N = \frac{1}{1-\beta^2+\beta^4}\left[\left(-1+2\beta^2\right)a^+a + \left(-2+\beta^2\right)aa^+ + \left(2-\beta^2\right)I\right] \tag{2.7}$$

(I being the identity operator). The constructed algebra turns out to be trilinear in the creation and annihilation operators. There are no bilinear algebras with the requirement property of small violation of the PEP. It can be demonstrated [30] that the lowest dimensionality of this representation of the algebra with $\beta \neq 0$ is equal to three, while in the case of $\beta = 0$ the algebra has a unique representation for a and N coinciding with the two-dimensional representation of the usual Fermi algebra.

The IK paper end with a detailed analysis of a toy Hamiltonian which includes a term that explicitly violates the PEP and from here one calculates a transition probability per unit time $W\left(|1\rangle \to |2\rangle\right)$, which obviously depends of the violation parameter β. The Ignatiev and Kuzmin toy Hamiltonian is:

$$H = H_{free} + H_{in} = E\,N + \varepsilon V \tag{2.8}$$

where N is the particle number operator, V is the (adimensional) interaction energy

$$V = a^2a^+ + a^+a^2 + aa^+a + \text{h.c.} \tag{2.9}$$

E is the energy of the one-particle state and ε is the coupling constant (which has the dimension of energy). For a convenient application of perturbation theory they assume that $\varepsilon << E$. In the representation where the operator N is diagonal, H takes the form:

$$H = \begin{pmatrix} 0 & \varepsilon & 0 \\ \varepsilon & E & \beta\varepsilon \\ 0 & \beta\varepsilon & 2E \end{pmatrix}. \tag{2.10}$$

Through the calculation of eigenvalues and eigenfunctions of the Hamiltonian, they obtain the transition matrix from state $|i\rangle$ to state $|j\rangle$, which has the form:

$$M = \begin{pmatrix} 1 & -\varepsilon/E & 0 \\ \varepsilon/E & 1 & -\beta\varepsilon/E \\ 0 & \beta\varepsilon/E & 1 \end{pmatrix} \tag{2.11}$$

and finally the transition probabilities W_{lk}. In particular they find:

$$\begin{cases} W_{01} = 2(\varepsilon/E)^2 (1-\cos Et) \\ W_{02} = 0 \\ W_{12} = 2\beta^2 (\varepsilon/E)^2 (1-\cos Et) \end{cases} \tag{2.12}$$

obviously, the probabilities of the oscillations violating the PEP (W_{02} and W_{12}) are suppressed at least by the factor β^2 and vanish if $\beta = 0$. It can be noted that although the interaction Hamiltonian does not contain any intrinsically small parameter (the smallness of the ε/E *ratio* is irrelevant here because it is used only to make perturbation theory applicable) the transitions between certain states are strongly suppressed. This suppression is controlled by a small parameter which enters the commutation relations.

Extending these considerations to the electrons one can interpret the states $|1\rangle$ and $|2\rangle$ as states occupied by one or more electrons. Experimentally this means that anomalous atoms could exist (atoms with an anomalous filling of the electron shells) and/or that instability conditions could emerge (an electron can pass from $|1\rangle$ to $|2\rangle$ through the emission of an X-ray).

The IK theory introduces, in a simple and seemingly natural way, the violation of the Pauli principle.

When it first appeared in 1987 the IK model was immediately criticized by Govorkov [31], who noticed that in a true field theoretic context the IK model produces

states with negative norm, and thus cannot be generalized to be a true relativistic theory. Biedenharn, Truini and van Dam (BTvD) [32] also noticed that one can introduce two states, called $|e\rangle$ and $|\mu\rangle$, and creation and destruction operators which act on the tensorial product of the states according to the relations:

$$\begin{cases} b^+|0\rangle = \cos\theta_B|e\rangle + \sin\theta_B|\mu\rangle = |1\rangle \\ b^+|1\rangle = 2\sin\theta_B\cos\theta_B|e\rangle|\mu\rangle = \sin 2\theta_B|2\rangle \end{cases} \tag{2.13}$$

If now one replaces $\beta = \sin 2\theta_B$, one obtains the same algebra of the IK model, meaning that the IK model is equivalent to a theory which contains two similar fermions which mix, but remain different one from the other. Thus the BTvD result shows how the IK algebra is related to distinguishability of particles.

Okun, in one of his famous articles [33], noted that the BTvD states are ruled out by experiments (since the actual distinguishability of the states would lead to a doubling of the Bhabha cross-section in electron-positron colliders) unless the states are degenerate in mass, i.e. the particles are actually indistinguishable and there is no mixing.

Thus we see that IK model is perfectly viable but cannot be extended to become a true field theory and is already excluded by experiments. Still it remains an important conceptual tool to put experimental results in a theoretical frame, and continues to be a simple benchmark and an example for any more complete theory.

2.4 Different statistics

Generalized statistics was introduced in physics in the form of parastatistics as an exotic possibility extending the Bose and Fermi statistics. The general principles of quantum theory do not require that all particles be either bosons or fermions. This restriction requires an additional postulate which A.M.L. Messiah [4] named the

"symmetrizzation postulate" (see Par. 1.2). Since the symmetrizzation postulate is not an intrinsic part of the quantum theory, this postulate must be subjected both to theoretical study and to experimental test. Quantitative tests require theories in which the symmetrizzation postulate does not have to hold and in which the violation of the postulate is reflected in a parameter that departs from its standard value at which the symmetrizzation postulate and the spin-statistics connection do hold. It is certainly possible that violations of statistics are extremely small and requires high-precision tests to be observed.

2.4.1 Gentile's Intermediate Statistics

The first attempt to go beyond Bose and Fermi statistics seems to have been made by G. Gentile [34] who suggested an "intermediate statistics" in which at most *n* identical particles could occupy a given quantum state. In intermediate statistics, Fermi statistics is recovered for $n = 1$ and Bose statistics is recovered for $n \to \infty$; thus intermediate statistics interpolates between Fermi and Bose statistics. However, Gentile's statistics is not a proper quantum statistics, because the condition of having at most *n* particles in a given quantum state is not invariant under change of basis. For example, for intermediate statistics with $n = 2$, the state $|\psi\rangle = |k,k,k\rangle$ does not exist; however, the state $|\chi\rangle = \sum_{l_1,l_2,l_3} U_{k,l_1} U_{k,l_2} U_{k,l_3} |l_1,l_2,l_3\rangle$, obtained from $|\psi\rangle$ by the unitary change of single-particle basis, $|k\rangle' = \sum_l U_{k,l} |l\rangle$ does exist. By contrast, parafermi statistics of order *n* (see next section) is invariant under change of basis. Parafermi statistics of order *n* not only allows at most *n* identical particles in the same state, but also allows at most *n* identical particles in a symmetric state. In the example just described, neither $|\psi\rangle$ nor $|\chi\rangle$ exist for parafermi statistics of order two.

2.4.2 Green's Parastatistics

H.S. Green [29] proposed the first proper quantum statistical generalization of Bose and Fermi statistics. Green noticed that the commutator of the number operator with the annihilation and creation operators is the same for both bosons and fermions:

$$\left[n_k, a_l^+\right]_- = \delta_{kl} a_l^+ \ . \tag{2.14}$$

The number operator can be written:

$$n_k = (1/2)\left[a_k^+, a_k\right]_\pm + const. \ , \tag{2.15}$$

where the anticommutator (commutator) is for the Bose (Fermi) case. If these expressions are inserted in the number operator-creation operator commutation relation, the resulting relation is trilinear in the annihilation and creation operators. Polarizing the number operator to get the transition operator n_{kl} which annihilates a free particle in state l and creates one in state k leads to Green's trilinear commutation relation for his parabose and parafermi statistics,

$$\left[\left[a_k^+, a_l\right]_\pm, a_m^+\right]_- = 2\delta_{lm} a_k^+ \tag{2.16}$$

Since these rules are trilinear, the usual vacuum condition,

$$a_k \left|0\right\rangle = 0 \tag{2.17}$$

does not suffice to allow calculation of matrix elements of the a's and a^+'s; a condition on one-particle states must be added,

$$a_k a_l^+ \left|0\right\rangle = p\delta_{kl} \left|0\right\rangle \ . \tag{2.18}$$

Green found an infinite set of solutions of his commutation rules, one for each integer p, by giving an ansatz which he expressed in terms of Bose and Fermi operators. Let

$$a_k^+ = \sum_{p=1}^{n} b_k^{(\alpha)+} \quad , \quad a_k = \sum_{p=1}^{n} b_k^{(\alpha)} \tag{2.19}$$

and let the $b_k^{(\alpha)}$ and $b_k^{(\beta)+}$ be Bose (Fermi) operators for $\alpha = \beta$ but anticommute (commute) for $\alpha \neq \beta$ for the "parabose" ("parafermi") cases. This ansatz clearly satisfies Green's relation. The integer p is the order of the parastatistics. The physical interpretation of p is that, for parabosons, p is the maximum number of particles that can occupy an antisymmetric state, while for parafermions, p is the maximum number of particles that can occupy a symmetric state (in particular, the maximum number which can occupy the same state). The case p = 1 corresponds to the usual Bose or Fermi statistics. Later, Messiah and Greenberg proved that Green's ansatz gives all Fock-like solutions of Green's commutation rules. Local observables have a form analogous to the usual ones. From Green's ansatz, it is clear that the squares of all norms of states are positive, since sums of Bose or Fermi operators give positive norms. Thus parastatistics gives a set of orthodox theories. However, the violations of statistics provided by parastatistics are necessarily large. Parafermi statistics of order 2 has up to 2 particles in each quantum state. High-precision experiments are not necessary to rule this out for all particles we think are fermions.

2.4.3 The Ignatiev and Kuzmin model and "Parons"

Interest in possible small violations of the exclusion principle was revived by the paper of Ignatiev and Kuzmin [30] (see Chap. 2.3 above). Stimulated by the IK paper, Mohapatra and Greenberg showed that the IK oscillator could be represented by a modified form of the order-two Green ansatz. They suspected that a field theory

generalization of this model having an infinite number of oscillators would not have local observables and set about trying to prove this. They found that they could construct local observables and gave trilinear relations which guarantee the locality of the current [35]. They also checked the positivity of the norms with states of three or less particles. At this stage, they named these particles "parons" since their algebra is a deformation of the parastatistics algebra, and thought they had found a local theory with small violation of the exclusion principle. They did not know that Govorkov [31] had shown that any deformation of the Green commutation relations necessarily has states with negative squared norms in the Fock-like representation. For their model, the first such negative-probability state occurs for four particles in the representation of S_4 with three boxes in the first row and one in the second. They were able to understand Govorkov's result qualitatively as follows: since parastatistics of order p is related by a Klein transformation to a model with exact *SO(p)* or *SU(p)* internal symmetry, a deformation of parastatistics which interpolates between Fermi and parafermi statistics of order two would be equivalent to interpolating between the trivial group whose only element is the identity and a theory with *SO(p)* or *SU(p)* internal symmetry. This is impossible, since there is no such interpolating group. Although Greenberg and Mohapatra failed here, their work stimulated a further research that led to the concept of quons.

2.4.4 "Quons"

Quons [36], labeled by the real parameter q, allow a continuous violation of Bose and Fermi statistics for identical particles, including possible small violations. For $q = 1$, only the one-dimensional symmetric representation of the symmetric group occurs. For $q = -1$, only the one-dimensional antisymmetric representation occurs. For $-1 < q < 1$ all representations of the symmetric group occur. As $q \rightarrow 1$, the representations with more horizontal (symmetrized) boxes in their Young graphs are more heavily weighted; for $q = 1$ only the one-dimensional symmetric representation survives. Analogously, as $q \rightarrow -1$, the representations with more vertical (antisymmetrized) boxes in their Young graphs are more heavily weighted; for $q = -1$ only the one-dimensional antisymmetric

representation survives. Thus the departure of q from 1 for bosons or from -1 for fermions is a measure of the violation of statistics. Outside the interval [-1, 1], squares of norms of states become negative. As far as we know, quons are the only case of identical particles in three-dimensional space that have small violations of Bose and Fermi statistics. Unfortunately, the quon theory is not completely satisfactory. The observables in quon theory do not commute at space-like separation, i.e. quon theory is nonlocal (noncausal). If they did, particle statistics could change continuously from Bose to Fermi without changing the spin. Since space-like commutativity of observables leads to the spin-statistics theorem, this would be a direct contradiction. Kinematic Lorentz invariance can be maintained, but without space-like commutativity or anticommutativity of the fields the theory may not be consistent. For non-relativistic theories, however, quons are consistent.

The quon algebra for creation and annihilation operators is [37]:

$$a_k \; a_l^* - q \; a_l^* = \delta_{kl} \quad . \tag{2.20}$$

The parameter q can be different for different particles. The Fock-like representation that we consider obeys the vacuum condition:

$$a_k \left| 0 \right\rangle = 0 \, . \tag{2.21}$$

These two conditions determine all vacuum matrix element of polynomials in the creation and annihilation operators. In the case of free quons, all non-vanishing vacuum matrix elements must have the same number of annihilators and creators. For such a matrix element with all annihilators to the left and creators to the right, the matrix element is a sum of products of "contractions" of the form $\left\langle 0 \right| a \; a^* \left| 0 \right\rangle$ just as in the case of Wick's theorem for bosons and fermions. The only difference is that the terms are multiplied by integer powers of q. The power can be given as a graphical rule: put ○'s for each annihilator and ×'s for each creator in the order in which they occur in the matrix element on the x-axis. Draw lines above the x-axis connecting the pairs that are contracted. The

minimum number of times these lines cross is the power of q for that term in the matrix element. The physical significance of q for small violations of Fermi statistics is that $q_F = 2v_F - 1$, where the parameter v_F that indicates the magnitude of the violation appears in the two-particle density matrix:

$$\rho_2 = (1 - v_F)\rho_a + v_F \rho_s . \tag{2.22}$$

For small violations of Bose statistics, the two-particle density matrix is:

$$\rho_2 = (1 - v_B)\rho_s + v_B \rho_a \tag{2.23}$$

where $\rho_{s(a)}$ is the symmetric (antisymmetric) two-boson density matrix. Then $q_B = 1 - 2v_B$. As already stated above, for q in the open interval (-1, 1) all representations of the symmetric group occur. As $q \rightarrow 1$, the symmetric representations are more heavily weighted and at $q = 1$ only the totally symmetric representation remains; correspondingly, as $q \rightarrow$ -1, the antisymmetric representations are more heavily weighted and at q = -1 only the totally antisymmetric representation remains. Thus for a general n-quon state, there are $n!$ linearly independent states for -1 < q < 1, but there is only one state for $q = \pm 1$.

We emphasize something that some people find very strange: there is no operator commutation relation between two creation or between two annihilation operators, except for $q = \pm 1$, which, of course, correspond to Bose and Fermi statistics. Indeed, the fact that the general n-particle state with different quantum numbers for all the quons has $n!$ linearly independent states proves that there is no such commutation relation between any number of creation (or annihilation) operators. (An even stronger statement holds: there is no two-sided ideal containing a term with only creation or only annihilation operators.)

It seems likely that the Fock-like representations of quons for $|q| < 1$ are homotopic to each other and, in particular, to the $q = 0$ case, which is particularly simple. All bilinear observables can be constructed from the number operator, n_k, defined by:

$$\left[n_k, a_l^*\right]_- = \delta_{kl}\ a_l^*, \tag{2.24}$$

or the transition operator, n_{kl}, defined by:

$$\left[n_{kl}, a_m^*\right]_- = \delta_{lm}\ a_k^*. \tag{2.25}$$

Clearly, $n_k = n_{kk}$. For $q \neq \pm 1$, these operators are represented by infinite series in the creation and annihilation operators. Once Eq.(2.24 or 2.25) holds, the Hamiltonian and other observables can be constructed in the usual way; for example,

$$H = \sum_k \varepsilon_k\, n_k\ . \tag{2.26}$$

As in the Bose case, the transition or number operator for quons defines an unbounded operator whose domain includes states made by polynomials in the creation operators acting on the vacuum. What is different for quons is that the number operator has an infinite degree representation in terms of the annihilation and creation operators. (As far as we know, this is the first case in which the number operator, Hamiltonian, etc. for a free field are of infinite degree. Presumably this is due to the fact that quons are a deformation of an algebra and are related to quantum groups.) Several authors [38,39,40,41] gave proofs of the positivity of the squares of norms of quon states. It is amusing to note that, despite the lack of locality or antilocality, the free quon field obeys the TCP theorem and Wick's theorem (with added factors of q as mentioned above) holds for quon fields.

The simplest way to detect small violations of statistics is to find a state which either Fermi or Bose statistics would not allow [42]. For Fermi (Bose) statistics, this would be a state in which identical particles are not totally antisymmetric (symmetric). The path-breaking high-precision experiment of Ramberg and Snow [2] searched for transitions to a state in which the electrons of the copper atom are not totally antisymmetric. The failure to detect such transitions (above background) led to the following upper bound on violation of the exclusion principle:

$$\rho_2 = \frac{1}{2}\left(1-\beta^2\right)\rho_a + \frac{1}{2}\beta^2\rho_s \quad , \quad \frac{1}{2}\beta^2 \le 1.7\cdot 10^{-26} \quad . \tag{2.27}$$

ρ_2 is the two-electron density matrix, $\rho_{a(s)}$ is the antisymmetric (symmetric) two-electron density

matrix. For two electrons in different states ρ_2 can be expressed in terms of q of the q-mutator as:

$$\rho_2 = \frac{1}{2}(1-q)\,\rho_a + \frac{1}{2}(1+q)\rho_s \tag{2.28}$$

so the Ramberg Snow bound is:

$$0 \le (1+q)/2 \le 1.7\cdot 10^{-26} \quad . \tag{2.29}$$

2.5 An interesting experimental suggestion

A very particular interpretation of the PEP was proposed by E. Corinaldesi [16]. He conjectured that the symmetry of integral spin particles and the antisymmetry of half-integral spin particles under exchange are not kinematic principles but are rather the time-dependent consequences of interactions among the particles. In this way antisymmetrization, and hence PEP, is established only after a dynamical equilibration, and then one is entitled to use the commonsensical and non-quantum mechanical locution "newly captured electron." Hence, a freshly constituted ensemble of electrons may exhibit violations of the PEP, but as the ensemble ages, the violations become more and more infrequent.

An experiment was proposed by A. Shimony to test Corinaldesi's conjecture [43]. In Shimomy's proposal a beam of Ne^{+} ions, accelerated in a linear accelerator to 100th the velocity of light, is crossed by a beam of electrons from an electron gun at variable positions along the direction of flow of the ions. Some of the ions capture electrons, at a rate monitored by detectors sensitive to the photons emitted in the capture process. A PEP violating electron can make a transition to the doubly occupied 1s level, emitting a photon of approximately 1 keV. A rate of detection of such photons, which diminishes with the distance of the detector from the point of capture, and hence with the age of the ensemble, permits in principle the calculation of the equilibration constant of Corinaldesi's conjecture.

Chapter 3

The VIP experimental method

3.1 Experimental methods to test PEP for electrons

Experimental tests of the Pauli Exclusion Principle for electrons are done, basically, in two different ways:

- the search for the so-called "non-Paulian" atoms, as the paronic helium for example, characterized by an electron in the fundamental level and another one in an excited one, such as the spin and the space components of the wave functions are both antisymmetric, resulting in a global state of the $1s2s\,^1S_0$ for which the total wave function is symmetric – bringing to a violation of the symmetrization principle [13];

- the search for "anomalous", Pauli prohibited, X-ray transitions inside atoms, i.e. electron transitions to states already occupied by the maximum allowed number of electrons compatible with the Pauli exclusion principle.

It is this second method, already used by Ramberg and Snow [2], and derived from the original experiment of Goldhaber and Goldhaber [20], which is used in VIP. The basic idea is to introduce "new" electrons in a copper bar (new in the sense that the already existing ones in the copper bar had already all the time to perform the allowed and "prohibited" transitions, so no transitions would occur to be measured "now") and try to measure the K-series X-ray transitions (2p → 1s in particular) for which the 1s level is

already occupied by 2 electrons. Such transitions, obviously, would only be possible if the PEP is violated, see Fig. 3.1 for a schematic drawing.

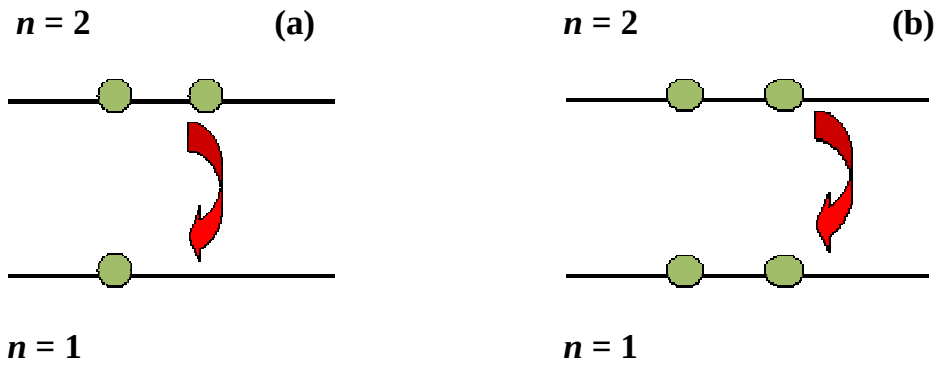


Fig. 3.1: (a) 2p → 1s allowed transition; (b) 2p → 1s transition violating the Pauli principle

The "anomalous" transitions are detected by their energy shift with respect to the normal transitions, because instead of the normal 8.05 keV energy of the 2p → 1s transition in Copper, one gets a value closer to the one corresponding to a $(Z - 1)$ atom (7.5 keV in this case) [44], which can be readily measured, especially by a good energy resolution detector such as VIP has.

Previous to VIP, this experimental method was applied with the goal to establish an upper limit for the probability of PEP violation for electrons in an experiment performed at the end of 80's: the Ramberg and Snow experiment, which we briefly present in the next section.

3.2 The Ramberg and Snow experiment

The first place to look for the violation of the exclusion principle was where it first made its appearance, inside atoms. In an atom with many electrons, part of the electrons are forced into high-energy orbits because all low-energy orbits are already occupied, following PEP. But if, violating PEP, two electrons could be in the same state, an electron in a high-energy orbit will occasionally get an opportunity to lower its energy by joining another electron in a lower-orbit, emitting the surplus energy as an X-ray. However, it is not as simple as that. Even if the PEP does not govern all electrons, QM will not allow an electron that is obeying the principle to start to disobey it. If we are to see electrons in the desirable inner orbits of atoms being joined by guests in defiance of Pauli's rule, the newcomers must already be lawbreakers and must come from outside the atom. One possible source of such non-paulian electrons might be an electric current flowing around the atom.

The first experiment for the research of the anomalous X-rays arising from a small violation of the PEP hwas done by Ramberg and Snow [2]. From a minimal set of assumptions they concluded that the probability that a new electron introduced into copper would form a mixed symmetry state with respect to the electrons already present in the copper sample, thus violating the Pauli Principle, is less than $1.7 \cdot 10^{-26}$. In order to quantify the proposed VIP improvements in sensitivity, let's briefly discuss the parameters of the Ramberg and Snow's experiment.

The experimental setup (Fig. 3.2) used as X-ray detector a proportional chamber, with an energy resolution of about 1200 eV of FWHM at 7 keV, placed above a copper bar in which a current of 30-40 A was circulated. The setup was installed in the basement of Muon building at Fermilab. The measurement lasted 2 months, during which data with and without (representing the background) circulating current was taken. The obtained spectra are shown in Fig.3.3.

According to Ramberg and Snow calculations the number of X-rays violating the Pauli principle expected is:

$$N_X \geq \frac{1}{2}\beta^2 N_{new}\frac{N_{\text{int}}}{10}\cdot\left(geometric\ factor\right), \tag{3.1}$$

where

$$N_{new} = \frac{1}{e}\int_T I(t)\,dt \tag{3.2}$$

represents the number of "new" electrons, and

$$N_{\text{int}} = diameter / m.f.p. = \frac{D}{\mu} \tag{3.3}$$

(*m.f.p.* = mean free path) is the number of scatters just below (in front of) the detector. Also they assume that the capture probability (aside from the factor $\sim\beta^2$) is greater than 1/10 of the scattering probability. Finally, geometrical factors arising from the circular aperture of the detector and the fact that it is situated on one side of the metal only yield a factor $\pi/8$. The fraction of current that is visible to the detector is λ/z, where z is the thickness of the copper strip and λ is the mean absorption length, in fact X-rays coming from any atomic transition in the metal may be absorbed before they reach the surface and are subsequently detected.

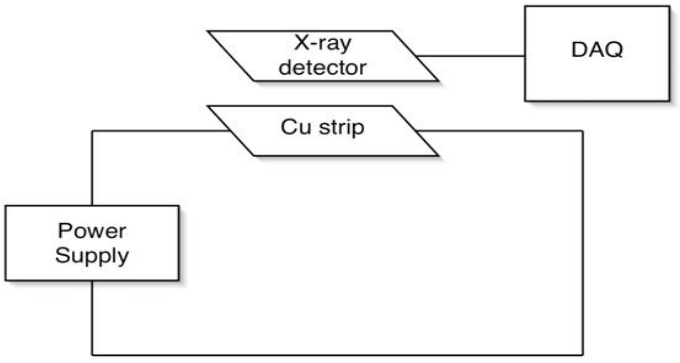


Fig. 3.2: The scheme of the experimental setup used in the Ramberg and Snow measurement

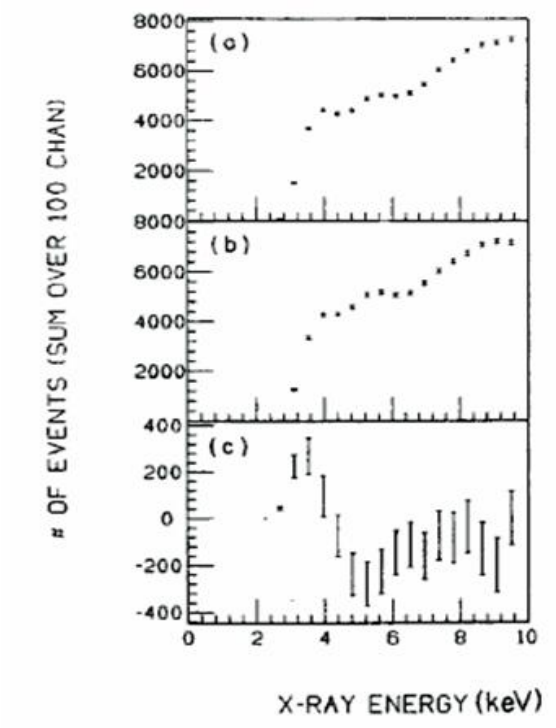


Fig. 3.3: The spectra measured by Ramberg and Snow [2]. (a) Number of triggers summed over 100 ADC channels, plotted versus equivalent X-ray energy with current-on in copper strip below X-ray counter. (Note the points are separated by 50 channels, so that only every other point is statistically independent). (b) Same as (a) but with no current passing through an identical strip of copper. (c) Difference between (a) and (b) after normalization at the 9.5 KeV point.

Taking into account the integrated circulated current ($\Sigma I \Delta t$ = 15.44·10^6 C), the geometry and the material characteristics (D = 2.5 cm, μ = 3.9·10^{-6} cm, λ = 11.16·10^{-4} cm, z = 0.15 cm, e = 1.6·10^{-19} C), the result of the measurement, in terms of the β-parameter yielded an upper limit for the probability of the Pauli principle violation of:

$$\frac{\beta^2}{2} \leq 1.7 \cdot 10^{-26} \,. \tag{3.4}$$

VIP plan is to improve on this limit by 3-4 orders of magnitude.

The energy of PEP violating transition in the RS experiment was estimated to be 7.6 keV – but not real calculation was performed to obtain this value. In the framework of VIP, we have performed accurate calculation of PEP violating transitions; in the next section these results are presented.

3.3 Calculation of the PEP violating transition energies

In our model we suppose that the non-paulian electron capture in Cu atoms occurs with a mechanism similar to normal X-ray decays. However, it is suppressed by the β^2 factor measuring the probability of Pauli-forbidden events, and shifted in energy towards the analogous decay of Ni atoms, due to the shielding of the extra 1s electron. Such a decay is in general a radiative process, whose cross-section can be expressed by the matrix element for matter-radiation interaction. In the dipole approximation we have:

$$\sigma_c = \frac{4\pi^2\alpha}{\hbar} E_{if} \left| \left\langle \Psi_f \left| \vec{\varepsilon} \cdot \vec{r} \right| \Psi_i \right\rangle \right|^2 \delta\left(\hbar\omega - E_{if}\right) \tag{3.5}$$

where α is the fine structure constant, $E_{if} > 0$ is the energy difference between the two levels, and $\hbar\omega$ is the photon energy. The energy conservation is expressed through the delta-function, and $\langle \Psi_f |$ and $| \Psi_i \rangle$ are the final and initial states, respectively. Quadrupole and higher order radial matrix elements are suppressed, in the X-ray region, by a factor 100 and more and are henceforth neglected. In our model $\langle \Psi_f |$ represents the final-decay state, which is always the 1s state, while $| \Psi_i \rangle$ can represent either a 2p state (K_α emission), a 3p state (K_β emission), or, finally, the Bloch state of the conduction electron, in which case the decay originates directly at K-*edge*.

Here we try to evaluate the X-ray energy in all three cases. We suppose that the orbitals have no time to relax during the transition process (the so-called “sudden approximation”), which is justified, as the energy differences involved in all transitions is

around $\Delta E \approx 8000$ eV. Thus, such transitions occur in a time of the order of $\Delta t \cdot \hbar \approx \Delta E \approx 10^{-19} s$, while the typical electron dynamics involves a much bigger time-scale, of the order 10^{-15} s. For what transition rates are concerned, a direct calculation is not possible, in the absence of any detailed information about the anomalous-electron wave-function. However, a rough estimate can be obtained for K_α - and K_β-emissions in the "normal" electron decay to a 1s hole in copper atoms. In this situation the transition rates are usually a factor 10 in favour of the K_α-line. More difficult is the direct comparison between K_α-emission and K-*edge* transition, as this latter is a characteristic of the solid state. One might argue that K_α-emission should be anyway favoured, because conduction electrons are at the Fermi energy, where the dipolar matrix element of Eq. (3.5) is strongly suppressed and a direct quadrupolar 3d→ 1s transition occurs, whose rate is usually negligible, compared to a K_α-line. For this reason, even though in the following we consider all transitions as possible, we take the K_α-emission as the most probable channel for the Pauli-violating-electron decay.

So we analyze the three possible decay channels of an eventual Pauli-principle-violating electron in a copper wire. The electron can reach the lowest 1s state either through a direct transition from Cu conduction band (*K-edge*), or by a 2p (3p) capture, with a subsequent K_α (K_β) emission. The energy of the produced X-ray depends on the path followed by the electron. Here we calculate the three different energy values and try to give a plausible estimate for the relative transition rates.

3.3.1 K_α emission

Referring to Fig.3.4, we are interested in the energy difference $\Delta E = E_3 - E_4$. If we consider the usual K_α transition, which is given by $\Delta E' = E_1 - E_2 \approx 8042$ eV[1], we can write

---

1 *Here we just performed a weighted average of the two values for $K_{\alpha 1}$ and $K_{\alpha 2}$ ($E_{K\alpha 1} = 8048$ eV and $E_{K\alpha 2} = 8028$ eV), neglecting spin-orbit splitting, with the usual statistical weight 2:1 = $K_{\alpha 1}$: $K_{\alpha 2}$.*

$\Delta E = \Delta E' - k$, where the term k takes into account of the extra correlations.

Such a term is necessarily positive because we expect a correction towards Ni K_α -line, due to the extra electron shielding of the nucleus potential. In order to evaluate k we express E_3 in terms of E_1 and E_4 in terms of E_2 as follows:

$$E_3 = E_1 + V_{1s\_Z} + V_{2p\_Z} + V_{1s\_e^-} + V_{2p\_e^-} + V_{1s\_1s} + V_{2p\_2p} + 2V_{1s\_2p} \tag{3.6}$$

$$E_4 = E_2 + V_{1s\_Z} + V_{2p\_Z} + V_{1s\_e^-} + V_{2p\_e^-} + 2V_{1s\_1s} + 2V_{1s\_2p} \tag{3.7}$$

Here $V_{nl\_n'l'}$ represent the Coulomb interaction between two electrons in the shells nl and $n'l'$, respectively, while V_{nl_Z} takes into account of the Coulomb attraction with the Z protons of Cu nucleus and $V_{nl\_e^-}$ counts the interactions with all electrons other than 1s and 2p, as represented in red in Fig.3.4. Notice that the two expressions (3.6) and (3.7) are not strictly correct, because orbitals are not frozen, and even neglecting the extra 1s and 2p electrons, the remaining electrons in the configuration E_3 are not exactly comparable to those of configuration E_1, because of the relaxation due to the extra shielding (and similarly for E_4 and E_2). However, as E_4 and E_3 are subtracted, and as both terms are characterized by this extra shielding, we can suppose that the neglected relaxation effects,
being of the same order of magnitude, cancel out, so that what is left is the correct X-ray energy. Thus, we get:

$$\Delta E = \Delta E' - (V_{1s\_1s} - V_{2p\_2p}) \approx 7669 \text{ eV} \tag{3.8}$$

where we used the values $V_{1s\_1s} \approx 483$ eV and $V_{2p\_2p} \approx 110$ eV for the direct Coulomb integrals. These latters, in their Slater F^{2n} form, are taken from the "Atomic Structure Calculations" [45]. The connection between F^{2n} and $V_{nl\_n'l'}$ can be found, e.g., in [46], and reads: $V_{1s\_1s} = F^0$ and $V_{2p\_2p} = F^0 - 2/25\ F^2$.

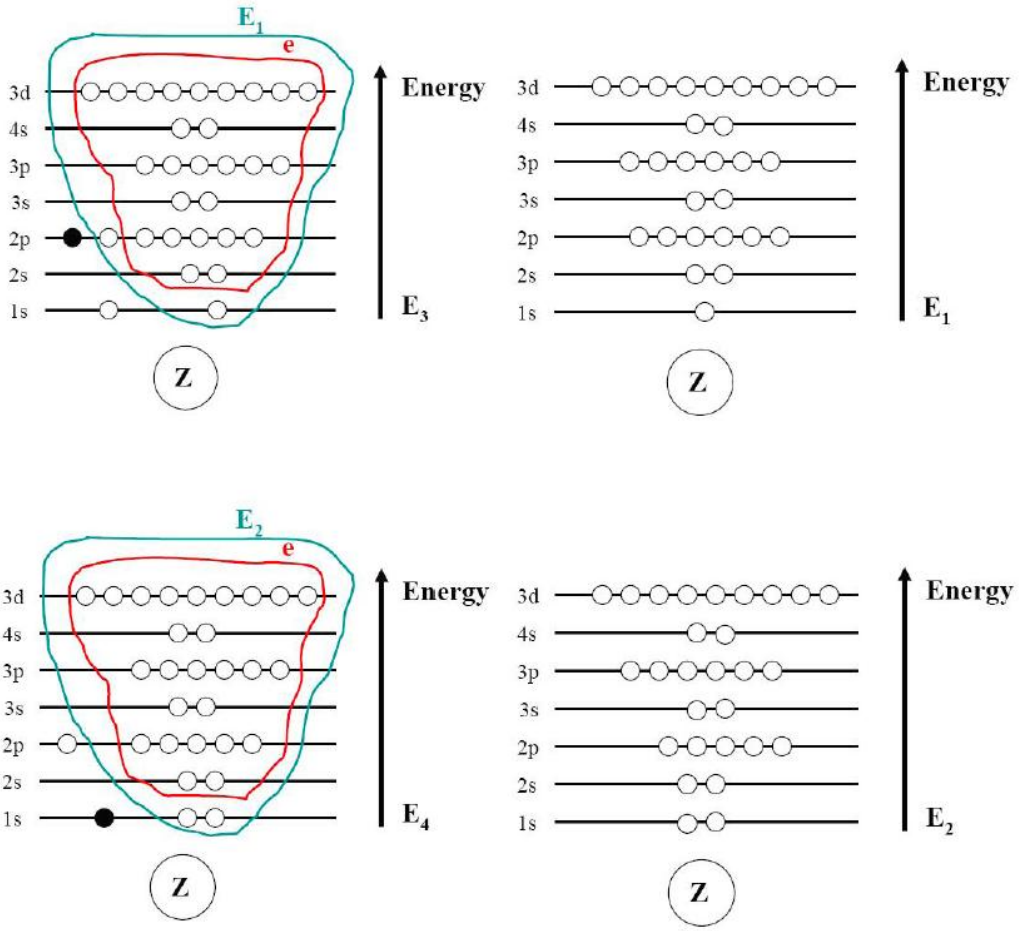


Fig. 3.4: Energy levels for K_α -emission. The ensemble of "spectator" electrons ($2s^2$); ($2p^5$); ($3s^2$); ($3p^6$); ($4s^2$); ($3d^9$) are called e. The subset of E_4 and E_3 corresponding, respectively, to E_2 and E_1 is also indicated.

3.3.2 K_β emission

We can follow exactly the same path as in the previous subsection, with the only difference that the"normal" energy for K_β emission is $E_{K\beta} \approx 8905$ eV, and that instead of V_{2p_2p}, one has to use $V_{3p\_3p} \approx 20$ eV. Thus in this case the corrected energy is:

$$\Delta E = \Delta E' - (V_{1s\_1s} - V_{3p\_3p}) \approx 8442 \text{ eV} . \quad (3.9)$$

3.3.3 *K – edge* decay

Referring to Fig. 3.5, we can perform the same calculation as before, with the warning that now we are dealing with Cu atoms, instead of $Cu1^+$ ions, when comparing with the "normal" case. Due to this and to the fact that the decaying electron comes from an outer valence shell and not from a core one, we expect a better result within our scheme of frozen orbitals.

Again, we are interested in the energy difference $\Delta\widetilde{E} \equiv \widetilde{E}_3 - \widetilde{E}_4$, while the usual K-*edge* transition is given by $\Delta\widetilde{E}' \equiv \widetilde{E}_1 - \widetilde{E}_2 \approx$ 8979 eV. If we write, like before, $\Delta\widetilde{E} \equiv \Delta\widetilde{E}' - \widetilde{k}$, where the term $\widetilde{k}$ takes into account of the extra correlations, and suppose again that the state of all electrons labelled by *e* (in red in Fig.3.5) is not much affected by the presence of an extra 1s electron, we can express $\widetilde{E}_3$ in terms of $\widetilde{E}_1$ and $\widetilde{E}_4$ in terms of $\widetilde{E}_2$ as follows:

$$\widetilde{E}_3 = \widetilde{E}_1 + V_{1s\_Z} + V_{1s\_e^-} + V_{1s\_1s} + V_{1s\_3d} \quad (3.10)$$

$$\widetilde{E}_4 = \widetilde{E}_2 + V_{1s\_Z} + V_{1s\_e^-} + 2V_{1s\_1s} \quad (3.11)$$

Thus, with the same idea of the previous two subsections, we get:

$$\Delta E = \Delta E' - (V_{1s\_1s} - V_{1s\_3d}) \approx 8534 \text{ eV}. \quad (3.12)$$

Again, we used the values $V_{1s\_1s} \approx 483$ eV and $V_{1s\_3d} \approx 38$ eV eV for the direct Coulomb integrals, as taken from [45]. The connection with F^{2n} in the case of V_{1s_3d} is taken from

[41], without exchange term, as appropriate for a Pauli-principle-violating electron: $V_{1s\_3d} = F^0$.

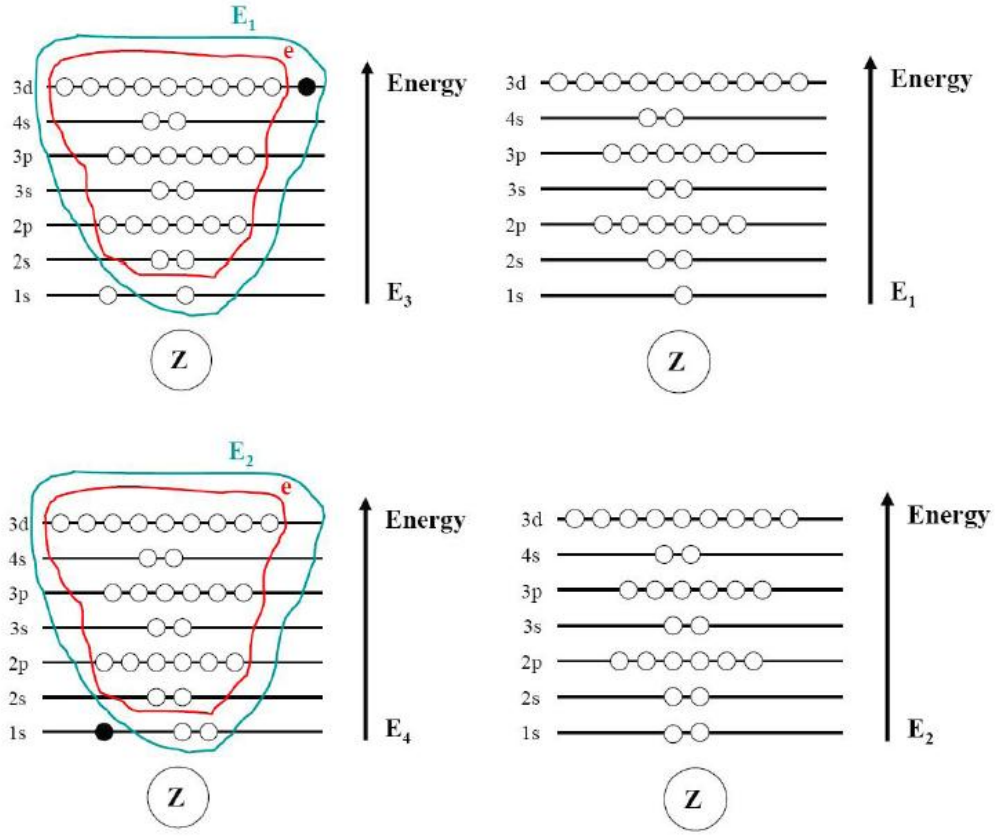


Fig. 3.5: Energy levels for K-edge transition. The ensemble of "spectator" electrons ($2s^2$); ($2p^6$); ($3s^2$); ($3p^6$); ($4s^2$); ($3d^{10}$) are called e. The subset of E_4 and E_3 corresponding, respectively, to E_2 and E_1 is also indicated.

3.3.4 Calculation results

Our results, for the anomalous K_α, K_β and *K-edge*, are, respectively: $\Delta E_{K\alpha}$ = 7669 eV; $\Delta E_{K\beta}$ = 8442 eV; $\Delta E_{K\text{-}edge}$ = 8534 eV. The approximations introduced here do not allow to get an uncertainty less than some tens of eV, and for this reason this calculation

should be considered just as a first step. However, we believe that the importance of the present approach lies in its handiness, that allows a simple comprehension of the related physical ideas, even though to the detritement of the precision. In the next paragraph we shall perform a more precise, ab-initio evaluation of the X-ray energies by means of a Dirac-Fock numerical code, which should improve the uncertainty of a factor of about ten (from several tens of eV to several eV).

3.3.5 Calculation using the Dirac – Fock method

Since we are considering a non-antisymmetrized electron then we can follow the case of a muonic atom. The reported values were obtained from Dirac-Fock calculations, as previously performed for a variety of muonic atoms by Mallow et al. [47]. Dirac-Fock approximation takes into account the relativistic corrections that are relevant for heavy ions, as we have considered our sample (Z_{Cu} = 29). Thanks to a numerical code it is possible to perform relativistic calculations for atoms, using the "multiconfiguration" approximation for the *N*-electron wave function. This scheme proceeds through the optimization of the parameters during a self-consistent process [48]. Muon and electron can be treated in analogous way through the self-consistent field theory, to obtain wave functions and energies. The most commonly used approximation to treat the Hamiltonian of an *N*-electron system is the so called "no pair" approximation that explicitly excludes electron-positron pairs. The effects of the Breit operator, the Lamb shift and all sort of radiative corrections are included as perturbations.

A lot of results from muonic atom experiments are explained, in first approximation, on the basis of two-body (muon and nucleus) "hydrogenic" models, with the muon so close to the nucleus that the effect of the atomic electrons could be ignored. Recent experiments show that these models are not entirely satisfactory (for example the transition intensities are dependent on the atomic charge state).

We can follow the self-consistent Dirac-Fock method to determine the structure of muonic atoms with all the electrodynamic corrections to the electronic energy, as

implemented in a numerical code by Desclaux [47,48]. Then we can put a different mass for the exchange between muon and electron.

We consider the energy eigenvalue equation for a many-electron atom:

$$H_I \Psi = E \Psi \tag{3.13}$$

where, in the relativistic case, H_I is the usual approximate Breit-Dirac Hamiltonian:

$$H_I = \sum_{j=1}^{N} \left[\vec{\alpha} \cdot c\vec{p}_j + V_n(r_j) \right] + \sum_{i \neq j} \left(\frac{1}{r_{ij}} + H_B(i,j) \right) \tag{3.14}$$

(N is the number of electrons). The first summation includes kinetic energy, spin-orbit interaction and the electron-nucleus Coulomb interaction. The potential V_n takes into consideration a finite-nuclear-charge distribution and not a non-physical nuclear point-charge one. The $1/r_{ij}$ term is the electron-electron Coulomb repulsion and $H_B(i,j)$ is the Breit operator which take into account magnetic interaction and retardation. This last is used only at the first-order perturbation to correct the relativistic interaction between the electrons. Another correction, as a perturbation for heavy atoms, is the Lamb-shift. Some approximations are made: antisymmetrizzation of the total wave function, central-field, application of the variational principle.

Adding to the N-electron atom a negative muon in a bound state, we had to add to the atomic Hamiltonian those terms which corresponds to the muon's kinetic energy and its Coulomb interaction with the nucleus and the electrons:

$$H_\mu = H_I + \vec{\alpha} \cdot c\vec{p}_\mu + \beta E_0(\mu) + V_n(r_\mu) + \sum_{K=1}^{N} \frac{1}{r_{\mu k}} \tag{3.15}$$

Since muon and electron are distinct particle there is no muon-electron exchange interaction. So, it is possible treat these systems as ordinary atoms.

To solve these coupled integro-differential equations a numerical code was developed by Desclaux and collaborators [47,48]. We used this code to perform our calculations, as shown in Tab.1.1:

Transition	Initial energy	Final Energy	Transition energy	Radiative transition rate (s-1)	Multipole order
core:	(1s)2(2s)2(3s)2(2p*)2(3p*)2(2p)4(3p)4(3d*)4(3d)6				
2p1/2-1s1/2	-45799	-53528	7729	2.63E+14	E1
2p3/2-1s1/2	-45780	-53528	7748	2.56E+14	E1+M2
3p1/2-1s1/2	-44998	-53528	8530	2.78E+13	E1
3p3/2-1s1/2	-44996	-53528	8532	2.68E+13	E1+M2
2p1/2-2s1/2	-44998	-45934	936	1.64E+12	E1
2p3/2-2s1/2	-44996	-45934	938	1.49E+12	E1+M2

Tab. 1.1 – Transition with "Pauli-violating electron" in Cu.

We recall now the values of the energy of emitted "Pauli-principle-violating" X-ray in the three most plausible decay channels calculatred with the previous method, respectively: $\Delta E_{K\alpha} = 7665$ eV, $\Delta E_{K\beta} = 8442$ eV, $\Delta E_{K\text{-}edge} = 8534$ eV.

Our results are in good agreement with results in Tab. 1. From the Tab.1 we can see that the K_α emission's transition energy, $(\Delta E_{K\alpha})_{DF}$, is about 7742 eV, given by its statistical ratio due to the spin ($2p_{1/2}$: $2p_{3/2}$). In an analogous way we determine $(\Delta E_{K\beta})_{DF}$ ≈ 8531 eV. Both $\Delta E_{K\alpha}$ and $\Delta E_{K\beta}$ differ from our values of some tens of eV, in particular:

$$\{[(\Delta E_{K\alpha})_{DF} - \Delta E_{K\alpha}] / (\Delta E_{K\alpha})_{DF}\} \approx 9.95 \cdot 10^{-3} \approx 1\ \% \qquad (3.16)$$

$$\{[(\Delta E_{K\beta})_{DF} - \Delta E_{K\beta}] / (\Delta E_{K\beta})_{DF}\} \approx 0.010 \approx 1\ \% \qquad (3.17)$$

Also, it is interesting to note the radiative transition-rate values, as an indication of the probability of the decay: K_α emission is more probable than K_β one (they differ of one order of magnitude). This induces us to draw our attention on K_α emission rather than

K_β one. For what concerns the *K-edge* transition, it is more difficult to calculate the radiative transition-rate because it is not a purely atomic process, but a solid state one, thus implying that an electron from infinity falls to 1s orbital. This makes extremely difficult the estimation of its transitions rate with the code of [48].

Chapter 4

VIP feasibility study

4.1 First experimental test of CCD use at the Neuchatel laboratory

The VIP aim is to improve the RS limit of the PEP violation for electrons by about 3-4 orders of magnitude. In order to do so, VIP taked advantage of the next two key-elements:

- much better X-ray detectrors: Charge-Coupled Device
- installation and data taking in a reduced-background environment: the Gran Sasso underground laboratories.

Before having built VIP, however, both these key-elements were experimentally tested, in the so-called VIP feasibility study, the results of which are presented in what follows.

A feasibility measurement, applying the same method of Ramberg and Snow, was performed in 1998 for a period of three months, in the basement of the Neuchatel laboratory [49]. The setup used 3 CCD's (Charge-Coupled Device) as X-ray detectors, with an energy resolution of 400-500 eV FWHM, already about 3 times better that the Ramberg and Snow setup. The total measurement time was of 1856 hours:

- 1524 hours with a circulating current of 5, 10 or 15 A;
- 332 hours without current – to measure the background.

In Figs. 4.1 and 4.2 details of the setup are shown. The measured spectra for one CCD as an example are shown in Fig. 4.3.

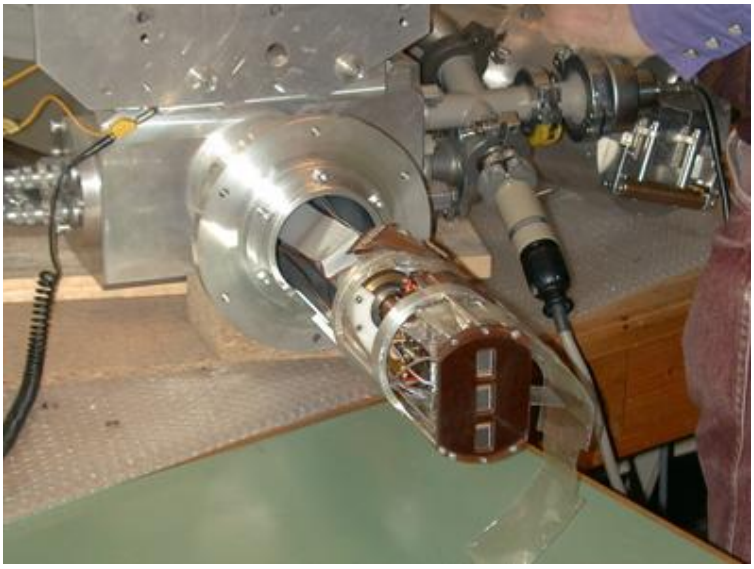


Fig .4.1: The test setup used at Neuchatel – detail of the CCD detectors.

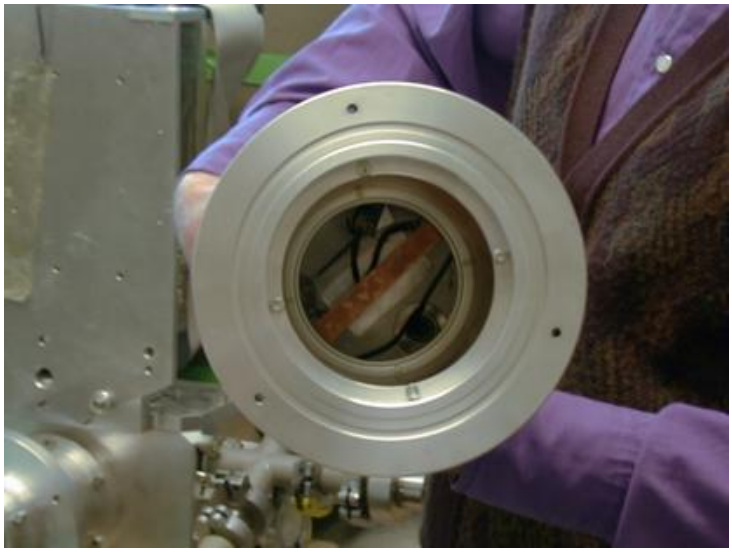


Fig 4.2: The test setup used at Neuchatel – detail on the copper bar.

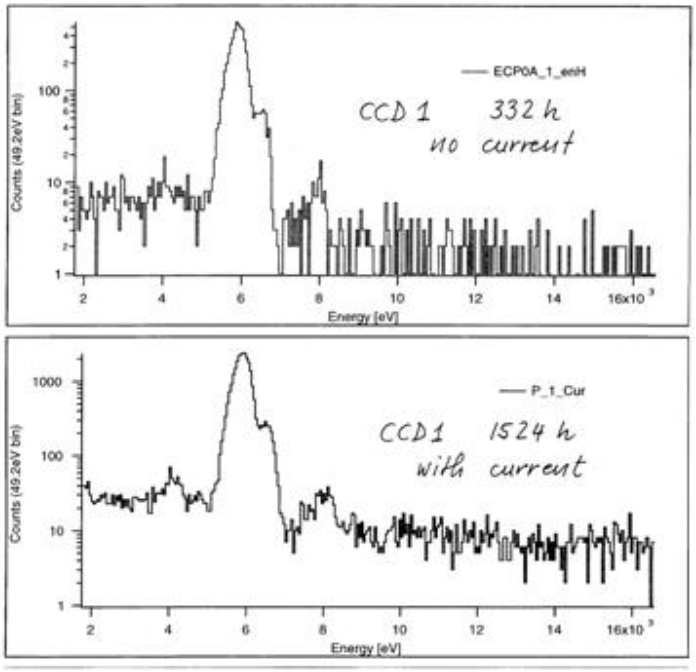


Fig. 4.3: The measured spectra with the test setup at Neuchatel.

With the improved detector resolution and geometry, taking into account the integrated circulated current, the geometry and the material characteristics, an upper limit of

$$\frac{\beta^2}{2} \leq 0.95 \cdot 10^{-27} \tag{4.1}$$

was obtained, about an order of magnitude better than the Ramberg and Snow measurement.

4.2 Background measurement at LNGS with a Test Setup

A low background is essential to VIP to achieve a high sensitivity, because the statistical significance of the subtracted spectrum is a function of the statistical fluctuations. Such a background reduction is not possible by use of trigger system since the CCDs (as will be shown in Chapter 5) are not triggerable detectors; then, what can be done is to install the apparatus in an already reduced background envirinment – namely in an underground laboratory.

In the period November 2004 – April 2005 an intense activity was dedicated to measurements of the background in the Laboratory at Frascati and at the LNGS Laboratories. A 2 CCD test setup was built, containing 2 CCD-55 of the same type as those used in the VIP setup. The CCDs are contained in an aluminium cylinder, with a quality (purity) very similar to the one of the VIP setup. In Fig. 4.4 and Fig. 4.5 details of the test setup are shown, while in Fig. 4.6 the 2 CCD test setup, as installed in the Laboratory in Frascati, is shown. Moreover, the materials of the setup are of the same quality and purity as those from the setup.

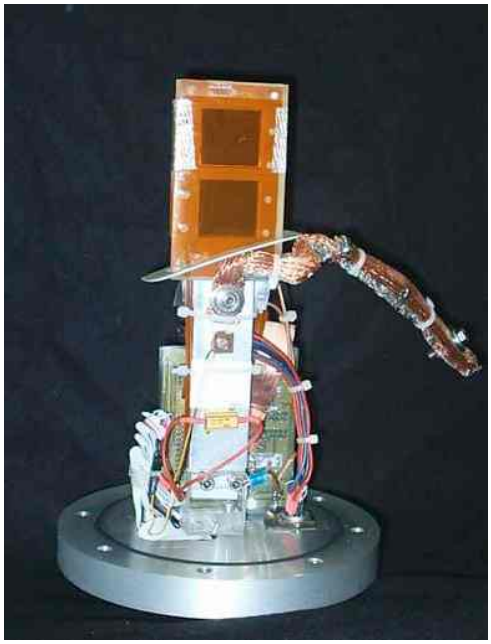


Fig. 4.4: The 2-CCD test apparatus used to take background data in the Gran Sasso underground laboratory in february 2005. The CCD's are visible in the upper part of the structure, and just below is the control electronics and the support structure that is maintained at low temperature (about 150 K).

Fig. 4.5: The 2 CCD housing in aluminium with teflon shielding inside.

Fig. 4.6: The 2-CCD test setup installed in the laboratory.

4.2.1 2 CCD Test Setup at LNF

In the period November-December 2004, a series of background measurements with the 2 CCD test setup was performed in the laboratory. The measurements were done without and with shielding, and were preceded by a calibration measurement. The calibration measurement was done with a Fe source, and the resulting spectrum, calibrated in energy, is shown in Fig. 4.7.

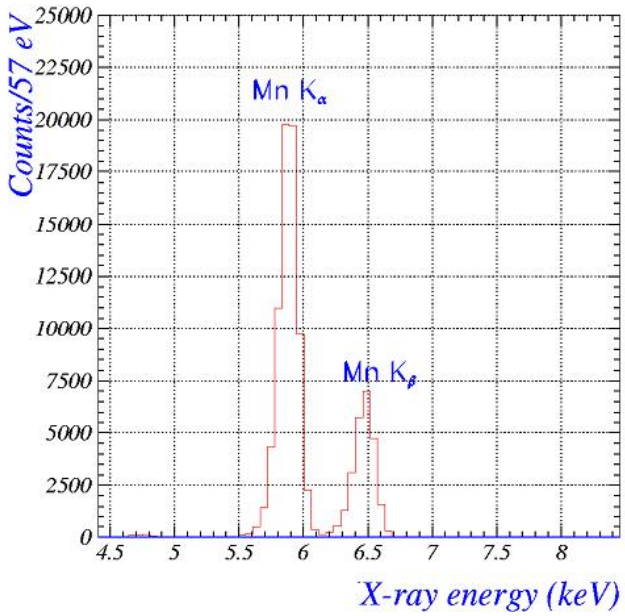


Fig. 4.7: Energy calibration – measurement with an Iron source performed in the LNF laboratory.

A second calibration measurement was performed at the end of the measurements in the laboratory, and the result was the same as the one reported in Fig. 4.7, checking in this way the stability of the energy calibration. The energy resolution was of 180 eV (FWHM) at about 6 keV.

The measurement performed with the 2 CCD test setup in the laboratory, without shielding, lasted 65 hours. The obtained spectrum is shown in Fig. 4.8.

The analysis of the spectrum shows a background reduction in the region of interest (7.4 – 7.9 keV) of a factor about 1.5 better than in Neuchatel. This reduction is a compound consequence of the lack of the Copper line from the spectrum; and a better energy resolution compared to the Neuchatel case (~500 eV in Neuchatel vs 160 eV in the current setup).

Soon after a measurement with an optimized shielding in the lab conditions was performed. The shielding was composed of an external layer of Lead (10 cm thick) and an internal layer of Copper (5 cm). The setup was enclosed in a plastic housing flushed with nitrogen, in order to remove possible Radon contaminations. The measurement lasted 325 hours, and the spectrum is shown in Fig. 4.9.

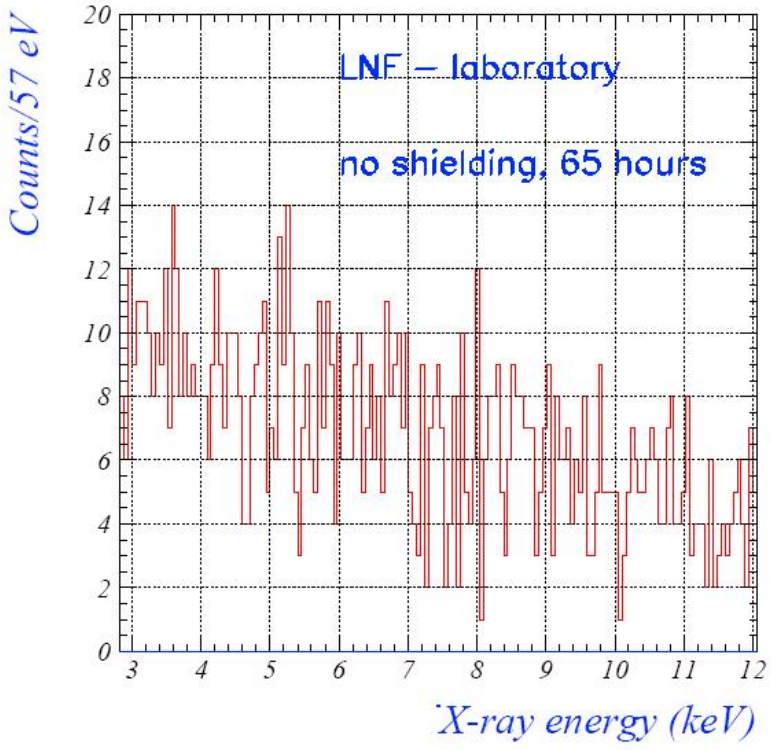


Fig. 4.8: X-ray spectrum obtained in the laboratory without shield.

A background reduction factor of about 5.2 with respect to no-shielding situation was obtained.

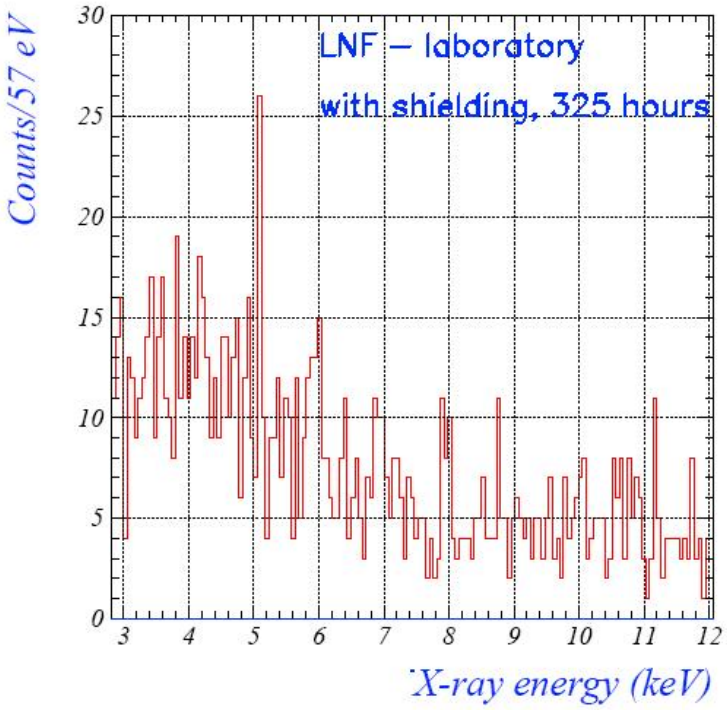


Fig. 4.9: X-ray energy spectrum obtained in the laboratory, with shielding.

4.2.2 2 CCD Test Setup at LNGS

In December 2004 the 2 CCD test setup was transported to the LNGS and installed in the barrack allocated to VIP.

The installation was followed by a 3-days period of DAQ – performed without shielding. In the first months of 2005, a measurement with a preliminary shielding was done.

The measurement, performed in the period 20-23 December, gave a background reduction factor of about 4.5 with respect to the situation in the laboratory without

shielding. A picture of the setup installed at LNGS without shielding is shown in Fig.4.10.

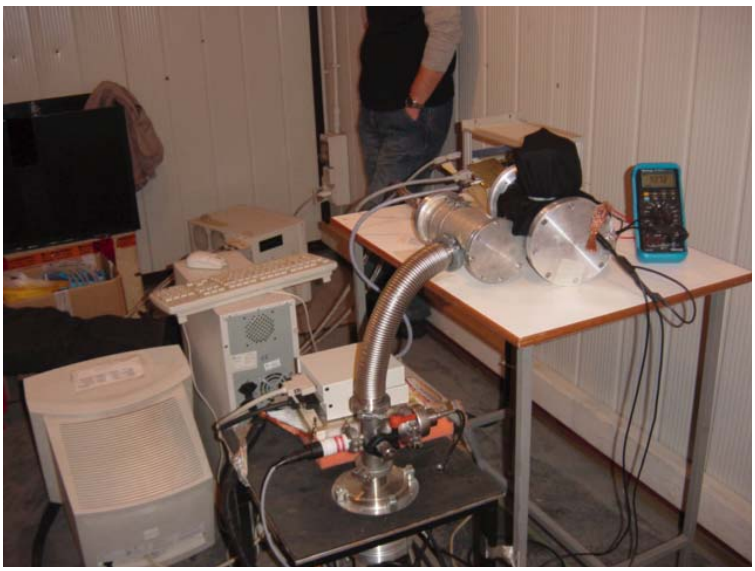


Fig. 4.10: The 2 CCD test setup, without shielding, installed at LNGS.

In the period 21 February – 28 March, a measurement of the background at LNGS with a preliminary shielding was performed. The shielding was done in an external layer of lead and an internal one of copper, 5 cm thickness each. In Fig. 4.11 and Fig. 4.12 some pictures of the shielded 2 CCD test setup are shown.

The measurement lasted 832 hours, and the obtained spectrum is shown in Fig. 4.1

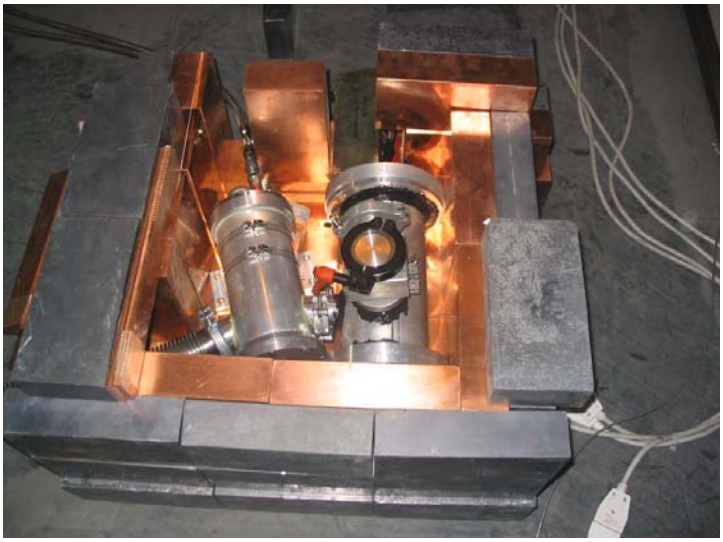


Fig. 4.11: The 2 CCD test setup with shielding at LNGS – detail during construction.

Fig. 4.12: The test apparatus used to measure the environmental background in the Gran Sasso laboratory in february 2005. The two-CCD prototype is housed inside the lead- and copper brick shielding. The CCD's are cooled at about 150 K, and the whole apparatus operates in vacuum: the vacuum pumps are visible on the right.

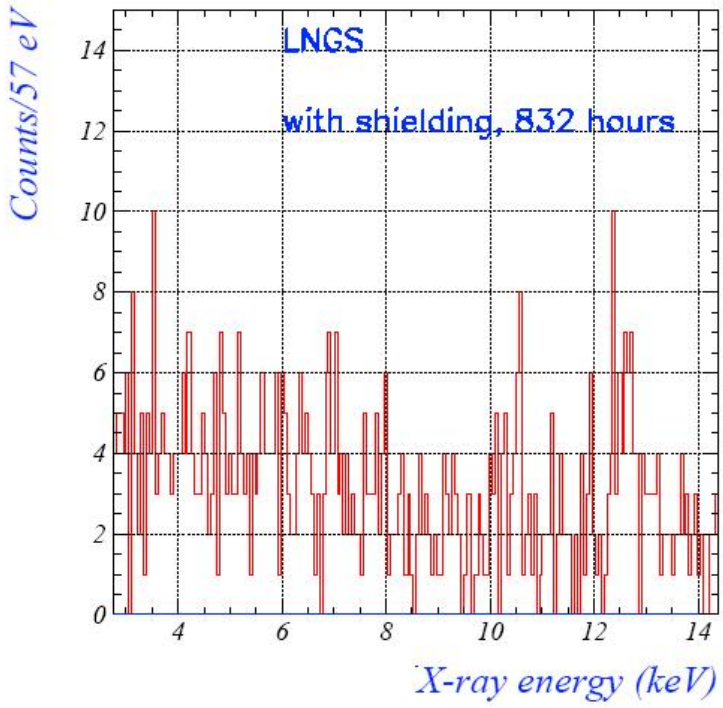


Fig. 4.13: X-ray spectrum obtained at LNGS with shielding.

To summarize a background measurements with a 2 CCD test setup were performed in the DEAR laboratory at Frascati and at the LNGS laboratories, without and with shielding. A comparison between the normalized spectra such obtained is realized in Fig. 4.14.

The results of the measurements was a factor about 50 in the background reduction, at LNGS with respect to the Neuchatel measurement, and a factor about 10 with respect to LNF laboratory. A more improvement of the background reduction is feasible, by:

- design of a special VIP shielding geometry, which better covers all the solid angle;
- use of specially treated shielding materials (lead and copper);
- flushing with nitrogen .

Moreover, by working at LNGS an improved background quality is obtainable: day/night and seasonal stability, not obtainable when working in the laboratory.

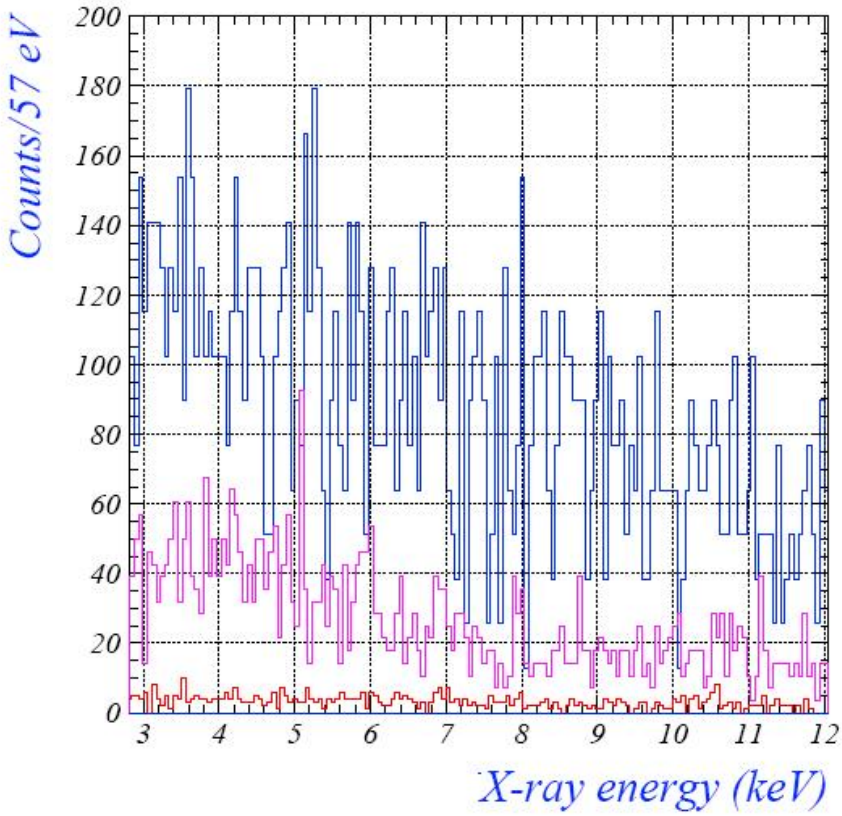


Fig. 4.14: Comparison between normalized background spectra obtained with the 2 CCD setup. Uppermost histogram (bleu): the result obtained in the laboratory without shielding; middle histogram (violet) the results obtained in the laboratory with shielding; lowermost histogram (red) the results obtained with a preliminary (partial) shielding at LNGS.

Chapter 5

The VIP experiment

5.1 The VIP setup

The VIP setup is schematically shown in see Fig.5.1.

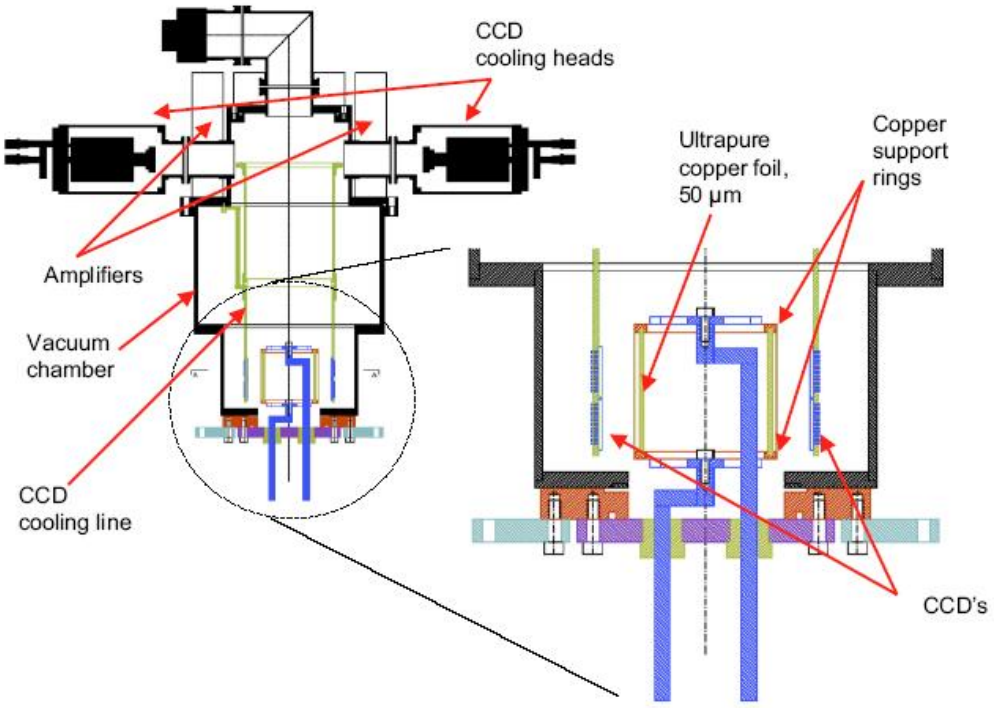


Fig. 5.1: Layout of the VIP vacuum chamber: the inset shows an enlargement of the copper cylinder and CCD layout.

As key-elements it contains a copper target (Fig. 5.2), where a current is circulated, and X-ray CCD detectors surrounding it – ready to measure the X rays generated in the Pauli violating transitions. The copper target is a thin layer with 45 mm radius, 88 mm height and 50 μm thickness, to avoid that the produced X-rays will be absorbed again by the copper (50 μm is comparable with the "absorption length" of the X-ray in the copper). This solution provides a uniform distribution of the surface current and, at the same time, a higher surface current density with respect to the copper bar "à la Ramberg and Snow", thus optimizing the acceptance and the detection efficiency for the X-rays. The target is surrounded by 16 equally spaced CCDs of type 55 made by EEV [50,51]; they are at a distance of 23 mm from the copper cylinder, grouped in units of two chips, one above the other.

The setup is enclosed in a vacuum chamber, and the CCDs are cooled to about 168 K by the use of a cryogenic system. The current flows in the thin cylinder made of ultra-pure copper foil at the bottom of the vacuum chamber. The CCD's surround the cylinder and are supported by cooling fingers that project from the cooling heads in the upper part of the chamber. The CCD readout electronics is just behind the cooling fingers; the signals are sent to amplifiers on the top of the chamber. The amplified signals are read out by ADC boards in the data acquisition computer. More details on the CCD-55 performance, as well on the analysis method used to reject background events, can be found later. Fig. 5.3 shows the CCD system together with its ancillary system (read-out electronics and cooling).

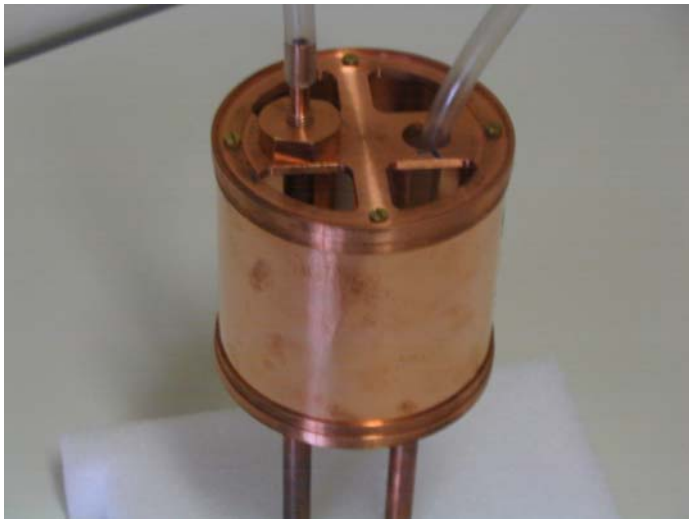


Fig. 5.2: The copper "target" used in VIP.

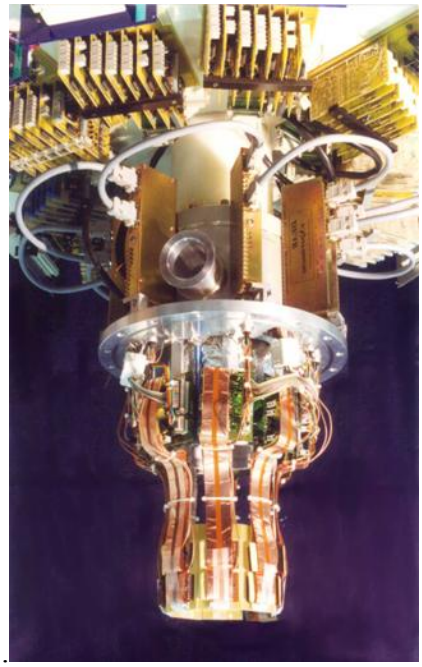


Fig. 5.3: The 16 CCD-55, together with the cryogenic and read-out systems.

5.2 CCDs detectors

The CCDs (Charge Coupled Devices) are used as soft X-ray detectors. The advantages of using them lie in their excellent intrinsic position resolution together with a good energy resolution, good efficiency and interesting background-rejection capabilities. CCDs are solid state detectors with a pixel structure, have to be operated at 150÷180 K and require a sophisticate electronic read-out system [52,53,54].

The CCD chip is an array of Metal-Oxide-Semiconductor capacitors (MOS capacitors), each capacitor represents a pixel. The separation of the columns is done with a channel stop (a NPN diode structure between each column). The rows are separated by potential wells which are produced by an array of electrodes mounted on top of the depletion layer. Three electrodes are needed for one pixel: a positive voltage is applied to two of them and a negative voltage to the third (see Fig. 5.4). For the read-out the accumulated electrons are shifted to a special read-out line at the end of the columns, electrode by electrode, by applying three AC voltages with the proper phases to the electrodes of the pixels. In this way the charges can be transferred row by row to a serial output register.

CCDs have one shortcoming: due to their short operation it is impossible to read them out in coincidence with a fast trigger signal. Instead they are operating in an accumulation mode where the read-out has to be done after regular time intervals that are usually several minutes long (for our detectors 5 minutes). Because events are collected into the CCD for a finite time and then read-out, this collection time must not be too long, to avoid that pixel or one of its neighbors has a second hit during that period.

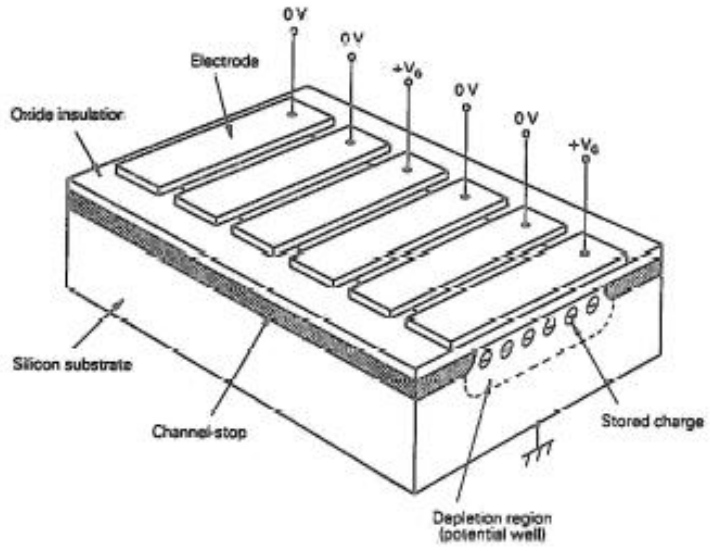


Fig. 5.4: Section of a CCD detector; charge signal transfer from one pixel to the next.

5.2.1 Software for X-ray/charge-particle identification and background rejection

How to eliminate the unwanted events and deduce the X-ray position spectra? Events are collected into the CCD for a finite time and then are read-out; this collection time must not be too long to avoid that a pixel may be hit twice during that period. The energy content of each pixel is analyze in order to identify the X-ray event of the proper energy thus to reject the background. In most of the cases the energy of an X-ray is deposited into one single pixel. Sometimes, however, the energy is split between two or more pixels (≈ 30% of cases). Background events like neutrons, gammas and charged particles, on the other hand, are usually split into many pixels. This feature allows for a rather powerful rejection of the background. Indeed, a good event is characterized by an isolated pixel containing the proper energy. In order to take into account the split of the

events into more than one pixel, we also consider two, three or four neighbor pixel hits surrounded by empty pixels. The sum of the energies of these pixels must also add to the correct energy. In this way large pixel assemblies are eliminated. Note that, in addition, every pixel has a low energy content corresponding to its noise and dark current (the thermal noise is negligible at the operating temperature for the CCDs). This noise can be seen as a Gaussian noise peak in the very-low energy region of the energy spectrum (the peak location defines the offset which is subtracted from each energy spectrum). Thus, an empty pixel means, actually, a pixel with an energy content below a given limit. So, for a single-pixel event, if all eight surrounding pixels are below this limit, the event is considered an X-ray.

As regards our analysis we have decided to consider as good events all single- and double-pixel events. Double-pixel events are X-ray events where the total charge produced by the X-ray is spread over two pixels; they are produced by X-rays that are absorbed at the edge of a pixel. Moreover, X-ray events with the charge distributed over more than two pixels are extremely rare [55].

5.3 VIP DAQ slow control and monitoring

The read-out of the several million channels of the CCDs is accomplished through a very reliable DAQ system, while a continuous monitoring will keep under control the relevant parameter needed to assure the good functioning of the VIP setup. Examples of important quantities to be monitored are:

- the CCDs temperature, which should be stabilized at about 170 K in order to minimize the dark current and to obtain the best energy resolution;

- the insulation vacuum in order to allow for an efficient cooling and to prevent the formation of ice, which would decrease the CCD quantum efficiency and could possibly produce irreversible damages.

Moreover, there are few controls that has been implemented to talk to the experimental setup (vacuum pump, cryogenic compressor, DAQ electronics). The DAQ, as well as the Slow Control and Monitoring, operating remotely through the WAN. In what follows we describe our adopted solution, based on commercial hardware and software.

5.3.1 Slow control and monitoring hardware

An IBM PC AT compatible computer is installed in the experimental area expanded with a National Instruments AT-MIO-16L-9 multifunction Data Acquisition (DAQ) board. The AT-MIO-16L-9 contains a 12-bit ADC with up to 16 analog inputs, two 12-bit DACs with voltage outputs, eight lines of TTL-compatible digital I/O, and three 16-bit counter/timer channels for timing I/O.

The AT-MIO-16L-9 is interfaced with a resistance temperature device (RTD) signal conditioning accessory for National Instruments DAQ boards, namely SC-2042-RTD. The SC-2042-RTD provides eight channels of current excitation (1 mA), and allows the measurement of the voltages on the PT-100 thermoresistors mounted on the CCD mechanical support. The temperature of the CCD chips is calculated using a polynomial fit of the standard PT-100 voltage vs. temperature dependence. The isolation vacuum pressure is monitored using a Balzers IKR 050 vacuum gauge together with a Balzers vacuum measurement and control unit of the type TPG 300 or IMG 300 connected to one of the AT-MIO-16L-9 ADC channels. Concerning the on/off switches, a custom made box, with logic-switched 220V relays and 0, 5V logic signals is interfaced with the Digital I/O lines of the AT-MIO-16L-9 board. A schematic of the Slow Control and Monitoring Hardware is shown in fig. 5.5.

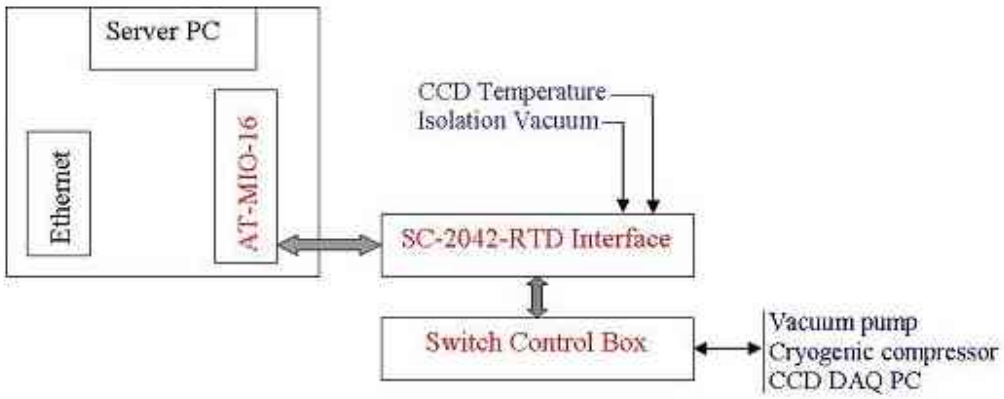


Fig. 5.5: Schematic representation of the VIP Slow Control and Monitoring Hardware.

5.3.2 Slow control and monitoring software

The software is based on National Instruments LabView graphical development environment and consistsin an graphical user interface (GUI) through which the user is able to monitor and control the CCD temperature and isolation vacuum, the status of alarms related with each monitored quantity, the switches associated with any device connected to the switch control box.

The GUI is fully remote controlled across the network using VNC software (Virtual Network Computing). Moreover, the variables monitored by the acquisition program is published in real-time on the VIP WWW page in the form of an HTML report, through the help of LabView Internet Toolkit. The web reports is available to any authorized member of the VIP experiment. In case of hardware failures or out of range values for the CCD temperature or isolation vacuum pressure, an automatic GSM-SMS procedure is foreseen. In fig.5.6 a schematic description of the remote communication with the Slow Control and Monitoring Server is presented.

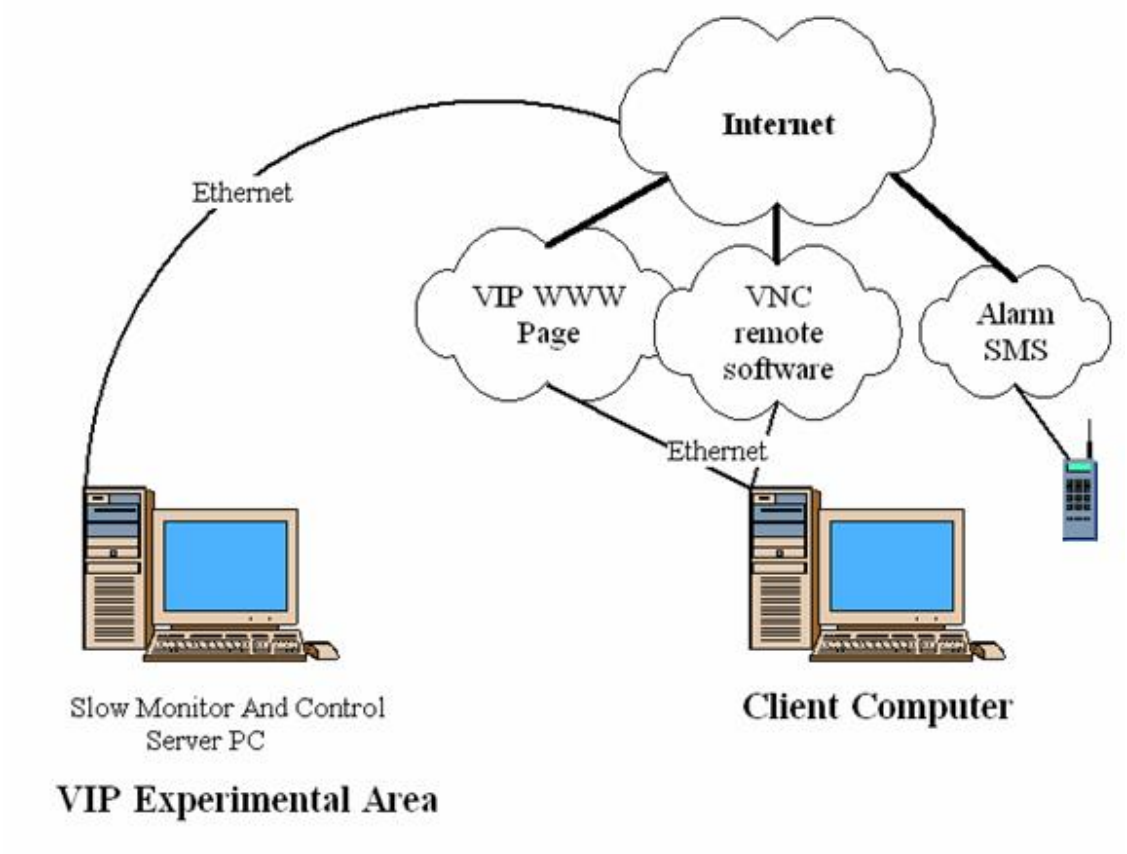


Fig. 5.6: Remote Communication with the Slow Control and Monitoring software.

Chapter 6

First VIP results

6.1 LNF- measurements

Previous to the transportation of the VIP setup at LNGS, a measurement was done at the LNF laboratory, in order to test the performance and stability of the setup. The measurements reported here have been performed in the period 21 November – 13 December 2005 [56]. Two types of measurements were performed:

- 14510 minutes (about 10 days) of measurements with a 40 A current circulating in the copper target;
- 14510 minutes of measurements without circulating current;

where CCDs were read-out every 10 minutes. The two resulting calibrated in energy X-ray spectra are shown in Fig. 6.1(a), with circulating current, and (b), without current. The spectra refer to 14 CCDs (out of 16), due to noise problems in the remaining 2. Frequent calibration runs with an X-ray tube activating the copper and a zirconium foil, resulted in an energy scale variation of less than 3 eV at 8 keV. An analogous behavior was shown by the stability of the line widths, confirming the stability of the detector resolution. An independent assessment of the stability of the energy scale and resolution, obtained by monitoring the position and the width of the peaks of the copper *K*-lines on temporally split data samples during the run, yielded comparable results. Both spectra show clearly the copper K_α and K_β lines superimposed to a continuous background. The

spectra, generated by the cosmic rays interactions and by natural radioactivity, show no evidence of further structures, as a consequence of the careful choice of the materials used in the setup.

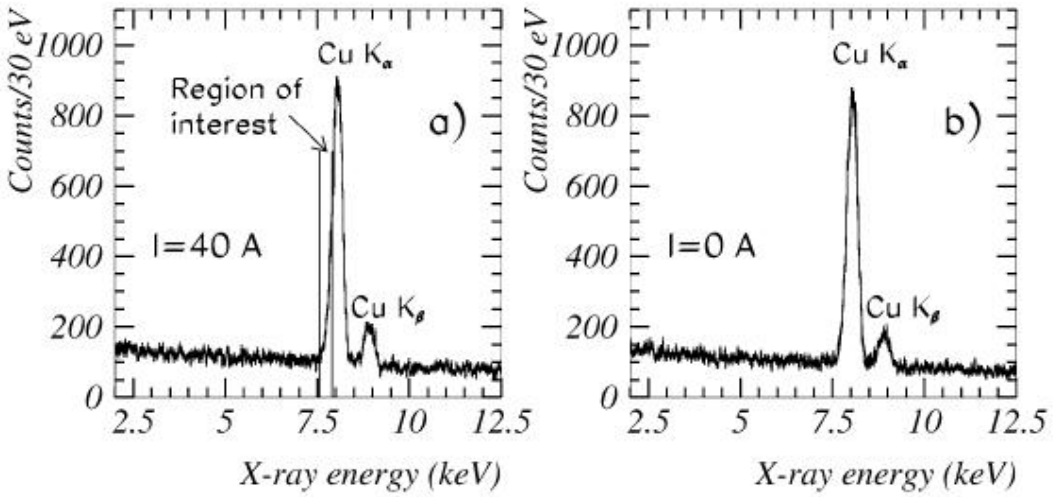


Fig. 6.1: Energy spectra for the VIP measurements: (a) with current (I = 40 A); (b) without current (I = 0 A).

6.2 PEP-violating X-ray spectrum

In order to obtain the number of X-rays due to the possible PEP violating transitions, the spectrum without current was subtracted from the one with current. The resulting subtracted spectrum is shown in Fig. 6.2(a) (whole energy scale) and (b) (a zoom on the region of interest). It is to be noticed that the subtracted spectrum fluctuates around zero within statistical error and it shows no structure. This is another consistency check of the stability of the energy scale. The region of interest, from 7.564 to 7.894 keV, is defined by the CCD energy resolution (320 eV FWHM) at the K_α copper transition (8.04 keV), with an additional uncertainty of 10 eV, to account for the theoretical

uncertainty in the calculation of the PEP violating transition energy. The numbers of X-rays in the region of interest were:

- $NX = 2721 \pm 52$ at $I = 40$ A
- $NX = 2742 \pm 52$ at $I = 0$ A
- $\Delta NX = -21 \pm 73$ for the subtracted spectrum.

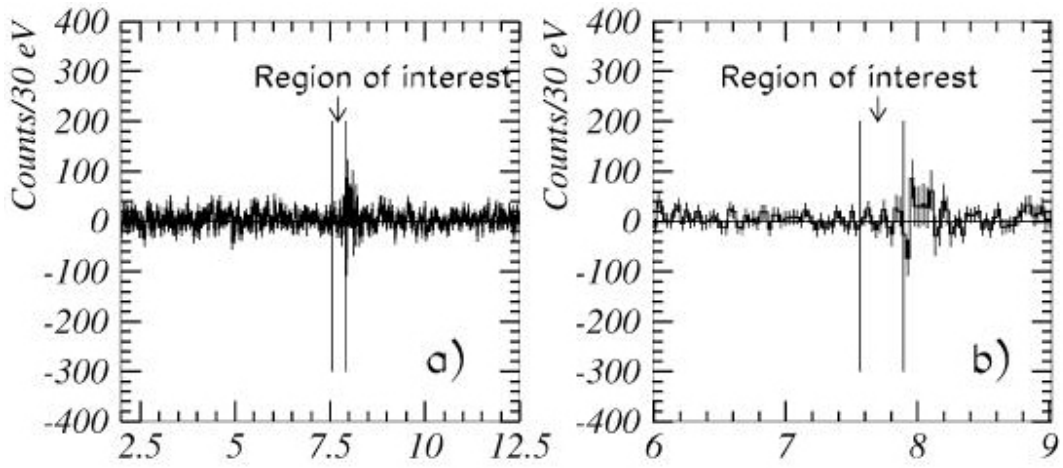


Fig. 6.2: The subtracted spectrum, current min us no-current, giving the limit on PEP violation for electrons: (a)whole energy range; (b) expanded view in the region of interest (7.564 – 7.894 keV). No evidence for a peak in the region of interest is found.

6.3 Determination of the PEP violation probability limit

For the parametrization of the results in a Pauli principle violating theory, we use the notation of Ignatiev and Kuzmin [9], which has been incorporated in the paper of Greenberg and Mohapatra [35]: even though the model of Ignatiev and Kuzmin has been later shown to be incompatible with quantum field theory [30], the parameter β that

measures the degree of PEP violation has stuck and is still found in the literature, also because it is easy to show that it is related to the parameter q of quon theory, by the relation: $(1 + q)/2 = \beta^2/2$ [42] (in quon theory, $-1 \leq q \leq 1$, where $q = -1$ corresponds to fermions and $q = 1$ corresponds to bosons, so that here q must be close to -1 and $(1 + q)/2$ must be very small, because we are dealing with electrons). Moreover, we used this parametrization for an easy comparison of our results with the previous Ramberg and Snow ones [2], since the same has been used in that paper. In [9] a pair of electrons in a mixed symmetry state has the probability $\beta^2/2$ for the symmetric component and $(1 - \beta^2/2)$ for the usual antisymmetric one. The parameter $\beta^2/2$ is related, then, to the probability that an electron violates PEP (see also [29] for further details). To determine the experimental limit on $\beta^2/2$ from our data, we used the same arguments of Ramberg and Snow, to compare the results. The number of electrons that pass through the conductor, which are new for this conductor, is:

$$N_{new} = (1/e) \sum I \, \Delta t \tag{6.1}$$

where e is the electron electric charge, I is the current intensity and Δt represents the time duration of the measurement with current on. Each new electron will undergo a large number of scattering processes on the atoms of the copper lattice. The minimum number of these internal scattering processes per electron, defined as N_{int}, is of order D/μ, where D is the length of the copper electrode (8.8 cm in our case) and μ is the mean free path of electrons in copper. The latter parameter is obtained from the resistivity of the metal. We assume that the capture probability (aside from the factor $\sim \beta^2/2$) is greater than 1/10 of the scattering probability. The acceptance of the 14 CCD detectors and the probability that an X-ray of about 7.6 keV, the energy of the possible anomalous transition generated in the copper target, is not absorbed inside the copper itself, were evaluated by a Monte Carlo simulation of the VIP setup, based on GEANT 3.21. This probability turns out to be 2.1%. Moreover, a CCD efficiency equal to 48% for a 7.6 keV X-ray was considered. All these factors built up the so-called *geometric factor* (~ 1%). The number of X-rays generated in the PEP violating transition, ΔNX, is then related to the $\beta^2/2$ parameter by:

$$\begin{aligned}\Delta N_X &\geq \frac{1}{2}\beta^2 N_{new}\,\frac{1}{10}\,N_{\text{int}}\cdot\left(geometricfactor\right)\\ &= \frac{\beta^2\left(\sum I\,\Delta t\right)D}{e\mu}\,\frac{1}{20}\cdot\left(geometricfactor\right)\end{aligned} \tag{6.2}$$

Then, for $\sum I\,\Delta t$ = 34.824×10^6 C, D = 8.8 cm, μ = 3.9× 10^{-6} cm, e = 1.602×10^{-19} C, we get:

$$\Delta N_X = 4.9\cdot 10^{-29}\cdot\frac{\beta^2}{2}. \tag{6.3}$$

The difference of events between the measurements with and without current, reported in the previous section, is $\Delta NX = -21\pm 73$. Taking as a limit of observation three standard deviations, we get for the PEP violating parameter:

$$\frac{\beta^2}{2} \leq \frac{3\cdot 73}{4.9\cdot 10^{29}} = 4.5\cdot 10^{-28} \quad \text{at } 99.7\ C.L. \tag{6.4}$$

We can interpret this as a limit on the probability of PEP violating interactions between external electrons and copper atoms: $\beta^2/2 \leq 4.5\cdot 10^{-28}$. We have thus improved the limit obtained by Ramberg and Snow by a factor about 40.

Chapter 7

VIP at LNGS

7.1 Installation at LNGS

In February 2006 the VIP setup was transported and installed at the LNGS underground laboratory. The first measurements, having the goal to check the stability of the setup, were performed without shielding, with the setup as showed in Fig.7.1. The setup is based on a double layer of copper and lead bricks, under a very thick layer of rock (about 2 km). It is placed into a cabin (Fig 7.2), with two air-conditioner systems, having the goal to preserve the low temperature in the room, on which depend the CCD temperatures, and the electronics performances.

The values of the DAQ parameters, as room, CCDs and target temperatures, vacuum pressure, and current circulated into the target, are constantly monitored and displayed on the "VIP Slow Monitor", on the VIP web page. The values of these parameters during the DAQ were:

- room temperature about 15° C;
- CCDs temperature about –115° C;
- target temperature 8° C without current and 12° C with current;
- vacuum pressure about 3×10^{-8} mbar
- current 0A (no current DAQ) or 40A (DAQ with current).

Fig. 7.1 : The VIP setup at LNGS, without shielding.

Fig. 7.2 : The VIP cabin at LNGS.

7.2 Data taking with shielding

In order to reduce the background, the apparatus is currently installed in the LNGS laboratories, to reduce cosmic rays interaction, while the effects of natural radioactivity are moderated by a massive shield built by low activity materials. On April 2006 it was realized a double-layer shielding with copper and lead bricks around the setup, as shown in Fig. 7.3 and 7.4. The shielding was done in an external layer of lead and an internal one of copper, 10 cm thickness each. In this final configuration we obtained the first preliminary updated results.

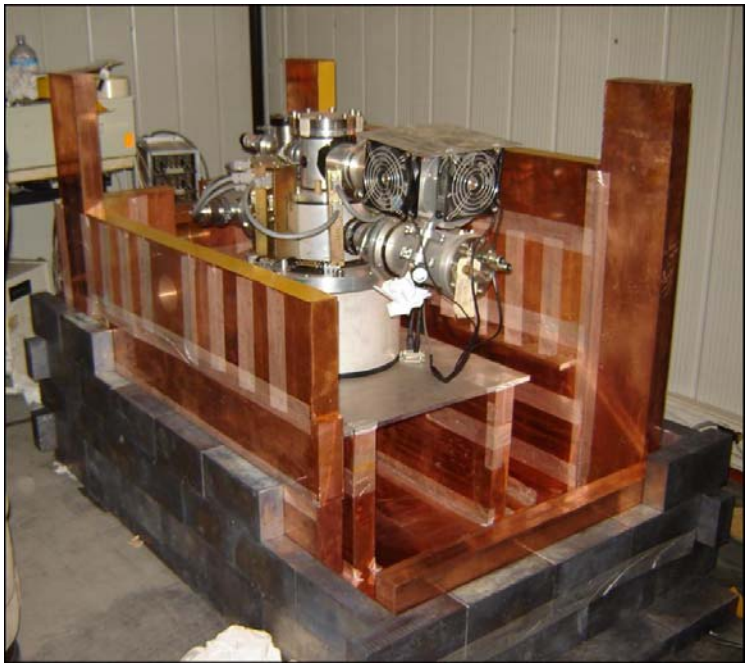


Fig. 7.3 : The VIP setup with a partial shielding (under construction).

Fig. 7.4: The VIP setup with total shielding into the cabin.

7.3 Results: the new experimental limit

At the time of writing, about one year of data have been analyzed, in the above mentioned acquisition conditions. The measurements reported here have been performed in the periods April – November 2006, January – May 2007. In between the setup was revised, for a failure of cryogenic system (of CCD detectors).

By a first analyses of the data and of the relative spectra, we have decided to put a cut to reject all the files of the first period of the DAQ (for each sets of data) in which the setup was still in stabilization phase. This cut reduced the period to be considered for further analyses. Our overall data set consist of two periods of data acquisition:

- 236005 minutes of measurements with a 40A current circulating in the copper target;
- 172685 minutes of measurements without circulating current (used for background evaluation);

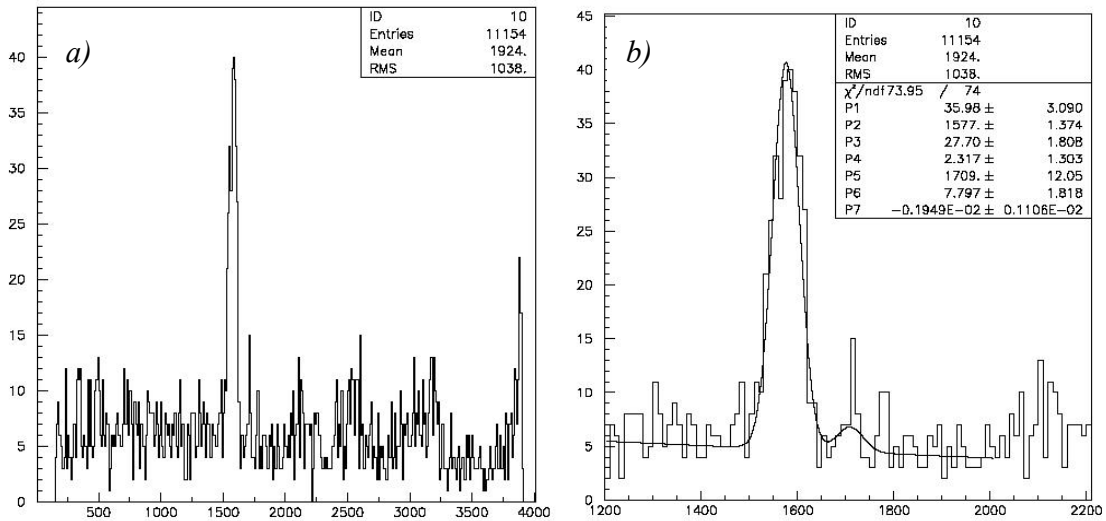
where CCDs were read-out every 5 minutes. Only 15 CCDs, out of 16, were considered for analyses, because the remaining 1 (number 1) is afflicted by noise problems.

Due to slightly different working conditions (CCD temperatures for example) we treated the two data blocks independently, performing for each one of them a separate data analysis. Then we have summed the obtained subtracted spectra, to deduce the PEP violation parameter from the whole available statistics. We analyzed then the data in this way:

- Block I
  - 142905 minutes of measurements with a 40A current
  - 84430 minutes of measurements without current

- Block II
  - 93100 minutes of measurements with a 40A current
  - 88255 minutes of measurements without current

As a first step of the analysis, each CCD was calibrated in energy. In Fig. 7.5 is shown, as an example, the sequence in which a single CCD's spectrum (a) is zoomed and fitted (b), and then calibrated in energy (c).



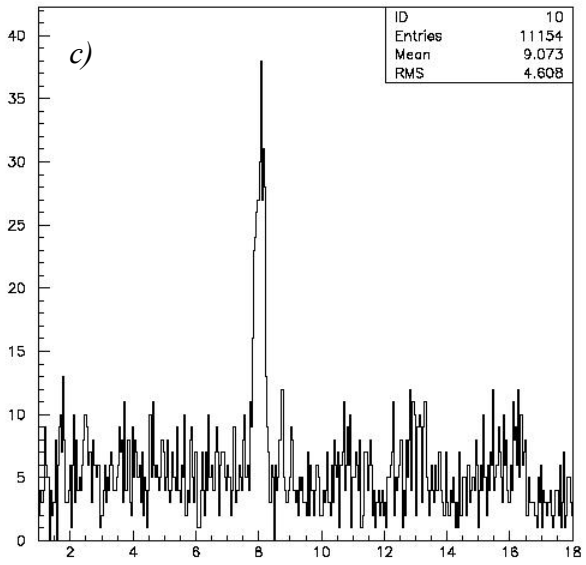


Fig. 7.5: Sequence in which a single CCD's spectrum (a) is zoomed and fitted (b), and then calibrated in energy (c).

This procedure allows to have 16 calibrated spectra in which all the energy scale are perfectly aligned.

For each data set (I=0 or I=40 A) the individual spectra were summed then into an unique spectrum. The two resulting X-ray spectra, for the block I are shown in Fig. 7.6 with circulating current and Fig. 7.7 without current, while the two resulting X-ray spectra, for the block II are shown in Fig. 7.8 with circulating current and Fig. 7.9 without current.

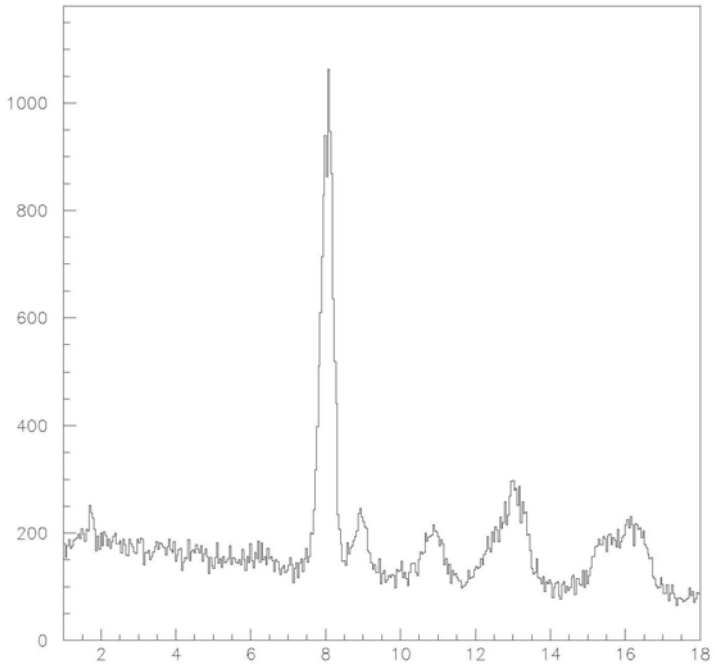


Fig. 7.6 : Energy spectra for the VIP measurement without circulating current (I = 0 A), for the block I.

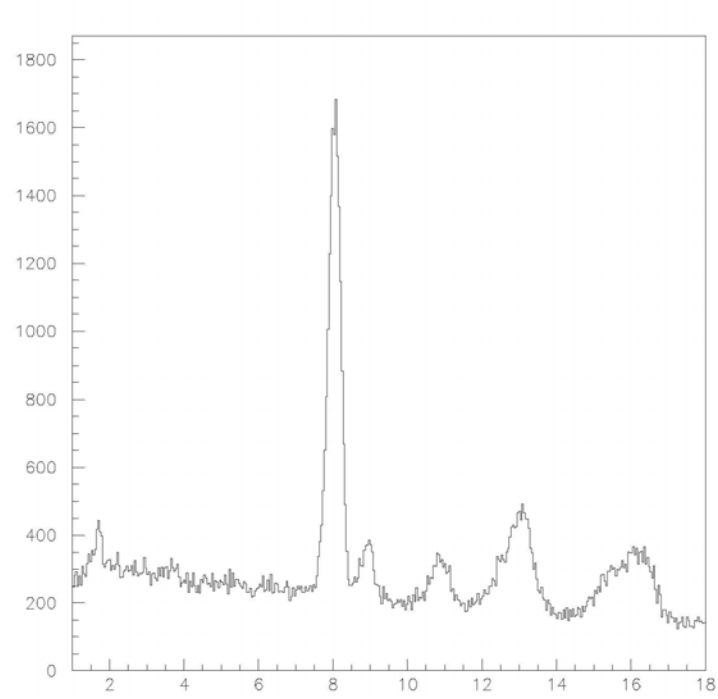


Fig. 7.7 : Energy spectra for the VIP measurement with circulating current (I = 40 A), for the block I.

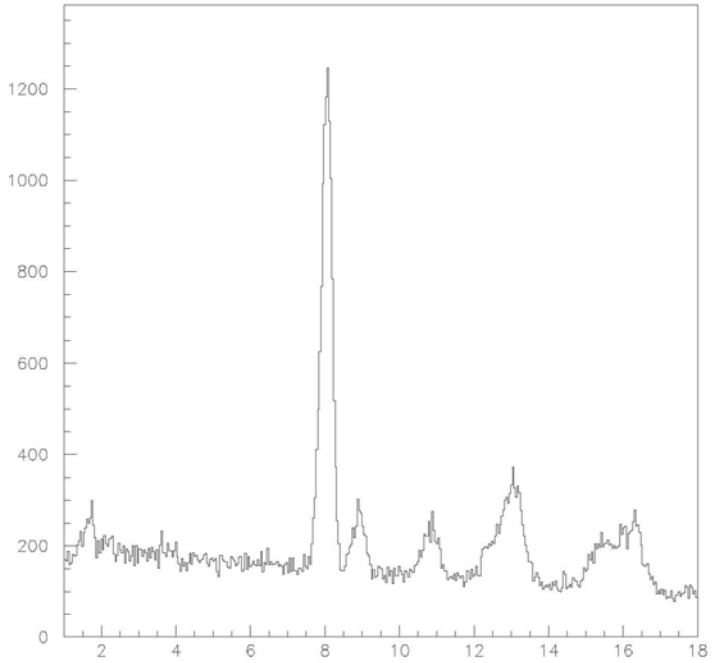


Fig. 7.8 : Energy spectra for the VIP measurement without circulating current (I = 0 A), for the block II.

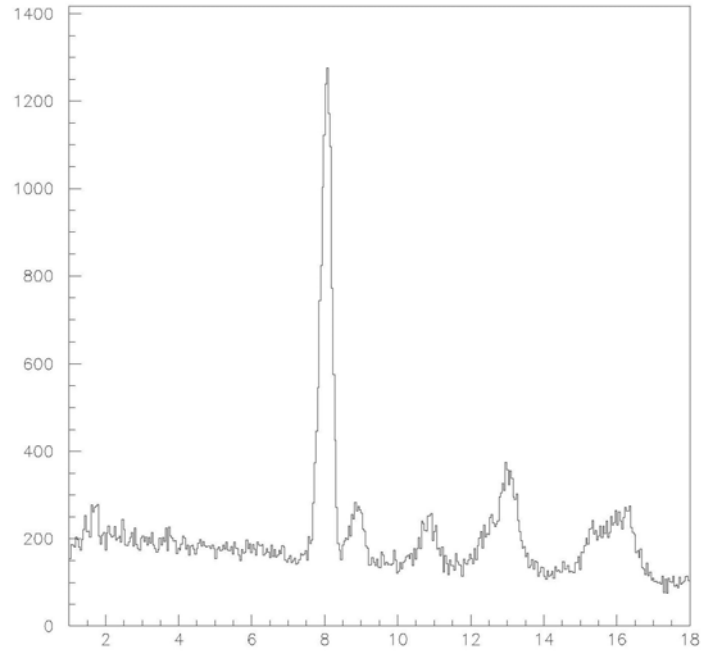


Fig. 7.9 : Energy spectra for the VIP measurement with circulating current (I = 40 A), for the block II.

Before going on and extract the probability of PEP violation – subtracting the two spectra – we performed an analyses of the structures present in the individual spectra.

On the histograms can be performed a fit with a function given by the sum of a linear polynomial (representing the background) and 5 gaussians. To reduce the number of the parameters in the fit, it is assumed that the first two Gaussians have the same σ, deduced from the Cu K_α. The aim of the fit is to verify the peaks positions to individuate all the elements that contribute to the background. But it is not so simple! In fact the last three peaks have a complex structure, probably given by the superposition of more adjacent peaks. A detailed analysis could give information about the elements that belong to the same setup or to the surrounding environment with the aim to shield the setup and the detectors and to reduce the background as more possible.

In Fig. 7.10 is showed an analysis performed on the "with current" spectra, for 142905 minutes of total time and 117300 events, in the energy range 1÷18 KeV. Several peaks are evident but their structures seem to be complex. Lines corresponding to some elements are plotted over each possible peak. For each line the elements and the corresponding energies are reported in Tab. 7.1.

Line	***Element***	***Energy (KeV)***
1	K_α (Si)	1.74
2	K_α (Ca)	3.69
3	K_α (Cu)	8.04
4	K_β (Cu)	8.91
5	L_α (Pb)	10.54
6	L_α (Bi)	10.83
7	L_β (Pb)	12.62
8	L_α (Th)	12.95
9	L_β (Bi)	13.01
10	L_α (U)	13.60
11	L_γ (Bi)	15.25

12	K_α (Zr)	15.75
13	L_β (Th)	16.04
14	L_β (U)	16.98
15	K_β (Zr)	17.70

Tab. 7.1: Elements found from the background analysis.

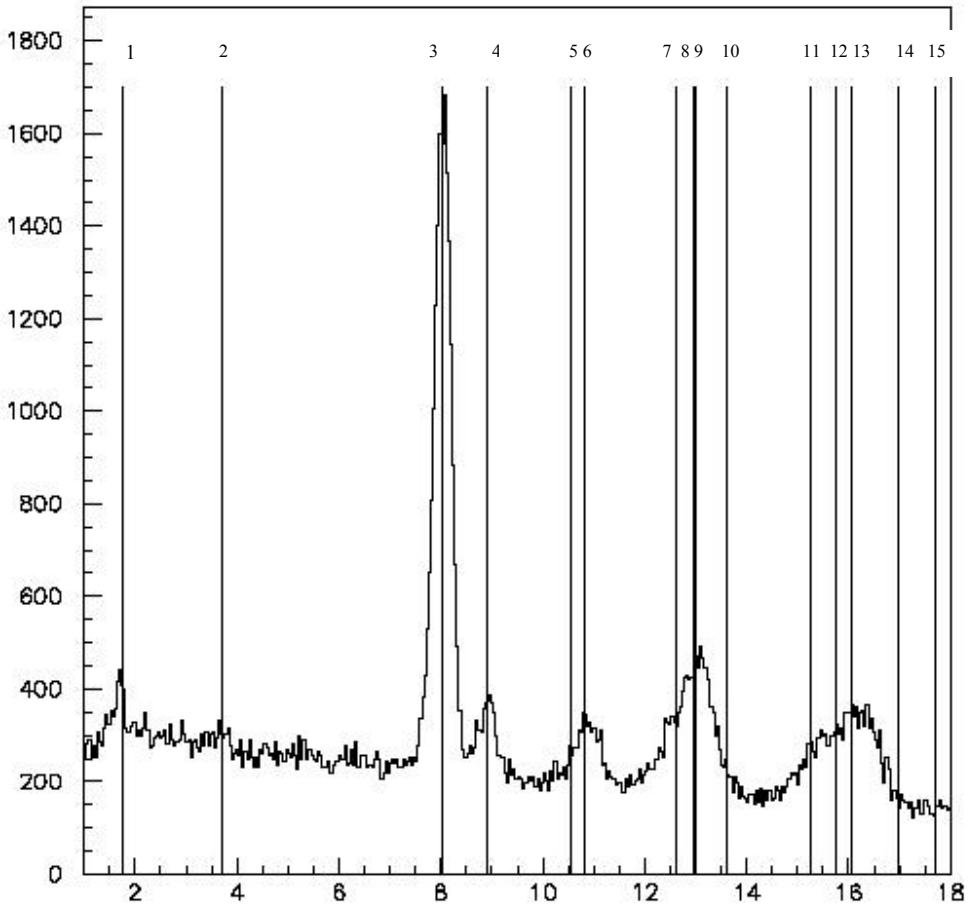


Fig. 7.10: Lines corresponding to some elements are plotted over each possible peak.

Some of these elements belong to the setup, for example: Si in the CCDs, Ca in the ceramics supports of the chips, Cu in the target, Pb in the shielding, etc… Towards

more high energies there are elements as Bi, Th, U, coming from the decay chain of Rn, which starts from U-238 and stops with Pb.

Another similar analysis was performed on "with current" summed spectra, for 142905 minutes of total time and 116970 events in the energy range 1÷18 KeV. This second analysis consists in the achievement of the best fit over a reduced energy range 6÷14 KeV. The best fit is realized through a function given by the sum of 5 gaussians and a third degree polynomial to draw the background, with the constraints to have the same σ for the first two peaks and for the last two (see Fig. 7.11). From the fit parameters there are no doubts on the presence of Bi and Pb in the surrounding environment.

An analogous study performed over a "without current" summed spectra shows the same result.

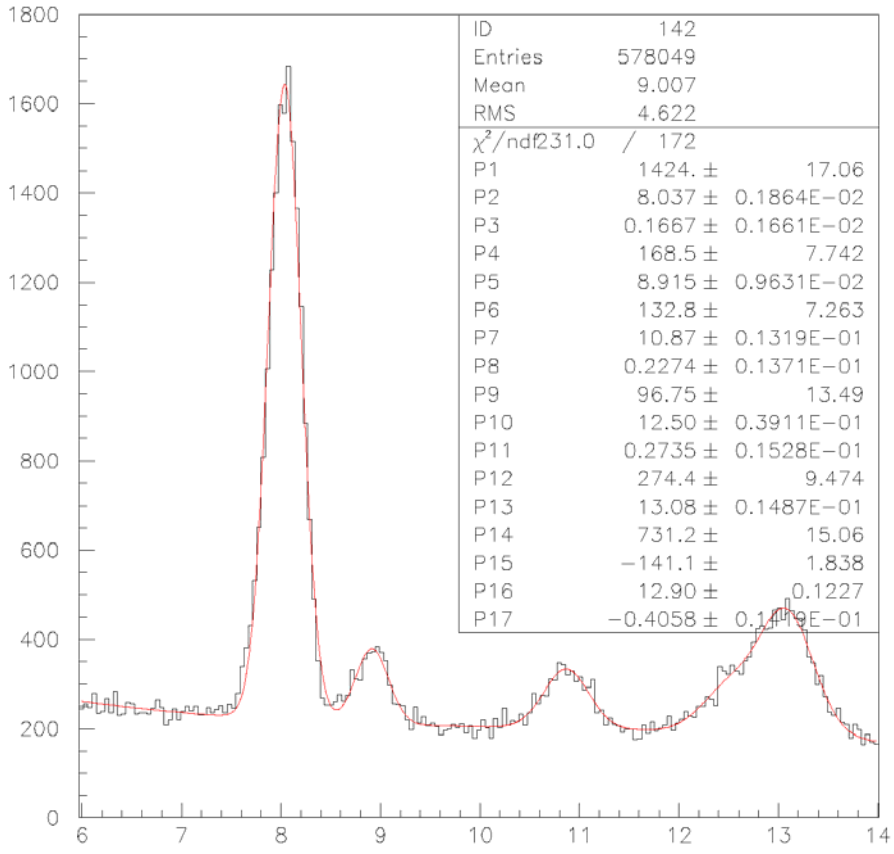


Fig. 7.11: The best fit for the background analysis.

We proceed now to extract the probability of PEP violation – following the same procedure as explained in Chapter 6. In both the two cases, the second spectrum (I=0A) was then normalized to the same DAQ time as the I=40A (with a normalization constants: C_I = 142905/84430 = 1.693 and C_{II} = 93100/88255 = 1.055). The two resulting subtracted spectra are shown in Fig. 7.12 (block I) and in Fig. 7.13 (block II). We obtained the "PEP violating" spectrum by summing the two subtracted spectra related to the two periods. The final spectrum is shown in Fig. 7.14. The region of interest is defined starting from the CCD energy resolution, to which we added 10 eV for accounting for the error in the PEP violating energy transition, and it is between 7.53 keV and 7.93 keV.

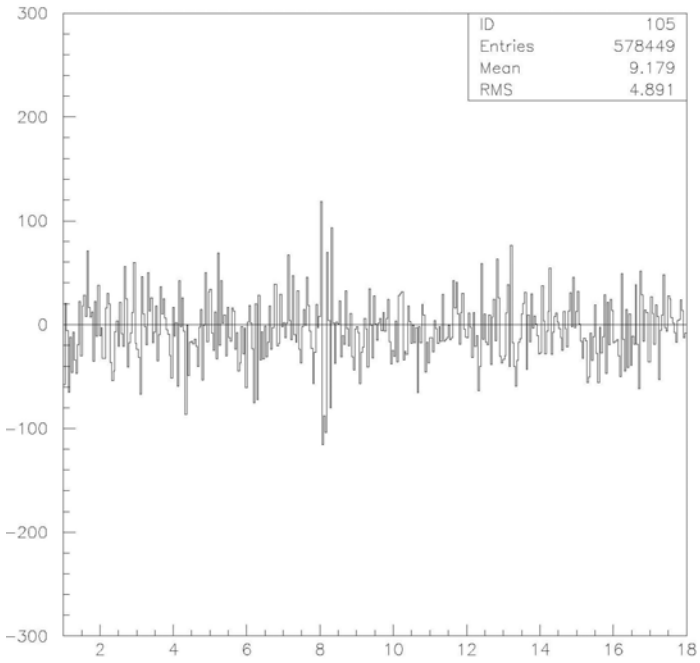


Fig. 7.12: The subtracted spectrum, current minus no current, for the block I.

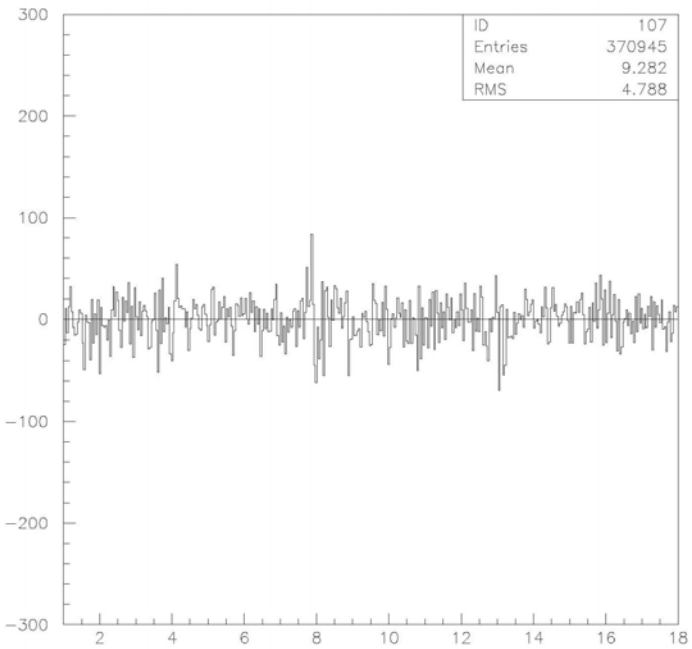


Fig. 7.13: The subtracted spectrum, current minus no current, for the block II.

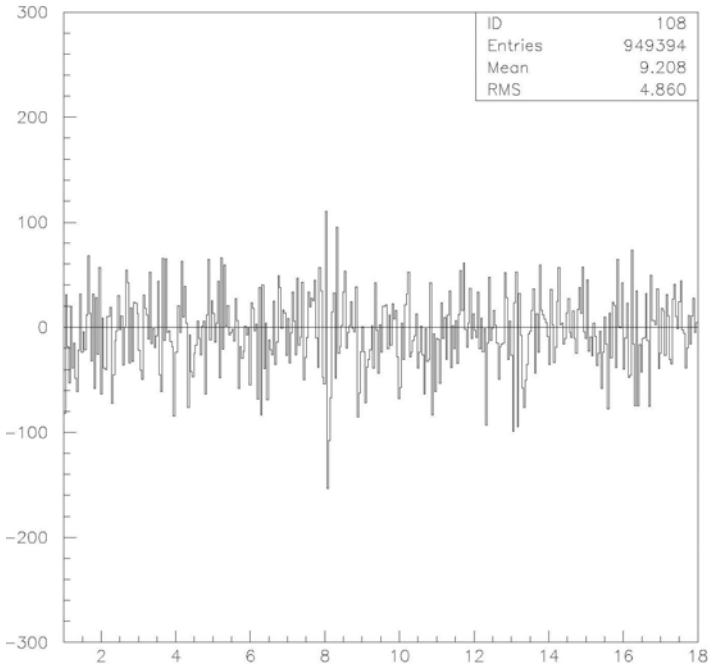


Fig. 7.14: The final spectra.

The numbers of X-rays in the region of interest are:

- N_x (I) = 3406 ± 58 at I = 0 A
- N_x (I) = 5751 ± 76 at I = 40 A
- N_x (II) = 3471 ± 59 at I = 0 A
- N_x (II) = 3866 ± 62 at I = 40 A
- ΔN_x = 190 ± 152 for the subtracted spectrum.

Following the same procedure to determine the experimental limit on $\beta^2/2$ from our data as the one in [3], and propagating correctly the errors, we found the number of X-rays generated in the PEP violating transitions, and taking as a limit of the observation three standard deviations, we get for the PEP violation probability:

$$\frac{\beta^2}{2} \leq \frac{3 \cdot 152}{79.72} \cdot 10^{-29} = 5.72 \cdot 10^{-29} \quad \text{at } 99.7\% \text{ C.L.} \tag{7.1}$$

In this way we have improved the limit obtained by Ramberg and Snow of about a factor 250. Today this is the best value ever reached on the probability of PEP violation for the electrons.

Conclusions

The Pauli Exclusion Principle was formulated to explain the regularities of both the Periodic Table and atomic spectra, and has a crucial role in our understanding of the structure and properties of atoms and nuclei, and of matter in general.

Despite its enormous success the absolute validity of this principle, as happens with other fundamental principles (CPT symmetry, Lorentz invariance, locality, just to name some) is still open to questions and some recent theoretical analysis have suggested that such small violations might be possible. This has inspired several experimental tests of its validity. Ramberg and Snow placed an upper limit on $\beta^2/2$ of $1.6 \cdot 10^{-26}$ by searching for K-series X-ray transitions when a current is passed through a copper strip; the X-ray would be emitted if the "new" electrons cascade down to an already fully occupied K-shell, so violating PEP.

Following the same experimental method the VIP experiment improved the aims limit on the violation of the Pauli principle for electrons by almost three orders of magnitude, $\beta^2/2 \leq 5.7x10^{-29}$, result which is presented in this thesis for the first time.

This result stimulated the theoretical work related to possible new physics, in which a small PEP violation is contemplated.

Bibliography

[1] W. Pauli, *Nobel Lecture*, December 13, 1946.

[2] E. Ramberg e G. A. Snow: *Experimental limit on a small violation of the Pauli principle*, Phys. Lett. B **238** (1990) 438.

[3] I. Duck and E. C. G. Sudarshan: *Towards an understanding of the spin-statistics theorem*, Am. J. Phys, **66** (1998) 284.

[4] A. M. Messiah and O. W. Greenberg: *Symmetrization postulate and its experimental foundation*, Phys. Rev. **136** (1966) B248.

[5] F. Mandl: *QM* (Wiley, 1992).

[6] W. Pauli: *The connection between spin and statistics*, Phys. Rev. **58** (1940) 716.

[7] I. Duck and E.C.G. Sudarschan: *Pauli and the Spin-Statistics Theorem* (World Scientific Publishing, 1997).

[8] J. Schwinger: *Spin, Statistics and the TCP Theorem,* Physics **44**, (1958) 223.

[9] K. Johnson and E.C.G. Sudarshan, Ann. Phys. **13** (1961) 126.

[10] G. Lüders and B. Zumino, Phys. Rev. **110** (1958) 1450.

[11] E. Fischbach, T. Kirsten, and O. A. Schaeffer, Phys. Rev. Lett. **20** (1968) 1012.

[12] V. M. Novikov et al., Phys. Lett. B **240** (1990) 227.

[13] K. Deilamian et al.: *Search for Small Violations of the Symmetrization Postulate in an Excited State of Helium*, Phys. Rev. Lett. **74** (1995) 4787.

[14] A. S. Barabash, et al., JETP Lett. **68** (1998) 112.

[15] E. Nolte et al.: *Accelerator mass spectroscopy for tests of the Pauli exclusion principle and for detection of ββ decay products*, J. Phys. G **17** (1991) S355.

[16] E. Corinaldesi, *Model of the Dynamical Theory of the Pauli Principle*, Supplemento al Nuovo Cimento, Vol. 5, Num. 3 (1967).

[17] F. Reines and H. W. Sobel: *Test of the Pauli Exclusion Principle for Atomic Electrons*, Phys. Rev. Lett. **32** (1974) 954.

[18] B. A. Logan and A. Ljubicic: *Validity of the Pauli exclusion principle for nucleons*, Phys. Rev. C **20** (1979) 1957.

[19] T. Kishimoto et al.: *Search for violation of the Pauli principle through spontaneous neutron emission from lead*, J. Phys. G **18** (1992) 443.

[20] M. Goldhaber and G. Goldhaber, Phys. Rev. **73** (1948) 1472.

[21] Y. Suzuki et al., Phys. Lett. **B311** (1993) 357.

[22] NEMO Collaboration: *Testing the Pauli Exclusion Principle with the NEMO-2 detector*, Nucl. Phys. B (Proc. Suppl.) **87** (2000) 510.

[23] H. Ejiri, H. Toki, Phys. Lett. **B306** (1993) 218.

[24] R. Bernabei et al.: *Search for non-Paulian transitions in ^{23}Na and ^{127}I*, Phys. Lett. B **408** (1997) 439.

[25] D. Kekez, A.A. Ljubičić, B.A. Logan, Nature **348** (1990) 224.

[26] D. Miljanić et al., Phys. Lett. **B252** (1990) 487.

[27] R. D. Amado and H. Primakoff: *Comments on testing the Pauli principle*, Phys. Rev. C **22** (1980) 1338.

[28] K. Akama et al., *Superficial Violation of the Pauli Principle Due to the Possibile Substructure of the Electrons*, Phys. Rev. Lett. Vol.68 **12** (1992) 1826

[29] H. S. Green: *A Generalized Method of Field Quantization*, Phys. Rev. **90** (1953) 270.

[30] A. Yu. Ignatiev and V. A. Kuzmin: *Is small violation of the Pauli principle possible?* , ICTP preprint IC/87/13 (1987); V. N. Gavrin, A. Yu. Ignatiev and V. A. Kuzmin: Search for small violations of the Pauli principle, Phys. Lett. B **206** (1988) 343.

[31] A.B. Govorkov, Phys. Lett. A**137** (1989) 7.

[32] L. C. Biedenharn, P. Truini and H. van Dam: *A remark on possible violations of the Pauli principle*, J. Phys. A **22** (1989) L67.

[33] L. Okun: *Comments on Testing Charge Conservation and the Pauli Exclusion Principle*, Comments Nucl. Part. Phys. **19** (1989) 99.

[34] G. Gentile, Nuovo Cimento **17** (1940) 493.

[35] O. W. Greenberg and R. N. Mohapatra: *Local Quantum Field Theory of Possible*

Violation of the Pauli Principle, Phys. Rev. Lett. **59** (1987) 2507.

[36] O. W. Greenberg: *Example of infinite statistics*, Phys. Rev. Lett. **64** (1990) 705.

[37] O.W. Greenberg and R.C. Hilborn, arXiv: hep_th/9808106 v1.

[38] D. Zagier, Commun. Math. Phys. **147**, (1992) 199.

[39] M. Boz˙ejko and R. Speicher, Commun. Math. Phys. **137**, (1991) 519.

[40] R. Speicher, Lett. Math. Phys. **27**, (1993) 97.

[41] D.I. Fivel, Phys. Rev. Lett. **65**, (1990); 3361 erratum, ibid **69**, (1992) 2020.

[42] O.W. Greenberg, arXiv: cond-mat/9301002 v1.

[43] A. Shimony, *Proposed Experiment to Test the Possible Time Dependence of the Onset of the Pauli Exclusion Principle*, Quant. Inf. Proc. Vol. 5, Num. 4 (August 2006) 227.

[44] P. Indelicato, private communication.

[45] J. B. Mann, *Hartree-Fock energy results for the elements Hydrogen to Lawrencium*, Los Alamos Scientific Laboratory report LA-3690, UC-34 Physics, TID 4500.

[46] R.D. Cowan, *The theory of atomic structure and spectra*, University of California Press (1981) 165.

[47] The adopted code is described in: J.V. Mallow, J.P. Desclaux, A.J. Freeman, *Dirac-Fock method for muonic atoms: transition energies, wave functions, and charge densities,* Phys. Rev. A**17** (1978) 1804.

[48] J.P. Desclaux, *Relativistic multiconfiguration Dirac-Fock package*, http://dirac.spectro.jussieu.fr/mcdf/autres/24_mettec.html .

[49] M. Augsburger, J.-P. Egger, "New experimental limit on a violation of the Pauli exclusion priciple", preprint, 19 August 1999, unpublished.

[50] J.-P. Egger, D. Chatellard and E. Jeannet, *Progress in soft X-ray detection: The case of exotic hydrogen,* Particle World 3 (1993) 139.

[51] EEV (English Electric Valve), Waterhouse Lane, Chelmsford Essex, CM1 2QU, UK.

[52] D. Varidel et al., *CCDs as Low-Energy X-ray Detectors*, Nucl. Instr. and Meth. in Phys. Res. A**292** (1990) 147-155.

[53] D. Sigg, *CCDs as X-ray Detectors in a High radiation environment*, Nucl. Instr. and Meth. in Phys. Res. A**345** (1994) 107-114.

[54] G Fiorucci et al., *CCDs as Low-Energy X-ray Detectors*, Nucl. Instr. and Meth. in Phys. Res. A**292** (1990) 141-146.

[55] T. Ishiwatari et al., *New Analysis Method for CCD X-ray Data*, Nucl. Instr. and Meth. in Phys. Res. A**556** (2006) 509.

[56] S.Bartalucci et al., *New experimental limit on the Pauli Exclusion Principle violation by electrons*, Physics Letters B - **641** (2006) 18–22

Abstract

The Pauli Exclusion Principle (PEP) represents one of the fundamental principles of the modern physics and is at the very basis of our understanding of matter: thus it is of foremost importance to test the limits of its validity.

The PEP is a manifestation of the Spin-Statistics relation: the half-integer spin particles (fermions) follow Fermi statistics, while the integer-spin particles (bosons) the Bose –Einstein one, and is intimately related with the notion of "identity" of particles. In its original form the PEP was an adjunct to Bohr's old quantum theory, which stated that an electron in an atom could only move to one of a discrete set of orbits; on the basis of experimental findings (X-ray atomic spectra for example) Pauli added the postulate that only one electron in the atom can fill each of the allowed orbits (including the spin in the orbit's definition). When Bohr's theory gave way to modern quantum mechanics, in which the traditional planet-like orbits of electrons in atoms are replaced by the more abstract notion of the electron states, the PEP survived as the statement that no two electrons could be in the same state (i.e. they cannot have the same quantum numbers). More generally, there is a strong connection between spin and symmetry class, as Pauli stressed in his Nobel Lecture "*...we want to stress here a law of Nature which is generally valid, namely, the connection between spin and symmetry class. A half-integer value of the spin quantum number is always connected with antisymmetrical states (exclusion principle), an integer spin with symmetrical states*".

In the same Nobel lecture however, Pauli himself says *"Already in my original paper I stressed the circumstance that I was unable to give a logical reason for the exclusion principle or to deduce it from more general assumption. I had the feeling and I still have it today, that this is a deficiency. The impression that the shadow of some incompleteness fell here on the bright light of success of the new quantum mechanics seems to me unavoidable".* This statement, more than 60 years after Pauli formulated it, can be repeated practically unchanged – not much progress has been achieved in the deep understanding of the physical origin of the spin-statistics relation. As a

consequence, even if today there are no compelling reasons to doubt the validity of the Pauli Exclusion Principle, it still spurs a lively debate on its limits, as testified by the abundant contributions found in the literature and in topical conferences.

Here I describe a method to observe possible small violations of the PEP for electrons, through the search for anomalous X-ray transitions in copper atoms, produced by new electrons in a copper block (introduced by a circulating current), which can be captured in a Pauli-forbidden transition to the 1S level, already occupied by two electrons. In 1990, an upper limit on the PEP violation was found using this method by Ramberg and Snow (RS) with a dedicated experiment. The search method is implemented in the VIP (Violation of the Pauli Exclusion Principle) experiment, an international collaboration among 6 Institutions from 4 countries, that has the scientific goal to improve by three-four orders of magnitude the RS's limit on the probability of PEP violation for electrons, bringing it into the 10^{-29}–10^{-30} region, which may be of particular interest for all those theories related to possible PEP violations, and that would come from new physics. Reaching such limits opens up some very interesting scenarios, as stressed by Duck and Sudarshan:

"... recently... membrane theorists have been speculating on a large compactification radius for one of their eleven dimensions, which could give a ratio (for PEP violation) of 10^{-30}*".*

VIP has performed the most precise measurement on the validity of PEP for electrons, establishing the limit on the probability of its violation at 5.7×10^{-29}. This result, presented here for the first time, improves on RS measurement by almost three orders of magnitude, and represents the reference value for all those theories that deal with possible violations of spin-statistics relation.

Keywords

Pauli Exclusion Principle; Quantum mechanics; Identical particles; Anomalous atomic transitions; X-rays; CCD; Symmetrization principle; Spin-statistics connection.

Brief abstract

The Pauli Exclusion Principle (PEP) represents one of the fundamental principles of the modern physics and is at the very basis of our understanding of matter.

The PEP is a manifestation of the Spin-Statistics relation and is intimately related with the notion of "identity" of particles. In its original form the PEP was an adjunct to Bohr's old quantum theory, which stated that an electron in an atom could only move to one of a discrete set of orbits; on the basis of experimental findings (X-ray atomic spectra for example) Pauli added the postulate that only one electron in the atom can fill each of the allowed orbits (including the spin in the orbit's definition). When Bohr's theory gave way to modern quantum mechanics, in which the traditional planet-like orbits of electrons in atoms are replaced by the more abstract notion of the electron states, the PEP survived as the statement that no two electrons could be in the same state (i.e. they cannot have the same quantum numbers). This statement, more than 60 years after Pauli formulated it, can be repeated practically unchanged. As a consequence, even if today there are no compelling reasons to doubt the validity of the Pauli Exclusion Principle, it still spurs a lively debate on its limits.

Here I describe a method to observe possible small violations of the PEP for electrons, through the search for anomalous X-ray transitions in copper atoms, produced by new electrons in a copper block (introduced by a circulating current), which can be captured in a Pauli-forbidden transition to the 1S level, already occupied by two electrons. In 1990, an upper limit on the PEP violation was found using this method by Ramberg and Snow (RS) with a dedicated experiment. The VIP (Violation of the Pauli Exclusion Principle) experiment has performed the most precise measurement on the validity of PEP for electrons, establishing the limit on the probability of its violation at 5.7×10^{-29}. This result, presented here for the first time, improves on RS measurement by almost three orders of magnitude, and represents the reference value for all those theories that deal with possible violations of spin-statistics relation.

Abstract breve

Il Principio di Esclusione di Pauli (PEP) rappresenta uno dei principi fondamentali della fisica moderna ed è alla base della nostra comprensione della materia. Il PEP è una manifestazione della relazione Spin-Statistica ed è intimamente correlato alla nozione di "identità" delle particelle. Nella sua forma originale il PEP fu una conseguenza della vecchia teoria quantistica di Bohr, secondo la quale un elettrone in un atomo avrebbe potuto muoversi solo attraverso un set discreto di orbite. Sulla base di osservazioni sperimentali (ad esempio i raggi-X negli spettri atomici) Pauli aggiunse il postulato secondo cui in un atomo un solo elettrone può occupare ciascuna delle orbite consentite (includendo la definizione di Spin).

Quando la teoria di Bohr cedette il posto alla meccanica quantistica moderna, in cui le tradizionali orbite tipo-pianeta degli elettroni nell'atomo vennero sostituite dalla più astratta nozione di stati elettronici, il PEP si tradusse nel fatto che due elettroni non avrebbero potuto trovarsi nello stesso stato (cioè i due non avrebbero potuto avere gli stessi numeri quantici). Tale stato di cose è rimasto invariato dopo oltre 60 anni dalla formulazione di Pauli. Di conseguenza, sebbene non ci siano motivi di dubitare la validità del Principio di Esclusione, oggi si dibatte sui suoi limiti.

In questa tesi descriverò il metodo per osservare possibili piccole violazioni del PEP per gli elettroni, attraverso la ricerca di raggi-X provenienti da transizioni anomale in atomi di rame. Tali transizioni sarebbero prodotte da elettroni "nuovi" introdotti attraverso un flusso di corrente in una lamina di rame, che potrebbero essere catturati e compiere transizioni (che violano il PEP) verso un livello 1S già occupato da due elettroni con spin opposto.

Nel 1990, un limite superiore alla probabilità di violazione del PEP fu trovato, sfruttando tale metodo, da Ramberg and Snow (RS). L'esperimento VIP (Violation of the Pauli Exclusion Principle) ha effettuato le misure più precise sulla validità del PEP per elettroni, fissando il limite della probabilità sulla sua violazione a 5.7×10^{-29}. Questo risultato, presentato qui per la prima volta, ha migliorato le misure di RS per almeno tre ordini di grandezza e rappresenta il valore di riferimento per tutte quelle teorie che sono correlate ad una possibile violazione della relazione spin-statistica.