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Planning Retail Distribution of Fuel Oils

Pasquale Carotenuto [a,*], Stefano Giordani [b], Daniele Celani [b]

[a] Consiglio Nazionale delle Ricerche - Istituto per le Applicazioni del Calcolo "M. Picone", via dei Taurini 19, 00185 Roma, Italy
[b] Università di Roma "Tor Vergata" – Dipartimento di Ingegneria dell'Impresa, via del Politecnico 1, 00133 Roma, Italy

Abstract

In this paper, we study the Periodic Petrol Station Replenishment Problem (PPSRP), an important real problem relative to logistics in the fuel oil distribution. It consists, under suitable assumptions and subject to certain operational constraints, of determining the fuel oil procurement plans of a set of petrol stations and planning the delivering routes of petrol products to the stations along a certain planning horizon. The considered problem belongs to the class of Inventory Routing Problems (IRP), of which it is a particularization. In particular, it draws its concepts from two different classes of distribution problems, the Periodic Vehicle Routing Problem (PVRP) and the Petrol Station Replenishment Problem (PSRP). We provide a mathematical formulation of the problem, and due to the large size of the real instances which in general an oil company has to deal with we heuristically solve it. We propose different heuristic strategies based on a common partitioning-then-routing paradigm, and in order to assess their performances we test them on a set of scenarios coming from Italian fuel oil distribution real cases and compare the results with those obtained by using other known heuristic approaches.

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Keywords: Fuel Oil Distribution; Freight Transport;Transportation Planning; Vehicle Routing; Heuristics.

1. Introduction

We consider the problem of planning the distribution of petrol products to a set of petrol stations faced by an oil company. This is a very complex problem and in order to reduce its complexity, the decision process is typically

* Corresponding author. Tel.: +39-06-49270926; fax: +39-06-4404306.
E-mail address: carotenuto@iac.cnr.it

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subdivided by the oil company into three phases: 1) (strategic phase) petrol stations are preassigned to one or a few storage depots from which it will be refuelled during the next medium-large term planning horizon; 2) (tactical phase) given a few days planning horizon (e.g., a week) and the subset of petrol stations covered by the same few storage depots, planning the distribution of fuel oil from depots to the assigned petrol stations; 3) (operational phase) day by day final tank truck routes are established by considering also specific operational constraints.

In this work, we specifically consider the tactical phase when the weekly fuel oil replenishment plan for each station is defined by the oil company, by determining the days when each petrol station will be replenished, along with the delivery amount of petrol products. Simultaneously, the tank truck (vehicle) routes from depots to stations are determined for each day of the week, in order to deliver the planned fuel oil replenishment amounts to petrol stations. At this level, for simplicity, oil company defines replenishment weekly plans for each petrol station by assuming a single undifferentiated product, and determines petrol stations visiting sequences (vehicle routes) for each day of the week, assuming a fleet of homogeneous vehicles. The main aim at this phase is to minimize the total route length traveled by the tank trucks during the considered week.

Typical operational conditions and contract agreements with petrol stations requires for the oil company to refurnish each petrol station by covering its petrol demand typically for a couple of days, forcing the company to fulfill the estimated weekly demand of a petrol station with a number of replenishments during the week, with the chance to select one out of a set of replenishment plan (or pattern) established in accordance with the petrol station owner, where a replenishment pattern specifies the set of visiting days (visiting pattern) along with the (possibly distinct) fuel oil amounts (demand pattern) to be delivered in these days, respectively. We model this tactical problem as a Periodic Petrol Station Replenishment Problem (PPSRP) by modeling it as a generalization of the Periodic (capacitated) Vehicle Routing Problem (PVRP) and provide its mathematical formulation.

The problem is NP-hard and, due to the large size of the real instances which in general an oil company has to deal with, we heuristically solve the problem. Starting from the multi-stage heuristic approaches proposed by Triki (2013), we propose new heuristic strategies based on the group-first-route-second paradigm. In order to assess their performances, we test the heuristics on a set of scenarios coming from Italian fuel oil distribution real cases, comparing also the results with those of an hybrid genetic algorithm proposed by Carotenuto *et al.* (2015).

The paper is organized as follows. In Section 2 we recall the relevant literature, in Section 3 we formally define the problem addressed and provide a mathematical formulation of the same. Section 4 describes the proposed heuristic procedures. Section 5 is devoted to the experimental analysis, and finally Section 6 gives some conclusions.

2. Literature review

Our problem belongs to the class of multi-period petrol station replenishment problems (MPPSRP) (see, e.g., Cornillier *et al.*, 2008b), where the aim is to optimize the delivery of several petrol products to a set of petrol stations over a given planning horizon. In the more general version, the MPPSRP consists in finding for each period of the planning horizon the amount of petrol product to be shipped to each petrol station, the assignment and loading of petrol products into the compartments of the tank trucks, the delivery routes, and the assignment of the routes to tank trucks and their time schedule, with the aim of minimizing the (routing and service) total cost. It can be viewed as an Inventory Routing Problem (IRP) with specific additional constraints such as the use of heterogeneous vehicles with compartments, also known as IRP in fuel delivery (see, e.g., Vidović *et al.*, 2014). Ng *et al.* (2008) study two small petrol distribution networks in Hong Kong, proposing a model for simultaneously assigning trips to tank trucks and stations, assuming stations inventories being managed by the vendor. Cornillier *et al.* (2008b) propose a heuristic approach to solve the case where the number of stations on any given route is limited to two. Popović *et al.* (2012) propose a variable neighborhood search heuristic for solving a multi-product multi-period IRP in fuel delivery with multi-compartment homogeneous vehicles, given a distinct deterministic petrol consumption for each fuel type and for each petrol station; they consider routes with at most three stations. Vidović *et al.* (2014) extend this limit to four and propose a mixed integer formulation that can be solved at optimum by commercial solvers only for very small instances (with 10 petrol stations and 3 days); they also propose some heuristics for solving larger instances up to 50 petrol stations and a time horizon of 5 days.

The periodic version of the petrol station replenishment problems (i.e., the PPSRP) is a special case of the MPPSRP. In the PPSRP a periodic service requirements is specified for each petrol station or frequency of service, i.e., the number of visiting days during a week. For this problem, Triki (2013) proposes four heuristics based on a three-phase approach, where first a replenishment plan is selected for each petrol station, then the routing for each day is determined, and, finally, a local improvement technique is applied to further reduce the routing length. Indeed, our problem is similar to the PPSRP but it generalizes the latter by considering replenishment plans that differ also upon demand patterns.

As for the single-period case, the problem is no longer an IRP but a Vehicle Routing Problem (VRP) with special vehicles. Avella *et al.* (2004) and Cornillier *et al.* (2008a) study this case proposing exact and heuristic algorithms. Cornillier *et al.* (2009) provide two heuristics for the more general case with time windows; the same authors extend this study by considering also multiple depots (Cornillier *et al.*, 2012). More recently, Benantar *et al.* (2016) propose a tabu search algorithm where specific attention is paid to tank trucks (or tankers) with compartments and customers with different types of demanded products and time windows.

The problem studied in this work is also a generalization of the Periodic (capacitated) Vehicle Routing Problem (PVRP), where the period (e.g., weekly) demand of a customer can be non-equally subdivided among the visiting days. This generalization is studied by Carotenuto *et al.* (2015) in the context of fuel oil replenishment problem. The authors propose a hybrid genetic algorithm similar to the one proposed by Vidal *et al*. (2012) for the multi-depot PVRP, that differently from the latter adopts some techniques and features tailored for the particular tactical problem of periodical fuel oil distribution to petrol stations. More recently, the same authors provide a generalization of their genetic algorithm to solve also the multi-depot case (Carotenuto *et al.*, 2017). As for the PVRP, recently Nguyen *et al.* (2014) develop a genetic algorithm for the variant of the PVRP with time windows, while Rahimi-Vahed *et al.* (2015) propose a modular heuristic algorithm to solve a particular application of the multi-depot case with the objective of determining the optimal fleet sizing, considering budget constraints in addition to standard constraints on vehicle capacity and route duration.

In particular, our work, starting from the PPSRP model proposed in Triki et al 2013, develops new heuristics and compares them with those proposed by Triki and with the genetic algorithm proposed by Carotenuto et al. (2015).

3. Problem definition

In order to reduce the complexity of the petrol station replenishment problem, we assume that: 1) petrol product is undifferentiated and consists of a generic fuel oil (this hypothesis follows from the fact that in general petrol stations are replenished with a single delivery using tankers with separated compartments for distinct products); 2) the tanker fleet is homogeneous; 3) other specific operational requirements such as time windows are not considered. In addition, we assume that a periodic service requirement is specified for each petrol station or frequency of service, along with specific visiting and demand patterns. Notwithstanding the problem remains NP-hard since it generalizes the PVRP.

Our PPSRP is defined as follows. Let C be the set of n petrol stations (customers) and let o be the storage depot. Let $G = (N, A)$ be a direct complete graph with $N = C \cup \{o\}$ being the set of $n + 1$ nodes, and $A = \{(i, j) \in N \times N : i \neq j\}$ being the set of arcs. Let arc $(i, j) \in A$ represents the direct-travel from petrol station (or depot) i to petrol station (or depot) j with travel cost (length) c_{ij}. Let T be the set of p time periods (days) of the planning horizon (week). Each petrol station $i \in C$ presents a visiting frequency (visiting days) $f_i \leq p$, with at most one visit per day, and a total weekly product demand q_i^{tot}. More specifically, let Π_i be the set of available replenishment plans (patterns) of station i, where a replenishment pattern $\pi \in \Pi_i$ is a visiting-day combination (i.e., a subset of f_i days of the week) along with the related specific demand combination of total amount q_i^{tot}; let d^{π}_{it} be the demand of station i in time period (day) $t \in T$, if pattern $\pi \in \Pi_i$ is chosen for station i; therefore, we assume $\sum_{t \in T} d^{\pi}_{it} = q_i^{tot}$, for each $\pi \in \Pi_i$. Note that with this assumption we generalize the basic PPSRP that assumes the total demand q_i^{tot} of station i during the planning horizon is equally subdivided among the f_i visiting days. Fuel oil is delivered with a fleet of homogeneous tank trucks, each one with capacity $Q_{\max}$: therefore, the total demand of petrol stations served by a tank truck cannot exceed $Q_{\max}$.

A feasible solution of the problem consists in selecting one of the available replenishment plans (patterns), for each petrol station, and routing a fleet of tank trucks (tankers), each one from the depot to a subset of petrol stations while respecting tanker capacity, in order to fulfil the selected station replenishment plans. The aim is determine the solution minimizing the total distance (length) travelled by tank trucks during the planning horizon.

Let $y_{i\pi}$ be a binary variable that takes value 1 if petrol station $i \in C$ is refurnished with replenishment plan (pattern) $\pi \in \Pi_i$, and 0 otherwise; let x_{ijt} be a binary variable that takes value 1 if arc $(i, j) \in A$ is traversed in time period (day) $t \in T$ by a tank truck, and 0 otherwise. Let $a_{\pi t}$ be a binary parameter equal to 1 if pattern π requires a visit in time period (day) t and 0 otherwise. The problem can be formulated as follows:

$$\min z = \sum_{t \in T} \sum_{(i,j) \in A} c_{ij} x_{ijt} \tag{1}$$

s.t.

$$\sum_{\pi \in \Pi_i} y_{i\pi} = 1 \qquad \forall\, i \in C, \tag{2}$$

$$\sum_{h \in N:(h,i) \in A} x_{hit} = \sum_{\pi \in \Pi_i} a_{\pi t} y_{i\pi} \qquad \forall\, i \in C, t \in T, \tag{3}$$

$$\sum_{j \in N:(i,j) \in A} x_{ijt} = \sum_{\pi \in \Pi_i} a_{\pi t} y_{i\pi} \qquad \forall\, i \in C, t \in T, \tag{4}$$

$$\sum_{i \in C:(i,o) \in A} x_{iot} = \sum_{j \in C:(o,j) \in A} x_{ojt} \qquad \forall\, t \in T, \tag{5}$$

$$\mathbf{x} \in \mathbf{X}(\mathbf{y}), \tag{6}$$

$$y_{i\pi} \in \{0, 1\} \qquad \forall\, i \in C, \pi \in \Pi_i, \tag{7}$$

$$x_{ijt} \in \{0, 1\} \qquad \forall\, (i, j) \in A, t \in T. \tag{8}$$

The objective function (1) is the total length traveled by tankers. Constraints (2) assure that exactly one replenishment plan (pattern) is assigned to each petrol station. Constraints (3) and (4) impose that exactly one tanker enters and (respectively) leaves each petrol station $i \in C$ during time period (day) $t \in T$ if the replenishment plan assigned to petrol station i establishes that the latter must be visited in time period t, and 0 otherwise. Constraints (5) impose that, for each time period $t \in T$, the number of tankers leaving the depot is equal to the number of tankers going back to the depot (at the end of their tour). Constraint (6) implicitly represents the set of subtour elimination and tanker capacity constraints. For example, by introducing a continuous variable u_{it}, for each $i \in C$, and $t \in T$, a family of constraints equivalent to (6) is:

$$u_{it} - u_{jt} + Q_{\max} x_{ijt} \le Q_{\max} - \sum_{\pi \in \Pi_j} d_{jt}^{\pi} a_{\pi t} y_{j\pi} \qquad \forall\, i, j \in C\colon (i, j) \in A, t \in T, \tag{9a}$$

$$\sum_{\pi \in \Pi_i} d_{it}^{\pi} a_{\pi t} y_{i\pi} \le u_{it} \le Q_{\max} \qquad \forall\, i \in C, t \in T. \tag{9b}$$

These constraints generalizes the (MTZ-like) subtour elimination constraints for the capacitated VRP (see, e.g., Toth and Vigo, 2002), ensuring that each tank truck route is connected to the depot and the total demand of the stations visited by a truck cannot exceed its capacity $Q_{\max}$.

The problem is *NP*-hard since it extends the capacitated VRP which is known to be *NP*-hard (see, e.g., Toth and Vigo, 2002).

4. Heuristic procedures

Due to the large size of the real instances which in general an oil company has to deal with, we heuristically solve the problem. We follow the same *group-first-route-second* heuristic approach used by Triki (2013), who proposes four heuristic strategies that solve PPSRP in two stages followed by a third local search improvement procedure: the first stage, namely the *assignment stage*, assigns replenishment patterns to petrol stations; the second one, namely the *routing stage*, (independently) solve heuristically the routing problem for each day $t \in T$, given the assigned replenishment pattern for each petrol station. The first two procedures by Triki (2013) balance the replenishment load during the planning horizon by minimizing: (TH1) the maximum total daily demand, and (TH2) the maximum daily number of replenished petrol stations, respectively. The other two heuristics start from two relaxations of the

PPSRP, and look at the minimization of the total traveled distance (cost) by minimizing: (TH3) the total distance between the depot and the stations to be served, and (TH4) the total distance of all the stations with respect to a set of T petrol stations, named hereafter “virtual centers” (one for each day of the planning horizon). Triki (2013) experimentally proved that heuristic TH4 outperforms TH3 which typically outperforms the other two.

We propose two new group-first-route-second heuristic strategies, called HA and HB, that solve our PPSRP in two stages (assigning/grouping and routing) by using two enhanced assignment procedures. The heuristic TH4 considers one virtual center for each day with the aim of grouping petrol stations, with the consequence of preferring to serve a subset of neighbor stations, for each day. Instead, our two novel heuristics consider multiple virtual centers for each day in order to look for a better replenishment-patterns-petrol-stations assignment that possibly groups petrol stations not only on a daily basis but also within the same assigned replenishment day on the basis of the tanker capacity. Therefore, contrarily to the routing stage of heuristics TH1–TH4 that requires to solve a capacitated VRP for each day, in our heuristics HA and HB the routing stage simply needs to solve a Traveling Salesman Problem (TSP) for each group of petrol stations. Moreover, due to the tanker capacity and petrol station demand amounts, the number of petrol stations to be replenished by a tanker is typically very small (on average 3, and at most 5) and, hence, we can solve optimally the TSPs in a very short time. Next, we give the details of the assignment stage of our heuristics.

4.1. Heuristic HA assignment stage

We consider a set V of petrol stations’ virtual centers and assume that each virtual center $j \in V$, for each time period (day) $t \in T$, represents a group of petrol stations to be replenished in day t presenting a total demand at most equal to the tanker capacity Q. For simplicity, we assume that the position of the virtual centers are chosen among those of the set of petrol stations and the depot, therefore let $V = C \cup \{o\}$. The problem addressed in the assignment stage consists in selecting one of the available replenishment plans (patterns), for each petrol station, and assigning petrol stations to virtual centers, respecting the above requirement and the selected station replenishment plans. The aim is to minimize the total distance of the stations from the assigned virtual centers.

Let $y_{i\pi}$ be again a binary variable that takes value 1 if petrol station $i \in C$ is refurnished with replenishment plan (pattern) $\pi \in \Pi_i$, and 0 otherwise; let w_{ijt} be a binary variable that takes value 1 if petrol station $i \in C$ is assigned to virtual centre $j \in V$ at time period (day) $t \in T$, and 0 otherwise; let z_{jt} be a binary variable, that takes value 1 if virtual centre $j \in V$ is used at time period (day) $t \in T$, and 0 otherwise. Moreover, let d_{it} be a (free) continuous variable representing the amount of demand of petrol station $i \in C$ at time period (day) $t \in T$, on the basis of the selected pattern. Let $a_{\pi t}$ be again a binary parameter equal to 1 if pattern π requires a visit in time period (day) t and 0 otherwise. Finally, let k be the maximum number of virtual centers that can be used during each time period (day) $t \in T$ (e.g., equal to the size of the tanker’s fleet). The problem can be formulated as follows:

$$\min z_{\text{HA}} = \sum_{t \in T} \sum_{i \in C} \sum_{j \in V} c_{ij} w_{ijt} \tag{10}$$

s.t.

$$\sum_{\pi \in \Pi_i} y_{i\pi} = 1 \qquad \forall\, i \in C, \tag{11}$$

$$\sum_{\pi \in \Pi_i} d_{it}^{\pi} a_{\pi t} y_{i\pi} = d_{it} \qquad \forall\, i \in C, t \in T, \tag{12}$$

$$\sum_{j \in V} w_{ijt} = \sum_{\pi \in \Pi_i} a_{\pi t} y_{i\pi} \qquad \forall\, i \in C, t \in T, \tag{13}$$

$$\sum_{j \in V} z_{jt} \le k \qquad \forall\, t \in T, \tag{14}$$

$$\sum_{i \in C} d_{it} w_{ijt} \le Q_{\max} z_{jt} \qquad \forall\, j \in V, t \in T, \tag{15}$$

$$y_{i\pi} \in \{0, 1\} \qquad \forall\, i \in C, \pi \in \Pi_i, \tag{16}$$

$$w_{ijt} \in \{0, 1\} \qquad \forall\, i \in C, j \in V, t \in T, \tag{17}$$
$$z_{jt} \in \{0, 1\} \qquad \forall\, j \in V, t \in T, \tag{18}$$
$$d_{it} \in \mathcal{R} \qquad \forall\, j \in V, t \in T. \tag{19}$$

The objective function (10) is the total distance of the petrol stations from the assigned virtual centers. Constraints (11) assure that exactly one replenishment plan (pattern) is assigned to each petrol station. Constraints (12) establish the amount of fuel oil demanded by each petrol station for each day on the basis of the assigned patterns. Constraints (13) assign exactly one virtual center to each petrol station for each planned replenishment day. Constraints (14) limit the number of used virtual centers for each day: these constraints are required for grouping petrol stations into groups of sufficiently large size and therefore avoiding as much as possible groups with only one stations. Constraints (15) limit the total demand of assigned petrol stations to each virtual center for each day to be at most equal to tanker capacity. Note that constraints (15) contains bilinear terms $d_{it}\, w_{ijt}$, , with $i \in C$, $j \in V$, $t \in T$, that can be linearized by introducing continuous variables δ_{ijt} in place of them, and adding the following constraints to ensure that $\delta_{ijt} = 0$ when $w_{ijt} = 0$ and $\delta_{ijt} = d_{it}$ when $w_{ijt} = 1$:

$$\delta_{ijt} \geq 0 \qquad \forall\, i \in C, j \in V, t \in T, \tag{20}$$
$$\delta_{ijt} \leq M_i w_{ijt} \qquad \forall\, i \in C, j \in V, t \in T, \tag{21}$$
$$\delta_{ijt} \leq d_{it} \qquad \forall\, i \in C, j \in V, t \in T, \tag{22}$$
$$\delta_{ijt} \geq d_{it} - M_i(1 - w_{ijt}) \qquad \forall\, i \in C, j \in V, t \in T, \tag{23}$$

with M_j being a sufficiently large scalar (e.g., $M_j = \max_{\pi \in \Pi_i, t \in T}\{d^{\pi}_{it}\}$). Hence, we finally replace constraints (14) with:

$$\sum_{i \in C} \delta_{ijt} \leq Q_{\max} z_{jt} \qquad \forall\, j \in V, t \in T, \tag{24}$$

and then obtaining a mixed integer program (MIP) formulation.

4.2. Heuristic HB assignment stage

The assignment stage of heuristic HB extends that of heuristic HA, by considering a new objective function that in addition considers also the total distance of the depot from the used virtual centers (i.e., those which are assigned to at least one petrol station), for each day of the week. Indirectly, this also limits the number of used virtual centers, with the consequence that we no longer require to limit such number. Therefore, the problem can be formulated as MIP (10)–(24) (obviously with constraints (24) in place of constraints (15)), with objective function (10) being replaced with the following new one:

$$\min z_{\mathrm{HB}} = \sum_{t \in T} \sum_{i \in C} \sum_{j \in V} c_{ij} w_{ijt} + \sum_{t \in T} \sum_{j \in V} c_{0j} z_{jt} \tag{25}$$

and without Constraints (14).

5. Computational results

We experimented the proposed heuristic algorithms on three real instances of an oil company with n = 49, 194 and 200 petrol stations and total amounts $\sum_{i \in C} q_i^{tot}$ of weekly stations' fuel oil demands equal to 1608, 4783, and 7255 Kiloliters (kL), respectively. Figure1 shows the map of the test with 200 petrol stations and 1 depot. The planning horizon is composed by p = 6 periods (days), tank truck capacity is $Q_{\max}$ = 39 kL, and when required by the assignment model (i.e, for our heuristic HA, and heuristics TH3 and TH4) we assume a fleet of $k = \lceil \sum_{i \in C} q_i^{tot}/(pQ_{\max}/2) \rceil$ tankers. The real data sets are the same used by Carotenuto *et al.* (2015), and assume that for each station i any pattern $\pi \in \Pi_i$ requires to replenish station i during a distinct combination Θ_π of $f_i \leq p$ days of the planning horizon, with an amount $d^{\pi}_{it} = q_i^{tot}/f_i$ of fuel oil for each day $t \in \Theta_\pi$. This assumption simplifies the HA assignment stage model formulation that becomes (10)–(18), without constraints (12) and with d_{it} fixed to q_i^{tot}/f_i. Analogue simplification can be done on the HB assignment stage model formulation. Moreover, the assumption on the demands also allows us to compare our heuristics with heuristics TH1, TH2, TH3, and TH4 of Triki (2013) which are designed only for this particular case. As for the latter heuristics we solve the routing stage by solving the capacitated VRP model formulation with a commercial solver (CPLEX). In addition, we also compare our results with the best solution value returned by the hybrid genetic algorithm (called HGA) proposed by Carotenuto *et al.* (2015), among 50 runs each one with at most 1200 seconds as time limit. Finally, we also solve the MIP formulation of our PPSRP for comparison. All the model formulations including those for the routing problems are solved with CPLEX 12.6 (on a PC with an Intel Core2 Duo CPU at 2.5 GHz and with 8GB of RAM) with CPU time limit fixed to 4000 seconds.

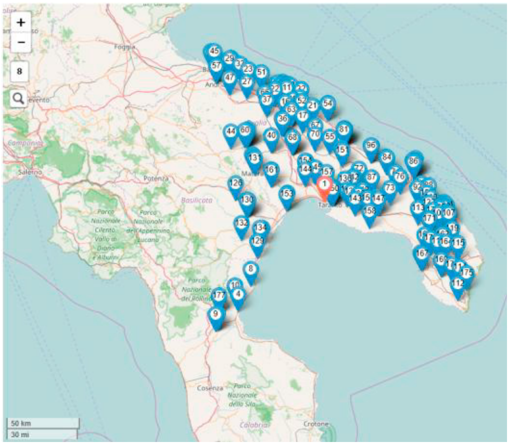

Fig.1 - The map of the test with 200 petrol stations and 1 depot

Table 1 and Table 2 list respectively the results in terms of total route length (Km) and CPU times, respectively, provided by our algorithms HA and HB in comparison with those of the other competing approaches. From the results listed in Table 1 (in bold the best results) we can see that our algorithms HB outperforms all the other two-stage (group-first-route-second) heuristic algorithms for all the three real cases, and also the hybrid genetic algorithm HGA for the cases with n = 194 and 200. Algorithm HA outperforms TH1, TH2 (except for the case with n = 49), and TH3. Finally, within the given time limit, CPLEX was able to find a feasible solution for the PPSRP, which results to be the worst one. According to Table 2 CPLEX was able to solve at optimum within the given time limit both the assignment and the routing stages only for algorithm HB and for the case with n = 49. In particular, the routing stage of heuristic HA and HB are done within one second: for these heuristics the routing stage requires to solve (approx.) n TSP instances, but each one with a few stations (on average 3 and at most 5). On the contrary algorithm TH1, TH2, TH3, and TH4 requires to solve p = 6 capacitated VRP instances with many stations (on average more than n/2) with the consequence that for many instances CPLEX was able to return only a feasible solution within the time limit. Finally, we note that CPLEX was unable to solve at optimum the assignment stages of algorithms HA, HB, and TH4, (except for algorithms HA and HB, with n = 49: indeed, for algorithm HA the assignment stage was solved at optimum in shorter time, i.e., within 88 seconds), whilst for the other heuristics this stage was solved at optimum within one second.

Table 1. Results on real instances.

	total route length (Km)							
n	HA	HB	TH1	TH2	TH3	TH4	HGA	MIP
49	3356.29	3136.41	3464.04	3352.37	3408.05	3290.43	**3134.38**	3586.63
194	25157.49	**23698.44**	26110.58	25474.63	26728.77	24778.32	24400.74	41756.47
200	24378.38	**23798.86**	24851.28	24933.36	25035.85	24088.62	23977.99	36289.38

Table 2. CPU times (in seconds) on real instances.

	HA		HB		TH1		TH2		TH3		TH4		HGA	MIP
n	*assign.*	*routing*	*assign.*	*routing*	*assign.*	*routing*	*assign.*	*routing*	*assign.*	*routing*	*assign.*	*routing*		
49	88	< 1	3800	< 1	< 1	11774	< 1	>6×4000	< 1	14410	633	16474	50×1200	>4000
194	>4000	< 1	>4000	< 1	< 1	>6×4000	< 1	>6×4000	< 1	>5×4000	>4000	>6×4000	50×1200	>4000
200	>4000	< 1	>4000	< 1	< 1	>5×4000	< 1	>6×4000	< 1	>6×4000	>4000	>6×4000	50×1200	>4000

6. Conclusions

In this work, we study the petrol station replenishment problem that consists in determining the fuel oil procurement plans of a set of petrol stations and planning the delivering routes of petrol products to the stations along a certain planning horizon, with the aim of minimizing the total routes' length travelled by tank trucks during the planning horizon. The problem is NP-hard since it generalizes the well known capacitated VRP. We propose two new group-first-route-second heuristics and evaluate their performances with respect to other known heuristics of the same class, and also with respect to a hybrid genetic algorithm recently proposed for the petrol station replenishment problem. Experimental results on a set of real instances show that in particular one of the proposed heuristics performs better than the other heuristics of the same class and often also with respect to the hybrid genetic algorithm used for comparison. Further studies will be devoted to the extension of the proposed approaches for solving the multi-depot version of the considered petrol station replenishment problem. Finally, the proposed model is scalable and transferable without excessive change to other similar case and applications. An example could be the case of reverse logistics or the differentiated waste collection when a visiting pattern and a planning time horizon is assigned and the amount of waste to pick up is known.

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