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A normative optimal strategy in intelligent transit networks

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Abstract. The paper focuses on unreliable dynamic transit service networks, on which, even if predictive info is available, trip planners should give dynamic strategy-based path suggestions, rather than provide a complete path up to destination. In the paper, the search for a travel strategy to be used as a path recommendation in innovative transit trip planners is analysed as a Markov decision problem, together with the relative solution approaches. Applicative examples of the procedure on a simple test network are then reported. Finally, some concluding remarks are made and future research perspectives outlined.

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Keywords: dynamic transit path choice modeling; dynamic normative transit travel strategy; Markov decision process; innovative transit trip planners; real-time predictive info system.

1. Introduction

Current transit trip planners, such as Moovit (2017) or GoogleTransit (2017), provide one or more origin-destination paths and, when transit services are regular and hence also route attributes, such paths allow users to reach their destinations with minimum travel time and cost (or maximum utility). By contrast, on stochastic (unreliable) transit networks, the path attributes are random variables and, even if trip planners use predicted travel attributes, experienced values can differ from those predicted, because forecasting methods do not allow attribute variability to be completely explained (see Section 3). Thus, it is not possible to define *a priori* exactly a complete path of maximum utility up to destination, as at certain nodes (*diversion nodes,* see Fig. 1a) some travel decisions should be made according to random occurrences (e.g. bus arrivals at stops, on-board crowding). Rather, following decision theory in uncertainty contexts (Von Neumann and Morgenstern, 1947), the objective should be to maximize long-run expected

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utility, following a *travel strategy*. A travel strategy (Spiess, 1983; Spiess and Florian, 1989) is a set of coherent decision rules which, at *diversion nodes*, establish specific behavior to be followed, and hence specific actions. An *optimal strategy* allows the traveler to obtain the long-run maximum expected travel utility up to destination Therefore, as suggested by Nuzzolo and Comi (2017), on unreliable networks with predictive info, transit trip planners should use an *optimal strategy or normative strategy approach.* Path suggestions should be provided, according to a *real-time dynamic decision-making*, with a sequence of choices in a set of diversion nodes, considering service occurrences which arise. Due to cognitive limitations, travelers without adequate support, such as an advanced trip planner, apply a strategy, here called *descriptive or subjective optimal strategy,* which can differ from the *objective optimal normative strategy*.

A new generation of transit trip planners has introduced travel strategy into path recommendation (Esztergár-Kiss and Csiszár, 2015; Comi *et al.*, 2017a). For example, a normative strategy-based approach is used in path suggestion by the transit route planner Hyperpath (SISTeMA, 2015), searching for the optimal strategy with scheduled travel times. Yet greater adherence is required between model assumptions and reality, as regards transit system functioning. In particular, forecasted bus arrival times should be used rather the scheduled ones used by Hyperpath.

The availability of a large quantity of data derived from automated data collecting allows more reliable path attribute forecasts (Moreira *et al.*, 2015). In this new context of intelligent transit systems, Nuzzolo and Comi (2017) and Nuzzolo *et al.* (2016) presented some real-time dynamic suggestion procedures, based on a normative strategy approach, suitable for the last generation of transit trip planners. An evolution of these procedures is the strategy path suggestion method presented in Section 3 and explored in the framework of a Markov decision problem – MDPm (Puterman, 2009), which allows both theoretical bases and optimal strategy search methods to be obtained.

In synthesis, this paper is organized as follows. Travel strategies on dynamic unreliable networks are defined in Section 2, while Section 3 describes the proposed advancement in dynamic travel strategy suggestion procedure. The optimal strategy search methods are explored in Section 4, in the framework of a Markov decision problem – MDPm. Section 5 reports some application examples of the procedure on a simple test network. Finally, in Section 6, some conclusions are drawn and some major research issues are pointed out.

In the following, unless otherwise reported, transit systems are assumed *unreliable, dynamic but steady* (no major disruptions are considered), with *individual real-time predictive information* (at least travel time components for specific origin-destination pairs and trip departure times).

2. Travel strategies

Line and run hyperpaths

Nguyen and Pallottino (1988) highlighted the underlying graph structure of Spiess' basic strategy concept, introducing a graph-theoretic framework and the concept of hyperpath. In this paper, we use two types of graph representation of a transit service network, *line graph* and *run graph*. While nodes of a *line graph* (see Fig. 1a) have only spatial coordinates, in a *run graph* (Fig. 1.b) the nodes have space-time coordinates. Hence, in the following we will refer to two types of hyperpath representations: *line hyperpaths* and *run hyperpaths.* For example, considering in Fig. 1b diversion node B at time $\tau_B^{5.1}$ when run 1 of line 5 arrives, two diversion links, representing two options, are present. One consists in boarding the approaching run 5.1, the other in waiting for the incoming run 1 of line 6 that is expected to arrive at time $\tau_B^{6.1}$.

Strategies and diversion rules

Given a *line service graph*, a travel strategy S is defined through:

- a *line hyperpath HP* from origin O and destination D, with a set of diversion nodes, each with a set of diversion links;
- a *diversion rule* dr_i, for each diversion node i, which determines the behavior and hence the actions, at that node.

For example, at diversion node B of *line hyperpath* of Fig. 1a, the set of attractive lines includes lines 5 and 6 and a diversion rule, which could be used, is to board the first arriving line. A different rule could be to board the arriving line if its expected travel time is less than or equal to the sum of the expected waiting time and the expected travel time of the other line.

It follows that a strategy S will be indicated as $S[HP;\boldsymbol{dr}]$, with $\boldsymbol{dr}$ the set of diversion rules dr_i. Note that on a service network, several hyperpaths can be considered. For example, with the graph in Fig. 1a, eleven hyperpaths can be found. Given a diversion rule and an objective of strategy-based decision making, an objective function Of can be considered, and the strategy $S^*[HP^*, dr]$ which optimizes this function is the *optimal strategy* conditional upon the diversion rule $\boldsymbol{dr}$ and the objective function Of, where HP^* is the optimal hyperpath among the alternative ones.

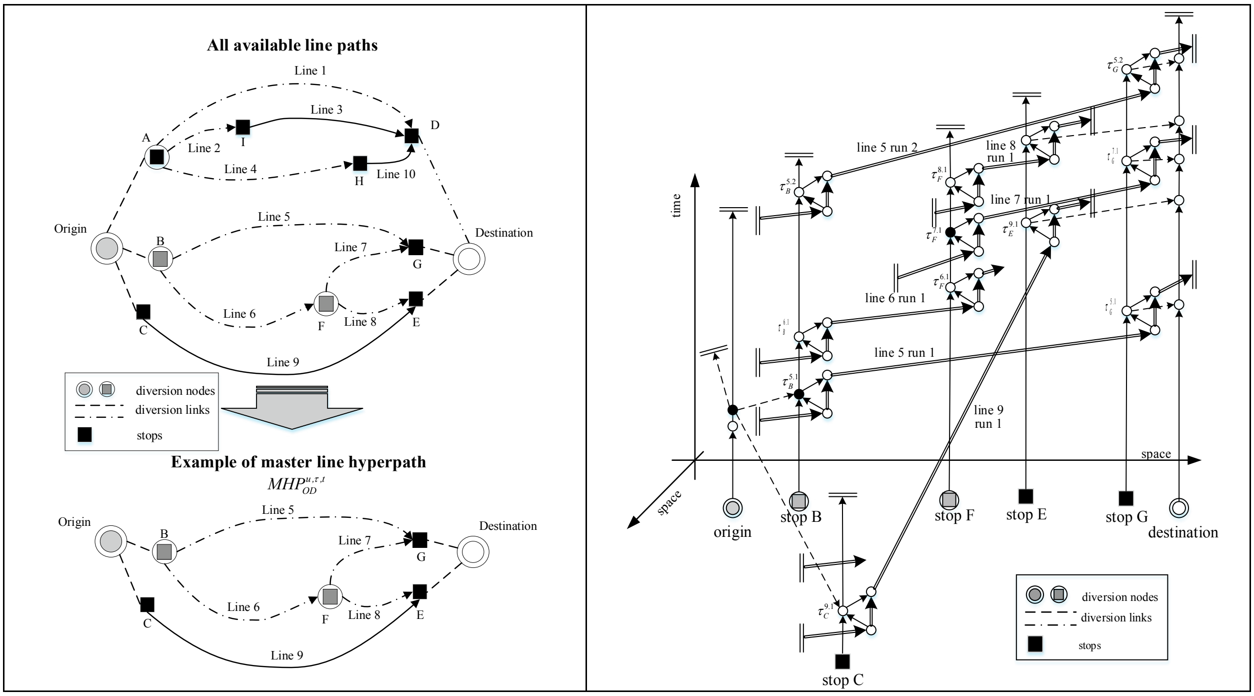

Figure 1. Example of a line graph and line hyperpath (*a*, left side) and run hyperpath (*b*, right side), corresponding to the line hyperpath

3. Real-time dynamic strategy-based path suggestion

A dynamic strategy-based execution of a trip makes up a sequence of diversion choices. A *service run graph* representation is here used and a travel strategy defines a *run hyperpath*, with at each *diversion node* the alternative outgoing links (*diversion links*), and how to choose among them *(diversion rule)*, according to transit services occurrences. For example, in Fig. 1b, at stop B when run 5.1 arrives, there is a diversion node with two diversion links, corresponding to two alternatives: to board run 5.1 or to wait for run 6.1. A possible diversion rule could be: compare the expected utilities up to destination of these two alternatives and choose the best one.

Upgrading the work of Nuzzolo *et al.* (2016) and Nuzzolo and Comi (2017), a new *real-time dynamic path suggestion* procedure for trip planning in uncertain contexts is reported below, after which the definitions of master hyperpath, line and run sub-hyperpaths and forecasted utility are recalled for the reader's convenience.

Master line hyperpath

Given an OD pair and a service line network at time τ, the trip planner considers a subset of feasible line paths, that is paths that satisfy some travel criteria, such as logical and behavioral constraints. We use *master line hyperpath* $MHP_{OD}^{u,\tau,t}$ to indicate a line hyperpath connecting origin O to destination D with this sub-set of feasible line paths (see Fig. 1.a). The criteria and constraints could take into account the attitudes of the specific traveler u and hence the prime u is used. Due to the randomness of transit services, we do not refer to time τ but to a time slice $\Delta\tau$ (e.g. $\Delta\tau = \tau$

± 5 minutes). Nevertheless, below, for the sake of simplicity, we continue to use τ. As a master line hyperpath can depend on departure time slice τ and day t, due to within-day and day-to-day dynamicity of transit services, primes τ and t are used.

The master hyperpath can be upgraded at each diversion node with respect to the service state at time τ of day t ($MHP_{OD}^{u,\tau,t}$). For example, info on disrupted lines allows such lines to be eliminated from the master hyperpath for time τ of day t.

Line sub-hyperpaths and run sub-hyperpaths

Given a master line hyperpath and a diversion node i, the sub-hyperpaths departing from this node and including the diversion link il are indicated as *line sub-hyperpaths* HP_{ilj} of the hyperpath set HP_{il}. The set of all the sub-hyperpaths departing from node i is indicated with HP_i. For example, at diversion node B of the master line hyperpath in Fig. 1.a, there is a line sub-hyperpath, including the diversion link B-G and three including the diversion link B-F. Corresponding to each line sub-hyperpath there is a run sub-hyperpath with the same spatial nodes, but also with temporal coordinates, as illustrated in Fig. 1b. The *optimal run sub-hyperpath* from node i to destination D is the *run sub-hyperpath of maximum forecasted utility* FHP_i^* among all the *run sub-hyperpaths of the set* HP_i.

Forecasted utility FU of a run sub-hyperpath HP_{ilj}

The *forecasted utility* $FU_{HP_{ilj}}^{u,\tau,t}$ at time τ of day t, of a run sub-hyperpath HP_{ilj} starting from the diversion node i as far as destination d with G_{ilj} elementary paths k, can be written as a function of its path utilities as follows:

$$FU_{HP_{il_j}}^{u,\tau,t} = \sum_{k \in G_{il}} p\left[k / HP_{il_j}^{u,\tau} \right] \cdot FU_k^{u,\tau,t}$$

where $p\left[k / HP_{il_j}^{u,\tau} \right]$ is the probability of using, at time τ, path k of sub-hyperpath HP_{ilj} and $FU_k^{u,\tau,t}$ is the forecasted utility of path k at time τ of day t, obtained using forecasted values FX of its attributes.

Forecasted path attributes

The forecasting methods to predict travel attributes (Moreira *et al.*, 2015), necessarily include forecasting errors. In some analyses performed by Comi *et al.* (2017b) on three months of automatic vehicle monitoring (AVM) data of two bus lines in Rome, values of the mean absolute percentage error were about 6%. This means that, for example, for a travel time of 1800 seconds, the mean absolute error is about 108 seconds. These types of results justify the application of strategy-based path suggestions on unreliable transit networks with diversion nodes.

Probability of path use

With the path attributes forecasted at time τ of day t, the relative run graph can be implemented. For example, refer to that reported in Fig. 1b and consider at origin the sub-hyperpath including stop B, run *5.1* and run 6.1. The forecasted arrival times at stop B of run 5.1 ($\tau_B^{5.1}$) and run 6.1 ($\tau_B^{6.1}$) derive from bus travel time forecasting, with known error probability density functions. These functions allow us to determine the probability of each run approaching as the first after the traveler's arrival. In general, assume a line Y with Gaussian forecasted arrival time FAT_Y (mean τ^Y and variance $\sigma^{2,Y}$) and a line X with Gaussian forecasted arrival time FAT_X (mean τ^X and variance $\sigma^{2,X}$), and let $\tau^X \geqslant \tau^Y$. The probability p[Y=1] that line Y arrives before line X, is: $p\left[Y=1\right] = p\left[\tau^X \geq \tau^Y\right] = p\left[Z = \tau^X - \tau^Y \geq 0\right]$ which can be easily obtained, given that Z is a Gaussian random variable with mean $z = \tau^X - \tau^Y$ and variance $\sigma^{2,Z} = \sigma^{2,X} + \sigma^{2,Y}$.

Diversion rule of the proposed strategy-based path suggestion

The diversion rule dr of the proposed strategy-based path suggestion can be reported as follows:

- given a master line hyperpath MHP_{OD}, at diversion node *i* and time τ of day *t,* the trip planner considers all the run sub-hyperpaths HP_{ilj} up to destination;
- the trip planner associates to each run hyperpath the forecasted run hyperpath utility, defined above, then compares these utilities and finally chooses the optimal run hyperpath, which is the hyperpath of *max forecasted utility*. In practice, this sub-hyperpath enumeration is avoided by the optimal strategy search reported in Sect. 4.

Real-time dynamic path suggestion.

As reported above, based on Nuzzolo *et al.* (2016) and Nuzzolo and Comi (2017), it is here proposed that a trip planner, in uncertain contexts, in order to optimize travel utility within the framework of Decision Theory (Von Neumann and Morgenstern, 1947), should apply the *real-time dynamic path suggestion* procedure:

"Given a master line hyperpath, a sequence of diversion choices is made at successive diversion nodes with the aim of maximizing the expected travel utility. When the traveler is at a diversion node, the diversion rule is to choose the run sub-hyperpath of max forecasted utility up to destination. This optimal run sub-hyperpath is used up to the next diversion node, where a new optimal sub-hyperpath up to destination is chosen, updating the forecasted utilities with the new path attribute forecasts and eventually upgrading the earlier master line hyperpath, including or deleting lines if particular events occur."

Therefore, at each diversion node, a set of diversion links is considered and an outgoing link has to be chosen (link diversion choice) with the predefined diversion rule. Note that, because of service unreliability, after the diversion choice only the path up to the successive diversion node is completely defined, with a temporary optimal sub-hyperpath from this node to destination, which could partially change when the path attribute forecasts are updated.

How to obtain the optimal run sub-hyperpath at a diversion node is reported below in Section 4.

4. Markov decision problems and optimal strategy search methods

In the following part of this paper, optimal strategy search is handled within the theoretical framework of a Markov decision problem-MDPm. Path choice in an unreliable service network entails decision making without comprehensive knowledge of all relevant factors and their possible future evolution. Hence, the outcomes of any decision depend partly on randomness and partly on the trip planner's decisions. Therefore, a general theoretical framework for optimal strategy search can be found in Stochastic Decision Theory. If path choice is considered as decision-making in a Markov decision process (MDPs), the Markov decision problem-MDPm (Puterman, 2009) approach can be used.

Markov decision problems - MDPm

A Markov decision process MDPs can be defined by the quintuple (T; SS^{τ}; $A_{s\tau}$; $p^{\tau}[j/s, a]$; $r^{\tau}[s, a]$), where:

- T is a *set of stages* τ at which the decision maker observes the state of the system and may make decisions;
- SS is the *state space*, where SS^{τ} refers to the possible system states for a specific time τ;
- $A_{s,\tau}$ is the set of possible actions that can be taken after observing state *s* at time τ;
- $p^{\tau}[j/s,a]$ are the *transition probabilities*, determining how the system will move to the next state. In particular, $p^{\tau}[j/s,a]$ defines the transition to state *j belonging to* $SS^{\tau+1}$ at time $\tau+1$, and, as a Markov process, only depends on state *s* and chosen action *a* at time τ;
- $r^{\tau}[s, a]$ is the *reward function*, which determines the consequence for the decision maker's choice of action *a* while in state *s,* and *R* is the reward set. In our cases, the value of the reward depends on the next state of the system, effectively becoming an "*expected reward*", expressed as:

$r^{\tau}\left[s,a\right]=\sum_{j\in SS^{\tau+1}} r^{\tau}\left[s,a,j\right]\cdot p^{\tau}\left[j/s,a\right]$; where $r^{\tau}\left[s,a,j\right]$ is the relative reward when the system is next in state *j*.

An MDPs with a specified optimality criterion (hence forming a sextuple) can be called *Markov decision problem MDPm*. Policies π are essentially functions that regulate, for each state, which actions to perform. The objective of MDPm is to provide the decision maker with an *optimal policy* π^* that associates to states *SS* actions *A* optimizing a predefined objective function. In most situations, a policy π that maximizes some cumulative function of the rewards is needed. A common formulation is the expected (discounted) sum over the given time horizon.

Optimal travel strategy as optimal policy of a MDPm

Given a run service network, the optimal travel strategy S^* can be seen as the optimal policy π^* of a finite and discrete *MDPm*, considering that:

- the set T is the set of times when the traveler is at a diversion node and the diversion link has to be chosen;
- the state space set SS is the set of diversion nodes among which travelers can move;
- the action set $A_{s,\tau}$ is set of sub-hyperpaths from the diversion node s to destination D;
- the change in the time of traveler location within the diversion node set consists in a Markov stochastic process;
- the transition probabilities $p^{\tau}[j/s,a]$ are the probabilities of going from a diversion node to each of the following diversion nodes if action a is applied;
- the reward function $r^{\tau}[s; a]$ is the expected utility of applying action a from a diversion node to destination node.

The optimal policy gives the best sequence of actions, considering the expected utility up to destination. To represent an MDPm, a *state-action tree* can be used, as reported in the next sub-section.

State-action tree

At every diversion node, each action can be represented with a set of outgoing links to the next diversion nodes. In Fig. 2 (left side), in relation to the master hyperpath (Fig. 1a), the decision tree is obtained. For example, at diversion node F three different actions are possible: 1) using run 7.1 (i.e. action a_7) and then stop G and destination; 2) using run 8.1 (action a_8), and then stop E and destination; 3) comparing the expected utility of boarding the first arriving run and the expected utility of the next run and then choosing the best (action a_{7+8}).

Transition probabilities

With regard to the transition probabilities, consider the case of diversion node F in Fig. 2. If action a_{7+8} is applied, the probability of moving onto node G is equal to the probability of using line 7, and the probability of moving onto node E is equal to the probability of using line 8. If action a_7 is applied, the probability of going onto node G is equal to 1. The same holds for action a_8 and node E. The probabilities $p^{\tau_F^{7.1}}\left[G/F,a_{7+8}\right]$ and $p^{\tau_F^{7.1}}\left[E/F,a_{7+8}\right]$ can be obtained from the knowledge of the arrival time probability density function of runs 7.1 and 8.1, as stated in Section 3, while the reward received after transitioning from state F to G or from state F to E can be computed from the forecasts of travel time from node F up to destination, including the expected waiting times.

MDPm solution methods

When the state transition function P and the reward function R are known, exact methods can be applied, for example dynamic programming (DP). The value or utility U_π of policy π starting from state s, which gives the overall expected value of the chosen policy from the current to the final state, may be expressed as follows:

$$U_\pi := E\left[\sum_{\tau=1}^{h-\tau} r_{\tau+i} \mid s_\tau = s\right]$$

The standard algorithms proceed iteratively to construct two vectors $U(s)\in R$ and $\pi(s)\in A$. $U(s)$ is the iterative version of the so-called Bellman equation, which determines a necessary condition for optimality to be obtained. The two most common DP algorithms to solve MDP are *value iteration* (Bellman, 1957) and *policy iteration* (Howard, 1960).

Optimal travel strategy search as a solution to a Markov decision problem

The optimal sub-hyperpath choice at diversion nodes, used in the proposed suggestion procedure, is equivalent to the resolution of an MDPm, using as expected reward, the forecasted utility of the sub-hyperpath, function of the forecasted path attributes. The attribute forecasted values, if obtained through statistical forecasting methods, are expected value estimations (Hyndman and Athanasopoulos, 2016). The probability of using each path k within a run sub-hyperpath HP_{ilj} , considered in Section 3, is used to determine transition probabilities.

The MDPm perspective allows us also to transfer to the travel strategy analysis the results of Markov process theory, as regards the existence and uniqueness of the solution and simulation algorithm convergence.

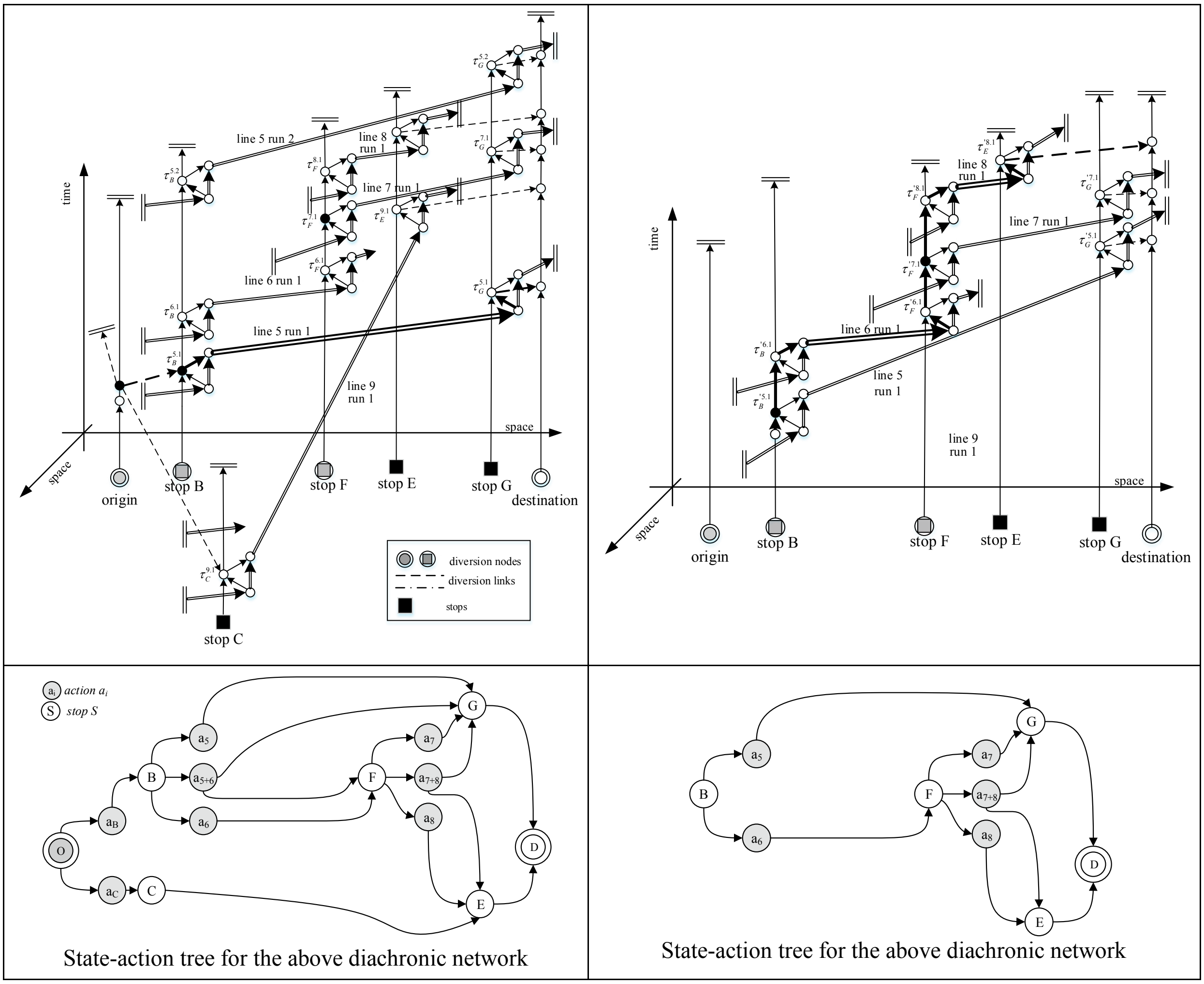

Figure 2. Examples of diversion choice at origin (left side) and sequential binary choice at diversion node (stop) B (right side).

5. Example of dynamic strategy-based path suggestion

A dynamic strategy entails a sequence of choices. Even if pedestrian and on-board diversion nodes could also be considered, for the sake of simplicity, diversion choices are analyzed below only at origin and at stops. At *origin*, according to the availability of real-time information and considering at departure time the values of forecasted path attributes, a run graph can be implemented, such as that represented in Fig. 2 (left side). Considering the state-action tree reported in the same Fig. 2 and applying the backward algorithm of Bellman (1957), the optimal policy and the optimal sub-hyperpath of max forecasted utility from origin O to destination D can be found. Suppose that this optimal policy includes stop B and action a5, or, in terms of strategy, the sub-hyperpath represented with a thick line in Fig. 2.

Moving onto this next diversion node, in our case stop B, while the traveler is waiting and when a line of the master hyperpath arrives, a new run graph up to destination is built with the updated forecasts (see Fig. 2 - right side). Now, the possible actions are: to board the arriving run or to wait for the next run of the other line. Given the new state-action tree reported in the same Fig. 2 and applying once again Bellman's backward algorithm, the optimal policy up to destination is found. If the traveler is suggested not to board the arriving run, the process is re-applied when the next run arrives.

Let us suppose that the optimal policy includes boarding line 6 up to node F and then using run 8.1, or, in terms of the optimal strategy, the sub-hyperpath of max forecasted utility up to destination represented with a thick line in Fig. 2- right side. In this way, stop F is reached and when a run of lines 7 or 8 arrives, a new run graph up to destination is built with the updated forecasts. The action of boarding is compared with that of waiting in terms of forecasted utilities, directly obtained as only elementary paths are available, and the best diversion link is found. Again, if the traveler is suggested not to board the arriving run, the process is re-applied when the next run arrives.

6. Conclusions

A normative strategy-based path suggestion method, which can be applied by advanced transit trip planners on unreliable service networks, was presented. The concepts of line and run hyperpath, master line hyperpath, dynamic link diversion choice rule and forecasted utility of run sub-hyperpath were explored within a dynamic normative travel strategy context. Further, the path suggestion method was examined in the framework of an MDPm, allowing the results of Markov process theory, in terms of existence and uniqueness of the solution and algorithm convergence, to be transferred to travel strategy analysis. Finally, the normative travel strategy search method with real-time predictive path attributes was applied to a simple test network to highlight some operative aspects.

Several research issues still need to be resolved. These include master line hyperpath choice modeling, development within theories other than that of expected utility, real-time optimal hyperpath search methods on large networks and path attribute forecasting methods using the large quantity of data that can be collected from transit networks.

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