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Maximin Fairness-Profit Tradeoff in Project Budget Allocation

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Abstract

Companies typically select those projects that maximize their profit as the primary criterion, within the limited budget at their disposal. This criterion may lead to some company departments getting an exceedingly large share of the overall budget and induce a negative perception of unfairness among the less favourite ones. We investigate how profit optimization can be sought after while achieving the desired level of fairness at the same time. Adopting a maximin approach to fairness and using an Integer Linear Programming solver, we show that a linear trade-off is possible, since fairness and profit exhibit a nearly perfect linear anticorrelation. Fairness can be improved by even a relatively small reduction of profit, especially in large companies (i.e., managing a large number of projects).

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1. Introduction

The budget allocation process determines how the (scarce) monetary resources devoted to investments into new projects are distributed among projects within a company [1]. Though the allocation is concerned with individual projects, the proposal of new projects and the subsequent money distribution are indeed carried out through the organizational structures that make up the company (e.g., departments or divisions): it is the departments that actually propose new projects to the company's apical management, and it is departments that actually receive the money and then manage it. Budget allocation (and the more general resource allocation) is a major issue in a company managing a portfolio of projects [2].

However, profit is not the only variable to consider when budget is allocated. An example of allocation in an aerospace company using artificial intelligence programming and a fuzzy analytic hierarchy process following a number of tangible and intangible criteria is reported in [3]. In particular, as shown in several papers, fairness has an

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impact on commitment levels and business-unit-level outcomes [4, 5, 6]. The individual employees within a department may feel frustrated and demotivated to work within a structure that is assigned only a thin slice of the budget pie. A company is therefore expected to distribute its budget in such a way as not to have any department unsatisfied and neglected due to budget allocation.

Fair allocation problems, such as that concerning the budget, arise naturally in various real-world contexts (mathematics, social choice, game theory and, more recently, computer science): resources are to be shared among several self-interested parties (players or agents), so that each party receives his/her due share. At the same time, the resources should be utilized in an efficient way from a central point of view. A wide variety of fair allocation problems have been addressed in the literature depending on the resources to be shared, the fairness criteria, the preferences of the agents, and other aspects, to evaluate the quality of the allocation.

From a wider perspective, many authors have dealt with the problem of balancing global efficiency and fairness either defining appropriate models or designing suitable objective functions or determining trade-off solutions (see for instance [7, 8, 9]). A recent survey of the trade-off between efficiency and equity in the operational research literature is conducted in [10]. The issue of the interaction of fairness policies and profit goals in a typical company setting is instead yet to be fully addressed; some early results have been provided in [11].

In this paper, we examine the budget allocation problem in a company that wishes to maximize its profit, but also to achieve fairness among its departments at the same time. We wish to investigate if (as expected) and how the fairness goal impacts on profit. We adopt an exact profit optimization procedure, imposing a threshold on the fairness achieved, so that any department obtains at least a pre-set share of the overall budget.

Our conclusions are that, in all cases examined, profit and fairness are linked by a nearly perfect linear anticorrelation: profit is traded off for fairness at a specific rate. The amount of profit that has to be sacrificed for fairness, however, appears quite limited, so that a fairer allocation policy can be safely adopted in place of a purely profit-maximizing one, with the trade-off being better for larger companies. In addition, raising the fairness threshold progressively reduces the dispersion of fairness values: as we seek for greater fairness, we get a value of the fairness index that is closer and closer to the threshold we have set, and independent of other conditions.

The paper is organized as follows. We define the budget allocation problem within a company in Section 2. In Sections 3 and 4 we then describe our modelling assumptions and the simulation experiment we conduct to assess the performance of budget allocation schemes in achieving fairness. The results of our experiments are finally reported in Section 5.

2. The budget allocation problem

Let's consider a company that has to allocate an overall budget B among its D departments. For the time being, we assume that the departments have the same size. Loosely speaking, we mean that they have the same working capacity and submit the same number of project proposals, which is N, so that the overall number of proposals under consideration to receive a slice of the budget is $D \cdot N$.

The j-th project ($j = 1 \dots N$) of the i-th department ($i = 1 \dots D$) requires a budget S_{ij}, which must be obtained in full for the project to be undertaken. If all the proposals were to be accepted, the needed overall budget would be

$$B_{\text{req}} = \sum_{i=1}^{D} \sum_{j=1}^{N} S_{ij} > B, \tag{1}$$

otherwise there would be no allocation problem, and all the proposals would be accepted.

The estimated return on the investment for the project with required budget S_{ij} is given by $R_{ij} \in (0, \infty)$.

Since $B_{\text{req}} > B$, the company has to select a subset of the proposals submitted for funding. If we employ an indicator variable X_{ij}, which is equal to 1 if the project is accepted and 0 otherwise, the budget allocated to the i-th department is

$$\tilde{B}_i = \sum_{j=1}^{N} X_{ij} S_{ij} \qquad i = 1, \dots, D, \tag{2}$$

and the overall allocated budget is

$$B_{\text{all}} = \sum_{i=1}^{D} \tilde{B}_i = \sum_{i=1}^{D} \sum_{j=1}^{N} X_{ij} S_{ij}, \tag{3}$$

while the overall profit (gain) for the company is

$$G = \sum_{i=1}^{D} \sum_{j=1}^{N} X_{ij} R_{ij} S_{ij}. \tag{4}$$

An obvious constraint is that the company cannot allocate a budget larger than B, i.e.

$$\sum_{i=1}^{D} \sum_{j=1}^{N} X_{ij} S_{ij} \leq B. \tag{5}$$

Here we consider the *budget allocation problem* as the problem of selecting the proposals to be funded (which in turn determine the amount of money to transfer to each department) with the goal of maximizing the overall profit G and the maximin index of fairness F, i.e.

$$\begin{aligned} &\underset{X_{ij}}{\text{maximize}} \quad G = \sum_{i=1}^{D} \sum_{j=1}^{N} X_{ij} R_{ij} S_{ij} \\ &\underset{X_{ij}}{\text{maximize}} \quad F = \min_i \tilde{B}_i = \min_i \sum_{j=1}^{N} X_{ij} S_{ij} \end{aligned} \tag{6}$$

It is to be noted that the maximum possible fairness value $F_{\max}$ is determined by the number of departments, since it corresponds to the case of perfect equipartition of the budget: i.e.

$$F \leq F_{\max} = \frac{B}{D}. \tag{7}$$

Here we do not follow the bi-objective approach, but instead maximize the overall profit while keeping the fairness index above a predetermined threshold given by an appropriately chosen parameter value $\lambda \in [0, 1]$. The budget allocation problem is therefore formulated as

$$\underset{X_{ij}}{\text{maximize}} \quad G = \sum_{i=1}^{D} \sum_{j=1}^{N} X_{ij} R_{ij} S_{ij} \quad : \; F \geq \lambda \cdot \frac{B}{D}, \tag{8}$$

where the value $\lambda = 0$ represents the case where fairness is not considered in the allocation.

Since the two goals conflict with each other, we expect that in practice a trade-off will be sought. Our goal is to explore the conflict between profit and fairness and determine what degree of trade-off is achievable. In order to look for the constrained optimization solution, we employ the Gurobi optimizer as an Integer Linear Programming (ILP) solver.

3. Modelling assumptions

So far we have described the budget allocation problem without any reference to what represents the input for the task at hand, i.e., the characteristics of the set of proposals submitted by the departments for funding. In this section, we describe our modelling assumptions, which will be used in the simulation experiments described in the following sections.

The characteristics of projects submitted to the company's top management for funding may be essentially described by two quantities: the size of the projects (here embodied by the amount of investment required) and the

Case	No. of departments (D)	No. of projects per department (N)
C1	5	50
C2	5	100
C3	10	50
C4	10	100

Table 1. Cases considered in the simulation

expected profit. As to the latter quantity, it may be alternatively described by the rate of return on the investment (ROI), which, when multiplied by the amount of investment, gives us the profit.

In this paper we assume that the individual project budgets can be considered as random variables drawn from a lognormal distribution

$$\mathbb{P}[S_{ij} < x] = \frac{1}{\sigma\sqrt{2\pi}} \int_0^{\ln x} e^{-\frac{1}{2}(\frac{y-\mu}{\sigma})} dy \tag{9}$$

This assumption has been validated through the analysis of more than 3000 projects spanning over a decade [12].

As to the rate of return, again we assume that the return is a random number, in this case drawn from a uniform distribution, whose lower bound a reflects that assumption that only projects with an estimated return positive and larger than a lower bound, e.g. the interest rate, are considered:

$$\mathbb{P}[R_{ij} < y] = \frac{y-a}{b-a} \qquad a \leq y \leq b. \tag{10}$$

4. Simulation experiments

In order to investigate the performance of the constrained optimization procedure, we have conducted a number of simulation experiments, following the modelling assumptions described in Section 3. As recalled in Section 2, we have used Gurobi (`www.gurobi.com`) as an ILP solver. In this section we describe our experiments.

We have considered the cases reported in Table 1. For each case we have generated 1000 test instances, with each instance consisting in a size and ROI value for each project submitted for funding. As described in Section 3, the size is generated according to a lognormal model, with $\mu = 5.2$ and $\sigma = 1.35$, as suggested in [12]. The ROI was set in the [5%-30%] range. In all experiments the overall available budget was set as 30% of the required budget, as defined in Equation (1): the available budget was therefore different for each instance.

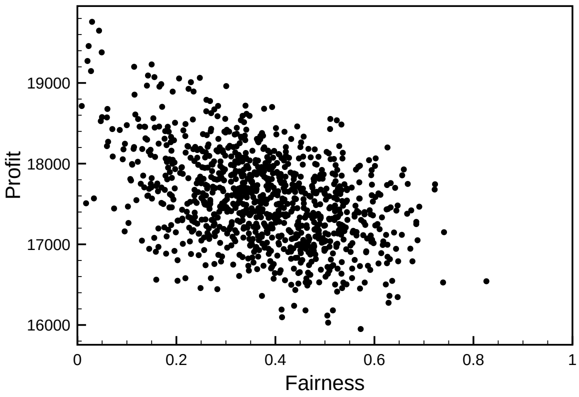
Since the cases considered in Table 1 span over different values for the number of departments, and the fairness index depends on the number of departments, as shown in Equation (7), in order to compare the results for the four cases on a level playing field, in the following we consider the normalized fairness

$$\tilde{F} = \frac{F}{F_{\max}} = \frac{D \cdot F}{B} \tag{11}$$

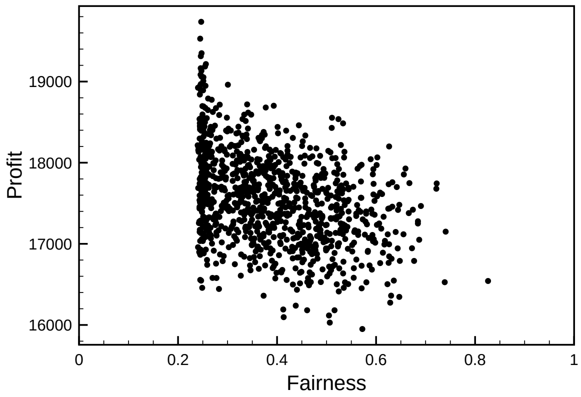
5. Profit vs. Fairness

After having defined the cases considered to examine the trade-off between profit and fairness, in this section we report the results. We first consider the impact of λ on the fairness actually achieved.

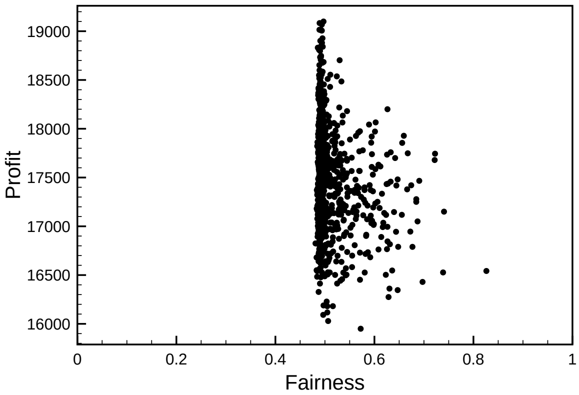
For every instance of the simulation sample we obtain a different profit-fairness combination. However, the constraint imposed on λ forces all the combinations into a limited area of the profit-fairness space, such that $\tilde{F} > \lambda$. In Fig. 1 we show the scatterplots resulting from the profit-fairness values for all the 1000 simulation instances in the C3 case (similar plots are obtained for the other cases). The pictorial representation of the shift of the lower bound on fairness is quite clear, but we also note that forcing the solution towards a higher fairness reduces the dispersion of fairness values at the same time, as can be verified by looking at Table 2, where the standard deviation of fairness reduces as we increase λ and drops dramatically when $\lambda \geq 0.5$. Instead, we can see in the same table that the dispersion of profit values reduces only moderately when we push solutions towards a higher fairness.



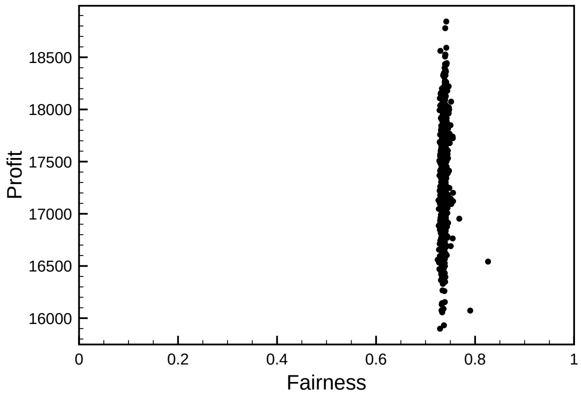
(a) $\lambda = 0$



(b) $\lambda = 0.25$



(c) $\lambda = 0.5$



(d) $\lambda = 0.75$

Fig. 1. Profit and Fairness in the C3 case

λ	Fairness σ	Profit σ
0	0.139	567.7
0.25	0.115	565.5
0.5	0.040	539.3
0.75	0.005	483.6

Table 2. Dispersion of fairness in Case C3

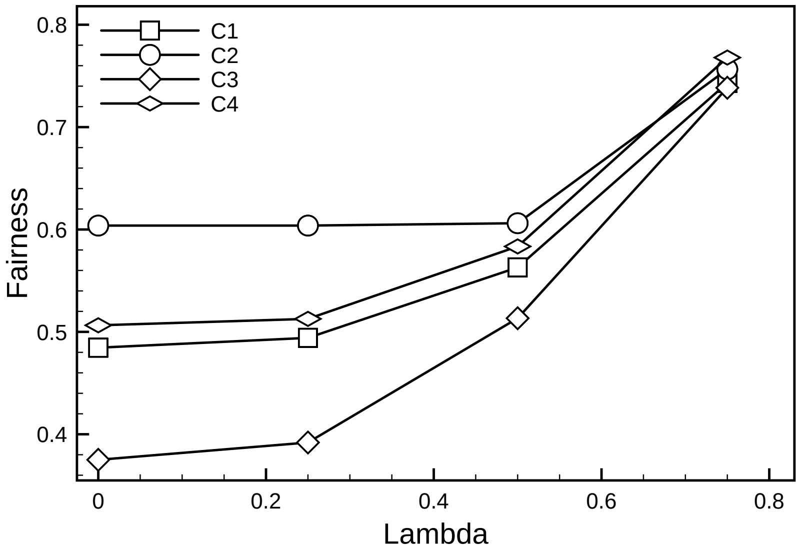


Fig. 2. Impact of the fairness threshold on the average fairness

Actually, we see that the minimum value of fairness is pushed up by the threshold λ, but what happens on the average, since the values of fairness appear to be progressively flattened on the threshold? In Fig. 2 we see that the average fairness grows nonlinearly when we set a higher threshold. However, as already noted, when the threshold gets quite high, the average fairness is practically equal to the threshold.

We can now examine the relationship between profit and fairness. In Fig. 3 we see how the average profit per project (defined as the overall profit divided by the number of submitted projects) reduces when the company opts for more fairness in the four cases of Table 1. The average profit is obtained by dividing by the number of submitted projects (rather than considering the profit itself or the average profit per approved project), to remove the influence of the size of the company and compare all the cases on a level playing field (companies with a larger number of submitted projects and an associated higher available budget will typically have a larger overall profit); it is to be noted that here the number of submitted, rather than approved, projects is considered as a measure of the size of the company. As can be seen, the average profit per project (as defined above) grows when the overall number of projects grows, though the increase is quite limited: as the number of submitted projects grows from $5 \times 50 = 250$ to $10 \times 100 = 1000$ (i.e., fourfold), the average profit moves in the [34.5-35.2] range, which means a change by a mere 2%. As to the exchange of profit for fairness, just observing the curves tells us that they get steeper as the number of projects grows, which is tantamount to saying that even a small loss in profit may provide a large increase in fairness. The relationship between profit and fairness can be represented by the incremental ratio E (the ratio of the change in fairness to the corresponding change in profit divided by the number of submitted projects), which acts as an exchange rate:

$$E = \frac{\Delta \tilde{F}}{\Delta \frac{G}{ND}} \tag{12}$$

This quantity gives us a precise indication of how much profit we have to sacrifice to improve fairness. Higher values of E correspond to a better trade-off, since large improvements in fairness can be achieved at the expense of a small sacrifice in profit. The results in Table 3 confirm that the trade-off improves as the number of projects grows (i.e., for larger companies). The differences in the results between cases C2 and C3 show that, for the same overall number of projects, the trade-off is better when projects are concentrated in the hands of fewer departments.

In all cases, the relationship between profit and fairness is nearly linear. The linear correlation coefficient between profit and fairness for the four cases is reported in Table 4. It takes absolute values in excess of 0.98 in three cases out of four (and a very large absolute value of nearly 0.98 in the C3 case): profit and fairness exhibit a nearly perfect

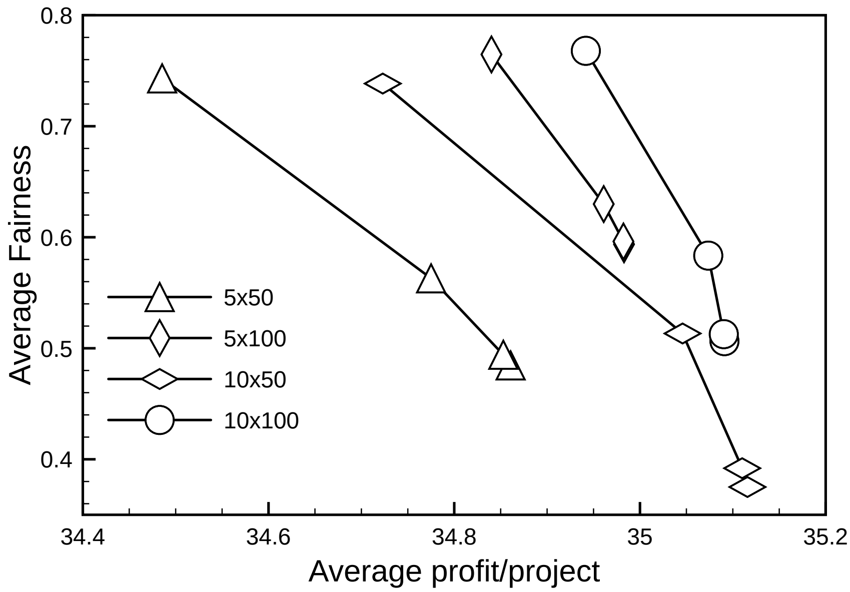


Fig. 3. Fairness-Profit trade-off

Case	No. of projects	E
C1	250	0.689
C2	500	1.198
C3	500	0.925
C4	1000	1.752

Table 3. Exchange of profit for fairness

Case	Profit-Fairness correlation
C1	-0.9972
C2	-0.9984
C3	-0.9789
C4	-0.9841

Table 4. Correlation between profit and fairness

linear anticorrelation.

6. Conclusions

The possibility of achieving fairness in budget allocation to departments within a company has been explored considering a maximin approach to fairness and an integer linear programming solver to maximize profit. The relationship between profit and fairness exhibits a nearly perfect anticorrelation: profit is exchanged linearly for fairness. The rate at which the trade-off takes place increases as the size of the company grows, so that in larger companies a smaller reduction in profit is needed to achieve a greater fairness. The trade-off is achieved by tuning the fairness threshold in the constraint adopted when performing profit optimization, but the relationship between the threshold and the average fairness is strongly nonlinear. A good degree of fairness can therefore be obtained without significantly sacrificing profit.

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