Prices and Locations in a Spatial Duopoly under Uniform Delivered Pricing

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Abstract

I analyse a two-stage location-price duopoly game under uniform delivered pricing when firms produce homogenous goods and are unable to ration the supply. Two tie-breaking rules (TBR) are studied: consumers either buy from the nearest firm or buy from either firms with equal probabilities. Under the first TBR, I find multiple single-price equilibria. Equilibrium locations are shown to be symmetric and to be such that the distance between firms increases (decreases) with the transportation cost (c) when c is high (low). Under the second TBR, firms cluster to the centre of the market line and choose the price that gives them zero profits. Surprisingly, when c is low, consumers are better off when they randomly select from which firm to buy. (JEL classification: L13, R30. Key words: Spatial duopoly, uniform delivered price, rationing.)

1 Introduction

Widespread use of uniform delivered pricing (UDP henceforth) policies is acknowledged in the literature (see, for instance, Greenhut [11] and Phlips [17]). The theoretical analysis of markets where this pricing policy is adopted has been however obstructed by the fact that UDP models suffer from problems of existence of the equilibrium even more seriously than other spatial models. Schuler and Hobbs [18] and Beckmann and Thisse [6] show that when products are homogeneous and consumers buy from the nearest firm in case of a price tie, no price equilibrium exists under UDP. De Palma *et al.* [8] give a similar result when the tie-breaking rule is that the consumer has a probability one half of purchasing from each firm.[1]

Recently, Iozzi [13] has shown that an equilibrium may exist with product homogeneity under both these tie-breaking rules when firms are assumed to be unable to ration the supply. However, it is assumed there that firms are sym-

[1] Different strategies have been used in the literature to overcome this problem. In some cases, it has been assumed that products sold by different firms are heterogeneous, although different approaches to product heterogeneity have been taken in the literature. (see, for instance, Anderson and De Palma [1], Anderson *et al.* [4], and De Fraja and Norman [7]). Using an alternative approach, Katz and Thisse [14] show the existence of a mixed strategy equilibrium in prices with perfectly homogeneous products.

metrically located along the market line. This paper extends this analysis by charactering the price equilibrium for any pair of firms' locations. The choice of locations by the two firms is also subject of analysis, using a standard two-period location then price game. By explicitly characterising the equilibria under the different tie breaking rules, the paper not only illustrates how results are deeply sensitive to market sharing rules but also provides a welfare ranking of the equilibria under the different tie-breaking rules.

Assuming 'no rationing' in a spatial market under UDP accounts to assuming that if firm i sells the good at price p to consumers located at location x along the market line, it has to supply at the same price also all consumers located at all locations different from x if these consumers are willing to buy from firm i.

It is rather common in the literature to assume that firms meet demand even if it is unprofitable to do so (see, for instance, Dixon [10] and Walbach [20]). One possible rationale for this behaviour is that firms face a cost to turning down customers. This may be costly in term of goodwill, reputation or offence caused. The existence of these costs has already been assumed in the literature (see Dixon [10]) and it is standard in Operational Research and inventory models (see, e.g. Taha [19]). Despite these costs are not explicitely modelled here, it is clear that a sufficiently high cost of turning customers away makes it preferable for the firm to serve even markets where revenues are smaller than the production and delivery costs.

Also, firms may be required not to ration supply by some kind of regulatory constraints. Indeed, regulatory requirements to satisfy all the demand at a uniform price are rather common in utilities. This is the case, for instance, of the domestic electricity markets in the United Kingdom. These regional markets, previously characterised by the presence of a single supplier, have been open to competition since 1998. In each region, all electricity suppliers are required by the regulator under the terms of their licences to publish their (uniform) prices and are not allowed to refuse to supply any customer in the region. Another example, less applicable to a spatial context, is given by Walbach [20]. He mentions that German car insurers are legally required to accept all customers for third party liability insurance.

The 'no rationing' assumption is crucial in restoring the possibility of equilibrium for the following reason. In a spatial duopoly under UDP when firms can ration the supply of the good, a firm may find it optimal not to supply all the customers along the market line. Then, the rival may prefer not to compete in price in the markets already served by the other firm but simply to supply at the monopoly price all the remaining markets. Clearly, this cannot be an equilibrium as the other firm would have an incentive to raise its price. When firms cannot ration the supply, they are obliged to serve all the customers when charging a price lower than the rival. This prevents each firm from leaving some markets to the rival, where the latter can charge its monopoly price.

The paper finds price-location equilibria which are deeply different according to the tie-breaking rule adopted. When consumers buy from the nearest firm, it is found that any pair of identical prices within a given range is a Nash equilibrium. The upper bound of this range is the price that makes the firm indifferent between matching and undercutting the rival. The lower bound of the interval is the price such that any firm make zero profits when matching the rival. Any pair of equal prices within this interval is a Nash equilibrium because no firms has an incentive to undercut the rival as it would have to serve the whole market line, facing too high a transportation cost. When consumers buy from either firm with probability equal to $\frac{1}{2}$, every locations is served by the lowest cost firm, that is by the firm which is located nearest to the centre of the market line. This firm charges a price slightly lower than the price that would give zero profits to the rival.

Equilibrium locations are also different under the different tie-breaking rules. When demand at each locations is equally shared between firms, it is found that firms choose to locate at the centre of the market line in the first stage of the game. The equilibrium in the pricing stage of the game is then with each firm patronising exactly half of the consumers at each locations and charging the price that allows it to make zero profits. On the other hand, when consumers buy from the nearest firm, firms symmetrically locate away from the centre of the market line. When the transportation cost is high, the higher is the unit cost of transport the more distant are the equilibrium locations. When the transportation cost is small, any pair of symmetric locations within a given range is an equilibrium. The intervals are smaller and closer to the center of the market line the higher is the unit cost of transport.

The paper also provides some indications on the welfare properties of the equilibria of the game under the different tie-breaking rules. A rather interesting welfare result is that for low values of the transportation cost, aggregate consumers' welfare is higher when consumers randomly select from which firm to buy, despite the ex-post inefficiency of this rule. On the other hand, the welfare of the society is always higher when consumers buy from the nearest firm. This is partly due to the zero profits obtained by the firms under the other tie-breaking rule.[2]

[2]The structure of the equilibria in the pricing game in the second

In order to relate these result to the existing literature, it is first necessary to note that the difficulty in the characterisation of the pricing game under UDP has prevented from a detailed study of the locational choice of firms under this pricing rule. Notable exceptions are Anderson *et al.* [3] and Katz and Thisse [14]. Anderson *et al.* [3] analyse a two-stage location then price game. They use a logit model to allow for heterogeneity of consumer preferences and assume the consumers have a positive probability of buying from either firm. They find that in equilibrium firms cluster at the centre of the market line and firms' profits are positive but decreasing with the degree of heterogeneity. Katz and Thisse [14] analyse the choice of locations and prices of two firms over a circular market assuming that consumers buy from the nearest firm in case of a price tie. When the firms actually compete over consumers, they show that only equilibria in mixed strategies may exist in the pricing game and that firms selects opposite locations over the circle.

The structure of the paper is as follows. The model is described in section 2. Sections 3 and 4 characterise the equilibrium of the game when consumers buy from the

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stage of the game and the welfare rankings under the different tie breaking rules are very consistent with those found in Iozzi [13], where firms' locations are exogenous and symmetric.

nearest firm and when consumers buy from either firm with equal probabilities respectively. Section 4 makes some welfare comparisons between the different market regimes. All the proofs are relegated to the Appendix. To save space, no details of the numerical analysis used to derive some of the results of the paper are provided here but are of course available from the author upon request.

2 The model

I assume a spatial linear market in which competition in prices between two profit-maximising firms takes place at each point on the market line. Consumers are evenly distributed over the line. Consumers' density and length of the market line are both normalised to 1. At each location along the line, consumers have elastic demand given by $q = 1 - p$.[3]

Each firm produces with constant (and identical) marginal and average cost that, without further loss of generality, is normalised to zero. Transportation cost (denoted by c) is assumed to be linearly increasing with quantity and

[3]Linearity of demand is not necessary for the characterisation of the equilibrium in the pricing stage of the games under both two tie-breaking rules under analysis. The linearity assumption is only needed in the study of the firms' choice of locations.

distance. It is assumed that the transportation cost is sufficiently low so that both firms are making nonnegative profits in equilibrium. Transport is under firms' control and no arbitrage can take place among consumers.

The two firms produce perfectly homogeneous goods; the firms are referred to as firm 0 and firm 1. The pricing policy adopted by both firms is uniform delivered pricing: the same price is charged to all customers, irrespective of their location, and firms deliver the good to customers' locations at their cost.

If the two firms charge the same price, two different rules on the resolution of the conflict over markets are studied:

- *efficient* tie-breaking rule: in case of both firms charging the same price at the same location, the market is supplied by the nearest firm.

- *random* tie-breaking rule: in case of both firms charging the same price at the same location, total demand in each local market is equally shared between the two firms.

These rules are usually interpreted in the literature as originating from different behaviour on the consumers' side.[4] As for the *efficient* tie-breaking rule, consumers' behaviour is usually defined as socially optimal because, given the quantities exchanged and the locations of the two firms, it minimises the total transportation cost.[5] As for the *random* tie-breaking rule, this may be the result of customers selecting randomly the firm from which to buy; then, if assigning an equal probability to buying from each firm, each local market is equally shared between the firms supplying that market (at least in expected terms).

I assume that firms cannot ration the supply of the good in any of the markets. This implies that, once a price has been set by one of the two firms, all the customers have the right to buy at that price from that firm, unless their demand is satisfied by the rival firm at the same or lower price. From this assumption, it follows that if one of the firms sets a price lower than the rival, it may end up serving all the customers along the unit line.

I model the strategic interaction between the two firms as a two-stage game. In the first stage, each firm chooses its location x_i, where $i = 0, 1$. In characterising the

[4]Gronberg and Meyer [12] makes tie-breaking rules dependent on firms' behaviour, with the *efficient* tie-breaking rule being the result of a cost-minimising collusive behaviour by the two firms over the locations they serve.

[5]See, for instance, Lederer and Hurter [15], and MacLeod *et al.* [16]

equilibrium of the price game in the second stage, I assume that firms can be located anywhere along the market line. On the contrary, when analysing the subgame perfect equilibrium of the game, locations are restricted so that $x_0 \in [0, \frac{1}{2}]$ and $x_1 \in [\frac{1}{2}, 1]$. This assumption causes some loss of generality. It implies that competition between firms is such that each firm always maintains at least some degree of local monopoly power. In spite of this loss of generality, this assumption allows for a greater simplification of an already rather complex technical analysis.

In the second stage, once locations are fixed and known to both firms, each firm sets its price p_i, where $p_i \in [0, 1]$. Given the nature of the game, the equilibrium concept I use is that of subgame perfection.

As to the players' payoffs, under the *efficient* tie-breaking rule firm 0's profits are given by

$$\Pi_0(p_0, p_1, x_0, x_1, c) = \begin{cases} \Pi_0^U(p_0, p_1, x_0, x_1, c) & \text{if } p_0 < p_1 \\ \Pi_0^M(p_0, p_1, x_0, x_1, c) & \text{if } p_0 = p_1 \\ \Pi_0^S(p_0, p_1, x_0, x_1, c) & \text{if } p_0 > p_1 \end{cases} \tag{1}$$

where

$$\Pi_0^U(p_0, p_1, x_0, x_1, c) \equiv \int_0^1 (1-p)(p - c|x - x_0|)dx;$$

$$\Pi_0^M(p_0, p_1, x_0, x_1, c) \equiv \int_0^{\frac{x_0+x_1}{2}} (1-p)(p - c|x - x_0|)dx.$$

and

$$\Pi_0^S(p_0, p_1, x_0, x_1, c) = 0$$

Under the *random* tie-breaking rule, firm 0's profits are given by

$$\Psi_0(p_0, p_1, x_0, x_1, c) = \begin{cases} \widetilde{\Psi} & \text{if } p_0 < p_1 \\ \frac{1}{2}\widetilde{\Psi} & \text{if } p_0 = p_1 \\ 0 & \text{if } p_0 > p_1 \end{cases} \tag{2}$$

where $\widetilde{\Psi} \equiv \int_0^1 (1-p)(p - c|x - x_0|)dx$.

Similar formulae as those in (1) and (2) apply for firm 1.

3 Equilibrium under the *efficient* tie-breaking rule

This section characterises the subgame perfect equilibrium of the two-stage location then price game under the *efficient* tie-breaking rule. Solving the game backwards, it is first necessary to characterise the equilibrium of the second stage of the game. This is the stage when firms choose optimal prices, taking the locations as fixed.

Proposition 1. *Let* $\underline{p}_i$ *be the smallest solution w. r. to* p_i *to* $\Pi_i^M(p_i, p_j) = 0$ *for* $i, j = 0, 1$ *and* $i \neq j$*. Let also* $\overline{p}_i$ *be the smallest solution w. r. to* p_i *to* $\Pi_i^U(p_i, p_j) = \Pi_i^M(p_i, p_j)$ *for* $i, j = 0, 1$ *and* $i \neq j$*. Then, any pair of prices* p_0 *and* p_1 *such that* $p_0 = p_1$ *and* $p_0, p_1 \in [max\{\underline{p}_0, \underline{p}_1\}, min\{\overline{p}_0, \overline{p}_1\}]$ *is a Nash equilibrium of the pricing game.*

In order to understand the meaning of the Proposition, I first discuss the nature of $\underline{p}_i$'s and $\overline{p}_i$'s. $\underline{p}_i$ is the lowest price consistent with firm i's nonnegative profits when this firm matches the price set by the rival and supplies only the customers in its market area. On the other hand, $\overline{p}_i$ is the price which makes firm i indifferent between undercutting the rival's price (and serving all the customers along the market line) and matching the price set by the other firm (and serving only the customers in its market area). For any price lower than $\overline{p}_i$, firm i prefers to match the rival to avoid delivering the goods also at remote locations when it undercuts the rival.

Note also that, whenever $|\frac{1}{2} - x_i| < |\frac{1}{2} - x_j|$, then: *i)* $\underline{p}_i > \underline{p}_j$, and *ii)* $\overline{p}_i > \overline{p}_j$ for $i, j = 0, 1$ and $i \neq j$. Condition *i)* says that, in case of matching prices, the lowest price consistent with firm i's nonnegative profits is higher than the price with similar properties for the rival when firm i is closer than firm j to the center of the market line. This is because firm i serves a larger market area and bears higher transportation costs.

Also, condition *ii*) states that the price which makes firm i indifferent between matching and undercutting the rival is smaller than the similar price for firm j when firm i is closer than the rival to the centre of the market line. The reason is that the gain in term of market area when undercutting the rival is relatively small.

I am now in the position to illustrate the result given in Proposition 1. An important feature of this result is that it does not hinge on the assumption of linearity of demand and total transportation cost. Indeed, the result holds as long as demand is decreasing in price and total transportation cost is increasing with the distance.

Proposition 1 is illustrated with the aid of Figure 1, which is plot for firms' locations equal to $x_0 = 0.3$ and $x_1 = 0.8$. Given these locations and following the previous discussion, it follows that $\underline{p}_0 > \underline{p}_1$ and $\overline{p}_0 > \overline{p}_1$. Hence, Proposition 1 reduces to saying that any pair of prices p_0 and p_1 such that $p_0 = p_1$ and $p_0, p_1 \in [\underline{p}_0, \overline{p}_0]$ is a Nash equilibrium of the pricing game.

First note that no pair of identical prices above $\overline{p}_0$ can be an equilibrium. This is because firm 0 would find profitable to undercut the rival and serve the whole market. This would also be the case for firm 1 if the price were above $\overline{p}_1$. Also, no pair of prices below $\underline{p}_0$ can be an equilibrium. Firm 0 would make negative profits even if only serving its market area and would then prefer to set a price high enough to leave the whole market to the rival and make zero profits. If the price were also lower than $\underline{p}_1$, this would also hold for firm 1.

On the other hand, if one of the two firm sets a price in $[\underline{p}_0, \overline{p}_0]$, it is easy to see that the best reply for the rival is to match this price. Indeed, if the rival firm sets a higher price, no consumer would buy from it and it would then make zero profits. If it charges a lower price, it ends up serving all the customers along the market line. Because of the high transportation costs, it makes lower profits than simply matching the rival.

I also state

Proposition 2. *If* $x_1 \geq \min\{x_0, x_0(4x_0 - 1)\}$ *and* $x_0 \leq \min\{x_1, x_1(7 - 4x_1) - 2\}$*, there always exists at least one Nash equilibrium price pair.*

This Proposition establishes the conditions on the firms' locations under which an equilibrium exist. It says that, for an equilibrium to exist, it is sufficient that the firms are located sufficiently far apart. In particular, note that Proposition 2 implies that a sufficient condition for the existence of an equilibrium is that the two firms are located in the two different halves of the market line. As already mentioned in the previous section, in the rest of the analysis I will assume that $x_0 \in [0, \frac{1}{2}]$ and $x_1 \in [\frac{1}{2}, 1]$.[6]

I turn now to characterising the equilibrium in the first stage of the game when firms choose locations. Firms anticipate the equilibrium of the price game that takes place in the next stage of the game and whose result in turn depends on the locations chosen in the first stage.

The existence of multiple equilibria in the pricing stage of the game and the particular structure of these price equilibria raises some problems in the full characterisation of the equilibrium of the whole game.

In the rest of the paper, I concentrate my analysis on those subgame perfect equilibria in which firms select in the second period the price pair that gives the highest joint profits amongst the set of Nash equilibrium price pairs. Note that this is also the price pair that grants highest profits to each firm when firms are symmetrically located, as it will be shown to occur in equilibrium. Hence, the Pareto optimality of these price pairs tend to make them a focal point on which it seems reasonable to concentrate the analysis.

Formally, this assumption implies that both firms choose

[6]See the previous section for a discussion on the restrictiveness of this assumption. Note also that, at a great cost of simplicity, most of the results of the paper could easily be obtained for players' strategy spaces such that their choices is $x_0 \in [0, 1-\alpha]$ and $x_1 \in [\alpha, 1]$, as long as α is sufficiently greater than 0.

in the second stage of the game the price which solves the following problem

$$\begin{array}{l} \max_p \int_0^{\frac{x_0+x_1}{2}} (p - c|\tilde{x} - x_0|)(1-p) d\tilde{x} + \\ \qquad \int_{\frac{x_0+x_1}{2}}^1 (p - c|\tilde{x} - x_1|)(1-p) d\tilde{x} \\ \text{s. t. } p \in [\max\{\underline{p}_0, \underline{p}_1\}, \min\{\overline{p}_0, \overline{p}_1\}] \end{array} \tag{3}$$

In principle, using the solution to (3), it is possible to solve analytically for the equilibrium locations. However, because of the mathematical complexity of the problem, it is not possible to obtain a complete analytical solution of the model. Then, I need to resort to a numerical simulation to solve the part of the problem that cannot be solved analytically.

The following Proposition gives details of the equilibrium locations for the first stage of the game under analysis when the firms select in the second period the Nash equilibrium price pair that brings about the highest joint profits to the two firms.

Proposition 3. *Let x_0^E and x_1^E be the equilibrium locations in the location stage of the game. Then,*

- *when $c \leq 1.347$, any pair of locations x_0 and x_1 such that $x_0 \in [\underline{x}_0^E, \overline{x}_0^E]$, $x_1 \in [\underline{x}_1^E, \overline{x}_1^E]$ and $x_0 = 1 - x_1$ are equilibrium locations, where actual values* *of* $\underline{x}_0^E, \overline{x}_0^E, \underline{x}_1^E$ *and* $\overline{x}_1^E$ *for a grid of values of* c *are given in Table 1.*

- *when* $c \geq 1.347$*,*

$$x_0^E = \frac{1}{8}c(14c - 2k_1) \tag{4}$$

$$x_1^E = \frac{k_2}{2k_3} \tag{5}$$

where

$$\begin{aligned}
k_1 &\equiv \sqrt{(39c^2 - 8c)} \\
k_2 &\equiv -6633678c^5 + 10670091c^4k + 4833808c^2 + \\
&\quad - 394480ck + 72797118c^4 - 10565050c^3k + \\
&\quad - 148992c - 33249040c^3 + 1664k + \\
&\quad + 4308248c^2k + 3072 \\
k_3 &\equiv -14756288c^4 + 2124604c^3k - 689600c^2 + \\
&\quad + 33120ck + 1024c - 768k + 6251320c^3 + \\
&\quad - 788392c^2k - 2308227c^4k + 14415501c^5
\end{aligned} \tag{6}$$

Proposition 3 is illustrated in Figure 2. In the Figure, the horizontal axis gives the market line while on the vertical axis I represent the transportation cost. When the transportation cost is low, there exist multiple equilibrium location pairs. These are given by any symmetric pair of locations that belongs to an interval that varies with c and whose extremes are given in Table 1 for a grid of values of the transportation cost. These intervals are also drawn in Figure 2 by a horizontal line for different values of c. The higher is the value of the transportation cost the closer to the centre of the market line are the intervals within which symmetric locations are equilibrium locations. Also, these intervals are narrower the higher is c.

When the transportation cost is high, there exists only one equilibrium location pair for any level of c. From Figure 2, it is easy to see that these equilibrium locations involve more distance between the firms the higher is c.

The reasons for these features of the equilibrium locations are the following.

When c is greater or equal than 1.347, the firms set the unconstrained joint profit maximising prices. This price is increasing with the distance between the firms. Hence, when c is sufficiently high, the firms prefers to locate further away from each other to induce a high equilibrium price in the following stage of the game. This reduces the quantity demanded by consumers and has a positive effect on profits. The opposite holds for low values of c, when the firms prefer to locate toward the centre of the market line to induce a low level of the equilibrium price in the

Table 1: The intervals for N.E. locations when $c \leq 1.347$

c	$\underline{x}_0^E$	$\overline{x}_0^E$	$\underline{x}_1^E$	$\overline{x}_1^E$
0.1	0.1343	0.2523	0.7477	0.8657
0.2	0.1446	0.2551	0.7449	0.8554
0.3	0.1559	0.2581	0.7419	0.8441
0.4	0.1683	0.2613	0.7387	0.8317
0.5	0.1818	0.2648	0.7352	0.8182
0.6	0.1963	0.2687	0.7313	0.8037
0.7	0.2115	0.2730	0.7270	0.7885
0.8	0.2273	0.2778	0.7222	0.7727
0.9	0.2444	0.2830	0.7170	0.7556
1.0	0.2595	0.2887	0.7113	0.7405
1.1	0.2754	0.2949	0.7051	0.7246
1.2	0.2908	0.3016	0.9684	0.7092
1.3	0.3057	0.3089	0.6911	0.6943

following stage of the game. In this case, because of the low level of the unit transportation cost, the increase in revenues outplays the increase in the total cost of transportation.

A rather different rationale is behind the equilibrium locations when the transportation cost is less or equal than 1.347. Under these conditions, firms set in the second period an equilibrium price which is lower than the unconstrained joint profit maximising price. Also, this equilibrium price increases the more distant are the two firms. Hence, the firms locates further away from each other so to be able to charge in the following period a price as close as possible to the price that gives them the highest profits. Note also that there exists multiple equilibria in locations in this first period. Given the discussion provided, it is clear that the locations that maximise the firms' profits are those at the more distant extremes of the intervals of equilibrium locations.

4 Equilibrium under the *random* tie-breaking rule

This section characterises the equilibrium of the two-stage game location-price game under the *random* tie-breaking rule. Solving the game backwards, it is first necessary to characterise the equilibrium of the second stage of the game. This is the stage when the two firms choose optimal prices, given the locations that have been selected in the previous stage. This is done in the following Proposition.

Proposition 4. *Let $\hat{p}_i^R$ be the minimum price consistent with non negative profits for firm i (with i = 0, 1), where $\hat{p}_i^R \equiv c(\frac{1}{2} - x_i + x_i^2)$. Let also p_i^m be the price firm i would set it if were a monopolist, where $p_i^m \equiv \frac{1}{2}(1 + \frac{1}{2}c - x_i(1 - x_i))$. Let p_0^R and p_1^R be the Nash equilibrium prices in the price stage of the game under the random tie-breaking rule. Then,*

$$p_i^R = \begin{cases} \min\{p_i^m, \hat{p}_i^R - \epsilon\} & \text{if } |\frac{1}{2} - x_i| < |\frac{1}{2} - x_j| \\ \hat{p}_i^R & \text{if } |\frac{1}{2} - x_i| \geq |\frac{1}{2} - x_j| \end{cases} \tag{7}$$

If firms' locations are symmetric, the only equilibrium is with both firms setting the same price $p_0^R = p_1^R$ and serving exactly half of the market at each location. Because of the nature of firms' profits, profits obtained undercutting the rival and serving the whole market line at each location are always twice as much the profits obtained matching the rival's price. This implies that each firm always finds profitable to undercut the rival's price unless the rival charges the price which brings about zero profits. If the rival sets this price, a further undercutting would generate negative profits. Hence, it is optimal to the firm to match the rival's price.

On the other hand, if the firms are asymmetrically located on the market line, the firm located closer to the centre of the market line has a comparative advantage over the rival in term of total transportation cost to be incurred in serving all customers located along the unit line.

Two different cases may then occur. A first case is when the price that the firm closer to the centre of the market line would charge if it were a monopolists is lower than the price that allows zero profits to the rival. In this case, the former firm can charge the monopoly price without any threat of price competition by the rival. Another possible scenario is when the monopoly price for the firm closer to the centre of the market line could be profitably undercut by the rival. In this case, the only equilibrium is with this firm charging a price infinitesimally lower than the price that allows zero profits to the rival. Hence, with asymmetric location, the resulting equilibrium is with only one firm serving all the locations on the market line.

The next Proposition characterises the equilibrium of the first stage of the game, when firms choose locations. In this stage of the game, firms anticipate the equilibrium of the price game that takes place in the next stage of the game and whose result in turn depends on the locations chosen in the first stage.

Proposition 5. *Let* x_0^R *and* x_1^R *be the equilibrium loca-*

tions in the location stage of the game under the random tie-breaking rule. Then,

$$x_i^R = \frac{1}{2} \quad \textit{for } i = 0, 1 \tag{8}$$

The Proposition simply says that the only equilibrium in the first stage is with both firms choosing to locate exactly in the centre of the market line.

The reason for this result is clear. Whenever a firm is located further away from the centre of the market line than the rival, it is profitably undercut by the rival and it is driven out of the market. On the other hand, if a firm locates closer than the rival to the centre of the market line, it obtains positive profits as it gains a cost advantage over the rival. This allows the firm to undercut the price which guarantees zero profits to the rival. Moreover, profits are higher the closer is the firm to the center of the market line. The only equilibrium locations are with both firm clustering at the centre of the market line. This is because no firm can do any better than locating at $\frac{1}{2}$ when the rival chooses the same location.

5 Comparisons of equilibria under the different tie-breaking rules

This section discusses the normative properties of the equilibria obtained for the two-stage location then price game under the different tie-breaking rules under analysis. The different market regimes are compared in term of aggregate consumers' surplus, industry profits and aggregate social welfare, as given by the unweighted sum of consumers' surplus and firms profits.

Given the multiplicity of equilibria under the *efficient* tie-breaking rule, I evaluate these welfare measures under two assumptions. First, as previously already highlighted, I concentrate on those subgame perfect equilibria in which the firms select in the second period the price pair that gives the highest joint profits amongst the set of Nash equilibrium price pairs. Second, when there exist multiple equilibria in locations, I concentrate on the equilibrium location pair that gives to the firms the highest profits. Also, when the equilibrium has been obtained by purely numerical methods, I need to resort to a welfare analysis of the same nature.

Proposition 6. *Assume transportation cost is such that equilibrium profits are nonnegative under both tie-breaking rules. Let* CS^E, Π^E *and* W^E *be the aggregate consumers'* *surplus, industry profits and social welfare respectively under the efficient tie-breaking rule. Let also* CS^R*,* Π^R *and* W^R *be the aggregate consumers' surplus, industry profits and social welfare respectively under the random tie-breaking rule. Then,* $\Pi^E > \Pi^R$ *and* $W^E > W^R$ *for any* c*, and* $CS^E < (>) CS^R$ *when* $c < (>) \frac{8}{3}$*.*

The Proposition illustrates three main findings of the comparative analysis of the equilibria under different tie-breaking rules.

First, consumers are better off under the *random* tie-breaking rule when the value of the transportation cost is low enough. This contrasts with the usual definition of the *efficient* tie-breaking rule being 'socially optimal'. The obvious reason for this definition is that consumers' behaviour under the *efficient* rule is socially optimal as it minimises total transportation cost for given prices and location. However, I show here that, differently from what is implied by the mentioned commonly used definition, consumers may be better off if they buy from a randomly selected firm instead of buying from the nearest firm.

In general, which one of the two equilibria obtained under the different tie-breaking rules is preferred by the consumers depends on the joint result of two different effects. A first effect is due to the different way in which competition between firms takes place when the different tie-breaking rules are assumed. Under the *efficient* tie-breaking rule, firms serve a given market area. Then, the price they choose is the one that (among the equilibrium prices) gives them the highest profits given the market area they supply. On the other hand, under the *random* tie-breaking rule, competition is much fiercer as firms are caught under the traditional Bertrand paradox; they undercut each other over the entire market line down to the price where both firms make zero profits. The other effect regards the level of total transportation cost borne by the firms and paid for by the consumers through prices. In this respect, it is clear that when consumers buy from the nearest firm, firms pay an overall total transportation cost lower than when they serve also consumers at remote locations. The total result of the two effects is that, when the transportation cost is low enough, the competitive effect prevails and the equilibrium price is lower under the random rule. The opposite holds for high enough values of c.

Secondly, the comparison of firms' profits under the different tie-breaking rules is made trivial by the zero level of profits obtained by both firms when the *random* tie-breaking rule is assumed.

Finally, from the comparison of the levels of welfare for the society under the different tie-breaking rules, it is clear that the equilibrium when consumers buy from the nearest firm is preferred to the equilibrium under the alternative tie-breaking rule.

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Appendix

Proof of Proposition 1. The proof proceeds along the following steps:

1. an equilibrium must be a single price equilibrium. If firm j charges $p_j < p_i$, it would serve all consumers. However, this cannot be an equilibrium as at least one of the following would apply. Either firm j would prefer to charge $p_j + \epsilon < p_i$ as $\Pi_j^U(p_j + \epsilon, p_i) > \Pi_j^U(p_j, p_i)$ (with $\epsilon > 0$ and sufficiently small); or firm i would have an incentive to undercut the price set by the rival charging $p_i = p_j - \epsilon$ as $\Pi_i^U(p_i - \epsilon, p_j) > \Pi_i^S(p_i, p_j)$.

2. any pair of prices p_0 and p_1 such that $p_0 = p_1$ and $p_0, p_1 \notin [\max\{\underline{p}_0, \underline{p}_1\}$, $\min\{\overline{p}_0, \overline{p}_1\}]$ cannot be an equilibrium. If $p_0, p_1 < \max\{\underline{p}_0, \underline{p}_1\}$ at least one firm makes negative profits. To see why, let $p_0 < p_1$. If $\underline{p}_0 < p_0, p_1 < \underline{p}_1$, then $0 = \Pi_i^S(\infty, p_j) > \Pi_i^M(p_i, p_j)$. Similarly, if $p_0, p_1 < \underline{p}_0 < \underline{p}_1$, then both $0 = \Pi_i^S(\infty, p_j) > \Pi_i^M(p_i, p_j)$ and $0 = \Pi_j^S(\infty, p_i) > \Pi_j^M(p_j, p_i)$ would hold. If $p_0, p_1 > \min\{\overline{p}_0, \overline{p}_1\}$ at least one firm prefers to undercut the rival and serve the whole market. To see why, let $\overline{p}_0 > \overline{p}_1$. If $\overline{p}_0 > p_0, p_1 > \overline{p}_1$, then $\Pi_i^U(p_i - \epsilon, p_j) >$

$\Pi_i^M(p_i, p_j)$. If $p_0, p_1 > \overline{p}_0 > \overline{p}_1$, then both $\Pi_i^U(p_i - \epsilon, p_j) > \Pi_i^M(p_i, p_j)$ and $\Pi_j^U(p_j - \epsilon, p_i) > \Pi_j^M(p_j, p_i)$ would hold.

3. any pair of prices p_0 and p_1 such that $p_0 = p_1$ and $p_0, p_1 \in [\max\{\underline{p}_0, \underline{p}_1\}, \min\{\overline{p}_0, \overline{p}_1\}]$ is a Nash equilibrium. Let firm i set $p_i \in [\max\{\underline{p}_0, \underline{p}_1\}, \min\{\overline{p}_0, \overline{p}_1\}]$. Firm j's optimal response is to charge $p_j = p_i$ as $\Pi_j^U(p'_j, p_i) < \Pi_j^M(p_j, p_i)$ for any $p'_j < p_j$ and $\Pi_j^S(p''_j, p_i) < \Pi_j^M(p_j, p_i)$ for any $p''_j > p_j$. This is because, charging $p'_j < p_i$ would cause firm i to serve all the customers, obtaining less profits than matching the rival and serving only half of the market; on the other hand, for $p''_j > p_i$ no consumer would buy from firm i.

□

Proof of Proposition 2. From (1)

$$\underline{p}_0 = \frac{c(5x_0^2 - 2x_0x_1 + x_1^2)}{4(x_0 + x_1)} \tag{9}$$

$$\underline{p}_1 = \frac{c(-8x_1 + 4 - 2x_0x_1 + x_0^2 + 5x_1^2)}{4(-2 + x_0 + x_1)} \tag{10}$$

$$\overline{p}_0 = -\frac{3}{4}cx_0 + \frac{1}{4}cx_1 + \frac{1}{2}c \tag{11}$$

$$\overline{p}_1 = -\frac{1}{4}cx_0 + \frac{3}{4}cx_1 \tag{12}$$

For an equilibrium to exist, it is necessary (and sufficient, for a low enough value of c) that $max\{\underline{p}_0, \underline{p}_1\} \leq min\{\overline{p}_0, \overline{p}_1\}$. This requires that: $i)$ $\underline{p}_i \leq \overline{p}_i$ for any $i = 0, 1$; and $ii)$ $\underline{p}_i \leq \overline{p}_j$ for any $i, j = 0, 1$ and $i \neq j$.

As for condition $i)$ for $i = 0$, from (9) and (11) it is easy to see that $\underline{p}_0 \leq \overline{p}_0$ when $x_1 \geq x_0(4x_0 - 1)$. When $i = 1$, from (10) and (12), it is easy to see that $\underline{p}_1 \leq \overline{p}_1$ when $x_0 \geq x_1(7 - 4x_1) - 2$.

As for condition $ii)$, using (10) and (11), it is easy to check that $\underline{p}_1 \leq \overline{p}_0$ when $-4x_0 + x_0x_1 + 4x_1 + x_0^2 - 3x_1^2 \geq 0$. The RHS of this inequality is a convex function of x_0 whose roots are $4 - 3x_1$ and x_1. As $4 - 3x_1 \geq x_1$ when $x_1 \leq 1$, $\underline{p}_1 \leq \overline{p}_0$ when $x_0 \leq x_1$. Using a similar argument, it is easy to show that also $\underline{p}_0 \leq \overline{p}_1$ when $x_0 \leq x_1$.

Combining these two findings gives the result in the Proposition. Note that an equilibrium always exists when $x_0 \in [0, \frac{1}{2}]$ and $x_1 \in [\frac{1}{2}, 1]$. Indeed, note that the RHS of the inequality that grants that condition $i)$ is satisfied is a convex function that has a global maximum at $x_0 = \frac{1}{8}$, it is equal to 0 when $x_0 = 0$ and is equal to $\frac{1}{2}$ when $x_0 = \frac{1}{8}$. Hence, when $x_1 \in [\frac{1}{2}, 1]$, the inequality weakly holds for any value of x_0. A similar argument establishes the result for $i = 1$. Lastly, when $x_0 \in [0, \frac{1}{2}]$ and $x_1 \in [\frac{1}{2}, 1]$, then $x_0 \leq x_0(4x_0 - 1)$ and $x_1 \geq x_1(7 - 4x_1) - 2$.

$\square$

Proof of Proposition 3. Consider the following problem. Let

$$\breve{p} \equiv \text{argmax}_p \left(\int_0^{\frac{x_0+x_1}{2}} (p - c|\tilde{x} - x_0|)(1-p)d\tilde{x} + \right. \\ \left. + \int_{\frac{x_0+x_1}{2}}^{1} (p - c|\tilde{x} - x_1|)(1-p)d\tilde{x} \right) \tag{13}$$

Then, consider the first derivatives of $\Pi_i^M(\cdot)$ w. r. to x_i (i = 0, 1) evaluated at $p_0 = p_1 = \breve{p}$,

$$\begin{aligned} \left.\frac{\partial \Pi_0^M(\cdot)}{x_0}\right|_{p_0=p_1=\breve{p}} &= -\frac{45}{128}c^2x_0^4 + \left(\frac{15}{16}c^2 + \frac{3}{32}c^2x_1\right)x_0^3 + \\ &+ \left(-\frac{9}{32}c^2 - \frac{3}{16}c^2x_1 - \frac{15}{64}c^2x_1^2\right)x_0^2 + \\ &+ \left(\frac{13}{16}c^2x_1^2 + \frac{5}{16}c^2 - \frac{11}{16}c^2x_1\right)x_0 + \\ &+ \left(-\frac{5}{32}x_1^3c^2 - \frac{5}{8}c\right)x_0 \\ &+ \frac{1}{8} + \frac{1}{16}c^2x_1 - \frac{1}{16}x_1^3c^2 + \frac{1}{8}cx_1 \\ &+ \frac{3}{128}x_1^4c^2 - \frac{1}{32}c^2x_1^2 - \frac{1}{32}c^2 \end{aligned} \tag{14}$$

and

$$\left.\frac{\partial \Pi_1^M(\cdot)}{x_1}\right|_{p_0=p_1=\breve{p}} = \frac{45}{128}x_1^4c^2 + \\ + \left(-\frac{3}{8}c^2 - \frac{3}{32}c^2x_0\right)x_1^3 + \\ + \left(\frac{15}{64}c^2x_0^2 - \frac{9}{32}c^2 - \frac{3}{8}c^2x_0\right)x_1^2 + \\ + \left(\frac{5}{32}c^2x_0^3 + \frac{9}{16}c^2 - \frac{1}{8}c^2x_0^2\right)x_1 + \\ + \left(-\frac{5}{8}c + \frac{9}{16}c^2x_0\right)x_1 + \\ -\frac{1}{8} + \frac{1}{2}c - \frac{1}{32}c^2x_0^2 - \frac{3}{16}c^2x_0 - \frac{7}{32}c^2 \\ -\frac{1}{8}c^2x_0^3 + \frac{1}{8}cx_0 - \frac{3}{128}c^2x_0^4 \qquad (15)$$

Let $\tilde{x}_0$ and $\tilde{x}_1$ be the solutions in x_0 and x_1 to the system of equations given by (14) and (15) both equated to zero. Clearly, $\tilde{x}_0$ and $\tilde{x}_1$ need not to be an equilibrium solution to the firms' problem for two reasons: *i*) the pair of price $\{\breve{p}, \breve{p}\}$ need not to be a Nash equilibrium price in the pricing stage of the game as p^{**} may be outside the interval $p_0, p_1 \in [max\{\underline{p}_0, \underline{p}_1\}, min\{\overline{p}_0, \overline{p}_1\}]$; and *ii*) $\tilde{x}_0$ and $\tilde{x}_1$ may be outside the strategy space of the firms.

I ignore for the moment these two problems and find the solutions to the system of equations (14) and (15), both equated to zero, using the mathematical software Maple. Due to the highly non-linear nature of (14) and (15), I find multiple solution pairs. The mathematical complexity of the problem prevents me from checking analytically the second-order conditions. Using numerical methods[7], I select the pair of solutions that *i*) guarantees the highest profits to the two firms, and *ii*) belongs to the firms' strategy space. These are the locations given in (4) and (5).

Now, I check for the resulting price in the first stage of the game to be a Nash equilibrium price. Plugging (4) and (5) into (9) – (12) and comparing these expressions with $\breve{p}$, I find that the 4-tuple $\{\breve{p}|_{x0=\tilde{x}_0,x1=\tilde{x}_1}, \breve{p}|_{x_0=\tilde{x}_0,x_1=\tilde{x}_1}, \tilde{x}_0, \tilde{x}_1\}$ is a subgame perfect equilibrium of the game if

$$c \in \left(\frac{128}{95}, 4 + \frac{6}{5}\sqrt{10}\right) \tag{16}$$

that is if c is approximately between 1.347 and 7.795.

Note now that no equilibrium can exist when $c > 4 + \frac{6}{5}\sqrt{10}$. In this case, from Proposition 1 the Nash equilibrium prices in the second stage of the game should be higher than p^* and equal to $\max\left\{\underline{p}_0, \underline{p}_1\right\}$. However, note

[7]Details are available from the author upon request. Numerical methods are also used to rule out the possibility of corner solutions.

that p^* (evaluated at equilibrium locations) tends to 1 as c approaches to 1.

When $c < \frac{128}{95}$, the problem becomes very difficult to solve as the payoff function varies with the relative position of the firms. Hence, the problem is solved by using a computer program which evaluates numerically equilibrium solutions for a grid of values of c, the only parameter of the model.[8] $\square$

Proof of Proposition 4. First note that $\hat{p}_i^R$ comes as the solution to $\Psi_0^M = 0$. Also, $p_i^m \equiv \text{argmax}(\Psi_0^U)$.

Take now the case $|\frac{1}{2} - x_i| < |\frac{1}{2} - x_j|$. It follows that $p_i^R < p_j^R$. Then,

1. no equilibrium price pair such as $p_i^R > p_j^R > \hat{p}_j^R$ or $p_j^R > p_i^R > \hat{p}_j^R$ can exist. Assume the first inequality holds. In this case, firm i sells zero output; it would be better off charging $p_j^R - \epsilon$ (with ϵ positive and small). This grants positive profits as long as ϵ is sufficiently small so that $p_j^R - \epsilon > p_i^R$. Similarly if the second inequality holds.

2. because of the assumption of firms never playing a dominated strategy, firm j never sets $p_j < \hat{p}_j^R$ as this is clearly a dominated strategy.

[8]Details are available from the author upon request.

3. when firm j sets p_j, the best reply for firm i is to set $p_j - \epsilon$ when $p_j < p_j^m$ and p_j^m when $p_j > p_j^m$. To see why, assume first that $p_j > p_j^m$. Then, firm i can optimally charge p_i^m and get monopoly profits. Assume now that $p_j < p_j^m$. Now, profits for firm i are monotonically increasing in p_i as long as $p_i < p_j$.

4. the pair of prices p_i^R and p_j^R is a Nash equilibrium. Assume first that $\hat{p}_j^R > p_i^m$. When firm j sets $\hat{p}_j^R$, the best reply for firm i is to set p_i^m (see step 3.). When firm i sets p_i^m, firm j gets zero profits if it charges $\hat{p}_i^R$ while it gets zero or negative profits if it charges any price $p_j \neq \hat{p}_i^R$. Now, assume $\hat{p}_j^R < p_i^m$. When firm j sets $\hat{p}_j^R$, the best reply for firm i is to set $\hat{p}_j^R - \epsilon$ (see step 3.). When firm i sets $\hat{p}_j^R - \epsilon$, firm j gets zero profits if it charges $\hat{p}_i^R$ while it gets zero or negative profits if it charges any price $p_j \neq \hat{p}_i^R$. .

5. the pair of prices p_i^R and p_j^R is the only Nash equilibrium. The reason is clear when $\hat{p}_j^R > p_i^m$ (see step 4.). When $\hat{p}_j^R < p_i^m$, if firm j sets any $p_j > \hat{p}_j^R$, the best reply for firm i is to set $p_j - \epsilon$ (see step 2.). However, p_j is not the best reply for firm j to firm i choosing $p_j - \epsilon$.

$\square$

Proof of Proposition 5. The proof proceeds along the following steps:

1. the pair of locations x_0^R and x_1^R is an equilibrium. If firm 1 chooses $x_1 = x_1^R$, then firm i gets weakly larger profits locating at $x_0 = x_0^R$. Indeed, it obtains zero profits selling zero output if locates away from the centre of the market line and gets zero profits serving half of each market at each location if it locates at $x_0 = x_0^R$. Similarly for firm 1.

2. the pair of locations x_0^R and x_1^R is the only equilibrium. Suppose firm 1 chooses location $x_1 \neq x_1^R$. Firm 0 gets positive profits as long as it locates anywhere at $x_0 \in (1 - x_1^R, \frac{1}{2})$.

□

Proof of Proposition 6. For values of c for which analytical expressions for equilibrium locations and prices are found, welfare comparisons is simply made plugging these equilibrium expressions into the expressions for aggregate consumers' surplus, firms' profits and social welfare, and comparing these welfare measures across the different regimes. However, because of the complexity of the expression for social welfare under the *efficient* tie breaking rule, the comparison is made by numerical methods, as detailed below. When the equilibrium variables are found by numerical methods, equilibrium welfare measures are obtained plugging, for a grid of values of c, the equilibrium values of the prices and locations into the expressions for consumers' surplus, firms' profits and social welfare, and then comparing the resulting figures.

□