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Existence and concentration of local mountain passes for a nonlinear elliptic field equation in the semi-classical limit. (English summary)

Topol. Methods Nonlinear Anal. **17** (2001), no. 2, 239–275.

Summary: “In this paper we are concerned with the problem of finding solutions for the following nonlinear field equation

$$-\Delta u + V(hx)u - \Delta_p u + W'(u) = 0,$$

where $u: \mathbb{R}^N \rightarrow \mathbb{R}^{N+1}$, $N \geq 3$, $p > N$ and $h > 0$. We assume that the potential V is positive and W is an appropriate singular function. In particular we deal with the existence of solutions obtained as critical (not minimum) points for the associated energy functional when h is small enough. Such solutions will eventually exhibit some notable behaviour as $h \rightarrow 0^+$. The proof of our results is variational and consists in the introduction of a modified (penalized) energy functional for which mountain pass solutions are studied and then are proved to solve our equation for h sufficiently small. This idea is in the spirit of that used in [M. A. del Pino and P. L. Felmer, *Calc. Var. Partial Differential Equations* **4** (1996), no. 2, 121–137; [MR1379196 \(97c:35057\)](#); *J. Funct. Anal.* **149** (1997), no. 1, 245–265; [MR1471107 \(98i:35183\)](#); *Ann. Inst. H. Poincaré Anal. Non Linéaire* **15** (1998), no. 2, 127–149; [MR1614646 \(99c:35228\)](#)], where ‘local mountain passes’ are found for certain nonlinear Schrödinger equations.”

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