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Continuous-time Spiking Neural Networks: general paradigm and *event-driven* simulation

University of Rome, “Tor Vergata”
Faculty of Engineering,
Department of Electronics

Mario Salerno
salerno@uniroma2.it

Gianluca Susi
gianluca.susi@uniroma2.it

Alessandro Cristini
alessandro.cristini84@gmail.com

Marco Re
marco.re@uniroma2.it

Giancarlo Cardarilli
g.cardarilli@uniroma2.it

Abstract

The aim of this research is to develop a simple and effective continuous-time Spiking Neural Network simulator, that takes into account basic biological neuron parameters, in which the latency time is the main effect for the spike generation.
A preliminary accurate analysis of the latency time has been developed, applying classical modelling methods to single neurons, by simulations on the most accurate biological model: the *Hodgkin-Huxley Model*. On the basis of the classical neuron theory, other fundamentals parameters of the systems are defined, such as subthreshold decay, refractory period, inhibitory behaviour, synaptic plasticity, etc.
Indeed, spike transmission and latency problems introduce the necessity of using continuous time simulation. Thus, direct use of digital computational methods, seem not completely appropriate. Due to the implicit high-sensitivity of the overall system to close events (conferred by the latency), and the high temporal dynamics of activity, an event-driven simulation method is necessary. In fact, for the proposed neural model, high precision and effectiveness are basically required.
A class of fully asynchronous Spiking Neural Networks with a high biological plausibility is definitively proposed, and networks with up to 100.000 neurons can be simulated in a quite short time with a simple MATLAB program. Is also possible to apply plasticity algorithms to emulate interesting global effects, as the *Neuronal Group Selection* or the *jitter-reduction*. Moreover, such a parallel processing system could be used for, but not only, engineering problems that involve the use of the classic artificial neural networks (e.g., pattern recognition) . Other applications concern the operation study of biological neural circuits and the exploration of chaotic dynamics in nervous system.

The *continuous-time* paradigm: an overview

In *Spiking Neural Networks* (*SNN*), the neural activity consists of spiking events generated by firing neurons [1], [2], [3], [4]. A basic problem to realize realistic *SNN* concerns the asynchronous times of arrival of the synaptic signals [5], [6], [7]. Many methods have been proposed in the technical literature in order to properly desynchronizing the spike trains; some of these consider transit delay times along axons or synapses [8], [9]. A different approach introduces the *spike latency* as a neuron property depending on the inner dynamics [10]. Thus, the firing effect is not instantaneous, but it occurs after a proper delay time which is different in various cases. This kind of neural networks generates apparently random time spikes sequences, since continuous delay times are introduced by a number of desynchronizing effects in the spike generation. In this work, we will suppose this kind of desynchronization as the most effective for *SNN* simulation. Spike latency appears as intrinsic continuous time delay. Therefore, very short sampling times should be used to carry out accurate simulations. However, as sampling times grow down, simulation processes become more time consuming, and only short spike sequences can be emulated. The use of the event-driven approach can overcome this difficulty, since continuous time delays can be used and the simulation can easily proceed to large sequence of spikes [11], [12].

The class of dynamical neural networks represents an innovative simulation paradigm, which is not a digital system, since time is considered as a continuous variable; this property presents a number of advantages. It is quite easy to simulate very large networks, with very high precision, using simple and fast simulation approach [13].

Spike latency: fundamental parameter for the system desynchronization

In this research work, a simple spiking neuron model is presented. The following biological inspired parameters are considered: spike threshold, subthreshold decay, synaptic integration [6]. The model is similar to the classic LIF (Leaky Integrate-and-Fire), and the basic difference from the latter is the presence of an expression, called *firing equation* (1), which qualitatively describes the behavior of the neuron suprathreshold: when the membrane potential reaches the spike threshold, the firing is not instantaneous, but has a variable continuous time delay, called *latency* [14].

$$t_f = 1 / (S - 1) \qquad (1)$$

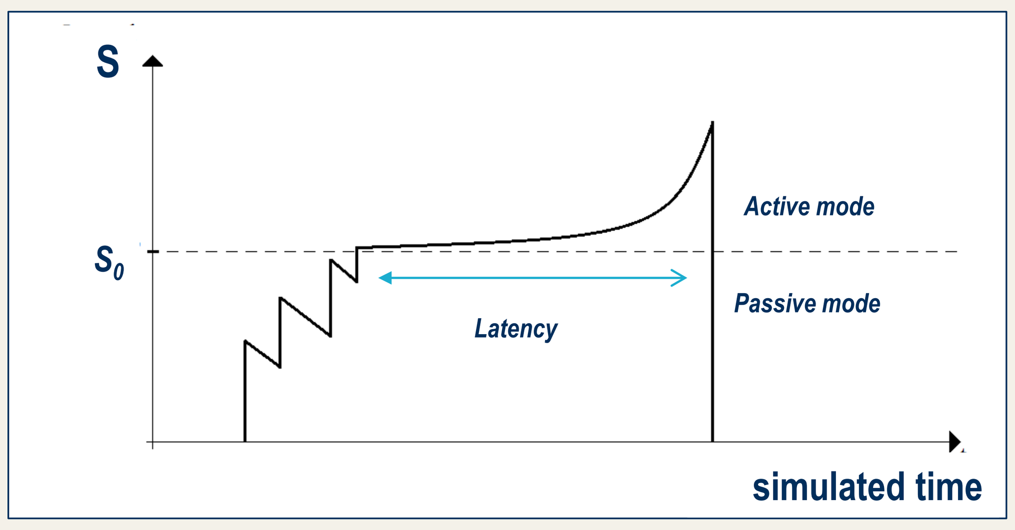

Figure 1. The behavior of the considered neuron is presented.
Once the threshold is reached, the firing is not instantaneous. Latency is the time involved in the process, depending on the state reached from the neuron.
In the figure, S represent the neuron internal state, and the system can switch from active to passive mode (or viceversa), depending to the inputs arriving to the considered neuron.
Note the behavior of the subthreshold decay: this parameter requires a quite synchony for incoming spikes to a target neuron. No spike could be generated otherwise.

The relation between latency and membrane potential approximately follows a branch of rectangular hyperbola (Figure 2) [15]. This relationship was accurately found by simulating a patch of neuronal membrane stimulated with brief current pulses, solving the Hodgkin-Huxley equations [16] through the simulator NEURON [17].
In the model, the variable *S* indicates the state of the neuron and can be linked to the membrane potential of the biological counterpart; the variable t_f, *time-to-fire*, can be linked to the spike latency. The passive mode of the neuron is expressed as:

$$S = S_a + P_r P_w - ld\ \Delta t, \text{ for } S < S_0 \qquad (2)$$

S_0 represents the spike threshold, defined as $S_0 = 1+d$, in which d denotes the threshold constant (necessary to bound the maximum value of t_f). S_a denotes the previous state.

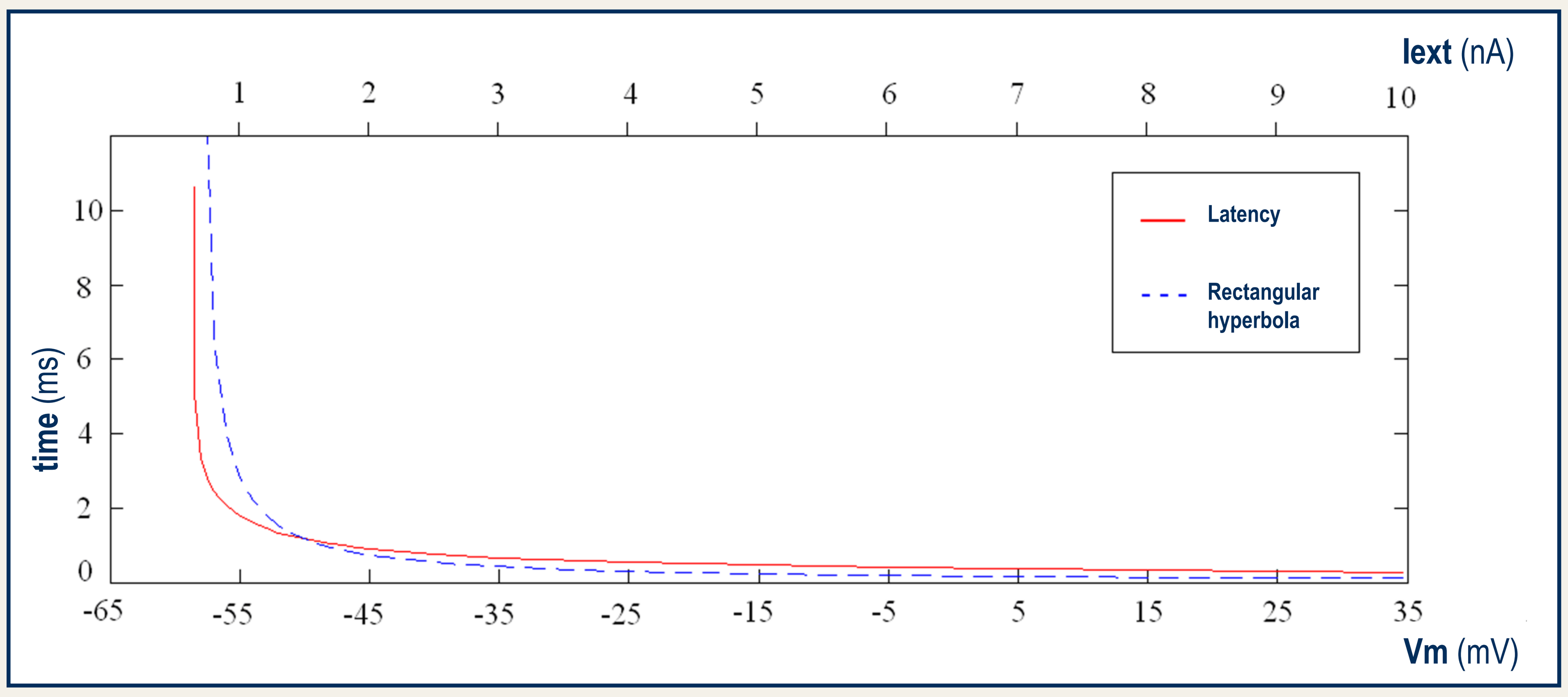

Figure 2. The red line indicates the latency as a function of the membrane potential V_m (or else of the current amplitude Iext, equivalently). The dashed blue line indicates the rectangular hyperbola.

The quantity P_r, presynaptic weight, denotes the signal transmitted from one neuron to a number of the other neurons. This quantity can be linked to the synaptic currents, as described by pulse trains. A negative value of P_r denotes that the firing neuron is an inhibitory one. The quantity P_w, postsynaptic weight, is associated to the connections. The latter indicates the strength among the connections. Moreover, *ld* is the linear subthreshold decay (not present for $S \geq S_0$). Finally, Δt represents the step interval. When a spike is generated the state *S* is reset to zero for a time corresponding to the *absolute refractory period* of a biological neuron.

The necessity of an event-driven approach

It can be seen that the proposed model is not suitable for a classical time-driven based simulation. Indeed, as larger the net is chosen, as firing close events are likely to occur in the whole net, in the same time interval. Thus, larger networks are simulated, smaller time intervals have to be chosen. Temporal continuity in the simulation is then necessary, in particular for very fast dynamics. On the other hand, the use of very little sampling times can make very slow the simulation process.
Due to the nature of spike events deriving from the illustrated model, is convenient to use an event-driven approach for the simulation of this class of neural networks. The proposed neural paradigm has been implemented in a simple MATLAB program, and the simulation method proceeds looking for the next firing event occurring in the whole network.
Moreover, by means of simulations this technique allows to investigate the properties of large networks, requiring a low computational cost [18-20].

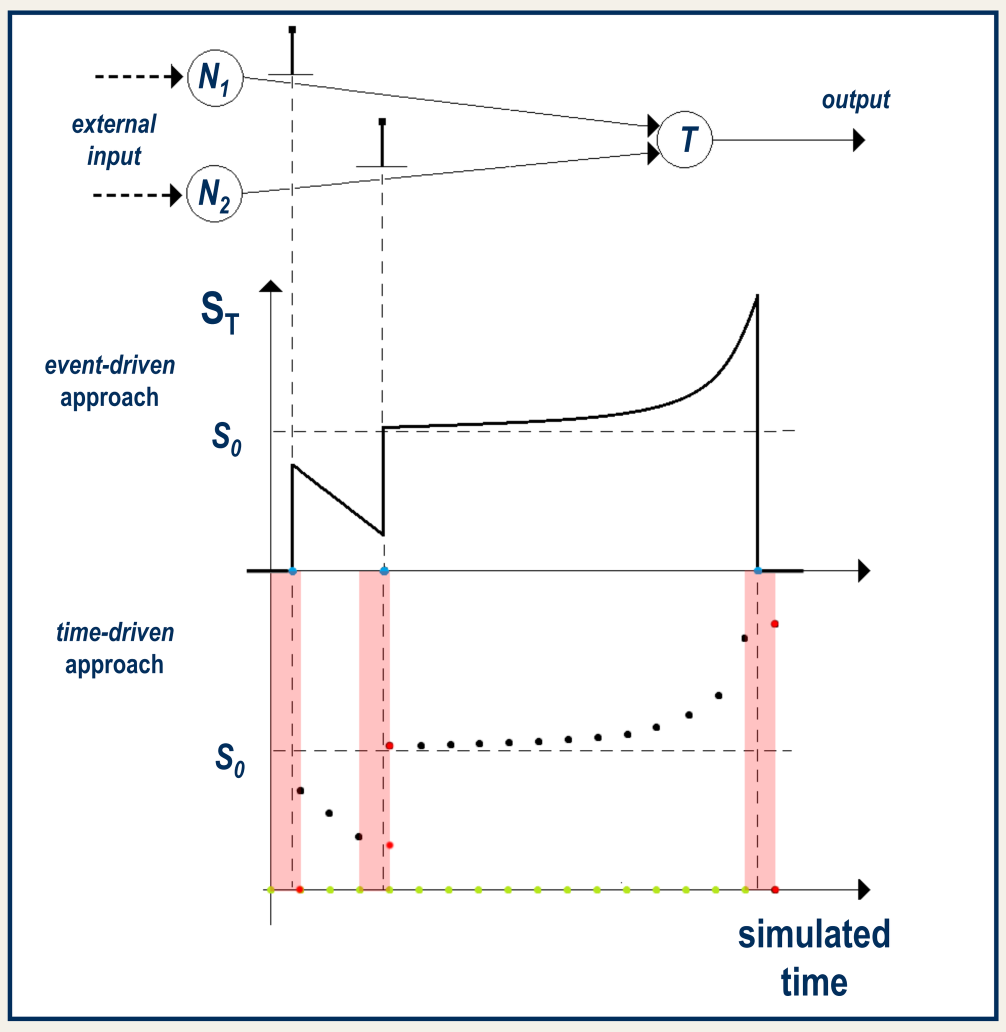

Figure 3. Differences between time- and event-driven approaches. In this case the summation of two contributes from N1 and N2 can cause the target neuron T to fire if the inner state of the latter overcomes the threshold level.
In the case of a time-driven approach, due to the time discretization an undesirable clusterization is arisen (light green dots). The latter involves uncertainty, and then errors. Because of the high-sensitivity of the system, this error can have impact to the behavior of the whole network, propagating through the downstream neurons.
In the case of an event-driven approach, the problem is overcome (blue dots).

Global effects and applications

By the simulation of a network designed as continuous-time system, it was possible to show the appearance of neural groups [21], or , under particular conditions, the jitter-reduction of statistical inputs [22]. The model can be able to work both for rate codes and temporal ones. It is also possible to use this model for some typical engineering problems (e.g., classification) [23].

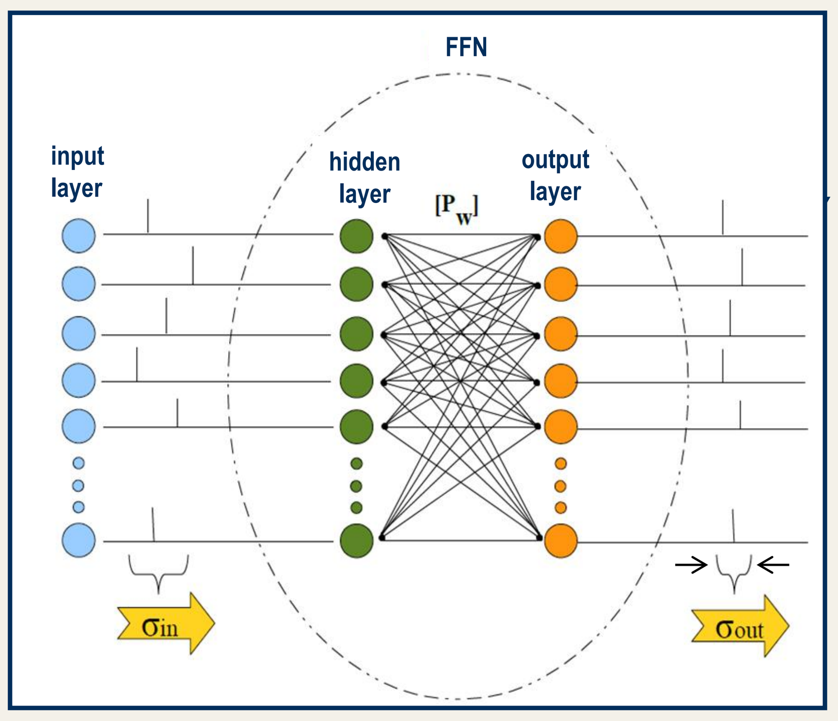

Figure 4. Jitter-reduction in case of fully connected feedforward network with weak synapses (P_w very low).

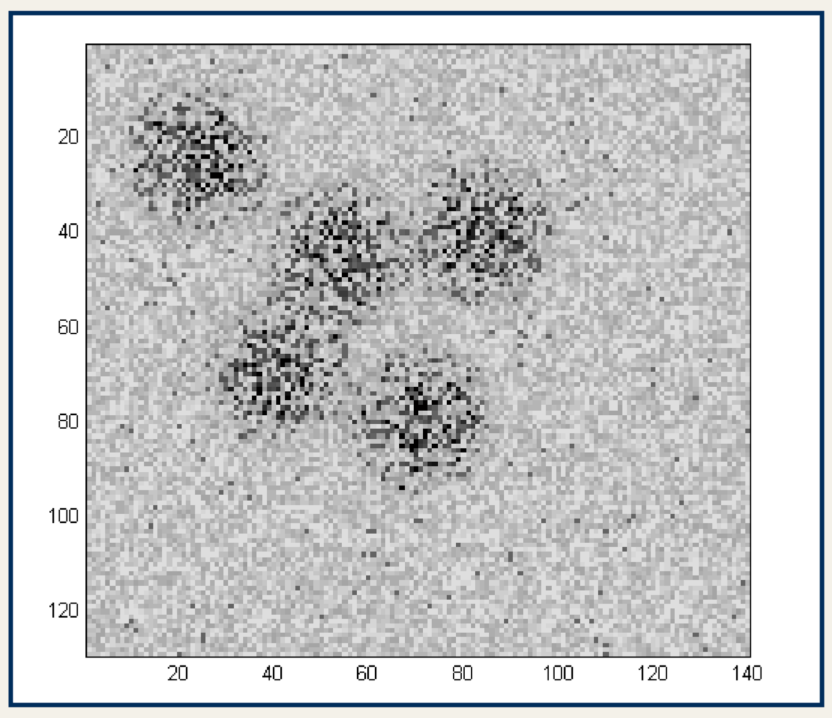

Figure 5: The Neuronal Group selection clearly appears in a network of neurons.

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Awards

Best Paper Category : “Spiking neural networks as analog dynamical systems: basic paradigm and simple applications” received at the International Conference *“Advances in Computer Engineering (ACE 2012)”*, Amsterdam, June 2012.

Place

University of Rome, “Tor Vergata” - Faculty of Engineering, Department of Electronics.