Simulating exogenous and endogenous technology when depletables, renewables and pollution co-exist: how to achieve sustainability?

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Abstract

In the attempt to mitigate climate change and the severe consequences of fossil fuel pollution, a number of different energy options and strategies are available to policy makers today. We construct a model accounting for pollution damages and the option to use a depletable and a renewable energy resource in production. We investigate whether this economy can optimally achieve weak and strong sustainability under different technology options and production and consumption functional forms. We start by setting up an economy including for pollution and the key distinguishing factors between depletable and renewable resources. A benchmark case is constructed against which the introduction of technology and the variation in key parameters is assessed. Endogenous technology is modelled as learning by doing in the renewable resource. Results under all options are presented and some energy policy conclusions are put forward. It will be key not to solely rely on changes in the energy mix but to move to an economy that can substitute renewable energy resources for fossil fuels.

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List of variables

C	Consumption
Z	Non-renewble
R	Renewable
F(.)	Production
E(.)	Energy production function
α	Share of depletable resource in the energy production function
β	Share of energy in the production function
K	Physical capital
δ	Depreciation rate
g	Renewable cost per unit
P	Stock of pollution
γ	Absorption rate of the environment
m	Exogenous technical progress
ω	Rate of exogenous technical progress
M	Stock of human capital
ϕ	Productivity of learning by doing
CD	Cobb Douglas
CES	Constant Elasticity of Substitution

Introduction

Today, as the world energy demand increases and pollution impacts grow, the key environmental concern is understanding climate change, mitigating its harmful effects and assessing how this can be done sustainably. As Nordhaus clearly puts it "Global warming poses a unique mix of problems that arise from the fact that global warming is a global public good, is likely to be costly to slow or prevent, has daunting scientific and economic uncertainties, and will cast a shadow over the globe for decades, perhaps even centuries, to come" (pg.11, Nordhaus, 2007). In this light, it will be crucial to diversify energy portfolios, in the effort to curb impacts of climate change. In fact, recent research suggests that the increase in greenhouse gas emissions following the increasing energy demand, is likely to cause major irreversible climatic changes and damages. Alternative energy options, including all forms of renewable energy such as wind energy and solar systems, exist and are needed energy substitutes for fossil fuel. These energy options offer low or zero emissions alternatives and have the potential to provide technological benefits (WEA 2004 and 2006, IEA 2006 and 2007). Many economists have tried to address the key question for environment, growth and sustainable development, namely whether economic growth can be compatible or reconciled with care for the environment, as discussed below. Thus, building on the existing literature, the concern in this paper is whether renewable energy resources can play a lead role in addressing climate change problems thus allowing economies to position themselves on a sustainable path.

Different modelling approaches have been proposed depending on the focus of the research at the time. Four components constitute the main building blocks of the models behind this research: utility, pollution, environment, and resources (depletable and non-depletable). Pollution has been modelled both as a flow and as a stock, but as clearly argued in Withagen (1995), it is necessary to include pollution as a stock to capture the negative externalities effect that this poses showing that these externalities can hamper optimal economic growth. In fact, initially utility was modelled as a function of consumption only but more recently authors have started introducing environmental quality, or lack of this, in the utility function (Withagen 1995, Pittel 2002). The literature on depletable resources and growth is extremely vast, stemming from the seminal papers of Hotelling (1931), Stiglitz (1974), Dasgupta and Heal (1974) to name but a few, in which technical progress is found to play a key role.

Few authors have introduced renewable resources and depletable energy resources into this framework at the same time. Tahvonen and Salo (2001) propose a historical perspective arguing that economies transition from renewable to non-renewable back to renewable energy forms. Our work differs from theirs, since we believe that renewable energy sources differ over time. For example wind mills of yesterday are not the wind turbines of today. Similarly for the case of solar energy that requires technological improvements embedded in the development of solar panel technology. Further, we look into the future and the potential for renewable energy development, while Tahvonen and Salo (2001) take a more historical perspective. Krautkramer (1995), although in a different setting, concludes that an economy may be able to prevent decline in consumption through the use of a backstop technology.

The issue of sustainable development and economic growth under ‘green’ constraints has been analyzed by a number of authors over the last decade. Aghion and Howitt (1998) find that innovation is a necessary but not sufficient condition to achieve sustainable development when environmental pollution and exhaustible resources are accounted for; however, conclusions on the adequate policy are not reached. Pezzey and Withagen (1998) conclude that if natural resources are exhaustible then consumption is single peaked, declining from some point in time onward. Smulders (2000) provides an overview of the different perspectives and finds that growth can be sustained indefinitely only if technological change is unbounded and if natural and man-made capital can be substitutes. In sum, technology and resource substitutability play a key role.

Under the first respect, we model technological innovation as Learning by Doing. Since the seminal paper by Arrow (1962) a considerable amount of authors have discussed the learning opportunities offered by new technologies. As time proceeds and a new technology is implemented we can learn to use it better, or in other words, knowledge accumulates which can make the resource more productive or reduce production costs (Romer 1986, Aghion and Howitt 1998). Many economists see an advantage in capturing the benefits from the learning path over time at the onset of production activities. New technologies such as renewable energies offer the potential for learning by doing benefits, although the policy side and implementation of this is debated (Isoard and Soria 2001 and Rivers and Jaccard 2006 and Kobos et al 2006, Rasmussen 2001).

Turning to resource substitutability, our starting point is the strand of literature focusing on substitutability between natural and physical capital (see Smulders, 2003 as a recent example). Here, in light of the debate, we focus on substitutability between depletable and renewable energy resources.

We build on a companion paper, which is also the previous chapter of this thesis and depart from it in several ways: based on the model constructed, we introduce endogenous technology; we release the assumptions made in the previous paper and let production and utility functional forms vary; finally, we explicitly test for the optimality of both strong and weak sustainability.

In the companion paper, we made three main assumptions that allowed to obtain a closed form solution: first of all, we assumed Cobb Douglas production for energy entailing that both the renewable and non-renewable resource are essential, secondly we analysed the case of exogenous technology and thirdly we restricted our attention to the objective of strong sustainability defined as non-increasing pollution (Pittel, 2002). Using these three assumptions allowed us to obtain a set of conditions under which an economy using both renewable and non-renewable resources would be strongly sustainable under a finite time horizon. Nevertheless we found that, as the planning horizon extended to infinity, the mere existence and availability of renewable energy resources would no longer be sufficient for the economy to be strongly sustainable.

In this paper, on the other hand, we move away from some of the restrictions posed by these three assumptions. Due to the complexity of the model it is not possible to obtain a closed form solution, therefore the analysis presented here is undertaken in GAMS[1]. The introduction of mathematical simulation allows us to add endogenous technology, to introduce Constant Elasticity of Substitution (CES) production, releasing the "essentiality condition"[2], to vary consumer preferences and to analyze both weak and strong sustainability. We use the definition of non-declining utility for weak sustainability and non-increasing pollution for strong sustainability. We model endogenous technology as learning by doing in the renewable energy. The learning by doing captures the idea that while a society produces it learns to carry out activities more efficiently. This does not occur due to direct R&D

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[1] Nordhaus (2007) has been a pioneer in mathematically simulating the effects of climate change by using economic and mathematical modelling tools and analyzing the efficient and inefficient strategies toward mitigating climate change damages.

[2] It is standard practice in the existing literature (Stiglitz 1974 for example) to start analysis with a Cobb Douglas functional form. This functional form entails that all inputs are essential for production. In this paper we release this assumption and analyze the implications of this.

investment decisions but as a side product of a certain activity, leading to the accumulation of knowledge in the economy (Arrow, 1962). This seems plausible in this case since renewable energies are still relatively new in the economies of today, i.e. their contribution to current energy demand is small, and offer a lot of space for learning how to use them better.

The paper proceeds as follows. We start off by recalling the structure of the companion paper model and building a baseline scenario. This baseline scenario will represent the benchmark against which to assess all simulations. We then introduce exogenous technology and simulate changes in the key parameters as identified in the companion paper. In addition to the companion paper here we look at implications for both weak and strong sustainability. We then add endogenous technology modelled as learning by doing in the renewable resource. Subsequently we release the assumption of the renewable and non-renewable energy resources being essential in energy production and introduce CES production. In the final set of simulations we generalize consumer preferences functional form. Conclusions are then drawn.

The model with no technological progress

The model in this paper builds on the structure and results obtained in the previous chapter of this thesis, including for a renewable and non-renewable energy resource and accounting for the effects on the environment. We set up an economy that can use both a renewable, R(t), and non-renewable energy resource, Z(t). Some key features differentiate these resources. The depletable energy resource is exhaustible and pollutes but costs less. On the other hand the renewable energy resource is not limited, does not pollute, offers learning by doing technological advantages but is more costly. Production is a function of physical capital, the renewable resource and non-renewable resource. Initially we model production in Cobb-Douglas form, for which both the renewable and non-renewable resource are essential, namely

$$F[K(t),R(t),Z(t)] = K(t)^{(1-\alpha)} EN(t)^{\alpha} = K(t)^{(1-\alpha)}[Z(t)^{\beta} R(t)^{(1-\beta)}]^{\alpha} \qquad (1)$$

The assumption of Cobb Douglas production will be released in the endogenous part of the analysis allowing us to move away from the essentiality condition embedded in Cobb Douglas specification for which $F(0,Z,R) = F(K,0,0) = 0$.

Pollution accumulates over time due to the use of the non-renewable resource in production, although partly re-absorbed naturally by the environment (see social planner's problem below). At the same time, pollution causes disutility to consumers. Therefore utility is a function of consumption and pollution, whereby individuals derive utility from consumption and are negatively affected by pollution[3], namely

$$u(C(t),P(t)) = \ln C(t) + \ln(\overline{P} - P(t)) \qquad (2)$$

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[3] Standard conditions on the utility function hold, namely $U_C > 0$ and $U_P < 0$ and concavity of the utility function, i.e. , $U_{CC} < 0$ and $U_{PP} < 0$. The assumption of separability in the utility function entails that the level of pollution has no impact on U_C and viceversa that consumption has no impact on U_P. Separability is a standard assumption (see for example Pittel (2002)).

where C(t) is consumption, P(t) is the pollution stock[4], and $\overline{P}$ is the uppermost pollution that society can withstand beyond which society would cease to exist[5].

A benevolent social planner maximizes utility subject to a pollution constraint, a depletable resource constraint and a capital accumulation constraint. Pollution accumulates based on the net amount of depletable resource used. The constraint on the depletable resource is the standard rule for decumulation of the stock of non-renewable resource. Physical capital accumulates based on output net of consumption, costs for the renewable resource and depreciation of physical capital. Structurally the problem is as follows:

$$\max_{\{C(t),R(t),Z(t)\}} \int_0^T u(C(t),P(t)) \cdot e^{-\rho t} dt \tag{3}$$

subject to

$$\dot{P}(t) = Z(t) - \gamma P(t) \tag{4}$$

$$\dot{S}(t) = -Z(t) \tag{5}$$

$$\dot{K}(t) = F[K(t),R(t),Z(t)] - C(t) - gR(t) - \delta K(t) \tag{6}$$

This is the structural form of the base case scenario which does not include for any technical progress. We will introduce exogenous and endogenous technical progress as we proceed in the analysis.

The baseline simulation[6]

Starting from the base model outlined above, we simulate an economy that is neither weakly nor strongly sustainable, which will represent the counterfactual for our set of

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[4] As outlined in the introduction, we consider pollution to be a stock since we refer to fossil fuels and the disutility generated through the consequent accumulation of carbon dioxide pollution. This is inline with the discussion in Withagen (1995)

[5] We assume that this threshold level of pollution can never be reached using all existing non-renewable resources in one same point in time.

[6] Details of the GAMS programmes are contained in the appendices.

simulation. This means that as time progresses and the economy grows, the stock of pollution increases over time, while utility decreases (Figure 1).

The share of renewable energy in the economy plays a key role in the whole analysis and will be one of the key parameters under investigation. Currently, renewable energy contributes approximately 6 percent to total energy supply worldwide[7]. Due to this we start by setting the share of non-renewable energy, beta, at 0.94. This will represent the value of beta for the base case scenario. We test two additional values of beta, 0.8 and 0.7. As beta decreases, the role of renewable energy in the energy production is increasing. Figure 1 illustrates how, also for these two values of beta, the economy is still unsustainable, both in a weak and in a strong sense.

After having set up the base scenario, we let technology, production and consumption functions and key variables vary at different stages of the analysis and assess the impact of these on sustainable development. We start off by adding exogenous technological progress and then move to endogenous technological progress. In the case of endogenous technology, as the analysis proceeds, we add CES energy production and introduce a more general isoelastic utility function[8].

The exogenous simulation set with exogenous technical progress

Following the detailed analysis and conclusions obtained in the previous chapter of this dissertation, we introduce exogenous technical progress as resource augmenting technical progress following, among others, Stiglitz (1974) and Valente (2005) whereby technical progress is driven exogenously and formally described as $m(t) = e^{\omega t}$. Accordingly, the production function in this case will be the following:

$$F[K(t), m(t)\cdot R(t), Z(t)] = K(t)^{(1-\alpha)} EN(t)^{\alpha} = K(t)^{(1-\alpha)}[Z(t)^{\beta}(m(t)R(t))^{(1-\beta)}]^{\alpha} \qquad (7)$$

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[7] This is the share of renewables in energy consumption in the EU-25 today based on the European Environment-State and Outlook 2005 report of the EU. This share encompasses all renewables. The target rate for 2030 is that this share increases to 20 percent, equivalent to a beta of 0.8 in the analysis presented here. Worldwide the current share of renewable energy is 0.5 percent, projected to increase under a business as usual scenario to 1.7 percent by 2030 (IEA 2006).

[8] In the current setting we are using a specific isoelastic function for utility, where the intertemporal elasticity of substitution is equal to 1.

We previously found that the rate of growth of the economy with exogenous technological progress was a function of the share of renewable resource, β, the share of the non-renewable resource, (1−β), and the rate of exogenous technological progress, ω, formally

$$\frac{\dot{Y}}{Y}^{EXO} = \frac{(1-\beta)}{\beta}\omega \tag{8}$$

We now run two sets of simulations. In the first set we vary the rate of technical progress. In the second set, we define a minimum and maximum rate of technical progress, and let the share of the renewable resources in the economy vary.

We start by setting the rate of exogenous technical progress at a very low level and then gradually increase it. We initially set ω at 0.1 and then increase it up to 0.5 and 0.9. Results are reported in Figure 2. We find that increasing the rate of technical progress has no significant effect on the optimal pollution trajectory over time. On the other hand, when technical progress is comparatively fast, in the long run utility increases. Thus faster exogenous technical progress, at low levels of renewable resource market share, does not allow to achieve strong sustainability, but can lead to weak sustainability in the long run.

The second set of simulations adds changes in the resource share to changes in the rate of technological progress, see Figures 3 and 4. We select two cases, one of relatively slow and one of relatively rapid technical progress, namely by setting ω=0.1 and ω=0.9. We start from the base case share of non-renewable resource, 0.94, in which renewables hold a small share of the market, and decrease it to 0.8 and 0.7, allowing for a more prominent role of the renewable energy resource.

We find that in all cases the economy is not strongly sustainable and that pollution always increases over time. For low levels of technological progress, as the share of the renewables increases, the level of pollution decreases, but when technological progress is fast, the contrary is true. This is because beta plays a double role, on the one hand as beta decreases the share of renewable resources increases, on the other, a decline in beta also coincides with an increase in the rate of technical progress which in turns makes all resources more productive. When the rate of technological progress is fast enough, it seems that the second effect overcomes the first, so that overall more pollution is optimal for the economy. For what concerns utility, the economy manages to reach a weakly sustainable path when technological progress is fast and the share of the depletable resource decreases. These results are in line with the conclusion achieved in the companion paper and summed up above. Weak sustainability is possible because of the exponential growth rate of technical progress that generates rapid output and consumption growth, counteracting the negative effects of increasing pollution stock in terms of utility. On the other hand, as shown in the companion paper, strong sustainability cannot be achieved.

The endogenous simulation set with learning by doing in the renewable resource

We introduce endogenous technical progress in the form of learning by doing. The idea here is that as time proceeds and the renewable energy source is used, society learns to use this resource better. Through the use of the renewable resource a knowledge base accumulates. The rate of accumulation of knowledge is directly proportional to the amount of renewable resource used. As time proceeds and knowledge accumulates, the renewable input in energy production becomes more productive. Formally, the evolution of knowledge, M(t), over time is defined as

$$\dot{M}(t) = \varphi M(t)R(t) \tag{9}$$

or in other words the rate of knowledge accumulation is proportional to the amount of renewable energy source used in society

$$\frac{\dot{M}(t)}{M(t)} = \varphi R(t) \tag{9'}$$

where φ represents a productivity parameter. The question we investigate now is whether the knowledge accumulation process directly linked to the use of the renewable energy resource, captured in (9) or (9'), can contribute toward achieving both weak and strong sustainability. More generally, what role does knowledge accumulation play in achieving sustainability?

The production function can be re-written as follows:

$$F(K(t), M(t)R(t), Z(t)) = [Z(t)^{\alpha}(M(t)R(t))^{\beta} K(t)^{1-\alpha-\beta}] \tag{10}$$

We add the knowledge constraint (9) to the previously shown social planner's problem , i.e. equations (3) to (6). In this case, the current value Hamiltonian is

$$H(t) = U(C(t), P(t)) + \lambda(t)[Z(t) - \gamma P(t)] + \xi(t)[-Z(t)] + \\ + \mu(t)\{F[K(t),(M(t)R(t)),Z(t)] - C(t) - gR(t) - \delta K(t)\} + \omega(t)[\varphi M(t)R(t)] \quad (11)$$

The first order conditions with respect to consumption, the amount of depletable resource extracted and the renewable resource, namely the control variables, are:

$$\partial H / \partial C = 0 \Rightarrow 1/C = \mu \quad (12)$$

$$\partial H / \partial Z = 0 \Rightarrow \mu F_z = \xi - \lambda \quad (13)$$

$$\partial H / \partial R = 0 \Rightarrow \mu F_R + \omega \phi M = \mu g \quad (14)$$

Specifically, while (12) is standard, condition (13) requires that the marginal benefits from depletable resource use equal marginal social costs. Further, condition (14) implies that marginal costs of renewable resources must be equal to marginal social benefits, given by the sum of marginal benefits in production *plus* marginal LBD benefits.

The costate equations for the state variables, namely pollution, the stock of depletable resource, physical capital and knowledge are as follows

$$\dot{\lambda} = (\rho - \gamma)\lambda - u_P(\cdot) \quad (15)$$

$$\dot{\xi} = \rho \xi \quad (16)$$

$$\dot{\mu} = \mu[\rho + \delta - F_K] \quad (17)$$

$$\dot{\omega} = (\rho - \varphi R)\omega - \mu F_M \quad (18)$$

And the following set of transversality conditions on the state variables need to hold for optimality to be met

$$e^{-\rho T}\lambda(T)P(T)=0 \tag{19}$$

$$e^{-\rho T}\xi(T)S(T)=0 \tag{20}$$

$$e^{-\rho T}\mu(T)K(T)=0 \tag{21}$$

$$e^{-\rho T}\omega(T)H(T)=0 \tag{22}$$

We now start to test whether the introduction of endogenous technology can lead to weak and strong sustainability. To this aim we set up a number of simulations. Initially we run a sensitivity analysis of the model to the initial level of knowledge in the economy and to the productivity of the knowledge sector. We call this set of simulations Set 1. Secondly, we substitute Cobb Douglas energy production with a CES energy production function. This allows us to move away from the assumption that both the renewable and depletable resource are essential in energy production. We test for varying levels of substitutability between the two resources. To this end, we investigate the sensitivity of the model to varying levels of the elasticity of substitution between the renewable and non-renewable energy source. We also vary the share of the renewable resource in the economy and combine this with a CES energy production function. These two sets of simulations are Set 2 and Set 3. In the final set of simulations, Set 4, we remove the assumption of unit intertemporal elasticity of substitution to evaluate how such assumption affects our results and include a wider range of consumers' preferences.

Endogenous simulation set with Cobb Douglas energy production (Set 1).

In an economy that can benefit from knowledge accumulation via the use of the renewable resource, we assess whether the productivity of the knowledge sector and the initial knowledge base play a role in achieving weak and strong sustainability. We then add the share of the renewable resource to the analysis to understand how the size of the resource can contribute toward sustainability when learning by doing from the renewable resource is present.

As shown in Table 1, we run 12 simulations subdivided in groups of four. Each group of simulations has a constant level of beta. The first group of simulations, group 1, uses the base case scenario share of depletable resources, namely sets beta equal to 0.94. Group 2 if for beta equal to 0.8 and group 3 for beta equal to 0.7. For each level of beta, we test the cases of a relatively high and relatively low initial knowledge base, M_0. In the case of a low initial knowledge base we set M_0 equal to 10, for the high case we set M_0 equal to 100. We increase the productivity parameter from 0.5 to 0.9. Results for pollution and utility are shown in figures 5 and 6.

In the case of pollution, we find that the optimal pollution trajectory is always increasing, showing learning by doing in the renewable resource is not sufficient to guarantee strong sustainability. Further, at higher levels of the initial knowledge base and a more productive knowledge sector, the pollution path continues to be increasing. When the resource share is added to the analysis, the optimal level of pollution is lower compared to the other cases, but still increasing. Thus, when both the renewable and depletable resource are essential in energy production, the mere existence of the resource and the technology benefits from learning to use the resource better, are no guarantee that an economy will be able to lower its emissions and move to a strongly sustainable path.

Introducing learning by doing does not lead to an optimally weakly sustainable path either. We find that for the base case of beta equal to 0.94, the initial knowledge base and the sector productivity, have no impact on the optimal path of utility. As the share of the deplatable resource decreases, the initial knowledge base and the sector productivity play a larger role.

Endogenous simulation set with CES energy production (Set 2 and Set 3).

We now release the crucial assumption of essentiality of both resources in energy production by introducing CES energy production. This type of production entails that the two resources can be substituted one for another at a varying rate. The extreme case, when the elasticity of substitution, sigma, is equal to 1, entails that Z and R can be fully substituted one for another.

$$F[K(t), m(t)\cdot R(t), Z(t)] = K(t)^{(1-\alpha)}\, EN(t)^{\alpha}$$

and where $EN(t) = \left[\beta Z(t)^{\sigma} + (1-\beta) R(t)^{\sigma}\right]^{1/\sigma}$

or in other terms $F[K(t), m(t)\cdot R(t), Z(t)] = K(t)^{(1-\alpha)} \left\{\left[\beta Z(t)^{\sigma} + (1-\beta) R(t)^{\sigma}\right]^{1/\sigma}\right\}^{\alpha}$ (23)

Sigma can vary from 1 to $-\infty$. For sigma=1 the resources are perfect substitute. Sigma=0 is a limit case when the CES production function tends to the Cobb Douglas function (this case has been analyzed above). As sigma moves toward $-\infty$, R and Z become less and less substitutable, complements in the limit. As listed in Table 2, we run 5 simulations setting the elasticity of substitution between R and Z, called sigma, equal to -3, -1, -0.5, 0.5. We use base values for the knowledge base and the productivity of the knowledge sector, respectively setting these at 10 and 0.5. The share of non-renewables in energy production is 0.94.

We find that substitutability between the inputs in energy production plays a key role in reducing pollution and achieving strong sustainability, as shown in Figure 7. When the depletable and renewable resource are not substitutable pollution increases. This holds true for all values tested with the exception of the perfect substitutability case. It is only when the two energy inputs are perfectly substitutable that the optimal path of pollution steadily declines over time leading to a strongly sustainable path for which pollution levels are also significantly lower.

The case of utility is somewhat different, see Figure 8. When resources are not substitutable, utility levels decline over time. In the case of perfect substitutability, utility initially increases but declines in the long run. Overall weak sustainability is not achieved in the long run in any case.

We now add the share of the renewable resource to the analysis, simulation Set 3 as shown in Table 3. We find a set of paths consistent with the results above. The share of the renewable resource plays no major role in modifying the pollution and utility paths, Figures 9 and 10. Strong sustainability is achieved when resources are perfectly substitutable and the resource share does not modify the path significantly. In the case of utility, when resources are not substitutable, utility declines. In the case of perfect substitutability, utility initially increases but declines in the long run. As the depletable resource share decreases, utility levels rise and increase for a longer period of time, but still decrease in the long run. This result is in line with the existing literature (see Pezzey and Withagen, 1998), although here a renewable resource and learning by doing have been added.

These results for utility and consequently weak sustainability are puzzling since we obtain strong but not weak sustainability. In order to shed some light on this puzzle, we do two things. First, we check if the value of the objective function, namely the Net Present

Value (NPV) of total flow of utility over time, increases as knowledge accumulation becomes more productive and the share of the renewable resource increases. We plot the value of NPV as a function of sigma and for two values of beta. We find that, as shown in Figure 11, increasing the substitutability between the resources increases the total value of the objective function. Furthermore, given the rate of substitution, higher shares of renewables also lead to higher values of the NPV. Therefore, although we still do not obtain weak sustainability, we find that for society overall increasing the energy mix and the substitutability among resources is positive. As a second step, Figure 12, we also drastically increase the productivity factor of the knowledge sector. In this case we find that in the very long run, utility increases for high enough levels of sigma. Nevertheless these results are still not conclusive on the utility side.

Endogenous simulation set with general isoelastic utility (Set 4).

Since we find that no other key parameters lead to steadily increasing utility over time, we now generalize consumer preferences by allowing the elasticity of substitution to vary, as discussed in Romer (2001) and Smulders (2000). We keep, however, the separability assumption. The Constant Relative Risk Aversion (CRRA) utility function is defined as follows:

$$u(C(t),P(t)) = \frac{C(t)^{1-\tau}}{1-\tau} + \frac{(\overline{P}-P(t))^{1-\tau}}{1-\tau} \tag{24}$$

where τ represents the household's willingness to shift consumption between different time periods. The smaller τ, the more willing the household is to allow its consumption to vary over time or in the words of Smulders, the more *flexible* the consumer. The intertemporal elasticity of substitution is defined as 1/ τ so as τ decreases the intertemporal elasticity of substitution increases showing how consumers become more willing to substitute consumption across time periods. It can be shown that when τ tends to 1 this functional form converges to the logarithmic functional form, namely, the functional form in (22) is a generalization of the previous preference function in (2).

Building on the conclusions from the previous set of simulations, we start from the case for which we had obtained an optimal strongly sustainable path, featuring a depletable resource share of 0.94, high productivity in the knowledge sector and perfect substitutability between the depletable and renewable resource.

We test two cases, one case in which consumers are relatively more flexible compared to the previous case and one in which consumers are comparatively less flexible. As discussed, in the limit logarithmic case used previously to describe consumer preferences, τ was equal to 1. Therefore we set τ equal to 0.9, the case of a relatively more flexible consumer, and 1.1, the case of a relatively less flexible consumer.

We plot the optimal utility path under the two cases, Figure 13. We find that when consumers are less flexible, utility declines over time. In the case of a more flexible consumer, utility increases in the long run. Therefore, flexibility of the consumer, ie the willingness of the consumer to substitute consumption across time periods, is important in reaching a weakly sustainable path.

Taking the case of a more flexible consumer, ie τ =0.9, as a second step we vary the productivity parameter and the share of the renewable resource, Figure 14. We find that the productivity of the knowledge sector is key for weak sustainability, once a more flexible consumer is introduced. A long run weakly sustainable path is obtained when the productivity factor reaches 100. The share of the renewable resource contributes to this. When we decrease beta from 0.94 to 0.7, the utility path is more steadily increasing.

Conclusions

In an attempt to contribute to the debate on climate change and the role for renewable energy resources, this paper presents a model that brings together a number of the building blocks in recent environmental economic literature, including for pollution, renewable and depletable energy resources, and technological progress. The scope of the paper is to assess how and if renewable resources can play a key role in achieving weak and strong sustainability, respectively defined as non-declining utility and non-inceasing pollution stock.

In the model we set up, society can choose whether to use a renewable energy resource or a depletable one, differentiated based on their contribution to pollution, cost, technology and availability. Renewable resources offer learning by doing potential since society can learn to use them better through renewable resource-related knowledge accumulation.

The key aspects we analyze in the paper are the roles played by technological improvement, substitutability between the renewable and depletable resource in energy production, and consumer flexibility.

With regards to technological progress we analyze two cases: firstly exogenous technology modelled as resource augmenting technological progress and, secondly, endogenous technology in the form of learning by doing in the renewable resource. Once endogenous technology is introduced, in order to account for the full range of substitution possibilities, we release the assumption of Cobb Douglas production and introduce CES production. With respect to consumer preferences, we initially use a logartithmic utility function and then replace it with a more general CRRA utility function. Due to the structural complexity in the endogenous case, it is not possible to obtain a closed form solution so the whole problem is mathematically simulated in GAMS. We therefore start by building a benchmark case that is neither strongly nor weakly sustainable whereby utility is declining and pollution is increasing. The effects of exogenous technological improvement, endogenous learning by doing, key parameters and functional forms, are then assessed in reference to this baseline scenario.

In the exogenous technology case, we investigate the role of two key parameters, namely the rate of technological progress and the share of the renewable resource in energy production. We find that increasing the rate of technological progress alone does not allow to achieve strong and weak sustainability. In the long run though, technological progress does lead to increasing utility if fast enough. When the effect of increasing the share of energy generated from renewable resources is added, this result is clearer. Fast technical progress and larger dependence on renewable resources leads to a weakly sustainable path. Nevertheless, the result for strong sustainability remains unchanged. The optimal pollution path is constantly increasing, so over time more and more pollution accumulates.

In the endogenous case, when energy production is Cobb Douglas, neither strong nor weak sustainability are achieved. We find that relaxing the essentiality condition of the energy inputs plays a crucial role. In fact, a declining optimal pollution path is obtained only when CES energy production is introduced and the resources are perfectly substitutable. In this case though, results for utility are puzzling, we therefore initially refer to the net present value (NPV) for some indication of preference. Based on the NPV of the simulations, we confirm that increasing the share of the renewable resources increases the NPV. Furthermore, when coupled with sigma, we find that consumers prefer an economy in which R and Z can be perfectly substituted for one another and where R is used more heavily in energy production.

Due to the puzzling nature of the utility results, we then move to a more general utility functional form capable of including a wider range of consumer preferences. We find that only when consumers are more flexible, i.e. more willing to substitute consumption across time periods, will this type of economy with endogenous technology be weakly, and strongly, sustainable over time.

The main message from our analysis is twofold:

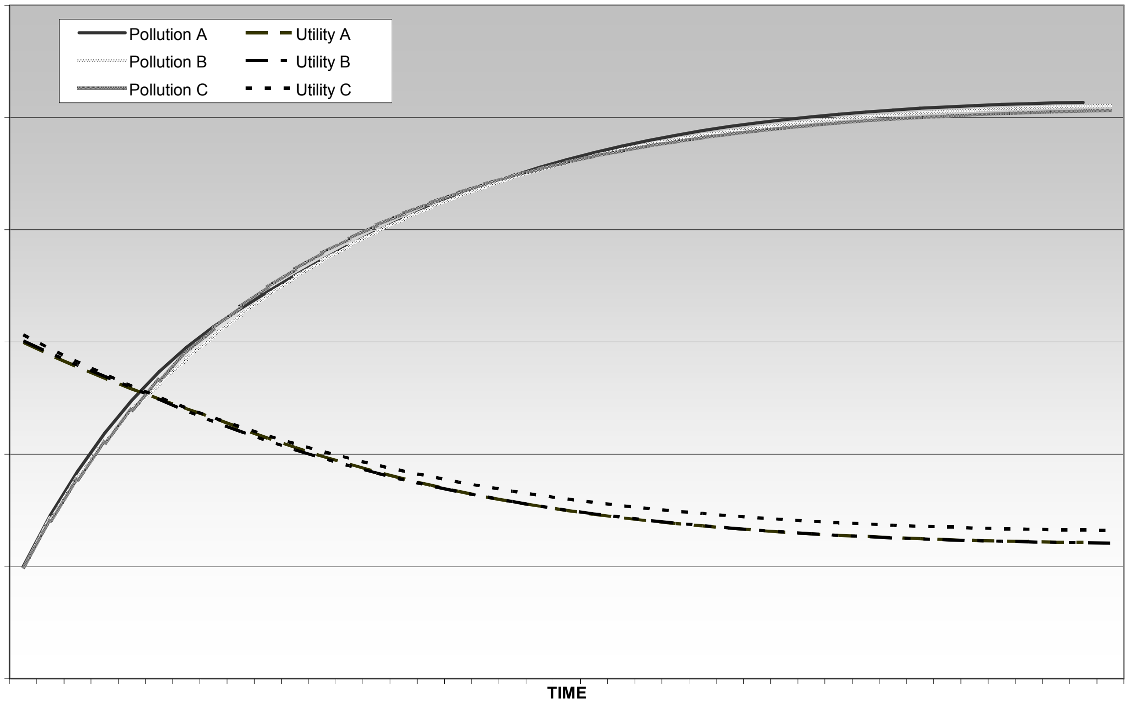
- a short run policy in terms of changes in the policy mix is indeed useful to take us on a more weakly sustainable path, if the role of renewables is sufficiently high. The level of pollution is, however, always increasing, so that we will be able to sustain such living standards only over a finite time span.
- Sustainable development in the long run, instead, requires a change in technology such that we can substitute between renewable and non renewable resources.

A strong political will is therefore needed. The change from a non renewable to a renewable energy system is indeed not an easy step but it is the only way to achieve strong sustainability in the long run. In fact, a change in the energy mix only moves doomsday a little forward.

Possible directions for further research are:

- Empirical analysis by applying this model to the case of a country's economy.
- Further investigate the way technical change and natural resources are modelled, for example by explicity adding the cost of the depletable resource, and/or by including a declining renewable resource cost function, resulting from technical progress.
- The inclusion of environmental policy in our setting which would imply solving a decentralized model and investigating optimal environmental policy in the presence of technical change.

Figure 1: Base simulation model with no technology.



Simulation A: Beta= 0.94; Simulation B: Beta= 0.80; Simulation C: Beta=0.70

Figure 2: Variation in pollution and utility optimal paths upon increases in the exogenous growth rate of technological innovation, at constant beta.

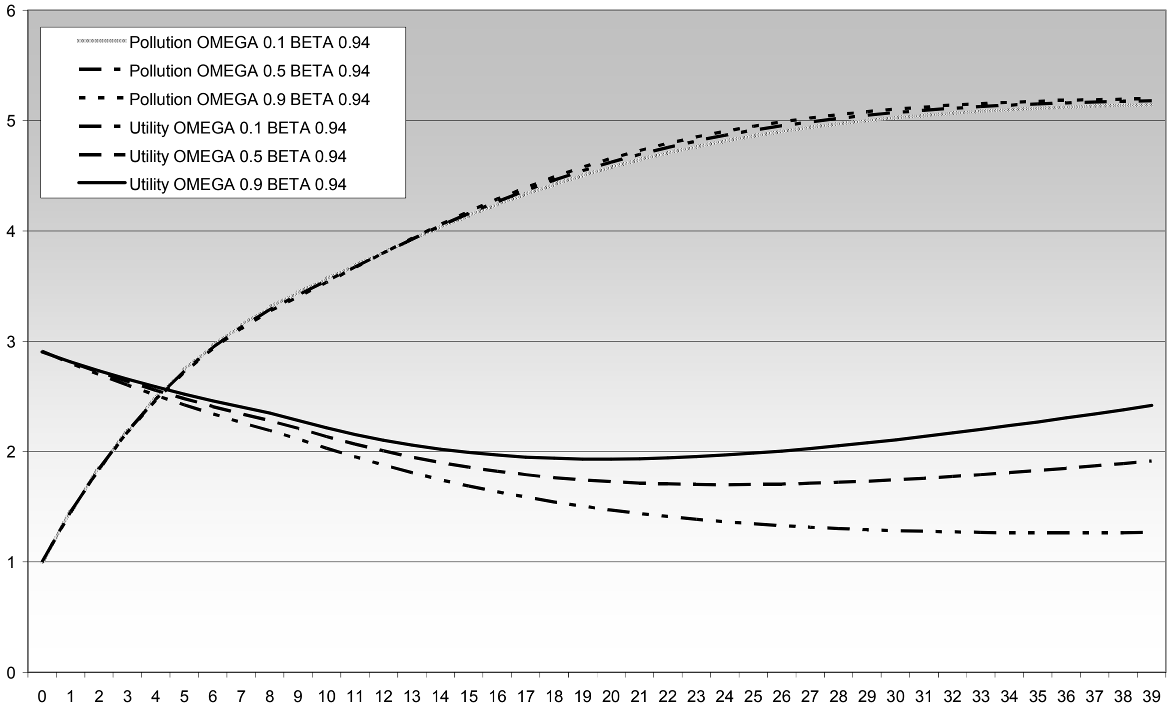


Figure 3: Optimal pollution trajectory upon increases in the exogenous growth rate of technological innovation and varying beta.

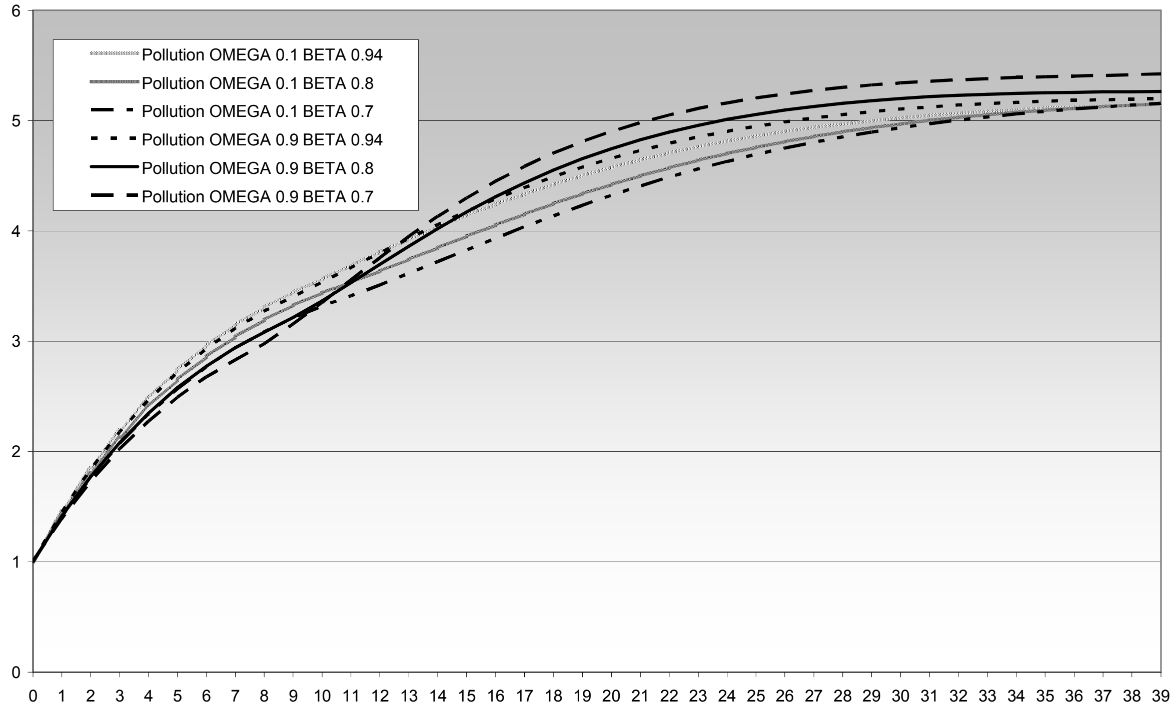


Figure 4: Optima utility trajectory upon increases in the exogenous growth rate of technological innovation and varying beta.

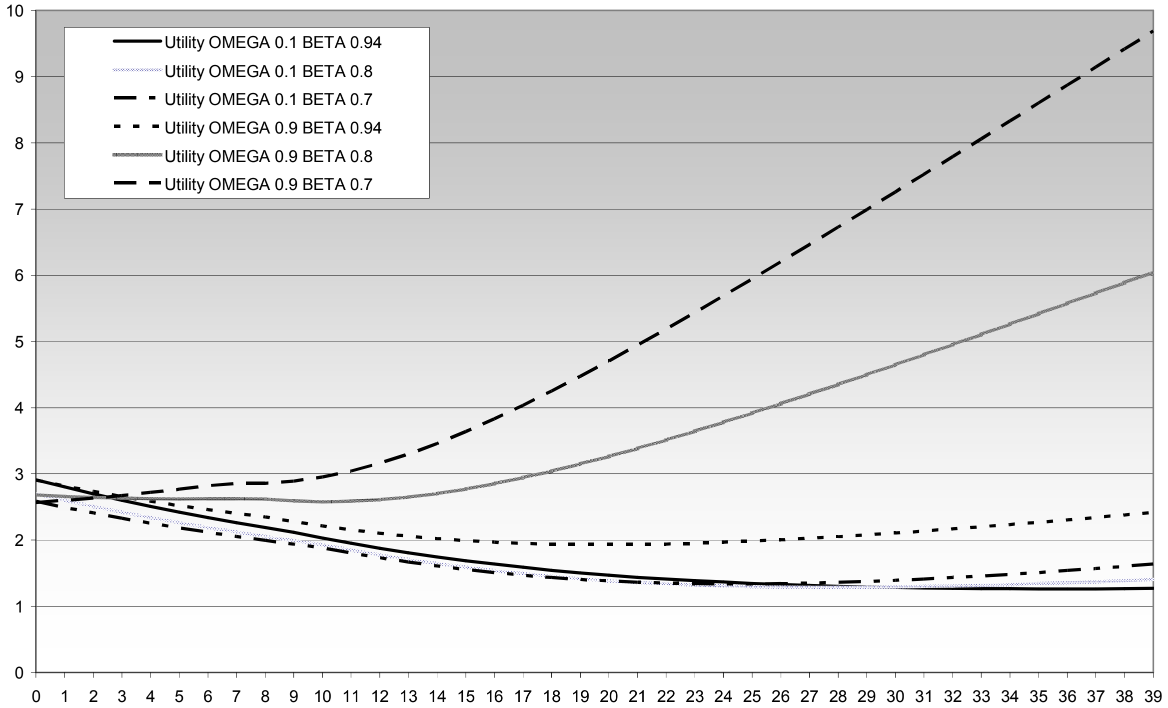


Table 1: List of simulations in SET1

Simulation	Simulation Group	Production	Beta	Mo	Phi
A	1	CD	0.94	10	0.5
B	1	CD	0.94	10	0.9
C	1	CD	0.94	100	0.5
D	1	CD	0.94	100	0.9
E	2	CD	0.8	10	0.5
F	2	CD	0.8	10	0.9
G	2	CD	0.8	100	0.5
H	2	CD	0.8	100	0.9
I	3	CD	0.7	10	0.5
J	3	CD	0.7	10	0.9
K	3	CD	0.7	100	0.5
L	3	CD	0.7	100	0.9

CD=Cobb Douglas, CES=Constant Elasticity of Substitution, Beta=Share of Depletable in the CD, Sigma=Elasticity of Substitution in CES, M0=Initial level of human capital, Phi=Productivity Factor,

Figure 5: Pollution levels for Set 1 simulations in the endogenous technology case.

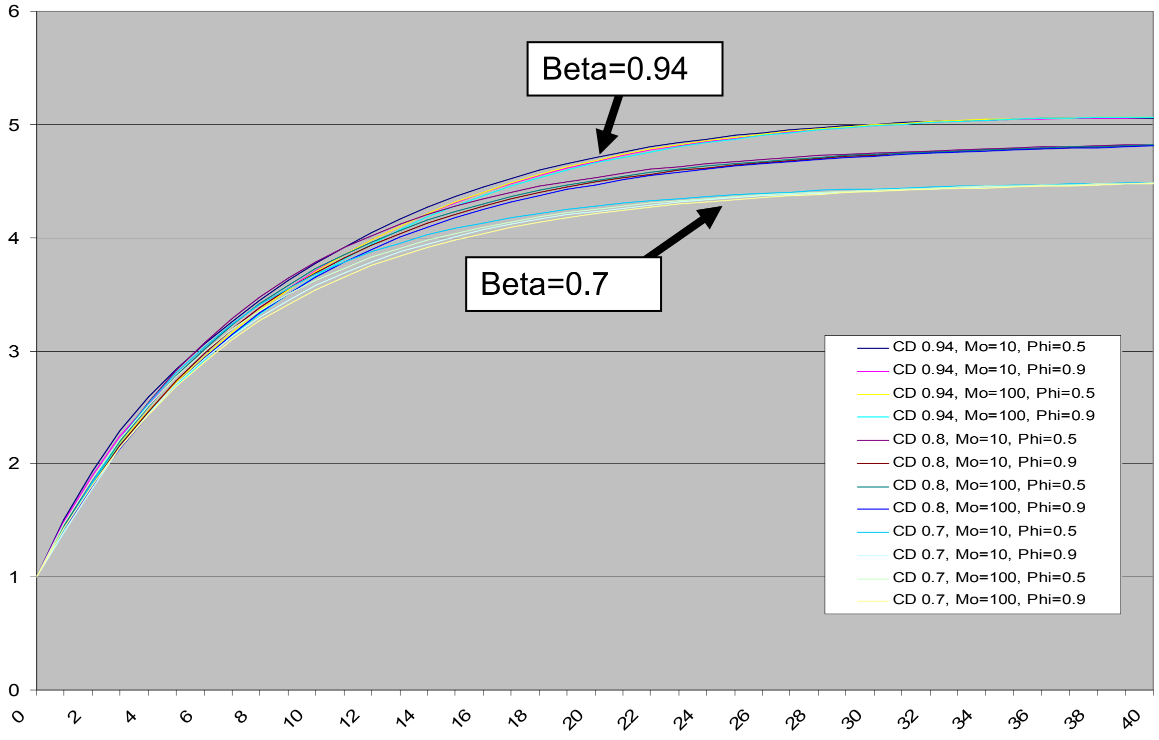


Figure 6: Utility levels for Set 1 simulations in the endogenous technology case.

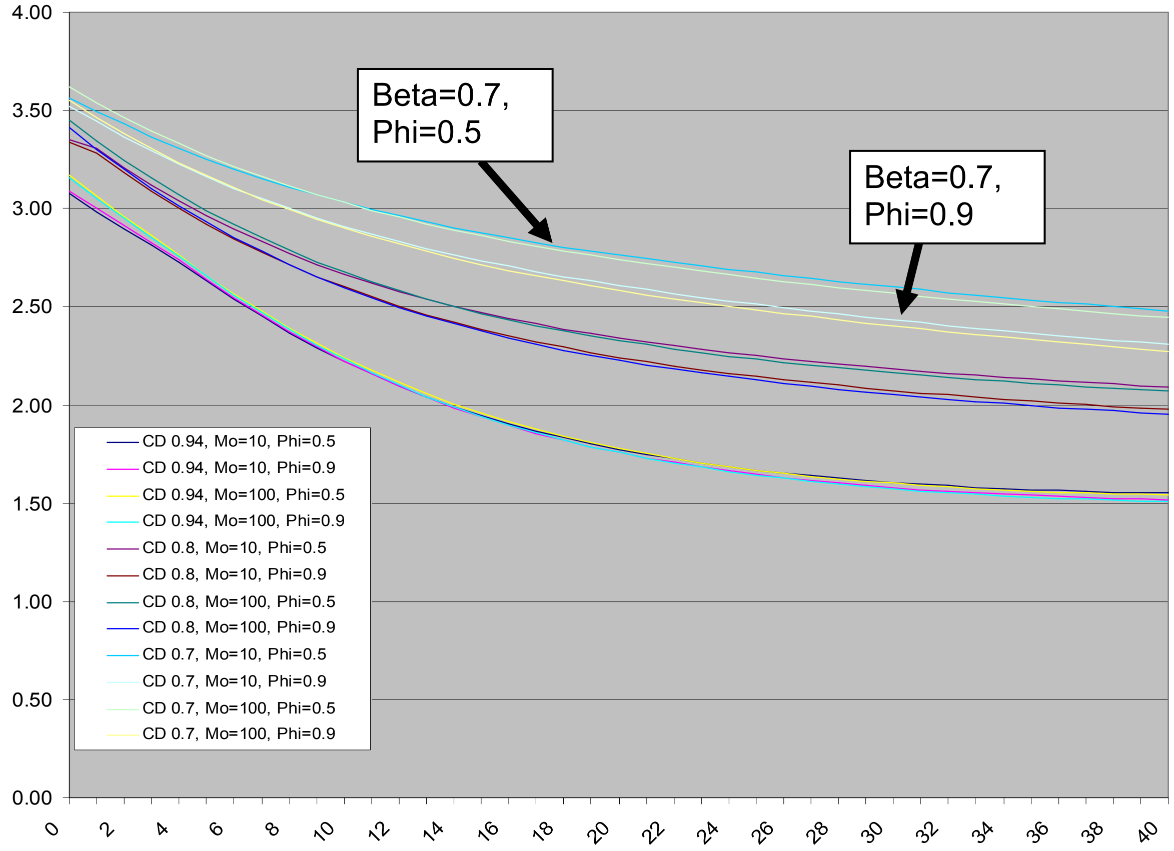


Table 2: List of simulations in SET2

Simulation	Production	Beta	Sigma	Mo	Phi
A	CES	0.94	1	10	0.5
B	CES	0.94	0.5	10	0.5
C	CES	0.94	-0.5	10	0.5
D	CES	0.94	-1	10	0.5
E	CES	0.94	-3	10	0.5

CD=Cobb Douglas, CES=Constant Elasticity of Substitution, Beta=Share of Depletable in the CD, Sigma=Elasticity of Substitution in CES, M0=Initial level of human capital, Phi=Productivity Factor,

Figure 7: Pollution levels for Set 2 simulations in the endogenous technology case.

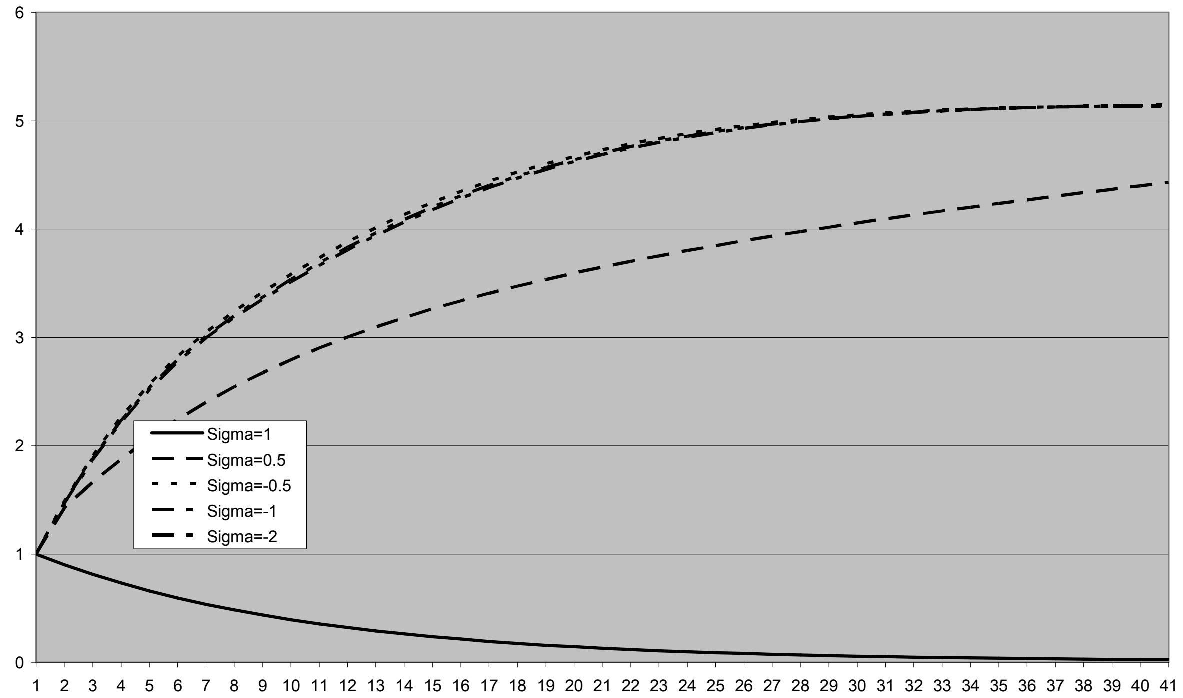


Figure 8: Utility levels for Set 2 simulations in the endogenous technology case

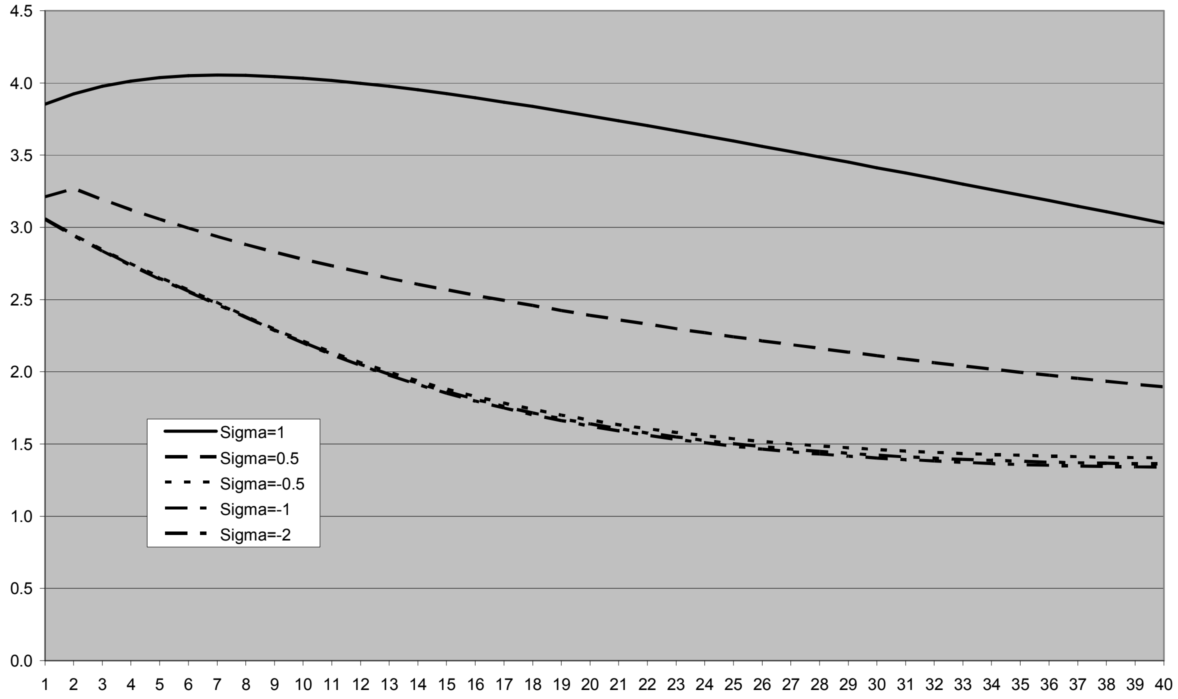


Table 3: List of simulations in SET3

Simulation	Production	Beta	Sigma	Mo	Phi
A	CES	0.94	1	10	0.5
B	CES	0.8	1	10	0.5
C	CES	0.7	1	10	0.5
D	CES	0.94	-2	10	0.5
E	CES	0.7	-2	10	0.5

CD=Cobb Douglas, CES=Constant Elasticity of Substitution, Beta=Share of Depletable in the CD, Sigma=Elasticity of Substitution in CES, M0=Initial level of human capital, Phi=Productivity Factor,

Figure 9: Pollution levels for Set 3 simulations in the endogenous technology case.

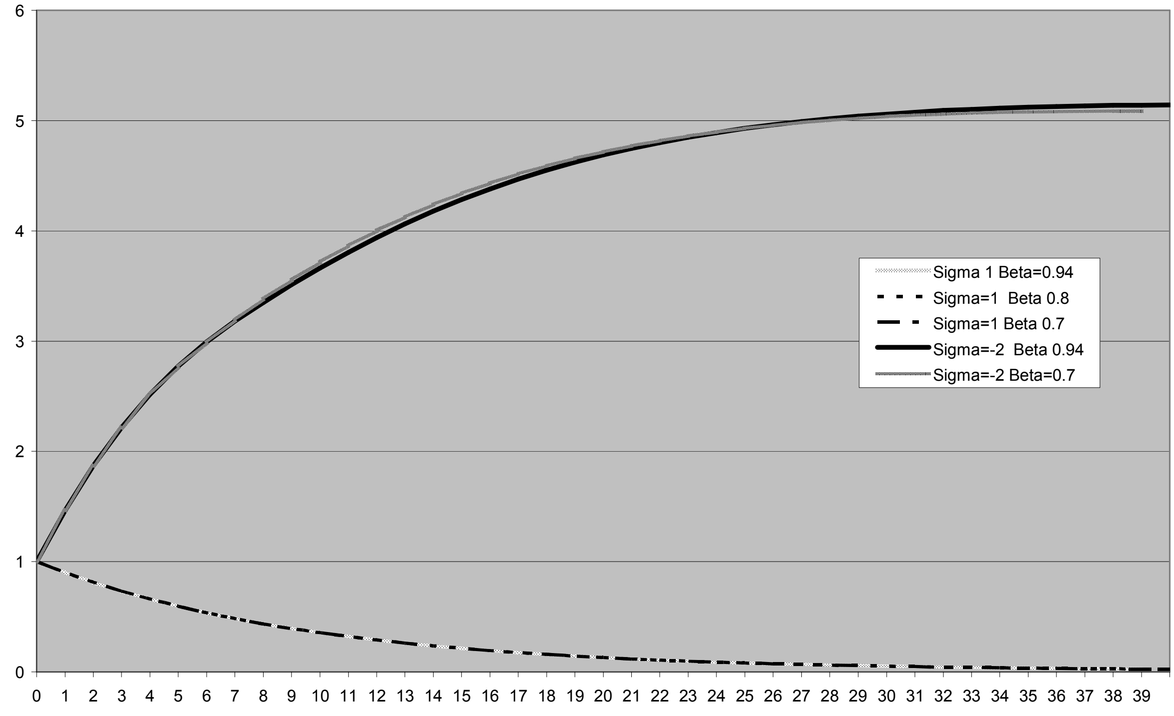


Figure 10: Utility levels for Set 3 simulations in the endogenous technology case.

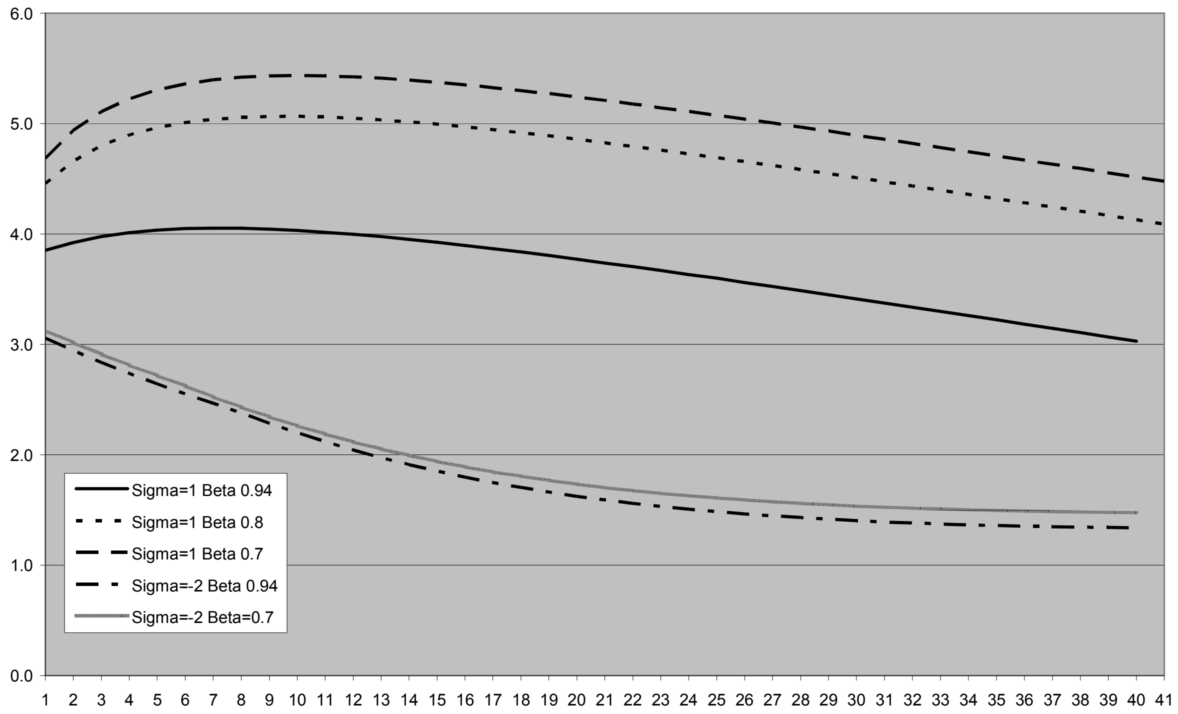


Figure 11: Net present value at varying SIGMA and for decreasing depletable resource shares.

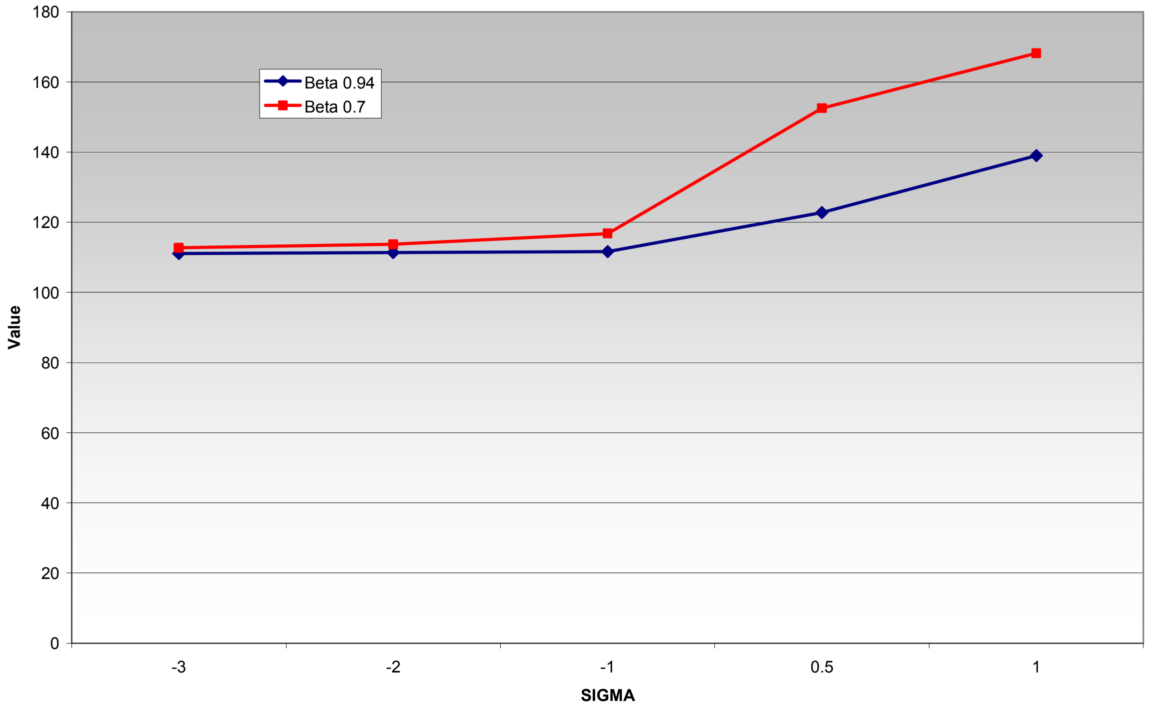


Figure 12: Utility at high levels of knowledge sector productivity.

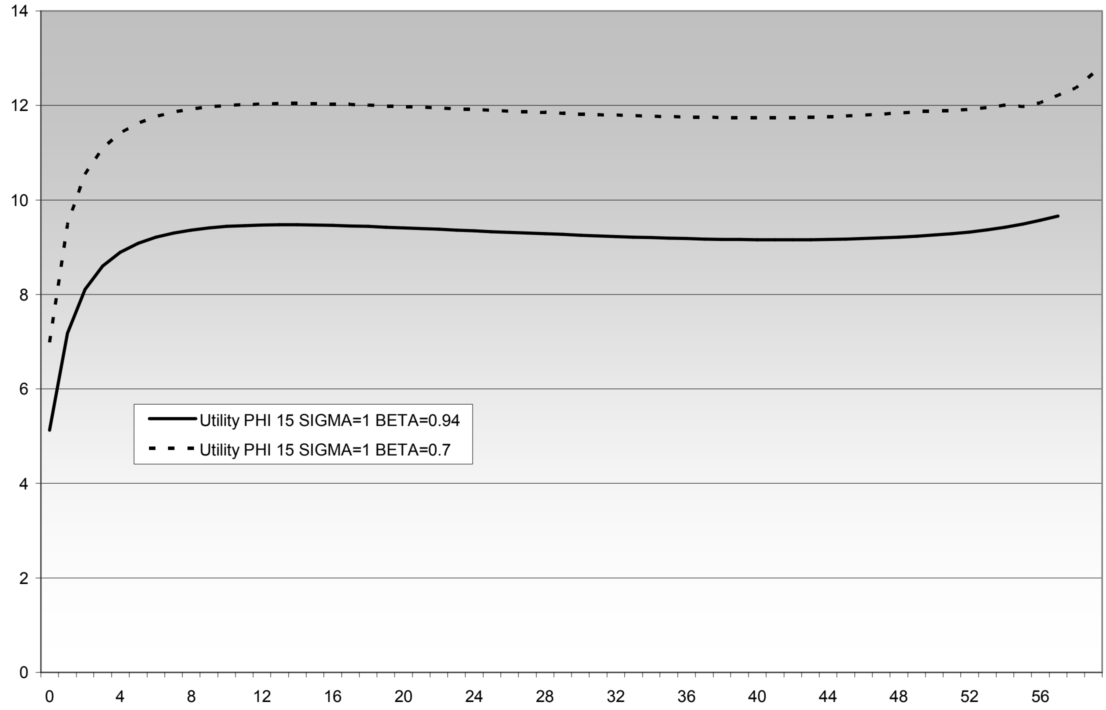


Figure 13: Optimal utility path with a CES preferences.

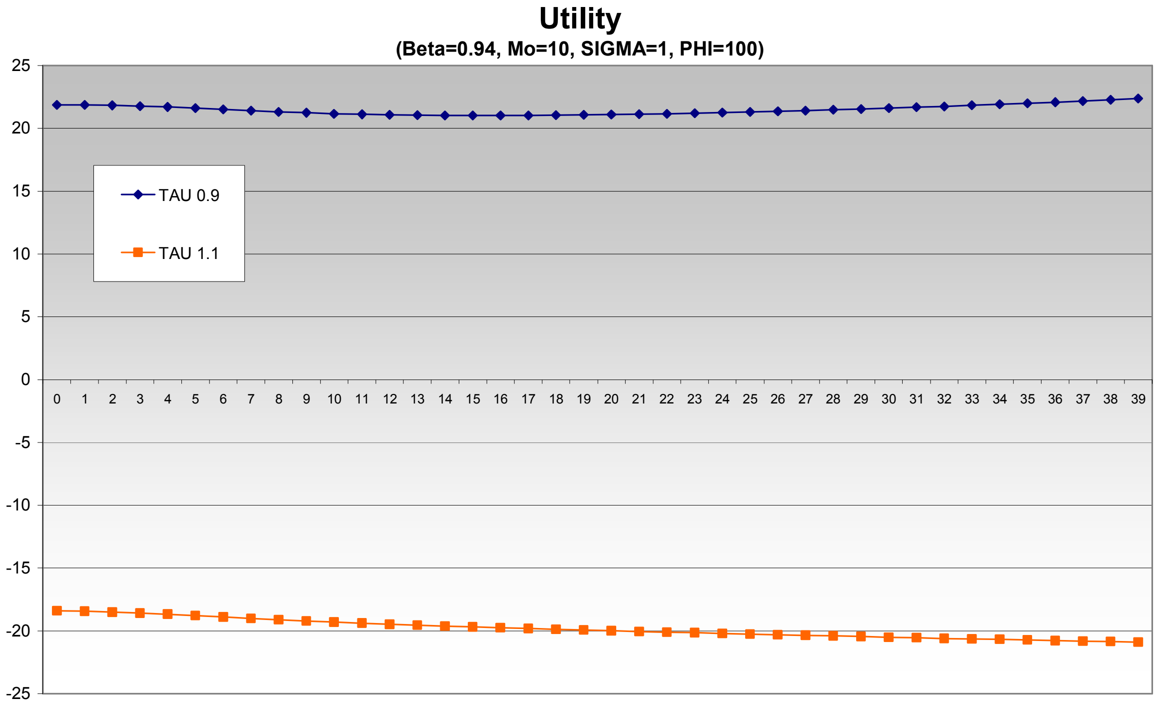
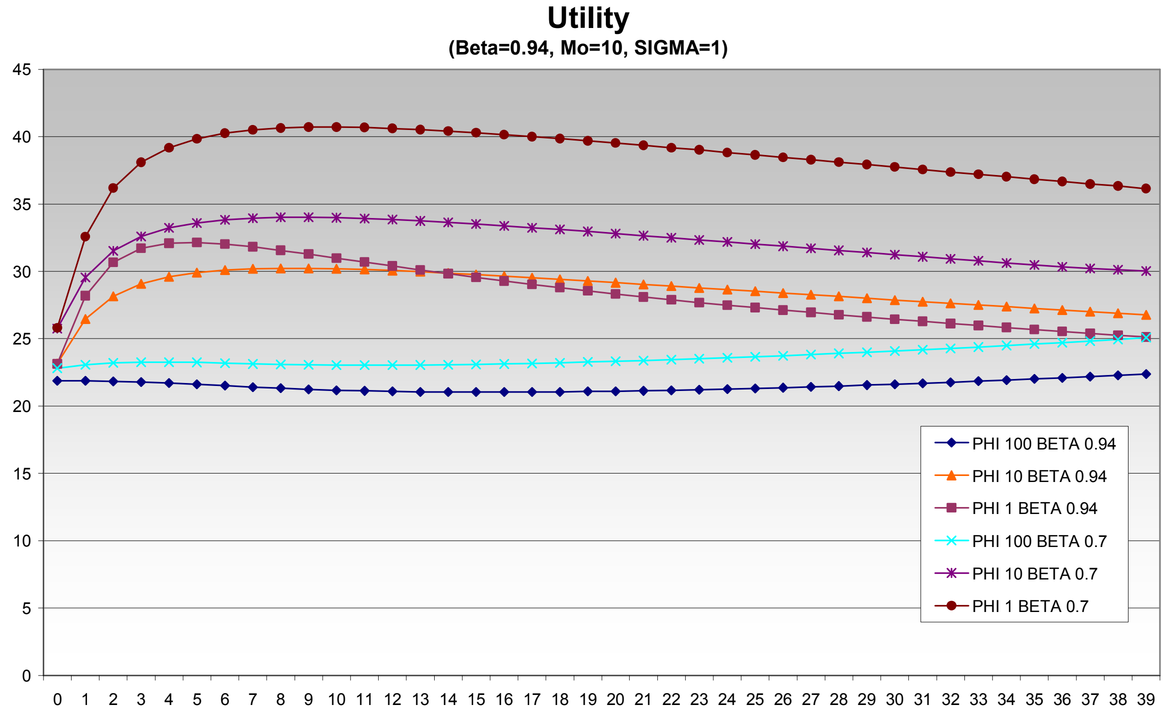


Figure 14: Optimal utility path at varying productivity levels of the knowledge sector and shares of the depletable resource.



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APPENDIX: GAMS PROGRAMMES

(A) Exogenous Technical Progress with Cobb Douglas Production Function

```
$ontext
OPTION NLP      = minos5 ;

KB(N,TT)..       R*K(N,TT) =L= I(N,TT);
$offtext

** REPRESENTATIVE AGENT MAXIMIZATION PROBLEM WITH
** RENEWABLE AND NON RENEWABLE ENERGY RESOURCES

SETS            T           Time periods                                          /0*60/;

SCALARS
** Preferences NEED TO GET VALUES FROM LITERATURE
rho             Rate of time preference per year                                  /.04/

** Technology
dk              Depreciation rate on capital per year                             /0.1/
alpha           Capital elasticity in production function                         /0.4/
beta            Non-renewable resource elasticity in energy production function   /0.2/
fi              Productivity of human capital sector                              /0.1/
theta           Renewable energy extraction costs                                 /0.2/

** Environment and emissions
gamma           Environmental absorption rate                                     /0.1/
pbar            Maximum pollution threshold                                       /10/;

* Definitions for outputs of no economic interest
SETS
tfir(t)
tfin(t);

PARAMETERS
dist(t)         Instantaeous rate of social time preference;
dist(t)=(1+rho)**(-ord(t));

* Defining Exogenous Learning by Doing (LBD)

PARAMETERS
GLbD            Yearly growth rate of LbD                                         /0.1/
EXM(T)          Exogenous LbD                                                     /0 1/
;

loop(t, EXM(t+1)=(1+GLbD)*EXM(T));

Display EXM;

* Unimportant definitions
```

```
tfir(t) = YES$(ORD(T) EQ 1);
tfin(t) = YES$(ORD(T) EQ CARD(T));

VARIABLES
y(t)                    Output UNIT
c(t)                    Consumption UNIT
p(t)                    Stock of pollution UNIT
r(t)                    Renewable energy resource UNIT
z(t)                    Non-renewable energy resource UNIT
k(t)                    Physical capital UNIT
s(t)                    Stock of non-renewable resource UNIT
x(t)                    Pollution change
I(T)                    Investment
EN(T)                   Total energy produced
*mu(t)                  Exogenous LBD multiplier
*m(t)                   Stock of human capital UNIT
*RATIO(T)               RATIO Z Q

UTIL                    Objective function;

POSITIVE VARIABLES Y, R, Z, P, C, K, S, I,EN;

EQUATIONS
** Equations of the model
QY(t)                   Output equation
QP(t)                   Pollution balance equation
QP_FIRST                Pollution at first point in time
QK(t)                   Man made capital balance equation
QK_FIRST                Capital at first point in time
QK_LAST                 Capital at last point in time
QS_FIRST                Depletable resource first period
QS(t)                   Depletable resource stock equation
QUTIL                   Objective function
QX(t)                   Pollution change maximum minus unit level
QC(T)                   Budget constaint
QEN(T)                  Total energy
;
QP_FIRST("0")..         P("0")=e=1;
QP(t+1)..               p(t+1)=E=z(t)+(1-gamma)*p(t);
QX(t)..                 x(t)=E=(pbar-p(t));

QK_FIRST("0")..         k("0")=E=20;
QK_LAST("60")..         RHO*k("60")=L=I("60");
QK(t+1)..               k(t+1)=E=K(T)*(1-DK)+I(T);

QS_FIRST("0")..         S("0")=E=32;
QS(t+1)..               s(t+1)=E=s(t)-z(t);

QEN(T)..                EN(T)=E=(Z(t)**BETA)*((EXM(t)*R(t))**(1-BETA)) ;
```

```
QY(t)..                  Y(t)=E=(K(t)**alpha)*(EN(T)**(1-alpha));

QC(T)..                  C(T)=E=Y(T)-I(T)-THETA*R(T);

QUTIL..                  UTIL=E=sum[t,dist(t)*(log(c(t))+log(x(t)))];

**  Lower bounds on variables
K.lo(t)        = 0.001;
C.lo(t)        = 0.001;
P.lo(t)        = 0.001;
R.lo(t)        = 0.001;
Z.lo(t)        = 0.001;
S.lo(t)        = 0.001;
*M.lo(t)        = 0.9;
X.lo(t)        = 0.001;
p.lo(t)        = 0.001;

**  Upper bounds on variables
*M.up(tfin(t))        = 1000;

** initial values
*P.l(T)      = 0.1;
*S.fx("60")     = 0.001;
*Z.FX("60")     = 0.0000001;
*Z.L("1")       = 0.2;
*R.fx("0")       = 2.5 ;
*K.L("0")       = 4 ;
*Y.L("0")       = 0.21 ;
*EN.L("0")       = 4.121 ;
*I.L("0")      =0.01 ;

** Solution options
option iterlim = 99900;
option reslim = 99999;
option solprint = on;
option limrow = 0;
option limcol = 0;
option nlp = minos;

* Optimal run
MODEL TEST /all/;
SOLVE TEST using nlp maximizing UTIL;
SOLVE TEST using nlp maximizing UTIL;
SOLVE TEST using nlp maximizing UTIL;
SOLVE TEST using nlp maximizing UTIL;
SOLVE TEST using nlp maximizing UTIL;
SOLVE TEST using nlp maximizing UTIL;
SOLVE TEST using nlp maximizing UTIL;
SOLVE TEST using nlp maximizing UTIL;
SOLVE TEST using nlp maximizing UTIL;
```

```
SOLVE TEST using nlp maximizing UTIL;
SOLVE TEST using nlp maximizing UTIL;
SOLVE TEST using nlp maximizing UTIL;
SOLVE TEST using nlp maximizing UTIL;

* Reporting of variables
PARAMETER REPORT SOLUTION SUMMARY;
report(t, "k") = k.l(t);  report(t, "c") = c.l(t);  report(t, "y") = y.l(t);
REPORT(T, "I") = I.L(T);  REPORT(T,"Z") = Z.L(T); REPORT(T,"R")=R.L(T);
REPORT(T,"P")=P.L(T); REPORT(T,"S")=S.L(T);  REPORT(T,"EN")=EN.L(T);
*REPORT(T,"M")=M.L(T);
display R.l, en.l;

DISPLAY util.l;
display R.M, en.M, EXM;
DISPLAY REPORT;
```

(B) Endogenous technology with varying consumer and producer functional forms

```
** REPRESENTATIVE AGENT MAXIMIZATION PROBLEM WITH
** RENEWABLE AND NON RENEWABLE ENERGY RESOURCES


 SETS         T          Time periods                          /0*60/;

 SCALARS
** Preferences
 rho           Rate of time preference per year                /.04/

** Technology
 dk           Depreciation rate on capital per year            /0.1/
 alpha        Capital elasticity in production function        /0.4/
 beta         Share of depletable resource                     /0.94/
 fi           Productivity of human capital sector             /10/
 theta        Renewable energy extraction costs                /0.2/
 tau          Intertemporal elasticity of substitution         /0.9/
 sigma        CES substitution elasticity                      /1.0

** Environment and emissions
gamma           Environmental absorption rate           /0.1/
pbar          Maximum pollution threshold              /10/ ;

** Environment and emissions
 gamma        Environmental absorption rate                     /0.1/
 pbar         Maximum pollution threshold                      /100/ ;

* Definitions for outputs of no economic interest
 SETS
 tfir(t)
 tfin(t);

 PARAMETERS
 dist(t)      Instantaeous rate of social time preference;
 dist(t)=(1+rho)**(-ord(t));

* Unimportant definitions
 tfir(t) = YES$(ORD(T) EQ 1);
 tfin(t) = YES$(ORD(T) EQ CARD(T));

 VARIABLES
 y(t)         Output UNIT
 c(t)         Consumption UNIT
 p(t)         Stock of pollution UNIT
 r(t)         Renewable energy resource UNIT
 z(t)         Non-renewable energy resource UNIT
 k(t)         Physical capital UNIT
```

```
 s(t)          Stock of non-renewable resource UNIT
 x(t)          Pollution change
 I(T)          Investment
 EN(T)         Total energy produced
 mu(t)         Exogenous LBD multiplier
 m(t)          Stock of human capital UNIT
 UTIL          Objective function;

POSITIVE VARIABLES Y, R, Z, P, M, C, K, S, I,EN;

 EQUATIONS
** Equations of the model
 QY(t)               Output equation
 QP(t)               Pollution balance equation
 QP_FIRST            POLLUTION AT FIRST POINT IN TIME
 QK(t)               Man made capital balance equation
 QK_FIRST            CAPITAL AT FIRST POINT IN TIME
 QK_LAST             CAPITAL AT LAST POINT IN TIME
 QS_FIRST            Depletable resource first period
 QS(t)               Depletable resource stock equation
 QUTIL               Objective function
 QX(t)               Pollution change maximum minus unit level
 QC(T)               BUDGET CONSTRAINT
 QEN(T)              TOTAL ENERGY
 QM_FIRST            Human capital first period
 QM(t)               Human capital equation ;

 QP_FIRST("0")..     P("0")=e=1;
 QP(t+1)..           p(t+1)=E=z(t)+(1-gamma)*p(t);
 QX(t)..             x(t)=E=(pbar-p(t));
 QK_FIRST("0")..     k("0")=E=20;
 QK_LAST("60")..     RHO*k("60")=L=I("60");
 QK(t+1)..           k(t+1)=E=K(T)*(1-DK)+I(T);
 QS_FIRST("0")..     S("0")=E=32;
 QS(t+1)..           s(t+1)=E=s(t)-z(t);
 QM_FIRST("0")..     M("0")=E=100;
 QM(t+1)..           M(t+1)=E=(1+fi*R(t))*M(t);

** CES ENERGY PRODUCTION FUNCTION with PRODUCTION SHARES
QEN(T)..      EN(T)=E=(beta*(Z(t)**sigma)+(1-beta)*((M(t)*R(t))**sigma))**(1/sigma);

** CD ENERGY PRODUCTION FUNCTION
*QEN(T)..     EN(T)=E=(Z(t)**BETA)*((M(t)*R(t))**(1-BETA)) ;

 QY(t)..      Y(t)=E=(K(t)**alpha)*(EN(T)**(1-alpha));

 QC(T)..      C(T)=E=Y(T)-I(T)-THETA*R(T);

** LOGARITHMIC UTILITY FUNCTION
 QUTIL..      UTIL=E=sum[t,dist(t)*(log(c(t))+log(x(t)))];
```

```
** CRRA UTILITY FUNCTION
* QUTIL..      UTIL=E=sum[t,dist(t)*(((c(t))**(1-tau)/(1-tau))+((x(t))**(1-tau)/(1-tau)))];

**  Lower bounds on variables
K.lo(t)        = 0.001;
C.lo(t)        = 0.001;
P.lo(t)        = 0.001;
R.lo(t)        = 0.001;
Z.lo(t)        = 0.001;
S.lo(t)        = 0.001;
M.lo(t)        = 0.9;
X.lo(t)        = 0.001;
p.lo(t)        = 0.001;

**  Upper bounds on variables
M.up(tfin(t))        = 1000;

** initial values
*P.l(T)      = 0.1;
*S.fx("60")     = 0.001;
*Z.FX("60")     = 0.0000001;
*Z.L("1")       = 0.2;
R.L("0")       = 2.5 ;
K.L("0")       = 4 ;
Y.L("0")       = 0.21 ;
EN.L("0")       = 4.121 ;
I.L("0")       =0.01 ;
*ratio.FX(T)$(ORD(T) GE 2)    = 0.3;

** Solution options
option iterlim = 99900;
option reslim = 99999;
option solprint = on;
option limrow = 0;
option limcol = 0;
option nlp = minos;

* Optimal run
MODEL TEST /all/;
SOLVE TEST using nlp maximizing UTIL;
SOLVE TEST using nlp maximizing UTIL;
SOLVE TEST using nlp maximizing UTIL;
SOLVE TEST using nlp maximizing UTIL;
SOLVE TEST using nlp maximizing UTIL;
SOLVE TEST using nlp maximizing UTIL;
SOLVE TEST using nlp maximizing UTIL;
SOLVE TEST using nlp maximizing UTIL;
SOLVE TEST using nlp maximizing UTIL;
SOLVE TEST using nlp maximizing UTIL;
SOLVE TEST using nlp maximizing UTIL;
```

```
SOLVE TEST using nlp maximizing UTIL;
SOLVE TEST using nlp maximizing UTIL;

* Reporting of variables
PARAMETER REPORT SOLUTION SUMMA RY;
report(t, "k")=k.l(t);  report(t, "c")=c.l(t);  report(t, "y")=y.l(t);
 REPORT(T, "I")=I.L(T);  REPORT(T,"Z")=Z.L(T); REPORT(T,"R")=R.L(T);
 REPORT(T,"P")=P.L(T); REPORT(T,"S")=S.L(T);  REPORT(T,"EN")=EN.L(T);
 REPORT(T,"M")=M.L(T);

display R.l, en.l, m.l;
DISPLAY util.l;
display R.M, en.M, m.M;
DISPLAY REPORT;
```