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## Design of a plasma discharge circuit for particle wakefield acceleration



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### ABSTRACT

Plasma wakefield acceleration is the most promising acceleration technique known nowadays, able to provide very high accelerating fields ( $10\text{--}100\text{ GV m}^{-1}$ ), enabling acceleration of electrons to GeV energy in few centimetres. However, the quality of the electron bunches accelerated with this technique is still not comparable with that of conventional accelerators; radiofrequency-based accelerators, in fact, are limited in the accelerating field ( $10\text{--}100\text{ MV m}^{-1}$ ) requiring therefore kilometric distances to reach the GeV energies, but can provide very bright electron bunches. Combining high brightness electron bunches from conventional accelerators and high accelerating fields reachable with plasmas could be a good compromise allowing to further accelerate high brightness electron bunches coming from LINAC while preserving electron beam quality. Following the idea of plasma wave resonant excitation driven by a train of short bunches, we have started to study the requirements in terms of plasma for SPARC-LAB [1,2]. In particular, here we focus on the ionization process; we show a simplified model to study the evolution of plasma induced by discharge, very useful to design the discharge circuit able to fully ionize the gas and bring the plasma at the needed temperature and density.

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### 1. Introduction

Particle wakefield acceleration (PWFA) is a technique developed to combine the high brightness electron bunches from conventional accelerators and the high accelerating field that can be reached from plasmas. The needs of a PWFA are two: a train of high quality electron bunches and a plasma. The plasma can be generated either by injecting high energy electrons in a gas or by producing a discharge in a gas, while the train of high quality electron bunches is generated by conventional accelerators. In particular, in our experiment, we will use a train of electron bunches coming from the SPARC linac [3,4]; the train of electron bunches are produced by the so-called Laser Comb Technique [5] proposed by the SPARC team few years ago and which has been recently tested with the SPARC photoinjector without the plasma [6,7].

The scheme of principle of the PWFA is shown in Fig. 1.

The train of electron bunches (propagating toward the right hand side inside the plasma) is composed – in the specific case of Fig. 1 – by three driver bunches, which create the plasma wakefield, and one witness bunch, which will surf on the wakefield prepared by the driver bunches and which will be further accelerated.

With such a train of  $N_T$  bunches a resonant excitation of plasma waves can be performed with the following scaling law [8–10]:

$$E_{acc} [\text{MV/m}] = \left( 244 \frac{N_b}{2 \times 10^{10}} \right) \left( \frac{600}{\sigma_z [\mu\text{m}]} \right)^2 N_T^2.$$

So, for example, with a train of 4 bunches with 16 pC/bunch separated by one plasma wavelength ( $160\ \mu\text{m}$ ), propagating in a plasma of density  $3 \times 10^{22}$  particles/ $\text{m}^3$  can generate an accelerating field in excess of 3 GV/m.

In this paper we focus on the production of the plasma for the accelerating technique described above. In particular, we will concentrate on the plasma discharge (mainly because the SPARC electron bunches are not sufficiently energetic to ionize the gas) and we will describe a simplified model that we have used to design the discharge circuit that we will use for our experiment. Because the model does not take into account some of the physical

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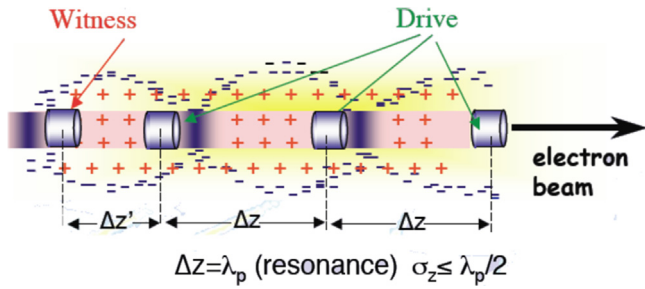


Fig. 1. Particle wakefield acceleration scheme.

mechanisms that exist during discharge (like heat transport and power losses), what we aim to find are the order of magnitudes of the discharge properties (like discharge duration, raising time and so on) and of the discharge circuit (maximum voltage and current, resistance, capacitor and so on).

## 2. Equations describing the gas ionization

In this section, we will list the plasma formulas that we have used to describe the ionization process. The most important formula that we use is Saha equations, which describe the degree of ionization of the plasma as a function of the temperature, density, and ionization energies of the atoms. However, this formula is only valid at thermal equilibrium and for weakly ionized plasmas (such that the Debye length is large and continuum lowering is negligible), but is sufficiently accurate for our purpose. For a gas composed of a single atomic species, the Saha equation is written as [11]

$$\frac{n_i}{n_n} = 2.4 \times 10^{21} \frac{T^{3/2}}{n_i} e^{-U_i/kT}$$

where  $n_i/n_n$  is the ionization fraction,  $n_i$  and  $n_n$  are the ionized and neutral particle densities respectively,  $T$  is the plasma temperature in K,  $U_i$  is the ionization potential and  $k$  is the Boltzmann constant. This formula is telling us that in order to bring the matter to the state of plasma, we need to increase its temperature until the density of ionized particle is much higher than the density of the neutrals. In the case of hydrogen ( $U_i \approx 13.6$  eV), the temperature that ensures that the gas is in the state of plasma is about 2 eV (20 000 K) [11].

This discussion is useful to understand that in fact to study the ionization process we have to study the temperature growth as a function of the time.

The first formula needed to describe the temperature growth is the resistance of the plasma column [12]:

$$R_p = \rho_e \frac{L_{cap}}{S_{cap}} = B_{res} Z \frac{\ln(\Lambda) L_{cap}}{T_e^{3/2} S_{cap}}$$

where  $\rho_e$  is the plasma resistivity which describes the evolution of the plasma resistivity due to the collisions between electrons and ions,  $Z$  is the ionization degree and  $\ln(\Lambda)$  is the Coulomb logarithm (which is 10 for most of the laboratory plasmas).  $L_{cap}$  and  $S_{cap}$  are the length and the cross-sectional area of the plasma holder, usually called capillary.

Next equation is the thermal capacity at constant volume:

$$C_V = \frac{3}{2} k n_a V = \frac{3}{2} \frac{PV}{T_{iniz}}$$

where  $k$  is the Boltzmann constant,  $P$  is the pressure,  $V$  is the volume,  $T_{iniz}$  is the initial temperature (room temperature  $\approx 300$  K) and  $n_a$  is the atom density, which is related to the

electron  $n_e$ , ion  $n_i$  and neutral  $n_n$  densities as follows:  $n_i = Zn_a$ ,  $n_e = n_i$  and  $n_n = n_a - n_e$ .

The temperature growth can then be written (neglecting the thermal power transferred to the capillary tube and the one emitted from the gas by radiation) as

$$\frac{dT}{dt} = \frac{1}{C_V} R_p I_p^2 = \frac{1}{C_V} B_{res} Z \frac{\ln(\Lambda) L_{cap}}{T_e^{3/2}} \frac{I_p^2}{\pi r_{cap}^2}$$

where  $I_p$  is the current used to trigger the discharge and  $Z$  is the ionization degree.

This equation can be easily integrated only assuming that the ionization degree and the discharge current are constant; under these strong hypothesis the solution is

$$T[t] = \left[ T_{iniz}^{5/2} + \frac{5}{2} B_{res} Z \frac{\ln(\Lambda) L_{cap}}{C_V} \frac{I_p^2}{\pi r_{cap}^2} t \right]^{2/5}$$

Fig. 2 shows the temperature growth obtained using a discharge peak current  $I_p = 20$  A and a ionization degree  $Z = 0.5$ .

However, in reality both the discharge current and the ionization degree are time dependent and this has to be taken into account to properly design the discharge circuit.

## 3. Ionization degree and discharge current

The result given before is not reliable because in reality ionization degree and discharge current are not constant and also they depend on the gas used.

Let us consider as gas to ionize the hydrogen, which has a ionization potential  $U_i \approx 13.6$  eV. The ionization degree  $Z$  for the hydrogen can be derived from the Saha equation [11]:

$$\frac{Z^2}{1-Z^2} \approx 2.4 \times 10^{15} \frac{T^{3/2}}{n_i} e^{-U_i/kT}$$

If now we use this equation inside the Spitzer equation of plasma resistivity, what happens is that the initial plasma resistivity is 0 – as shown in Fig. 3 – which is not physical.

In fact, a non-ionized gas is not a conductive medium but it is an insulator, which means that the initial plasma resistivity should be infinity and not zero. This behavior is due to the fact that the Spitzer formula for plasma resistivity is only valid for weakly/fully ionized gas and not for non-ionized gas. Therefore, we need to write a Spitzer-like formula for the plasma resistivity which is valid at the really beginning of the discharge, when the gas is non-ionized. To do so, we use the same approach used by Spitzer to write the plasma resistivity for a weakly ionized gas. Spitzer, in fact, estimates the plasma resistivity by calculating the rate of collisions between ions and electrons. For a gas in its neutral conditions, the collisions happen only between atoms and electrons. In gases, in fact, there is always a very low percentage of free electrons, coming from natural reasons like heat, light and so on.

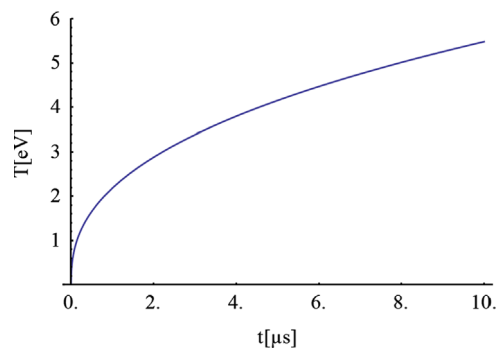


Fig. 2. Temperature growth as function of the time for  $I_p = 20$  A,  $Z = 0.5$ .

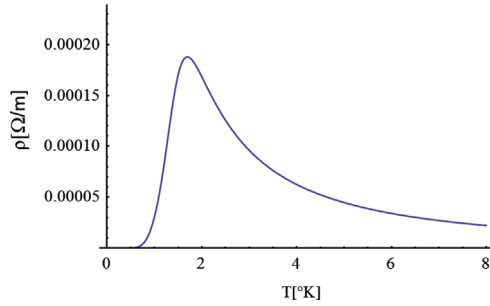


Fig. 3. Plasma resistivity as function of the temperature using the Saha formula for the ionization degree.

Therefore we can write the plasma resistivity as

$$\rho_{tot} = \rho_{ei} + \rho_{ae} = \frac{m}{n_e e^2} (\nu_{ei} + \nu_{ae})$$

where  $\rho_{ie} = \rho_e$  is the resistivity due to collisions between electrons and ions (happening with a frequency  $\nu_{ie}$ ) and  $\rho_{ae}$  is the resistivity due to the collisions between atoms and electrons (happening with a frequency  $\nu_{ae}$ ). The frequency of collisions between atoms and electrons is the inverse of the time of flight:

$$\nu_{ae} = \frac{1}{\tau} = \frac{v}{mfp}$$

where  $v$  is the velocity of the electrons and  $mfp$  is the mean free path. For collisions between atoms and electrons, the mean free path is [13]

$$mfp = \frac{kT}{\pi r_a^2 p}$$

where  $p$  is the pressure of the gas and  $r_a$  is the Bohr radius. Therefore, considering a Maxwellian distribution for the electron velocity, we have

$$\nu_{ae} = \frac{\pi r_a^2 p}{m_e^{1/2}} (kT)^{-1/2}.$$

The total plasma resistivity is therefore

$$\rho_{tot} = \rho_e + \frac{m}{n_e e^2} \frac{n_n \pi r_a^2 p}{n_0 m_e^{1/2}} (kT)^{-1/2}$$

where  $n_n$  is the density of neutrals,  $n_0$  is the initial density,  $n_e$  is the electron density and  $n_i$  is the ion density; these densities are related between each other as follows:

$$n_i = n_0 Z, \quad n_e = n_i, \quad n_n = n_0 - n_e.$$

Using the formulas listed above, we can now write the plasma resistance as

$$R_p = (\rho_e + \rho_{ae}) \frac{L_{cap}}{\pi r_a^2} = \left( B_{res} Z \frac{\ln(\Lambda)}{T_e^{3/2}} + \frac{m}{n_e e^2} \frac{n_n \pi r_a^2 p}{n_0 m_e^{1/2}} (kT)^{-1/2} \right) \frac{L_{cap}}{\pi r_a^2}.$$

Fig. 4 shows the plasma resistance due to collisions atom–electron and electron–ion, which now has a physical meaning: at the beginning, in fact, the gas is a pure resistivity medium (resistance is infinity) and becomes more and more conductive as the temperature grows; at some point, the collisions between electrons and atoms become negligible because the gas becomes a plasma and at that point only the collisions between electrons and ions become predominant.

The other equation missing is the discharge current. For this equation, let us consider the circuit shown in Fig. 5.

In this example there are a thyatron to charge a capacitor, a resistance to limit the current inside the plasma, an inductance

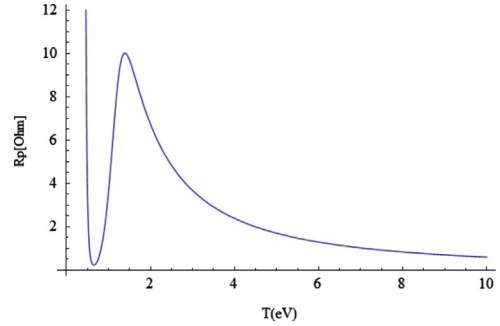


Fig. 4. Plasma resistance with contribution of collision atom–electron and ion–electron.

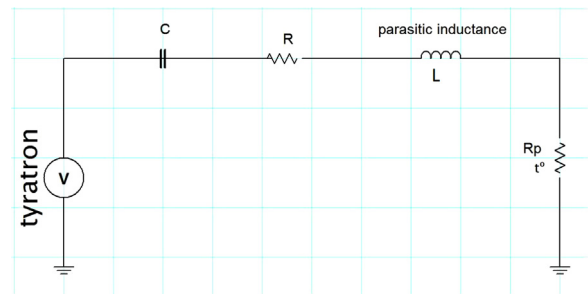


Fig. 5. Simplified discharge circuit.

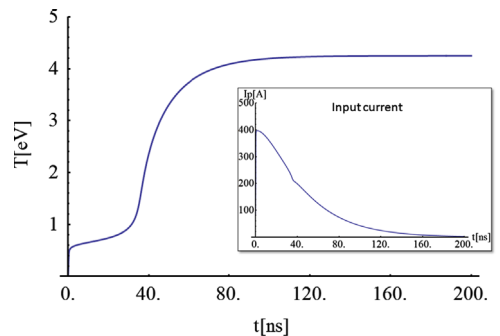


Fig. 6. Temperature growth as a function of the time for the input current shown in the inset plot (400 A peak current, 200 ns duration).

which represent the parasitic inductance of the circuit and eventually there is the plasma resistance.

The current flowing into the circuit can be found using the formulas for the RLC series circuit which is a second differential order equation:

$$\frac{d^2 I_p}{dt^2} + 2\alpha \frac{dI_p}{dt} + \omega^2 I_p = 0$$

where  $\alpha = (R + R_p)/2L$  is the circuit damping factor and  $\omega = 1/\sqrt{LC}$  is the circuit resonance frequency.

The temperature growth, considering the two new formulas introduced for the discharge current and the plasma resistance, is now very complicated and cannot be easily integrated: a numerical solution needs to be found. For this reason, we have used the Mathematica 8.0 Runge Kutta numerical solver at the 5th order (already integrated in Mathematica) to find a solution, which is reported in Fig. 6.

For this example, the following circuit parameters have been used:

$$C = 1 \text{ nF}, \quad R = 50 \text{ } \Omega, \quad V = 20 \text{ kV}, \quad L = 600 \text{ nH}$$

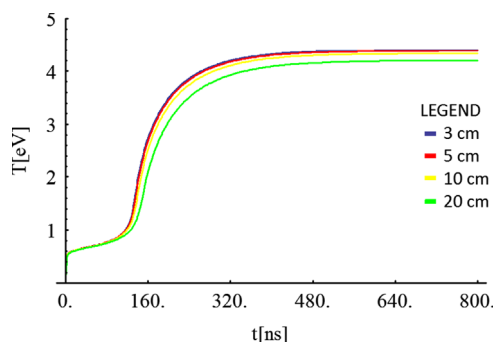


Fig. 7. Temperature growth as function of the time for different capillary lengths.

and a capillary – with a cylindrical section – with an inner diameter of 1 mm for a total length of 3 cm filled by 10 mbar of hydrogen.

The result illustrated in Fig. 6 shows that the temperature growth is strongly influenced by the plasma resistance trend, revealing a low growth in the region where there is a minimum in the plasma resistance.

#### 4. Conclusions

For our experiment, we have chosen to use rectangular cross-sectional area capillaries with a section of  $1 \text{ mm}^2$  and a length of 3 cm. Using the model described in this paper, we have found that the discharge that we need (considering a parasitic inductance of 600 nH and a desired final plasma temperature of 4 eV – temperature needed to ensure that the gas is in the fully ionized state) has the following parameters:

*total discharge duration* : 800 ns;

*voltage* : 20 kV;

*peak current* : 500 A;

*capacitor* : 6 nC.

As we can notice, for the rectangular cross-sectional area capillaries, the duration of the discharge needs to be much longer than that with cylindrical section capillaries (which is just due to the fact that the capillary volume is bigger). This discharge parameters can be used for different capillary lengths (considering to have a capillary section of  $1 \text{ mm}^2$ ), as shown in Fig. 7.

As we can see from this plot, the temperature is always growing, also after the discharge: this is due to the fact that in the model the power dissipated has not been taken into account.

#### 5. Future development

At the moment, we are starting to buy all the circuit components and we will assemble our circuit as soon as all the equipments will arrive. We will test the discharge off line, as well as all the vacuum pumps that will then be installed on the linac together with the new C-band cavity (which will replace the last S-band cavity used at the moment). The experiment is foreseen to be done in the next two years.

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